Steady Sate Analysis of Self-Excited Induction Generator using Phasor-Diagram Based Iterative Model

KANWARJIT SINGH SANDHU & DHEERAJ JOSHI
Department of Electrical Engineering
National Institute of Technology
Kurukshetra,Haryana
INDIA
sandhukjs@rediffmail.comT, dheeraj_joshi@rediffmail.com

Abstract: – Induction generators in self excited mode are found to be suitable for remote and windy locations. Prior to installation there is a need to predict the behaviour of machine under all possible operating conditions. This is possible through steady state modeling of such generators. Steady state analysis of self excited induction generator (SEIG) needs an estimation of generated frequency and magnetizing reactance under all possible operating conditions. So far most of the researchers have used loop impedance, nodal admittance or iterative techniques to determine the steady state performance of such machines. In this paper a new model based upon phasor diagram of induction generator has been proposed to analyze the behaviour of self excited induction generator. Modeling results in a third order equation in generated frequency and a simple expression for magnetizing reactance. Complete mathematical analysis to derive the different expressions is presented here. Computed results have been compared with experimental results on test machines. Closeness between the two proves the validity of proposed modeling.

Keywords: - Non-conventional sources, Renewable energy, Steady state analysis, Self-excited induction generator (SEIG), Wind energy.

Nomenclature

a	per unit frequency
b	per unit speed
C	excitation capacitance per phase
E_1	air gap voltage per phase at rated frequency
Err1	error in successive values of generated frequency
Err2	error in successive values of magnetizing reactance per phase
Err3	error in successive values of stator voltage per phase
I_1	stator current per phase
I_2	rotor current per phase, referred to stator
I_C	capacitor current per phase
I_L	load current per phase
I_{rc}	core loss current per phase
I_m	magnetizing current per phase
R	load resistance per phase
R_1	stator resistance per phase
R_c	core loss resistance per phase
R_2	rotor resistance per phase, referred to stator
V_t	stator voltage per phase
X	load reactance per phase
X_1	stator reactance per phase

X_2 rotor reactance per phase, referred to stator

X_C capacitive reactance due to C at rated frequency

X_m magnetizing reactance per phase at rated frequency

1. Introduction

A rapid increase in power demand and continuous depletion of fossil fuels has diverted the attention of scientists from conventional energy sources to non-conventional energy sources. Wind energy is emerging as an potential source among various non conventional energy sources. Most of the countries across the world are promoting such wind energy generating units.

Induction generators with cage rotor are found to be most suitable for wind energy conversion due to their advantages such as simple and rugged construction, low cost and no need of synchronization with existing grid. These machines can be operated in grid connected as well as in self-excited mode. Induction generator in self-excited mode is found to be capable to generate the power even in the absence of power grid. This makes it to be most useful machine for the remote windy locations.

Various methodologies adopted for the analysis of SEIG by researchers [1-9] are;

- Loop impedance technique
- Nodal admittance technique
- Iterative technique

Above methods require either lengthy derivations or solution of nonlinear equations. In this paper an attempt has been made to estimate the generated frequency and magnetizing reactance for self-excited induction generator using a new strategy based upon phasor diagram of the machine. Proposed modeling results in the third order equation in generated frequency and a simple expression for magnetizing reactance. Comparison of computed and experimental results on test machines confirms the validity of proposed modeling.

2. Steady-State Analysis

The steady-state operation of the self-excited generator may be analyzed by using the equivalent circuit representation as shown in Fig.1. In this circuit all parameters are assumed to be constant except magnetizing reactance.

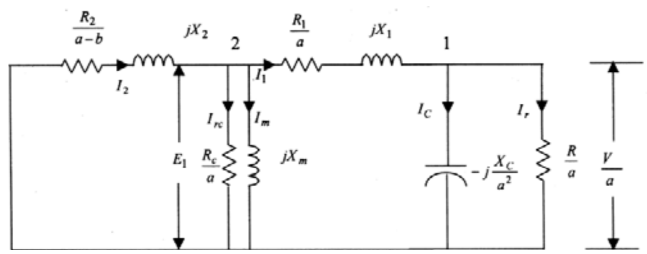

Fig.1 Equivalent circuit representation.

