Alternative Technique of Classifying PMUs at Optimal Environment in Power System

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Abstract: This paper presents an alternative technique of classifying measuring units in a measuring optimal environment of a power system. Each measurement is given a certain weight depending on its effect on state estimation solution. The technique used is to fit curve fitting to determine the optimal approximation in the absence of other measurements. Absence of measurement may be due to the lack of communication links, being identified as a bad measurement and that it has to be removed, or for any other reason that may apply. To obtain the measurement unit placement, singular value decomposition (SVD) technique was used to solve the problem of minimum number of measurements placement while maintaining accuracy as well as network observability. The relative error is used to compare the change in estimated solution related to the true value. Finally, the critical and non-critical measurements are identified in the system. The proposed method is applied to determine the measurement weights of the measurements weights of optimal PMU weights in state estimator solutions for the IEEE 14-bus, IEEE 30-bus and Tanzania Electric Supply Company (TANESCO) systems.

Key-Words: Measurement placement, accuracy, weight, state estimation, coefficient of determination

1 Introduction

A state estimator uses information from various measurement units and monitoring systems to estimate the states of a power system. Due to the current large and complexities in power systems, the demand of modern energy management systems (EMS) has also increased. However, it is impractical to install each measuring unit at each location; therefore, several locations will be unmetered. The role of the state estimation is to obtain the best estimate of the system with the available measuring units in the network [1]. Errors (noise) from the meter readings received at the control room can severely impact the quality of state estimation. Therefore, knowledge of their impact in the state estimation solution will draw the attention to the operators to monitor those measuring units very closely so as to minimize the state estimator’s errors. Due to the limited number of measuring units as well as minimization of the installation cost, the minimum number of measuring units is kept as the objective. This research intends to workout both the minimization of state estimation errors, that is, to maintain the accuracy as well as to obtain the effect of each measurement in the power system.

1.1 Background

The basic goal of placement algorithms is to achieve full system observability with a minimum number of measurement units so as to minimize the installation cost. A reliable estimate of the state of the system must be determined before any security assessment or control actions taken. In order to obtain the optimal number of measurements while maintaining the guarantee the accuracy for measurement to establish energy management system, the placements of these measurements is the critical in the power system network.

The aim of minimal measurement placement is to obtain a metering distribution system that is observable with established accuracy and cost. There are several proposed measurement placement methods. In [1], the importance of bus injection measurements over the line measurements is explained. As it has been presented in [2] and [3], algorithm used for measurement placement for power system nonlinear state estimation has a reduced number of possible combinations to be considered using condition number method. The gain or measurement matrix has to be formulated first. Each row of the gain matrix is temporarily
removed one at a time to obtain the objective function of measurement matrix. The possible location that has a minimum objective function is removed so as to reduce the possible location. Other methods such as the use of binary genetic algorithm [5], tabu search algorithm, an iterative search that starts from some initial feasible solution and attempt to determine a better solution in the manner of hill-climbing algorithm [8]. Particle swarm optimization (PSO) is one of the global optimization method expressed in [12] where the basic assumption behind the algorithm is the birds finding food by flocking and not individually. In [13] the exploitation of the optimization using genetic algorithm (GA) is explained.

1.2 Paper Organization
This paper is organized as follows. Section 1, is the introduction. Section 2 provides the background of power system state estimation. Section 3 is the illustration of the algorithm used. Section 4 gives results obtained and section 5 provides relevant conclusions.

2 Power System State Estimation
The most common algorithm used in achievement of state estimation is weighted least-squares criterion where the objective is to minimize the sum of the squares of the weighted deviations from the estimated measurements from the actual ones [9], [13], and [14]. Variable vector $x$ include the magnitude voltage and phase angle of the bus voltage measurement that are utilized to describe the operation state of a power system. The measurement vector can be expressed in (1) where $e$ is a random measurement error vector while $z$ and $H(.)$ are measurement vector received from the available measurement units and nonlinear measurement function vector respectively.

$$z = H(x) + e$$

The estimated state is obtained through minimization of objective function $J(x)$ given in (2) with respect to the variable vector $x$, given that $\sigma_{i}$ is the standard deviation of error of the $i^{th}$ measurement.

$$J(x) = \sum_{i=1}^{m} \left( \frac{|z_i - f_i(x)|^2}{\sigma_i^2} \right)$$

where $m$ is the number of measurements within the power system, $f_i(.)$ is the function used to calculate the value being measured by $i^{th}$ measurement.

