# Comprehensive Analysis of Electrical Distribution Network Operation – An Approach of Electrical Load Modelling

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*Abstract:* The electrical distribution network operation depends considerably on network busloads, which are formed by active and reactive power of the substations. Therefore, the network operation variables such as power flows, currents, voltages, etc. change similarly as do loads. It is possible to notice periodicity, temperature dependency, stochasticity etc. This paper describes the distribution network monitoring methodology based on the mathematical model of load, i.e. in described approach, the network busloads are modeled using the mathematical model of load and afterwards, modeled load values in different conditions are used for network monitoring. Monitoring is understood as network operation forecasting, simulation, and analysis. Success of monitoring is guaranteed by an adequate mathematical model, which describes the changes of the load during the entire observable period, both in the past and in the future. The results, obtained through network monitoring process could be used for long- and short-term planning and dispatching, considering adequate changes in time, temperature dependency, probable deviations of the state variables etc. Based on the results it is possible to assess network operation and possible future scenarios and their effect on the network and apply appropriate solutions to enhance system reliability and security. An application of network analysis, modelling and estimation but also validation of results with real measurements obtained from real distribution network is presented.

*Key-Words:* busloads, distribution network, mathematical model of load, load monitoring, network operation monitoring, *SCADA*, state estimation.

## 1 Introduction

The nature of a power system's operation depends on its load. Load changes regularly, depends on weather conditions, and has a stochastic nature [1]. The purpose of generation, transmission and distribution system is to cover the load in all conditions. The balance between generation and consumption must exist at all times. The power system load is controllable to some extent, for example, through electrical tariffs or using the load dependency of operational parameters (voltage and frequency). Nevertheless, possibilities of load control remain secondary as adjustable features of the power system operation. The behaviour of the load and the regularities of the load change are primary features of the power system operation.

In power system analysis load values are basically needed for three main purposes: system security studies, short- and long-term planning of system operational dynamics and short- and long-term system planning. Each of those applications need different approach. For example, in long-term system planning, country's development situation, forecasts and other socio-economic criteria are considered, but in power system security studies the load is modelled considering the imminent effect of system behaviour to contingencies. In [2] a long-term peak forecast possibility based on general modified exponential model is presented. In this paper, the attention is paid to the modelling and handling of the load from the perspective of modelling and analysis of the distribution network operational dynamics.

Over the decades, power system engineers have looked into the world of forecasting of the electrical system load. Different methods based on regression analysis, time-series models, neural networks [3], fuzzy logic [4], expert systems etc. have been developed and applied. Comprehensive overview of different load forecasting methods is presented in [5].

The above mentioned methods could be gathered into notion of conventional approach of load forecast, i.e. the needed load forecast is found using formal approach based directly on load data, which is given in time series. Load forecast models are chosen according to the nature of initial data (amount of data) and on the required result (e.g. forecast lead time). In case of forecast models the main attention is paid to the application of formal mathematical methods according to certain circumstances, not to consider the nature of the load.

In the distribution network, close connection between the electrical network operation and the change of network busloads prevails. The distribution network operation is considerably influenced by network busloads, which are formed by active and reactive power of the substations. The availability of information is somewhat modest in distribution networks and therefore comprehensive methods for network monitoring should be used. Over the years different methods for distribution network power flow calculations [6], [7] and state estimation [8], [9] have been composed and implemented for different studies.

The outline of this paper is as follows: in Section 2 an overview and principles of monitoring of the distribution network operation are presented, in Sections 3 and 4 basic principles and methods for modelling of load and state estimation of the distribution network are presented, respectively. In Section 5 results of the conducted study and correspondence of applied methodology to real network operation is presented. Section 6 present conclusions obtained from the study.

## 2 Monitoring of Distribution Network Operation

The principles of monitoring of the distribution network operation are presented in Fig. 1. Monitoring consists of network representation, gathering of information, modelling of load, estimation of load model, network operation modelling and estimation, verification of results and models, and adequate decision making considering the results obtained in different conditions. The results of the monitoring process can be used for network dispatching and for short- and long-term planning purposes.

The medium voltage (MV) distribution network can be represented as a set of feeders going out from a transmission substation (HV/MV), consisting of line parts and distribution substation connected to the end of MV feeders. Switching disconnectors at the disconnection points can change the configuration of a feeder (Fig. 2). Hence, distribution network has radial structure and its functioning is directly specified by loads and their regularities. In addition, MV/MV substations and voltage regulators may be incorporated.