Analysis of Fig. 1 in the absence of any power source & with 'V_t & E_1' as potential of node 1 &2 gives.

$$\frac{E_1}{jX_m} + \frac{aE_1}{R_c} + \frac{E_1 - \frac{V_t}{a}}{Z_1} + \frac{E_1}{\frac{R_2}{a-b} + jX_2} = 0 \quad \text{------------(1)}$$

Where, $Z_1 = (R_1/a) + j\,X_1$

$$\frac{E_1 - \frac{V_t}{a}}{Z_1} + \frac{\frac{V_t}{a}}{-j\frac{X_C}{a^2}} + \frac{\frac{V_t}{a}}{\frac{R}{a}} = 0 \qquad (2)$$

Equation (1) and (2) gives;

$$\frac{E_1}{jX_m} + \frac{aE_1}{R_c} + \frac{\frac{V_t}{a}}{-j\frac{X_C}{a^2}} + \frac{\frac{V_t}{a}}{\frac{R}{a}} + \frac{E_1}{\frac{R_2}{a-b} + jX_2} = 0 \quad (3)$$

Separation of real and imaginary parts of (3), results in the following;

$$\frac{\frac{E_1 R_2}{a-b}}{\left(\frac{R_2}{a-b}\right)^2 + X_2^2} + \frac{aE_1}{R_c} + \frac{V_t}{R} = 0 \qquad (4)$$

and

$$-\frac{E_1 X_2}{\left(\frac{R_2}{a-b}\right)^2 + X_2^2} + \frac{aV_t}{X_C} - \frac{E_1}{X_m} = 0 \qquad (5)$$

Simplification of (4) gives a simple expression in *a* as;

$$A_3 a^3 + A_2 a^2 + A_1 a + A_0 = 0 \qquad (6)$$

Where

$$A_3 = RX_2^2 E_1$$

$$A_2 = -2bRX_2^2 E_1 + V_t R_c X_2^2$$

$$A_1 = E_1 RR_2^2 + RE_1 b^2 X_2^2 + R_c RE_1 R_2 - 2V_t R_c bX_2^2$$

$$A_0 = V_t R_c R_2^2 - R_c RE_1 R_2 b + V_t R_c b^2 X_2^2$$

Solution of (6) gives the generated frequency for known values of operating speed and load resistance. Further exclusion of core loss branch leads to a quadratic equation in unknown frequency, in contrast to higher order polynomial equation in '*a*' as obtained by other research persons.
Equation (5) gives the unknown magnetizing reactance as;

$$X_m = \frac{E_1}{a\omega CV_t - \frac{E_1 X_2}{\left(\frac{R_2}{a-b}\right)^2 + X_2^2}} \qquad (7)$$

Analysis of phasor diagram of induction generator as shown in Fig.2 gives;

$$V_t = a_0\sqrt{E_1^2 - E_{1y}^2} - I_1 R_1 \cos\theta + a_0 I_1 X_1 \sin\theta$$

-------------(8)

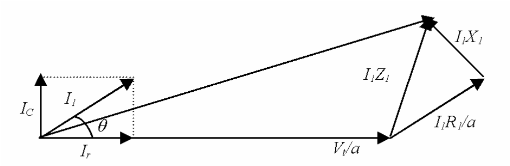

Fig.2. Phasor diagram of induction generator

Where

$$E_{1y} = \frac{I_1 R_1 \sin\theta}{a} + I_1 X_1 \cos\theta,$$

$$I_1 = \sqrt{I_L^2 + I_C^2}$$

E_{1y} is resolved component of E_1 along an axis perpendicular to terminal voltage.

3. Iteration Technique

Generated voltage and frequency for SEIG can be estimated using the following proposed iteration technique.

Step1. Computation of initial values of generated frequency and magnetizing reactance using following expression;

$$X_{m0} = \frac{1}{\dfrac{a_0}{X_C} - \dfrac{X_2(a_0 - b)^2}{R_2^2 + X_2^2(a_0 - b)^2}}$$

Initial values as;

$E_{10} = V_{t0}$ =1pu, $a_0 = 0.9999b$

Step2. Computation of air gap voltage, E_1 from magnetization characteristics (see Appendix-1 and Appendix-2).

Step3. Computation of modified values of stator voltage from (8).

Step4. Estimation of generated frequency 'a' and magnetizing reactance 'X_m' from (6) and (7) after using the modified values of E_1 and V_t.

Step5. Comparison of the new value of generated frequency 'a' with previous value i.e. a_0 as used in step 1.

If *Err1*=$|a - a_0|\langle\varepsilon$, Where ε =0.000001, a is treated as generated frequency and modified value of V_t may be treated as terminal voltage for SEIG.