The sum of the squares of the deviation of the $f_i(.)$ from their mean $\bar{f}_i(x)$ can be expressed by $\epsilon$ which is explained more in [15] and expressed as

$$\epsilon_x = \sum_{i=1}^{m} \left[ f_i(x) - \bar{f}_i(x) \right]^2$$

Equation (3) is used to compute the coefficient of determination, or $r$-squared ($r^2$) value, given in (4) which is a measure of quality of the curve fit.

$$r^2_x = 1 - \frac{J(x)}{\epsilon_x}$$

For perfect fit, $J(x) = 0$ and thus $r^2_x = 1$. Thus closer $r^2$ to 1, which express the better fit. The value $\epsilon$ indicates how much the data spread around the mean and the value $J(x)$ indicates how much data spread is unaccounted for by the model. Thus the ratio $\frac{J(x)}{\epsilon_x}$ indicates the fractional variation accounted for by the model [15]. For the model in which $r^2$ is negative indicates a very poor model that should not be used. As a rule of thumb [15], $r^2 \geq 0.99$.

Equation (2),(3) and (4) can be applied to individual estimated variables $x$.

$$J(x) = \sum_{i=1}^{m} \left[ x_i - \bar{x}_i \right]^2$$

where $J(x)$ is sum of the squares of the deviation of the $x_i$ from their estimated values, $\bar{x}$, and $\epsilon_x$ is the sum of the squares of the deviation of the estimated variables $\bar{x}_i$ from their mean $\bar{x}$.

Solution of state variable $x$ in the iterative algorithm to (1) is obtained by solving (7).

$$\Delta x = G^{-1}H^T R^{-1} \Delta z$$

where $R^{-1} = \begin{bmatrix} 1 & \sigma_1^2 & \sigma_2^2 & \cdots \\ \sigma_1^2 & 1 & \sigma_2^2 & \cdots \\ \sigma_2^2 & \sigma_1^2 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$, $G = H^T H$, and $H$ is the measurement Jacobian matrix, while $\sigma_i$ is a measurements unit standard deviations.

The study of singular value decomposition is used to obtain a minimum rank matrix. The numerical rank of a $(m \times n)$ matrix is the number of singular value of the matrix that is greater than $\sigma_{max} (m,n)e$, where $\sigma_{max}$ is the largest singular value of a matrix and $e$ is a machine epsilon. In this study, the network system measurements will include injected active power, injected reactive power and voltage measurements. For the $N$ bus
system, there will be; \( N \) possible locations for active power measurements, \( N \) possible locations for reactive power measurements, and \( N \) possible locations for voltage measurements. Therefore, there will be a total of \( 3N \) possible locations which forms the number of matrix rows. Similarly, there will be \( 2N-1 \) state variables which will form the column of the matrix. Therefore, the measurement matrix \( H \) will be \((m \times n)\) where \( m \) is a total number of possible locations, and \( n \) is the number of state variables. In this case, for a minimum measurement placement while maintaining observability the rank of the measurement matrix is considered. The results obtained in this paper are based on the status of measurements matrix \( H \) formed by inspecting its error effect through the quality of fitness.

3 Illustration of the Method Used

In this paper, the algorithm used is shown in Fig. 1. In order to obtain the optimum measurement unit placement, the algorithm proposed in [2] is used. However, the number of measurements to be obtained is \( 2N \) measurements instead of the proposed \((2N-1)\) so that the removal of the measurement to obtain the coefficient of determinant does not affect the observability of the network, and scale of the matrix. That is, the remaining \((2N-1)\) is still available in the network. Other methods can be used for the similar purpose.

The next step is to remove each measurement and the coefficient of determinant is calculated. Finally, classify measurements according to their coefficient of determinants, and measurements are classified such as a critical and non critical. As stated earlier in section 2, coefficient of determination \( \sim 1.00 \) shows that the low measurement index (weight) of the particular measurement removed. Negative shows the critical measurements and non critical measurements are categorised by their coefficient of determination, i.e. how close to 1.00. The more deviation from 1.00, the higher the measurement weight.

In Fig. 6, it is shown that, the weight of the measuring unit is observed by their effect of accuracy and that the critical measuring units have more effect in the accuracy of the system estimation. Loss of critical measurements can severely affect the state estimation accuracy, therefore the reading from these locations must be obtained.

3.1 Application Example

The algorithm has been tested using IEEE 14-bus test system shown in Fig. 2 and TANESCO mirror network shown in Fig. 4.