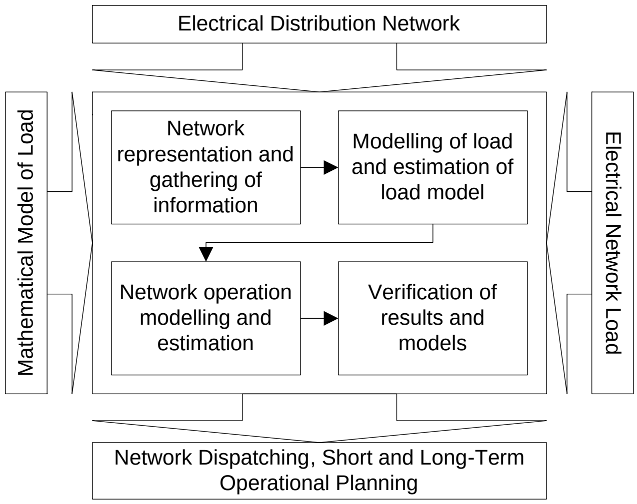


Fig. 1. Principle diagram of monitoring of distribution network operation

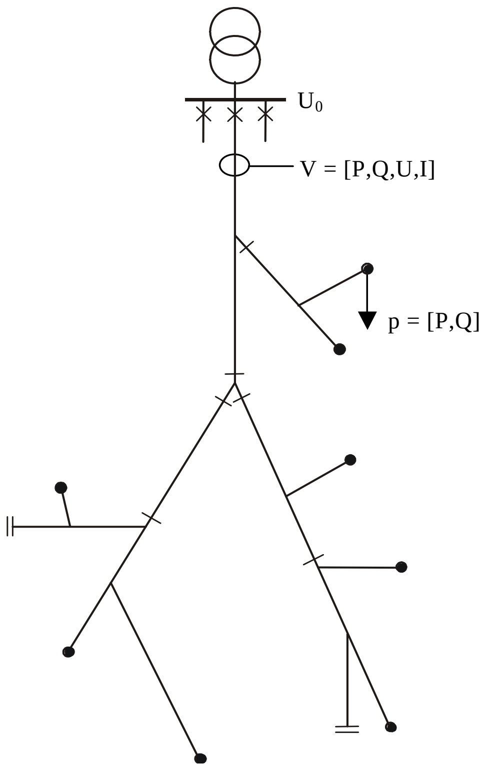


Fig. 2. Principal scheme of electrical distribution network

The load of a MV feeder is formed by active and reactive loads of distribution substations. The load of a distribution substation is formed as the sum of LV loads and losses of distribution transformers.

Dispatch system SCADA meters state parameters of a feeder periodically. Measured data may include values of active and reactive power, currents and voltages at various points of the feeder. Mainly these variables are measured only in HV/MVsubstations.

From the viewpoint of monitoring and availability of the measurement data, the distribution network should be divided into operatively observable and unobservable part (Fig. 3). In the observable part of the distribution network, the data, which can be used for handling the network operation, is obtained by the dispatching system (*System Control and Data Aquicition* - *SCADA*). *SCADA* gathers active and reactive power, currents and voltages of a feeder in HV/MV substations and at various points of the feeder. The sampling frequency in the *SCADA* is usually once or more times per minute.

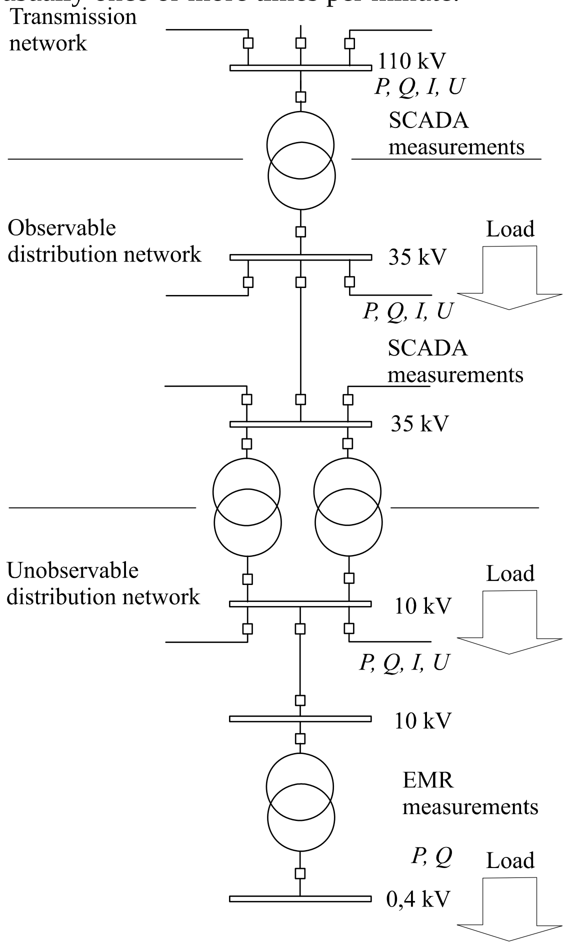


Fig. 3. Loads on the various levels of electrical network

Similar data for the unobservable part of the distribution network, e.g. LV distribution network, is not available. Although, measuring hourly electricity consumption of consumers for commercial purposes of the electricity market (energy meter reading, *EMR)* has recently expanded significantly, but unfortunately, the operative connection between different measuring systems is missing. However, the commercial data may also be used for estimating the load models. When even the commercially measured data is missing, load models can be estimated according to the data from the client information system (*CIS*).

A simple, two-way iteration process is used in the computation of the distribution network operation (Fig. 4). First, the power losses and power transmission in lines are found starting from the bus-loads at the end of the feeder. As an initial value of the bus voltage, the nominal voltage may be used. In the second phase, the bus voltages are calculated, starting from the HV/MV substation bus, the voltage of which is considered as given. In the second phase of the calculation, the load dependency on the voltage and the effect of the voltage regulators are considered. The described algorithm usually converges in 2…5 iterations.

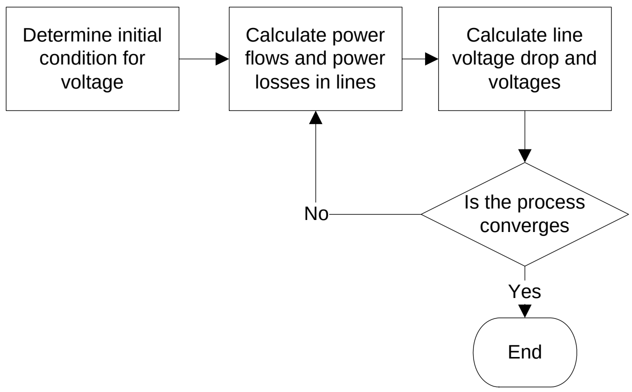


Fig. 4. Scheme of steady-state calculation of radial distribution network

In distribution network following simplifications are made: shunt conductances of lines are neglected, transformer shunt conductances are neglected and transformer losses are only considered when determining network active power and energy losses, inductances of cables are also neglected.