If it is not so, process may be repeated by replacing 'a_0' with 'a' until difference in the successive values for generated frequency comes out to be ε.

It is also possible to compute the generated voltage and frequency by comparing the successive values of X_m and V_t obtained after each iteration. For such comparison iterative procedure terminates only if following expressions are satisfied.

Err2=$|X_m - X_{m0}|\langle\varepsilon$

Err3=$|V_t - V_{t0}|\langle\varepsilon$

4. Results and Discussions

Modeling proposed in the paper is found to be useful for estimation of generated frequency and magnetizing reactance of SEIG. Further, inclusion of core loss branch makes the analysis more realistic.

TABLE 1. COMPARISON OF EXPERIMENTAL AND SIMULATED RESULTS (MACHINE-1).

b=1.0133, Cpu=0.8448										
Sr. No.	*Zpu*	Experimental Results		Simulated Results using Phasor Diagram Based Analysis (PDA)						
				Err1		*Err2*		*Err3*		
		a	V_t	a	V_t	a	V_t	a	V_t	
1.	2.3632	0.9820	0.9078	0.9820	0.9721	0.9820	0.9720	0.9820	0.9720	
2.	2.5004	0.9830	0.9210	0.9835	0.9858	0.9835	0.9858	0.9835	0.9858	
3.	2.6628	0.9856	0.9526	0.9851	1.0002	0.9851	1.0002	0.9851	1.0002	
4.	2.8312	0.9864	0.9684	0.9866	1.0134	0.9866	1.0133	0.9866	1.0133	
5.	3.0247	0.9870	0.9868	0.9881	1.0266	0.9881	1.0265	0.9881	1.0265	
6.	3.5013	0.9892	1.0131	0.9912	1.0527	0.9912	1.0527	0.9912	1.0527	
7.	4.4193	0.9922	1.0578	0.9953	1.0869	0.9953	1.0868	0.9953	1.0868	
8.	6.1056	0.9960	1.0921	0.9997	1.1224	0.9997	1.1224	0.9997	1.1224	
9.	6.6759	0.9968	1.1000	1.0007	1.1303	1.0007	1.1303	1.0004	1.1285	
10.	7.1224	0.9984	1.1052	1.0013	1.1356	1.0013	1.1356	1.0013	1.1356	

Table 1 shows the comparison of experimental and simulated results on Machine-1 [Appendix-1] using iterative process as explained in section-3.

It is observed that irrespective of error function (in terms of $a / X_m / V_t$) the final values for generated voltage and frequency turns out to be same up to 3rd digit after decimal.

However minimum numbers of iterations are required for final results in case error function is in terms of frequency. Table 2 shows the simulated results for Machine-2. The closeness between the experimental and simulated results as shown proves the validity of modeling adopted.

Simulated results may be obtained for any type of load connected across the machine provided equations (6) and (7) are modified as given in appendix-2. Fig 3 and Fig 4 shows the variation of generated frequency and terminal voltage with load for different load power factors. Unity power factor load seems to be justified for better frequency and voltage regulation.

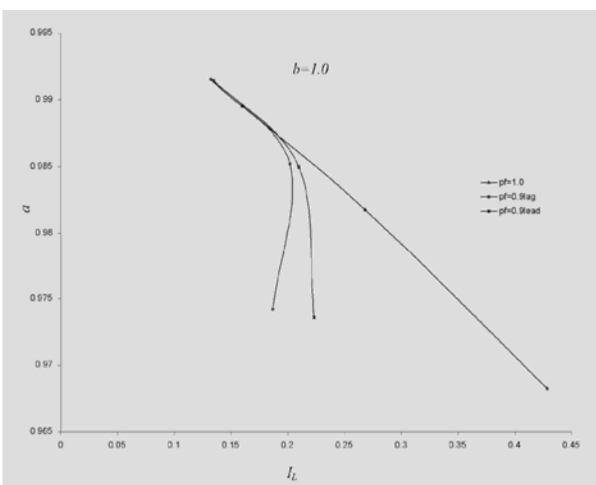

Fig. 3. Variation of generated frequency with load.