The measurement covariance matrix in (7) is determined by assuming Gaussian distributions with standard deviation of 0.001 for error in injection measurements and 0.04 for voltage measurements.

4 Results

In this section, obtained results from simulations of IEEE 14- bus, IEEE 30-bus testing systems and Tanzania Electric Supply Company (TANESCO) mirror network are presented.

4.1 IEEE 14-Bus Testing System

Table 1 shows the true and estimated values of the variables obtained when IEEE 14-bus test system was in test. Three measuring units are considered (i.e. active injected power, reactive injected power and voltage measuring units). It is observed that, the accuracy and better estimation depends on the type of measurements and number of different measurements used. This is illustrated in Fig. 5 where different types of measurements are used to obtain the residual in magnitude voltage. In determining the optimum measurement locations, measurements locations were as follows: Thirteen (13) injected active power measurements at bus 1-3, and 5-14. Eight (8) injected reactive power
measurements at bus 1-2, and 8-13. Six (6) voltage measurements at bus 3-5, 7, 12, and 14. The voltage measurement at bus bar number 13 was obtained as a 28th measurement. It was also observed that, when some measurements are removed, the Matrix becomes close to singular or badly scaled, here the weight of these measurements are given weight (index) to identify them as critical measurements. For 28 measurements in optimal measurement environment, the coefficient of determinant obtained in 0.9998. Tables 2 to 4 provide the coefficient of measurement when measurements are removed.

4.1.1 Injected active measurement units
In this simulation, injected active power measurements, five measurements were identified to be critical measurements: measurements at bus 3, 7, 8, 9 and 10. These are shown by negative coefficient of determination in Table 2. Other measurements may affect the state estimation based on their coefficient of determination and in this paper, they are considered to be non critical measurements. For example, injected active measurement in bus 5 is more sensitive than the rest of non critical measurements, while injected active power measurement in bus 13 is the least sensitive in the system measurements, thus low measurement index.

4.1.2 Injected reactive measurement units
For injected reactive power measurements, four measurements were identified critical measurements: at bus 1, 2, 8 and 11 as shown in Table 3. Injected reactive power at bus 13 is less sensitive than other injected reactive power measurements.

4.1.3 Voltage measurement units
Voltage measurement at bus 4 has been identified a critical measurement as shown in Table 4. Voltage measurements at bus 13 and 14 are the least sensitive, hence, low measurement index.

The relative error caused by the removal of a relative measurement is shown in Fig. 6 where the numbers of measurements are given in a sequence from the injected active measurement in bus 1, through injected reactive measurements to the voltage measurement in bus 14.

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Table 1: IEEE 14-bus system, true values for bus mag. / phase voltage, estimated mag. / phase voltage, and errors between true and estimated values

<table>
<thead>
<tr>
<th>Bus #</th>
<th>True voltage</th>
<th>Est. voltages</th>
<th>estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mag (p.u)</td>
<td>Angle (deg)</td>
<td>Mag (p.u)</td>
</tr>
<tr>
<td>1</td>
<td>1.060</td>
<td>0.00</td>
<td>1.034</td>
</tr>
<tr>
<td>2</td>
<td>1.045</td>
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<tr>
<td>3</td>
<td>1.010</td>
<td>-12.74</td>
<td>1.010</td>
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<tr>
<td>4</td>
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<td>-10.28</td>
<td>1.019</td>
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<td>1.053</td>
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<td>-14.78</td>
<td>1.055</td>
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<tr>
<td>13</td>
<td>1.050</td>
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<tr>
<td>14</td>
<td>1.036</td>
<td>-16.02</td>
<td>1.036</td>
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Table 2: IEEE 14-bus, injected active meas.; Coefficient of determinants in case of meas. loss

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<th>Injected Active Power Measurement Units</th>
<th>r²</th>
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<td>5</td>
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<td>6</td>
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<td>12</td>
<td>0.9988</td>
</tr>
<tr>
<td>13</td>
<td>0.9537</td>
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Table 3: IEEE 14-bus, injected reactive meas.; Coefficient of determinants in case of meas. loss

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<th>Injected Reactive Power Measurement Units</th>
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</tr>
<tr>
<td>3</td>
<td>-2.4285</td>
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</table>

Table 4: IEEE 14-bus, voltage meas.; Coefficient of determinants in case of meas. loss