Due to the fact that the difference of input and output voltage angle $\delta$ of any element of distribution network is small, it is therefore considered that when calculating voltages only real component of voltage drop is considered $\Delta U_i = \Delta U_i^{'}$ and the imaginary component $\Delta U_i^{''}$ is neglected (Fig. 5). If for *i*-th segment of line (Fig. 6) voltage $U_i$ and value of active power $P_i$ and reactive power $Q_i$ is known at the beginning of line, and also resistances $R_i$ and $X_i$, then voltage drop is

$$\Delta \underline{U}_i = \underline{I}_i \underline{Z}_i = (I_i' - jI_i'')(R_i + jX_i) = \\ = (I_i'R_i + I_i''X_i) + j(I_i'X_i - I_i''R_i) \qquad (1)$$

which real part is considered as voltage drop.

$$\Delta U_i \cong (I_i'R_i + I_i''X_i) \qquad (2)$$

Multiplying and dividing the result with voltage $U_i$, we obtain the equation with active and reactive power values

$$\Delta U_i = U_i - U_{i+1} \cong \frac{P_i R_i + Q_i X_i}{U_i} \qquad (3)$$

In the back-loop, when voltages are given and we search for power flows, then power losses in *i*-th line segment are found

$$\Delta P_i = \frac{P_{s\,i+1}^2 + Q_{s\,i+1}^2}{U_{i+1}^2} R_i, \quad (4)$$

$$\Delta Q_i = \frac{P_{s\,i+1}^2 + Q_{s\,i+1}^2}{U_{i+1}^2} X_i \quad (5)$$

and active and reactive power values at the beginning of line may be expressed as

$$P_i = P_{s\,i+1} + \Delta P_i \quad (6)$$

$$Q_i = Q_{s\,i+1} + \Delta Q_i \quad (7)$$

Here, active and reactive power $P_{s\ i+1}$ and $Q_{s\ i+1}$ are the sum of branches active and reactive power values leaving from node $i$ + 1 and load.

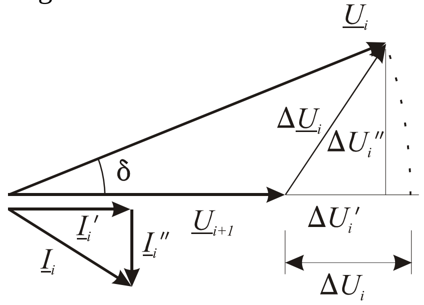


Fig. 5. Voltage drop and voltage loss at different network elements

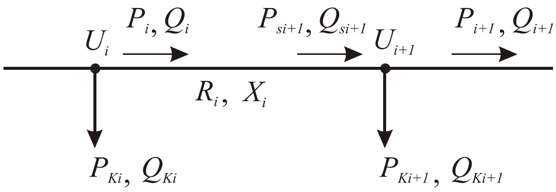


Fig. 6. Line segment calculation

The calculation of the unobservable distribution network operation does not differ essentially from the calculations in the observable part of the network. The network configuration is radial, and the necessary information concerning the load is received from similar load models, as in the higher level of the distribution network.

Among the measurements some errors and mistakes are presented. Therefore, data estimation is needed, especially for clearing up bad data. As the redundancy of data required for estimation is usually not available, traditional estimation methods are not directly applicable in distribution networks. To get additional data load models are used. The last stage of the distribution network monitoring is the assessment of models and network operation.

Modelling and analysis of distribution network operation is needed for short- and long-term operational planning of the electrical network. It is necessary to describe adequately changes in time, temperature dependency, probable deviations of the state variables and cover different cases and scenarios. Based on the results obtained it is possible to make more economical and technical decisions to obtain more efficient operation of the distribution network and enhance network reliability.

# 3 Mathematical Model of Load

Different methods based on regression analysis, time-series models, neural networks, fuzzy logic, expert systems etc. have been developed and applied to model the electrical network load [10]. Those methods can be named as forecasting models in where the needed load forecast is found using formal approach based directly on load data, which is given in time series. Forecasting models are formal data models, considering only the data. Nature of the load is not considered. More comprehensive solution is obtained by composing the mathematical model of load.

The mathematical model, describing changes of load (active power, reactive power, or current) consists of three basic components [1], [11]:

$$P(t) = E(t) + \Gamma(t) + \Theta(t) \quad (8)$$

where $E(t)$ is mathematical expectation of the load

$\Gamma(t)$ is temperature-sensitive part of the load

$\Theta(t)$ is stochastic component of the load.

Mathematical expectation $E(t)$ describes regular changes of a load, such as general trend and seasonal, weekly, and daily periodicity. Mathematical expectation is principally non-stochastic and corresponds to the normal temperature.

The temperature-sensitive part of a load $\Gamma(t)$ describes load deviations, caused by deviations of outdoor temperature from the normal temperature [12]. The normal temperature (mathematical expectation of the temperature) is the average outdoor temperature of the last 30 years on any given hour of the year. If the real outdoor temperature corresponds to the normal temperature, the value of the temperature-sensitive part of the load is zero. To compare the temperature dependencies of different loads, the component $\Gamma(t)$ is normalized

$$\Gamma(t) = R(t)\gamma(t) \quad (9)$$

where $R(t)$ is rate of the temperature dependency of the load and $\gamma(t)$ is normalized temperature dependency component. Rate $R(t)$ represents the load increase when the temperature rises by 1 °C. Component $\gamma(t)$ describes other regularities of the temperature dependency, e.g. delay etc.