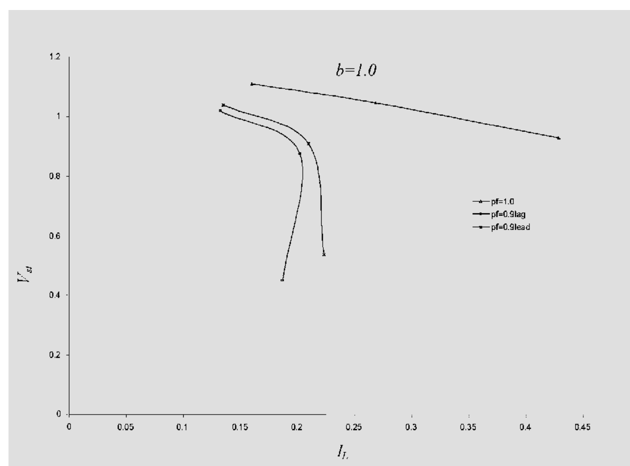

Fig. 4 Variation of stator voltage with load.

TABLE 2:- COMPARISON OF EXPERIMENTAL AND SIMULATED RESULTS (MACHINE-2)

Sr. No.	Experimental Results			Simulated Results using Phasor Diagram Based Analysis (PDA)	
	b	a	V_t	a	V_t
Zpu=3.4542 , Cpu=36					
1.	0.9533	0.9438	0.5826	0.942680	0.6644
2.	0.9780	0.9660	0.6869	0.967044	0.7675
3.	0.9986	0.9870	0.7652	0.987454	0.8400
4.	1.0106	0.9984	0.8217	0.999305	0.8818
5.	1.0286	1.0148	0.8826	1.017080	0.9440
6.	1.0466	1.0338	0.9434	1.034854	0.9975
7.	1.0640	1.0508	0.9913	1.051969	1.0371
Zpu=3.4542,Cpu=51					
8.	0.8533	0.8434	0.7217	0.8437	0.7605
9.	0.8806	0.8700	0.8130	0.8707	0.8434
10.	0.9020	0.8934	0.8739	0.8917	0.8920
11.	0.9266	0.9182	0.9347	0.9161	0.9479
12.	0.9600	0.9480	1.0086	0.9490	1.0231
Zpu=4.7495,Cpu=36					
13.	0.9613	0.9528	0.6695	0.9534	0.7417
14.	0.9780	0.9656	0.7434	0.9699	0.8010
15.	0.9973	0.9848	0.8173	0.9891	0.8692
16.	1.0266	1.0156	0.9130	1.0181	0.9714
17.	1.0420	1.0226	0.9739	1.0333	1.0095
Zpu=4.7495,Cpu=51					
18.	0.8566	0.8486	0.7565	0.8495	0.7981
19.	0.8766	0.8682	0.8130	0.8694	0.8521
20.	0.9000	0.8900	0.8782	0.8925	0.9057
21.	0.9240	0.9140	0.9391	0.9163	0.9605
22.	0.9373	0.9282	0.9695	0.9295	0.9909
23.	0.9533	0.9524	1.0304	0.9453	1.0272

5. Conclusions

In this paper attempt has been made to estimate the steady-state performance of self-excited induction generator using a new iterative technique approach based upon phasor diagram of the machine. The proposed technique has not been used by any other research person so far. Iterative technique is found to be very simple and effective. Simulated results are verified using experimental results on two test machines with different ratings. Close agreement between simulated and experimental results proves the validity of proposed modeling.

In future this research work may be extended to estimate the excitation capacitance and operating speed to obtain a constant voltage constant frequency operation. This may be helpful to promote the applications of self excited induction generators in remote and windy areas. In turn it may be helpful to preserve the conventional fuels which are likely to be finished with time.

Appendix 1

The details of the induction Machine-1 used to obtain the experimental results are;

- Specifications

3-phase, 4-pole, 50 Hz, star connected, squirrel cage induction machine
750W/1HP, 380 V, 1.9 A

- Parameters

The equivalent circuit parameters for the machine in pu are

$$R_1 = 0.0823, R_2 = 0.0696, X_1 = X_2 = 0.0766$$

- Base values

Base voltage =219.3 V
Base current =1.9 A
Base impedance=115.4 Ω
Base frequency=50 Hz
Base speed=1500rpm

- Air gap voltage

The variation of magnetizing reactance with air gap voltage at rated frequency for the induction machine is as given below.

$$X_m \langle 169.2 \qquad E_1 = 512.69 - 2.13 X_m$$
$$179.42 \rangle X_m \geq 169.20 \qquad E_1 = 891.66 - 4.37 X_m$$
$$184.46 \rangle X_m \geq 179.42 \qquad E_1 = 785.79 - 3.78 X_m$$
$$X_m \geq 184.46 \qquad E_1 = 0$$

Appendix 2

The details of the induction Machine-2 used to obtain the experimental results are;

- Specifications

3-phase, 4-pole, 50 Hz, delta connected, squirrel cage induction machine
2.2kW/3HP, 230 V, 8.6 A

- Parameters

The equivalent circuit parameters for the machine in pu are

$$R_1 = 0.0723, R_2 = 0.0379, X_1 = X_2 = 0.1047$$
.