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<tr>
<th>Voltage Measurement Units</th>
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<td>3</td>
<td>0.3293</td>
</tr>
<tr>
<td>4</td>
<td>0.7824</td>
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<tr>
<td>5</td>
<td>0.9998</td>
</tr>
<tr>
<td>6</td>
<td>0.9998</td>
</tr>
<tr>
<td>7</td>
<td>0.9993</td>
</tr>
</tbody>
</table>
4.2 IEEE 30-Bus Testing System

IEEE 30-bus system is shown in Fig. 3. For optimum measurement locations, measurements locations were as follows: Twenty-nine (29) injected active power measurements at bus 1-11, and 13-30. twenty (20) injected reactive power measurements at bus 1-6, 10, 14, 16, and 19-29. Eleven (11) voltage measurements at bus 6-7, 9, 11-13, 15, 17-18, 28 and 30.

Table 5 shows the true and estimated values of the variables obtained when IEEE 30-bus test system was in test. The voltage measurement at bus bar number 6 was obtained as a 60th measurement.

For 60 measurements in optimal measurement environment, the coefficient of determinant obtained in 0.9975. Tables 5 to 7 provide the coefficient of measurement when measurements are removed.

4.2.1 Injected active measurement units

Eleven (11) injected active power measurements were identified critical measurements. These measurements are at bus 3-5, 10, 13-14, and 16-20 shown by negative coefficient of determination in Table 6. Other measurement units in Table 6 are considered non-critical. In this measurement system
used, measurement unit at bus 28 is the least sensitive among the injected active measurement units in IEEE 30 bus testing system network.

Table 5: IEEE 30-bus testing system, true values for bus mag. /phase voltage, estimated mag. /phase voltage, and errors between true and estimated values

<table>
<thead>
<tr>
<th>Bus</th>
<th>True voltage</th>
<th>Est. voltages</th>
<th>estimated error</th>
</tr>
</thead>
<tbody>
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<td>#</td>
<td>Mag (p.u)</td>
<td>Angle (deg)</td>
<td>Mag (p.u)</td>
</tr>
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<td>0.859 0.000</td>
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<td>0.040 0.139</td>
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<td>0.0002 2.079</td>
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Table 6: IEEE 30-bus, injected active meas.; Coefficient of determinants in case of meas. loss

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<th>Bus</th>
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<th>4</th>
<th>5</th>
<th>6</th>
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<table>
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<table>
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<td>*</td>
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</table>

- Matrix is close to singular or badly scaled

Table 7: IEEE 30-bus, injected reactive meas.; Coefficient of determinants in case of meas. loss

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<th>Bus</th>
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<td>t²</td>
<td>0.9965</td>
<td>0.9966</td>
<td>0.9968</td>
<td>0.9968</td>
<td>0.9959</td>
<td>0.9973</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus</th>
<th>10</th>
<th>14</th>
<th>16</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>t²</td>
<td>0.7738</td>
<td>-4.5951</td>
<td>*</td>
<td>-1.3135</td>
<td>-0.3412</td>
<td>0.9803</td>
<td>0.9853</td>
</tr>
</tbody>
</table>

- Matrix is close to singular or badly scaled

4.2.2 Injected reactive measurement units

For injected reactive power measurements, four (4) measurements were identified critical measurements: at bus 14, 16, 19, and 20 as shown in Table 7. Injected reactive power at bus 6 is less sensitive than other injected reactive power measurements.

4.2.3 Voltage measurement units

Three (3) voltage measurements at buses 11, 13 and 18 have been identified a critical measurement as shown in Table 9. Voltage measurement at bus 6 is the least sensitive. Other measurements’ coefficients of determination are shown in Table 8.
Table 8: IEEE 30-bus, voltage meas.; Coefficient of determinants in case of meas. loss

<table>
<thead>
<tr>
<th>Voltage Measurement Units</th>
<th>Bus 6</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^2$</td>
<td>0.9975</td>
<td>0.9965</td>
<td>0.9959</td>
<td>*</td>
<td>0.9952</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>17</td>
<td>18</td>
<td>28</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9454</td>
<td>0.6721</td>
<td>*</td>
<td>0.9965</td>
<td>0.9975</td>
<td></td>
</tr>
</tbody>
</table>

* - Matrix is close to singular or badly scaled

4.3 Case Study: TANESCO Network

This method is also applied to the Tanzania Electric Supply Co. (TANESCO) network shown in Fig. 4. The system includes 32 which imply that there are 96 measurement locations. Transmission lines use pylons made of steel. Almost all the transmission lines are radial single circuit lines. The transmission lines are estimated to comprise of 2,624.36 km of system voltages 220 kV; 1441.50 km of 132 kV; and 486.00 km of 66 kV, totalling to 4551.86 km. The system is all alternating current (AC) and the system frequency is 50 Hz [10].