In Fig. 7 an example of load temperature dependency with daily values is presented. For comparison, temperature and normal temperature is also presented (scaled on the right axis). Here the load long-term forecast $E(t) + R(t)\gamma(t)$ is achieved by adding temperature dependency to mathematical expectation.

The stochastic component $\Theta(t)$ describes stochastic deviations of the load [13]. Due to the autocorrelation, the deviations are stochastically dependent on each other. It is practical to normalize the stochastic component. The proper rate is the standard deviation of the load $S(t)$. The result is

$$\Theta(t) = S(t)\left[\zeta(t) + \xi(t) + \pi(t)\right] \quad (10)$$

where $\zeta(t)$ is expected deviation – conditional mathematical expectation of the stochastic component. The expected deviation of the load may be used for short-term forecasting of the load. In Fig. 8 an example of real value of the load and additionally the values of long-term forecast $E(t)+\Gamma(t)$ and short-term expected value $E(t)+\Gamma(t)+S(t)\zeta(t)$ are presented. $\xi(t)$ is normally distributed non-correlated residual deviation (white noise) of the load. $\pi(t)$ is peak component that considers the existence of large positive or negative deviations (peak deviations) that do not correspond to the normal distribution. In practice those deviations may be observed in low- and medium voltage networks, where they cause distortion of load distribution.

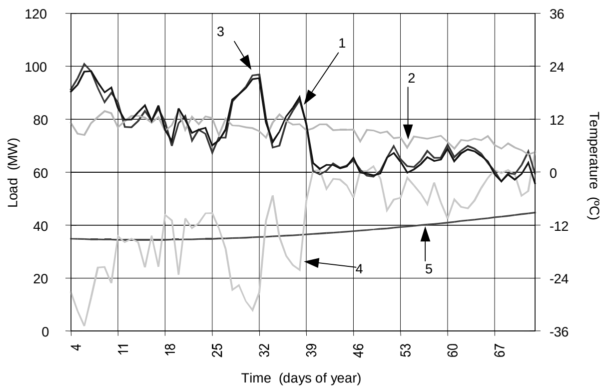


Fig. 7. Load real value (1), mathematical expectation (2), long-term expected value (3), temperature value (4), and normal temperature (5), daily values

The structure of the mathematical model is the same for all loads. In order to use the load model to describe the specific load the model parameters must be estimated. In the estimation process all regular data and other available quantitative and qualitative information about the load is used. If the existing data is not enough to evaluate all parameters of the model, then type-models (i.e., a typical set of model parameters) may be used. In such cases, the suitable amount of load parameters are determined by load statistical data, and the rest of the parameters will comply with the type-model. The result of estimation will be a complete model for all loads independently from the amount of used data.

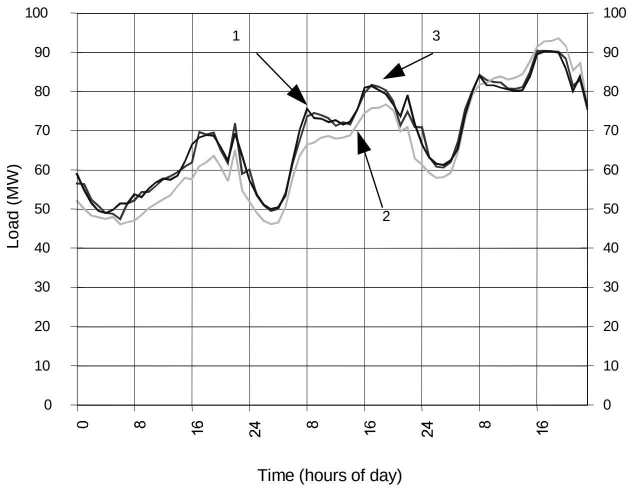


Fig. 8. Real value of the load (1), long-term expected value (2) and short-term expected value (3), hourly values

Mathematical model describes the load, but it does not directly determine the values needed in practice (load characteristics). But it is possible to find these load characteristics on the base of the load model. Load characteristics may be divided into initial and derived characteristics. Initial characteristics will be obtained from the mathematical model directly (load real values, mathematical expectation, standard deviation, temperature dependency etc.). Derived characteristics are achieved with combinations of primary characteristics by simple arithmetic relations (short- and long-term expected values, simulated load etc.).

During the research and planning of electrical power system security and reliability the load changes in time must be considered. Those changes are principally expressed by the mathematical expectation of the load. The mathematical expectation can be found based on load model for any time interval in the future or in the past. It must be emphasised that the mathematical expectation is calculated based on the parameters of the load model without using the currently available load data.

The temperature dependency has an important role in evaluation of possible load deviations. From the aspect of power system security and reliability the load increase during extreme temperatures is especially interesting. For example, in Finland, Lapland, where the electrical heating has a substantial role, the load increase caused by low temperature may reach up to 100 % and more compared to the load in normal temperature. In case of load analysis and short-term forecasting the temperature dependency is calculated on the base of real temperature data. For the purpose of long-term forecast and analysis when required it is possible to simulate the temperature. For example, it is possible to add deviation to the normal temperature or use some earlier year data that correspond to the exceptional weather conditions (cold winter etc.).

The influence of load standard deviation and thus also load stochastic deviation to the electrical transmission network busloads and regional loads is relatively small. The stochasticity is more of a problem of smaller loads, a problem of distribution networks. However, the stochastic component of the model enables to find the expected deviation of load and accordingly the short-term forecast of the load.