- Base values

Base voltage =230 V
Base current =4.96 A
Base impedance=46.32 Ω
Base frequency=50 Hz
Base speed=1500rpm

- Air gap voltage

The variation of magnetizing reactance with air gap voltage at rated frequency for the induction machine is as given below.

$$X_m \langle 82.292 \qquad E_1 = 344.411 - 1.61 X_m$$
$$95.569 \rangle X_m \geq 82.292 \qquad E_1 = 465.12 - 3.077 X_m$$
$$108.00 \rangle X_m \geq 95.569 \qquad E_1 = 579.897 - 4.278 X_m$$
$$X_m \geq 108.00 \qquad E_1 = 0$$

Appendix-III

Case I: For R load (excluding R_c)

Expression in *a* can be written as;

$A_2a^2+A_1a+A_0=0$

Where

$$A_2 = V_t X_2^2$$
$$A_1 = E_1 R R_2 - 2bV_t X_2^2$$
$$A_0 = b^2 V_t X_2^2 - bE_1 R R_2 + V_t R_2^2$$

The expression of magnetizing reactance can be written as;

$$X_m = \frac{E_1}{a\omega C V_t - \dfrac{E_1 X_2}{\left(\dfrac{R_2}{a-b}\right)^2 + X_2^2}}$$

Here stator current, I_1 expression is same as written in the IV section.

Case II: For *RL* load (including R_c)

Expression in a can be written as;

$A_5a^5+A_4a^4+A_3a^3+A_2a^2+A_1a+A_0=0$

Where;

$$A_5 = E_1X^2X_2^2$$
$$A_4 = -2E_1bX^2X_2^2$$
$$A_3 = E_1R_2R_cX^2 + E_1R^2X_2^2 + X^2R_2^2E_1 + E_1X^2X_2^2$$

$$A_2 = -E_1R_2R_cbX^2 - 2bX_2^2 + V_tRR_cX_2^2$$
$$A_1 = E_1R_2R_cR^2 + E_1R^2R_2^2 + E_1b^2X_2^2R^2 - 2V_tRR_cbX_2^2$$

$$A_0 = -E_1R_2R_cR^2b + V_tRR_cR_2^2 + V_tRR_cb^2X_2^2$$

$$X = R\sqrt{\left(\frac{1}{pf}\right)^2 - 1}$$

The expression of magnetizing reactance can be written as;

$$X_m = \cfrac{E_1}{a\omega CV_t - \cfrac{E_1X_2}{\left(\cfrac{R_2}{a-b}\right)^2 + X_2^2} - \cfrac{\cfrac{V_tX}{a}}{\left(\cfrac{R}{a}\right)^2 + X^2}}$$

Case III: For *RL* load (excluding R_c)

Expression in a can be written as;

$A_3a^3+A_2a^2+A_1a+A_0=0$

Where;

$$A_3 = E_1R_2X^2$$
$$A_2 = V_tRX_2^2 - E_1R_2bX^2$$
$$A_1 = -2bV_tRX_2^2 + E_1R_2R^2$$
$$A_0 = V_tRR_2^2 + b^2V_tRX_2^2 - E_1R_2R^2b$$
$$X = R\sqrt{\left(\frac{1}{pf}\right)^2 - 1}$$

The expression of magnetizing reactance can be written as;

$$X_m = \cfrac{E_1}{a\omega CV_t - \cfrac{E_1X_2}{\left(\cfrac{R_2}{a-b}\right)^2 + X_2^2} - \cfrac{\cfrac{V_tX}{a}}{\left(\cfrac{R}{a}\right)^2 + X^2}}$$

Here stator current, I_1 expression for case III and case IV, is modified and written as;

$$I_1 = \sqrt{(I_C - I_L\sin\varphi)^2 + (I_L\cos\varphi)^2}$$

Case IV: For *RC* load (including R_c)

Expression in a can be written as;