Table 9 is the simulation results for TANESCO power system. In this network thirty-one (31) injected active power measurements at bus 1-21 and 23-32. Twenty-six (26) injected reactive power measurements at bus 1-8, 10-11, 13-19, 22-26, 28, and 30-32. Six (6) voltage measurements at bus 9, 12, 16, 18, 21, 27 and 29. The voltage measurement at bus bar 16 was obtained as a 64th measurement. For 64 measurements in optimal measurement environment, the coefficient of determinant obtained in 0.9387.

Fig. 4 TANESCO power network

Fig. 5 IEEE 14-bus system magnitude voltage error in each bus
Table 9: TANESCO Network, true values for bus mag. /phase voltage, estimated mag. /phase voltage, and errors between true and estimated values

<table>
<thead>
<tr>
<th>Bus</th>
<th>True voltage</th>
<th>Est. voltages</th>
<th>estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>Mag (p.u)</td>
<td>Angle (deg)</td>
<td>Mag (p.u)</td>
</tr>
<tr>
<td>1</td>
<td>1.017</td>
<td>0.00</td>
<td>0.817</td>
</tr>
<tr>
<td>2</td>
<td>1.022</td>
<td>-5.11</td>
<td>0.962</td>
</tr>
<tr>
<td>3</td>
<td>1.009</td>
<td>-2.41</td>
<td>0.969</td>
</tr>
<tr>
<td>4</td>
<td>1.001</td>
<td>-7.02</td>
<td>1.001</td>
</tr>
<tr>
<td>5</td>
<td>1.032</td>
<td>-2.17</td>
<td>1.032</td>
</tr>
<tr>
<td>6</td>
<td>1.008</td>
<td>-2.91</td>
<td>1.007</td>
</tr>
<tr>
<td>7</td>
<td>1.017</td>
<td>-5.71</td>
<td>1.017</td>
</tr>
<tr>
<td>8</td>
<td>1.028</td>
<td>-3.82</td>
<td>1.027</td>
</tr>
<tr>
<td>9</td>
<td>1.033</td>
<td>-4.01</td>
<td>1.023</td>
</tr>
<tr>
<td>10</td>
<td>1.041</td>
<td>-7.77</td>
<td>1.041</td>
</tr>
<tr>
<td>11</td>
<td>0.997</td>
<td>-4.04</td>
<td>0.987</td>
</tr>
<tr>
<td>12</td>
<td>1.012</td>
<td>-8.98</td>
<td>1.012</td>
</tr>
<tr>
<td>13</td>
<td>1.029</td>
<td>-3.19</td>
<td>1.029</td>
</tr>
<tr>
<td>14</td>
<td>1.030</td>
<td>-2.99</td>
<td>1.030</td>
</tr>
<tr>
<td>15</td>
<td>0.951</td>
<td>-3.86</td>
<td>0.931</td>
</tr>
<tr>
<td>16</td>
<td>0.968</td>
<td>-9.81</td>
<td>0.967</td>
</tr>
<tr>
<td>17</td>
<td>1.011</td>
<td>-3.86</td>
<td>1.011</td>
</tr>
<tr>
<td>18</td>
<td>1.029</td>
<td>-8.01</td>
<td>1.028</td>
</tr>
<tr>
<td>19</td>
<td>1.082</td>
<td>-9.45</td>
<td>1.081</td>
</tr>
<tr>
<td>20</td>
<td>0.988</td>
<td>-8.81</td>
<td>0.988</td>
</tr>
<tr>
<td>21</td>
<td>1.011</td>
<td>-1.59</td>
<td>1.011</td>
</tr>
<tr>
<td>22</td>
<td>1.007</td>
<td>-7.64</td>
<td>1.006</td>
</tr>
<tr>
<td>23</td>
<td>1.02</td>
<td>-9.08</td>
<td>1.020</td>
</tr>
<tr>
<td>24</td>
<td>1.039</td>
<td>-1.03</td>
<td>1.039</td>
</tr>
<tr>
<td>25</td>
<td>1.027</td>
<td>-6.05</td>
<td>1.026</td>
</tr>
<tr>
<td>26</td>
<td>1.024</td>
<td>-5.94</td>
<td>1.004</td>
</tr>
<tr>
<td>27</td>
<td>1.048</td>
<td>-4.01</td>
<td>1.048</td>
</tr>
<tr>
<td>28</td>
<td>1.047</td>
<td>-7.05</td>
<td>1.067</td>
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<tr>
<td>29</td>
<td>1.03</td>
<td>-9.82</td>
<td>1.140</td>
</tr>
<tr>
<td>30</td>
<td>1.044</td>
<td>-6.20</td>
<td>1.044</td>
</tr>
<tr>
<td>31</td>
<td>1.018</td>
<td>-3.42</td>
<td>1.018</td>
</tr>
<tr>
<td>32</td>
<td>1.013</td>
<td>-3.05</td>
<td>1.013</td>
</tr>
</tbody>
</table>