The parameters of the mathematical model are estimated during the load research using the available load data and other information. It is possible to estimate the model parameters for different load cases, which are caused, for example, by switchings in lower lever network or by connecting or disconnecting large consumers to or from electrical network in time and space domain. While in reality, where different loads may change simultaneously, it is possible to talk about load scenarios. Different load scenarios and also the simulated load originated from different temperatures could be one of the sources for conducting the electrical system contingency analysis. An example of load different load cases is presented in Fig. 9. It is possible to observe that the load values in the substation may be on two different level. That characteristic should be considered when calculating and analysing the network operation.

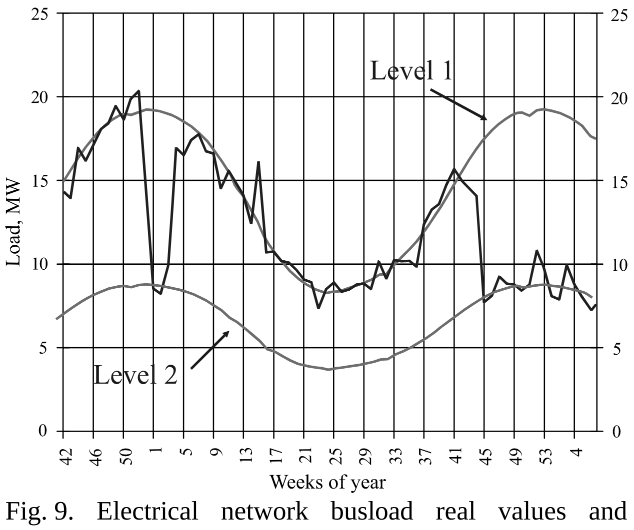


Fig. 9. Electrical network busload real values and mathematical expectation on two different levels (load cases)

# 4 Distribution Network State Estimation

The target of state estimation is to refine measured data, but it is especially important to clarify significant measuring errors or mistakes. As the redundancy of data required for the estimation is usually not available in distribution networks, traditional estimation methods used in the main grid are not directly applicable in distribution networks. For getting required additional data, load models are used. The estimation procedure on the base of load models is observed at [14], [15].

The calculating of feeder state parameters is based on network equations through main operating parameters – supply voltage and node loads

$$V_j(U_0, p_1, p_2, \ldots, p_n) \quad (11)$$

Symbol $p_i$ corresponds here to both active and reactive power. So the general number of loads $n$ is equal to double number of nodes and so there are $n$ + 1 main operating parameters $U_0$, $p_1$, $p_2 \ldots p_n$.

Supposing that for considered moment there are $m$ measurements ($P$, $Q$, $U$ or $I$), it is possible to obtain the refined operating parameters (refined measurements) $\tilde{V}_j$ from the criterion

$$\sum_j \left[\tilde{V}_j - V_j\right]^2 = \min, \; j = 1 \ldots m \quad (12)$$

Because of the needed data redundancy does not exist in distribution networks an additional condition is applied. It is needed that the increments of state parameters to be as small as possible

$$\sum_i \Delta^2 p_i = \min, \; i = 1 \ldots n \quad (13)$$

Let us see the network equations being linearized over main operating parameters, when marking $p_0 = U_0$

$$\Delta V_j = \beta_{0j}\Delta p_0 + \beta_{1j}\Delta p_1 + \beta_{2j}\Delta p_2 + \ldots + \beta_{nj}\Delta p_n \quad (14)$$

where

$$\beta_{ij} = \frac{\partial V_j}{\partial p_i}, \; i = 0 \ldots n \quad (15)$$

In the form of matrix

$$\mathbf{\Lambda} = \mathbf{B\Delta} \quad (16)$$

where

$$\mathbf{\Lambda} = \left[\Delta V_0, \Delta V_1 \ldots \Delta V_m\right] \quad (17)$$

is the vector of increments of measuring data

$$\mathbf{\Delta} = \left[\Delta p_0, \Delta p_1 \ldots \Delta p_n\right] \quad (18)$$

is the vector of increments of main state parameters

$$\mathbf{B} = \begin{vmatrix} \beta_{10} & \beta_{11} & \ldots & \beta_{1n} \\ \beta_{20} & \beta_{21} & \ldots & \beta_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \beta_{m0} & \beta_{m1} & \ldots & \beta_{mn} \end{vmatrix} \quad (19)$$

is the sensitivity matrix (Jacobian).

Here the increments $\Delta p_i$ are found on the base of the values calculated by a load model. It is possible to compose the mathematical model also for the supply voltage $U_0$. For simplicity it is the trivial model, where mathematical expectation and standard deviation of voltage are constant and deviation is described by means of *Box-Jenkins* model.

For finding elements of the sensitivity matrix B it is necessary, at first, to assign the increments of the model parameters $\Delta p_0 \ldots \Delta p_n$. Then to find the corresponding load values and at last to calculate the increments $\Delta V_j$ of considered state parameters on the base of non-linearized network equations. As a result

$$\beta_{ij} = \frac{\Delta V_j}{\Delta p_i}, \quad i = 0 \ldots n, j = 1 \ldots m \tag{20}$$

Let us see the extended vector of increments of measuring data

$$\tilde{\mathbf{\Lambda}}_0 = \left[\Delta\tilde{V}_1, \Delta\tilde{V}_2 \ldots \Delta\tilde{V}_m, 0, 0 \ldots 0\right] \tag{21}$$

in which the $n$ last components are zeros.

Next, matrix $\mathbf{B_0}$ with $n + 1$ columns and where on the first $m$ rows there are elements of sensitivity matrix $\mathbf{B}$ and on the next $n$ there is a zero vector and a unit matrix, is composed.

$$\mathbf{B}_0 = \begin{vmatrix} \beta_{10} & \beta_{11} & \ldots & \beta_{1n} \\ \beta_{20} & \beta_{21} & \ldots & \beta_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \beta_{m0} & \beta_{m1} & \ldots & \beta_{mn} \\ 0 & 1 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 \end{vmatrix} \tag{22}$$

As mentioned above, conditions are simultaneously satisfied, if

$$\left(\tilde{\mathbf{\Lambda}}_0 - \mathbf{B}_0\mathbf{\Delta}\right)^T \left(\tilde{\mathbf{\Lambda}}_0 - \mathbf{B}_0\mathbf{\Delta}\right) = \min_{\mathbf{\Delta}} \tag{23}$$

From here a system of linear equations over the increment vector $\mathbf{\Delta}$ follows

$$\mathbf{B}_0^T\mathbf{B}_0\mathbf{\Delta} = \mathbf{B}_0^T\tilde{\mathbf{\Lambda}}_0 \tag{24}$$

The increment vector $\mathbf{\Delta}$ allows to refine all state parameters – it is possible to calculate both the specified measured parameters (estimates) and whatever other operating parameters.

The results of state estimation are authentic, if there are no bad data and exclusive states are not observed. If bad data are obtained, they have to be removed and the estimation procedure must be repeated. In case of existence of an exclusive state that do not correspond to the load model, estimation is not possible. Measurements of this moment are not used. But that does not obstruct the process of estimation in the next moments.

## 5 Editing of the load models

For considering the possible changes of the load character it is necessary to edit (specify) the model parameters. In this connection the model components are fixed.

Let us see the node loads $p_i$ with parameters of the model $a_{i1}, a_{i2} \ldots a_{ir}$ at the moment $t_k$

$$p_i(t_k, a_{i1}, a_{i2} \ldots a_{ir}), \quad i = 1 \ldots n \tag{25}$$

where $r$ is the number of parameters that have to be estimated.

Next the network equations are presented

$$V_j = \mathrm{V}_j(U_0, p_1, p_2, \ldots p_n) \tag{26}$$

describing the state parameters via the parameters of the load model

$$V_{jk} = \mathrm{F}_{jk}(a_1, a_2, \ldots a_l), \quad j = 1 \ldots m \tag{27}$$

where $l = nr$ is the overall number of the parameters of all load models. The function $\mathrm{F}_{jk}$ corresponds to the moment $t_k$ on which the load values and supply voltage depend.

The parameters of the load model are calculated from criterion

$$\sum_j \sum_k \left[\tilde{V}_{jk} - V_{jk}\right]^2 = \min \tag{28}$$

where $\tilde{V}_{jk}$ is the measuring data at the moment $k$.

Network equations, linearized over the model parameters, are

$$\Delta V_{jk} = \alpha_{1jk}\Delta a_1 + \alpha_{2jk}\Delta a_2 + \ldots + \alpha_{ljk}\Delta a_l \tag{29}$$

where

$$\alpha_{sjk} = \frac{\partial V_{jk}}{\partial a_s}, \quad s = 1 \ldots l \tag{30}$$

In the form of matrix

$$\mathbf{\Lambda}_k = \mathbf{A}_k\mathbf{\Delta} \tag{31}$$

where

$$\mathbf{\Lambda}_k = \left[\Delta V_{1k}, \Delta V_{2k} \ldots \Delta V_{mk}\right] \tag{32}$$

is the increment vector of measured parameters

$$\mathbf{\Delta} = \left[\Delta a_1, \Delta a_2 \ldots \Delta a_l\right] \tag{33}$$

is the increment vector of model parameters

$$\mathbf{A}_k = \begin{vmatrix} \alpha_{11k} & \ldots & \alpha_{1lk} \\ \ldots & \ldots & \ldots \\ \alpha_{m1k} & \ldots & \alpha_{mlk} \end{vmatrix} \tag{34}$$

is the sensitivity matrix (Jacobean)

If the vector of metering deviations is

$$\tilde{\mathbf{\Lambda}}_k = \left[\Delta\tilde{V}_{1k}, \Delta\tilde{V}_{2k} \ldots \Delta\tilde{V}_{mk}\right] \tag{35}$$

then the increments for model parameters are calculated from the criterion

$$\sum_k \left(\tilde{\mathbf{\Lambda}}_k - \mathbf{A}_k \mathbf{\Delta}\right)^T \left(\tilde{\mathbf{\Lambda}}_k - \mathbf{A}_k \mathbf{\Delta}\right) = \min_{\mathbf{\Delta}} \quad (36)$$

that gives the system of linear equations over the vector $\mathbf{\Delta}$

$$\sum_k \left(\mathbf{A}_k^T \mathbf{A}_k\right)\mathbf{\Delta} = \sum_k \mathbf{A}_k^T \tilde{\mathbf{\Lambda}}_k \quad (37)$$

The base for linearizing the network equations is the load values, calculated by load models (short-term forecasts). Relative to the same values metering deviations are found. Linearization is acceptable, if the metering deviations are not too large. Otherwise it is necessary to use the iterative process, according to which the increments are added to the model parameters and new state parameters and metering deviations are calculated. In this connection repeated linearization (calculation of the new sensitivity matrix) is not needed and it is possible to continue with the old matrix. The elements of sensitivity matrix are calculated as follows

$$\alpha_{sjk} = \frac{\Delta V_{jk}}{\Delta a_s}, \; s = 1...l, j = 1...m. \quad (38)$$

The number of measuring data must be larger than the number of parameters being estimated. For example, if the number of the loads is 10 (5 distribution substations), parameters 10 and measurements 10, then the number of the required values is $l$ = 100 and so more than 10 metering hours are needed. Therefore the daily metering is enough to get the results. But the authenticity of results is a separate question. It is clear that the model, estimated on the base of the data of one or some days, is not usable for a longer period. For example, the data of winter loads is not representative for summer loads. Generally it is an adaptation problem, which can be resolved taking into account the essential meaning of the model parameters.

The degree of the equation system, obliged to be resolved, is high (for the above-mentioned example it is 100). It means that the volume of calculations is too large and it may mean that the equation system is ill-conditioned. The way out is to use the results of state estimation, by means of which the node loads are always found independently from the structure of measured state parameters. Consequently, it is possible to receive all load values, state parameters and edit the parameters of all load models. If $m$ = 1 and $l = r$ we can get

$$\mathbf{A}_k = \left|\alpha_{1k} \; ... \; \alpha_{rk}\right| \quad (39)$$

$$\mathbf{\Delta} = \left[\Delta a_1, \Delta a_2 \; ... \Delta a_r\right] \quad (40)$$

$$\tilde{\mathbf{\Lambda}}_k = \left[\Delta \tilde{V}_k\right] = \tilde{p}_k \quad (41)$$

$$\sum_k \left(\mathbf{A}_k^T \mathbf{A}_k\right)\mathbf{\Delta} = \sum_k \tilde{p}_k \mathbf{A}_k^T \quad (42)$$

where $\tilde{p}_k$ is the estimated value of the observed load at the moment $k$. The degree of the equation system is now $r$ or for the above-mentioned example it is now 10, and problems about calculation volumes are removed.

## 6 An Application of Distribution Network Monitoring

Described distribution network monitoring methodology was applied and tested based on real 35 kV radial distribution network. The purpose of the research was to assess the measurements and compose adequate load and network models for further research.

The network consisted of three HV/MV supply substations, six MV/LV substations connected by ten OHL (Fig. 10). The system was monitored by *SCADA*. The measurements received through *SCADA* were currents, active and reactive power in transmission lines, voltages in substations and transformer currents at low-voltage winding.

Before applying the above mentioned estimation methodology it is practically rational to perform preliminary estimation to determine large errors and exclude very bad data. Preliminary estimation is performed based on Kirchhoff´s current law.

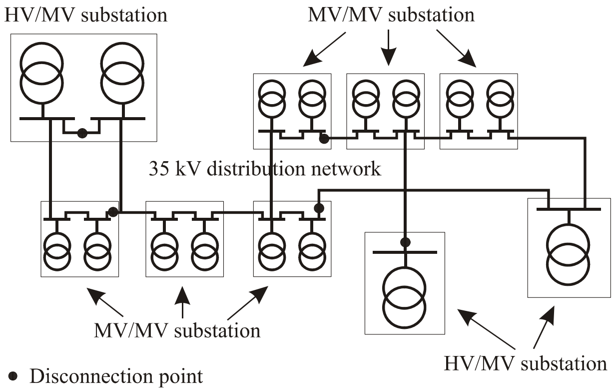


Fig. 10. Diagram of observed distribution network

The results of the preliminary estimation are presented in Fig. 11. The solid line shows that the measurements received were correct and they can be used for further research and calculations of the network, dashed line shows that the measurements were slightly out of predetermined correct threshold values and dotted line shows that the measurements are completely wrong and they must not be used.

Utilizing the composed substation busload models, voltage values and other parameters it is possible to calculate the distribution network operation. Based on the above-mentioned monitoring methodology it is possible to identify if the network state is compatible with the load and network models. In Fig. 12 the results of modelling the transmission line current compared to real measurements are presented. It is possible to observe that the obtained results of modelling the network state variables are sufficiently acceptable and comparable to the measured values of the network state variables and therefore, it may be concluded that based on the described methodology it is possible to model the network operation and use different initial conditions to investigate the network behaviour on different conditions.

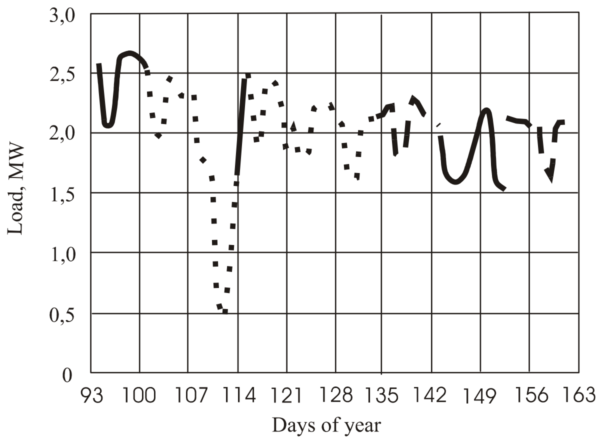


Fig. 11. Measurement values according to threshold values (solid line – reliable values, dashed line – doubtful values, dotted line – unreliable values), daily values

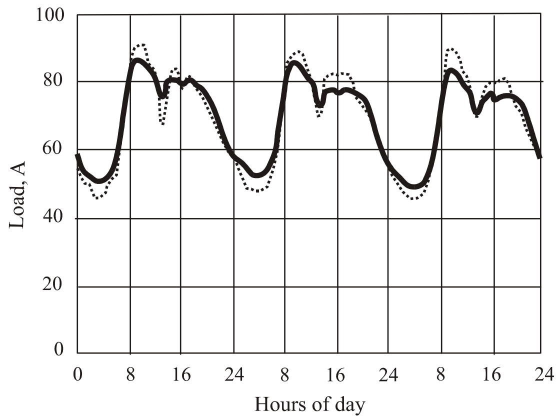


Fig. 12. Measured (dotted line) and calculated (solid line) values on transmission line current, hourly values

In Fig. 13 an example of long-term and short-term forecasts of load are presented. Here, the long-term forecast is achieved by adding temperature dependency to mathematical expectation. The short-term forecast is acquired by adding the loads expected deviation to the long-term forecast.

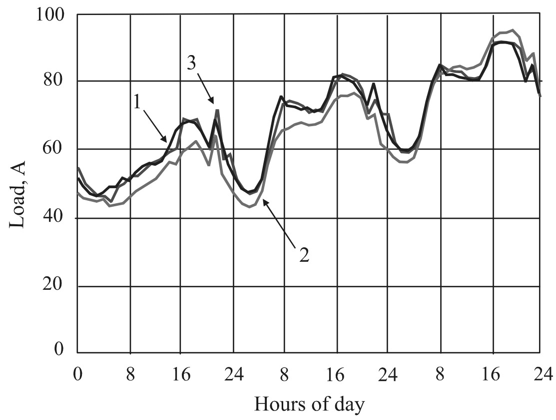


Fig. 13. Actual load (1), long-term forecast (2), and short-term forecast (3), hourly values

## 7 Conclusion

The distribution network monitoring methodology described in this paper is based on the physically well-grounded mathematical model that considers time-dependencies, stochasticity, temperature dependency and also voltage and frequency dependencies of load. Based on the mathematical model of the load it is possible to calculate, forecast and analyse the network operation for shorter and longer time periods and use the obtained results for network planning and dispatching.

The effectiveness of modelling and estimation of the distribution network operation depends on the accuracy of the load model. In the observable part of the distribution network the described methodology enables to determine if the network operation is compatible with the load and network models, regular operation or not. Although, due to the deficiency of available initial data the adjustment of the measurements is not feasible, it is possible to clarify the faulty measurements. There are no computational limitations to employ the methodology because laborious computations are absent. Furthermore, it is important that during the analysis both state estimation and editing of load model are performed. As a result, the load models should always correspond to the real situation.

The modelling offers similar possibilities of handling the operation variables, as in the case of the loads. The operation forecasting, analyzing, and simulation are possible for different given conditions. The results are obtained for any given time interval, not just single states like in traditional calculations.

*References:*

[1] Meldorf, M. Electrical Network Load Monitoring. *TUT Press*, 2008, 144 pp.

[2] Petros. C., Pandelis, B. Michail, P., Manthos, V. Long-Term Peak Forecast in the Greek Power System. *In Proc. 2006 IASME/WSEAS International Conference on Energy & Environmental Systems*. Chalkida, Greece, May 8-10, 2006. pp. 243-248.

[3] Hayati, M., Karami, B. Application of Neural Networks in Short-Term Load Forecasting. In Proc. *7th WSEAS Int. Conf. On Mathematical Methods and Computational Techniques in Electrical Engineering*. Sofia, 27-29. Oct. 2005. pp. 37-41.

[4] Mallesham, G. A Fine Short-Term Load Forecasting Using Neural Networks and Fuzzy Neural Networks. In Proc. 4th WSEAS International Conference on Power Engineering Systems (ICOPES`05). Rio de Janeiro, Brazil, April 25-27, 2005, 8 pp.

[5] Alfares, H. K., Nazeeruddin, M. Electric load forecasting: literature survey and classifcation of methods. *International Journal of Systems Science*, vol. 33, no. 1, pp. 23-34, 2002.

[6] Thukaram, D., Wijekoon Banda, H. M., Jerome, J. A Robust Three Phase Power Flow Algorithm for Radial Distribution System. *Int J Electric Power Systems Research*, Vol. 50, 1999, pp. 227-236.

[7] Wu, W. C., Zhang, B. M. A Three-Phase Power Flow Algorithm for Distribution System Power Flow Based on Loop-Analysis Method. *Int J Electrical Power and Energy System*, Vol. 30, 2008, pp. 8-15.

[8] Thukaram, D., Jerome, J., Surapong, C. A Robust Three-Phase State Estimation Algorithm for Distribution Networks. *Int J Electric Power System Research*, Vol. 55, 2000, pp. 191-200.

[9] Pereira, J., Saraiva, J. T., Miranda, V. An Integrated Load Allocation/State Estimation Approach for Distribution Networks. *Proc of 8th Int Conf on Probabilistic Methods Applied to Power Systems*, Iowa State University, 2004, pp. 180-185.

[10] Alfares, H. K., Nazeeruddin, M. Electric load forecasting: literature survey and classifcation of methods. *International Journal of Systems Science*, Vol. 33, no. 1, pp. 23-34, 2002.

[11] Meldorf, M., Kilter, J. Pajo, R. Comprehensive Modelling of Load. *Proc. of CIGRE NRCC Regional Meeting*, 2007, Tallinn, 18-20 June, pp. 145-150.

[12] Meldorf, M., Treufeldt, Ü., Kilter, J. Temperature Dependency of Electrical Network Load. *Oil Shale*, 2007, Vol. 24, No. 2 Special, pp. 236-248.

[13] Meldorf, M., Täht, T., Kilter, J. Stochasticity of Electrical Network Load. *Oil Shale*, 2007, Vol. 24, No. 2 Special, pp. 225-235.

[14] Meldorf, M., Treufeldt, Ü., Kilter, J. State Estimation Technique for Distribution Networks. *IEEE Conference Proceedings, PowerTech'05*. St. Petersburg. June 2005. [CD-ROM], paper no. 374.

[15] Meldorf, M., Treufeldt, Ü., Kilter, J. Estimation of Distribution Network State on the basis of a Mathematical Load Model. *Oil Shale*, 2005, Vol. 22, No. 2 Special, pp. 161 – 170.