$A_5a^5+A_4a^4+A_3a^3+A_2a^2+A_1a+A_0=0$

Where;

$$A_5 = E_1R^2X^2$$

$$A_4 = V_tRR_cX_2^2 - 2bE_1R^2X_2^2$$

$$A_3 = -2bV_tRR_cX_2^2 + E_1R^2R_2^2 + b^2E_1X_{se}^2X_2^2 + E_1R_2R_cR^2$$

$$A_2 = V_tRR_cR_2^2 + b^2V_tRR_cX_2^2 - 2bE_1X_{se}^2X_2^2 - E_1R_2R_cR^2b$$

$$A_1 = E_1 X_{se}^2 R_2^2 + b^2 E_1 X_{se}^2 X_2^2 + E_1 R_2 R_c X_{se}^2$$

$$A_0 = -E_1 R_2 R_c X_{se}^2 b$$

$$X_{se} = aR\sqrt{\left(\frac{1}{pf}\right)^2 - 1}$$

The expression of magnetizing reactance can be written as;

$$X_m = \frac{E_1}{\frac{aV_t}{X_C} - \frac{E_1 X_2}{\left(\frac{R_2}{a-b}\right)^2 + X_2^2} - \frac{aV_t X_{se}}{(aR)^2 + X_{se}^2}}$$

Case V: For *RC* load (excluding R_c)

Expression in *a* can be written as;

$$A_4 a^4 + A_3 a^3 + A_2 a^2 + A_1 a + A_0 = 0$$

Where;

$$A_4 = V_t R X_2^2$$

$$A_3 = -2V_t R X_2^2 + E_1 R_2 R^2$$

$$A_2 = V_t R R_2^2 + b^2 V_t R X_2^2 - E_1 R_2 b R^2$$

$$A_1 = E_1 R_2 X_{se}^2$$

$$A_0 = -bE_1 R_2 X_{se}^2$$

References

[1] K.S.Sandhu, “Steady state analysis of isolated induction generators”, *WSEAS Transactions on Environment and Development*, issue 1, volume 4, pp 66-77, Jan 2008

[2] K.S.Sandhu & Dheeraj Joshi, “A Simple Approach to Estimate the Steady-State Performance of Self-Excited Induction Generator”, *WSEAS Transactions on Systems and Control*, issue 3, volume,3, pp 208-218, March 2008

[3] K.S.Sandh & Shelly Vadhera, “Reactive Power Requirements of Grid Connected Induction Generator in a Weak Grid”, *WSEAS Transactions on Circuits and Systems*, issue3, volume7, pp 150-159, March 2008.

[4] K.S. Sandhu & S.P.Jain, “Steady State Operation of Self-Excited Induction Generator with Varying Wind Speeds”, *International Journal Of Circuits, Systems And Signal Processing* ,Issue 1, Volume 2, 2008.

[5] S.S.Murthy,O.P.Malik, and A.K.Tandon, “Analysis of self-excited induction generators,” *Proc. IEE*,vol. 129,pt. C, no. 6,1982, pp. 260-265.

[6] L.Quazene and G. McPherson, “Analysis of the isolated induction generator,” *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, no. 8, 1983, pp. 2793-2798.

[7] G. Raina and O. P. Malik, “Wind energy conversion using a self-excited induction generator,” *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, no. 12,1983,pp. 3933-3936.

[8] A. K. Tandon, S. S. Murthy and C. S. Jha, “New method of computing steady-state response of capacitor self-excited induction generator,” *IE(I) Journal-EL*, vol. 65,1985, pp. 196-201.

[9] N. H. Malik and S. E. Haque, “Steady-state analysis and performance of an isolated self-excited induction generator,” *IEEE Trans. Energy Conversion*, vol. EC-1, no.3, 1986,pp.134-139.

[10] T. F. Chan, “Steady-state analysis of self-excited induction generators,” *IEEE Trans. Energy Conversion*, vol. 9, no. 2, 1994, pp.288-296.

[11] T. F. Chan, "Analysis of self-excited induction generators using an iterative method," *IEEE Trans. Energy Conversion*, vol. 10 no. 3, 1995,pp. 502-507.

[12] K. S. Sandhu, "Iterative model for the analysis of self-excited induction generators," *Electric Power Components and Systems*, vol. 31, no. 10, 2003, pp. 925-939.

[13] Dheeraj Joshi, K.S. Sandhu and M.K. Soni, "Performance analysis of three-phase self excited induction generator using genetic algorithm", *Electric Power Components and Systems*, vol. 34, no. 4, 2006, pp. 461-470.