Table 10: TANESCO mirror network system, injected active meas.; coefficient of determinants in case of meas. loss

<table>
<thead>
<tr>
<th>Bus</th>
<th>Injected Active Power Measurement Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r²</td>
</tr>
<tr>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>7</td>
<td>0.6749</td>
</tr>
<tr>
<td>8</td>
<td>0.5414</td>
</tr>
<tr>
<td>9</td>
<td>0.9036</td>
</tr>
<tr>
<td>10</td>
<td>0.551.54</td>
</tr>
<tr>
<td>11</td>
<td>0.2822</td>
</tr>
</tbody>
</table>

4.3.1 Injected active measurement units

For 64 measurements in optimal measurement environment, the coefficient of determinant obtained in 0.9380. Tables 6 to 8 provide the coefficient of measurement when measurements are removed.

Seven (7) injected active power measurements were identified critical measurements. These measurements are at bus 9, 23, 25, 26, 27, 29, and 32 shown by negative coefficient of determination in Table 10. Other measurement units in Table 7 are considered non-critical. Measurement unit at bus 16 is the least sensitive among the injected active measurement units.

4.3.2 Injected reactive measurement units

For injected reactive power measurements, five (5) measurements were identified critical measurements: at bus 23, 24, 25, 26 and 28 as shown in Table 11. Injected reactive power at bus 31 is less sensitive than other injected reactive power measurements.

4.3.3 Voltage measurement units

Voltage measurement at bus 27 has been identified a critical measurement as shown in Table 12. Voltage measurement at bus 16 is the least sensitive. Other measurements’ coefficients of determination are shown in Table 12.
Table 11: TANESCO mirror network system, injected reactive meas.; coefficient of determinants in case of meas. loss

<table>
<thead>
<tr>
<th>Injected Reactive Power Measurement Units</th>
<th>Bus</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^2 )</td>
<td>0.9308</td>
<td>0.9370</td>
<td>0.9331</td>
<td>0.9382</td>
<td>0.9292</td>
<td>0.9286</td>
<td>0.9246</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>11</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>0.9336</td>
<td>0.9379</td>
<td>0.9378</td>
<td>0.9431</td>
<td>0.9293</td>
<td>0.9289</td>
<td>0.9289</td>
<td>0.9369</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>19</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>0.8537</td>
<td>0.8982</td>
<td>0.7450</td>
<td>-0.2317</td>
<td>-62.40</td>
<td>-56.06</td>
<td>-62.19</td>
<td>-3.512</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>31</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9312</td>
<td>0.9383</td>
<td>0.9354</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 12: TANESCO mirror network system, voltage meas.; Coefficient of determinants in case of meas. loss

<table>
<thead>
<tr>
<th>Voltage Measurement Units</th>
<th>Bus</th>
<th>9</th>
<th>12</th>
<th>16</th>
<th>18</th>
<th>21</th>
<th>27</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^2 )</td>
<td>0.9331</td>
<td>0.9379</td>
<td>0.9387</td>
<td>0.9364</td>
<td>0.6101</td>
<td>-48.50</td>
<td>0.9158</td>
<td></td>
</tr>
</tbody>
</table>

5 Conclusion

It is observed that, in the optimal measurement placement environment, critical measurements readings must be observed so as to estimate the power system state. In case the measurement is not observed other alternatives such as using pseudo readings, or previous readings to be used.

Weight of the measurement in optimal measurement placement can be used to identify measurement unit effect in state estimation. That is, the state estimation is more affected when bad readings come from the high weighted measuring units. Operators may take a special measure to ensure that, the high weighted measuring units are given priority in obtaining their readings accurately.

References:


