# Complete Dynamic Behaviour Mathematical Modelling of Hydromechanical Equipment. Case study: Hydro Power Plant Raul Mare-Retezat, Romania

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*Abstract:* - The paper is presenting an application of “HYDRO” mathematical model for simulating the dynamics of hydraulic equipment in the special case of the fast loading with electric power of Raul Mare-Retezat, hydro power plant, Romania. Based on the mathematical model described in the paper, a software application is developed in Delphi environment. This software tool is designed for studying the behaviour of the model in the event of perturbations. The results of the mathematical simulation are compared with those experimentally obtained through SCADA monitoring system with which the hydro-power plant is equipped.

*Key-Words:* - Hydro power plant, Dynamic behaviour, Mathematical model, Hydraulic parameters, Hydromechanical equipment, Hydro turbine.

## 1 Introduction

The „HYDRO” model was elaborated many years ago, as a necessity of detailing the mathematical description of the elements composing the hydromechanical equipment of hydro-power plants in long-term dynamics [1], [2]. Then the model was experimentally tested using concrete data of hydro-power plant Crainicel-Resita. The „HYDRO” model is presently used as an example of detailed mathematical expression of this equipment in the course of „Modelling and Identifying the Elements of Power Systems” taught by Prof.F.D. Surianu in the cycle of Master courses. The model has remarked and proposed its application to the Hydro-Power Plant „Raul Mare-Retezat” [3]. The results obtained in this applications have been compared with the experimentally registered ones and have proved the validity of the HYDRO mathematical model as well as its usefulness in simulating the dynamic behaviour of the hydro-power plants in long- therm dynamic stability.

## 2 Brief description of „Hydro” mathematical model

In simulating long-term dynamics of power systems, an initial perturbation can be the source of new perturbations which can appear, especially, in the hydromechanical equipment of the power plants, with the corresponding delays of the big time constants of these equipment and they may lead to an important worsening of post-damage states through which the system passes. That is why a simplified mathematical representation of the hydro-mechanical equipment is inadequate, there being necessary to detail the mathematical description of the working and regulating elements of these equipment for all types of power plants.

So, for hydro-power plants they have given up the well-known classical mathematical descriptions of the hydro-mechanical equipment shown in Fig. 1, a, b in favour of a complex mathematical representations of the composing elements of the hydro-unit system (feed pipe, water tower, forced-pipe, hydraulic turbine) as well as of the corre-sponding regulating systems (valve regulating system, speed governing system etc.), as shown in Fig.2.

As hydraulic systems cannot be standardized, they depending on the geographical situation of the area, it is necessary to detail the mathematical description of their composing units. Thus, to get a mathematical modelling of the hydraulic system we have to establish the characteristic working equations for each unit in common working conditions, then assemble them in a mathematical system able to be solved through a known numerical method.

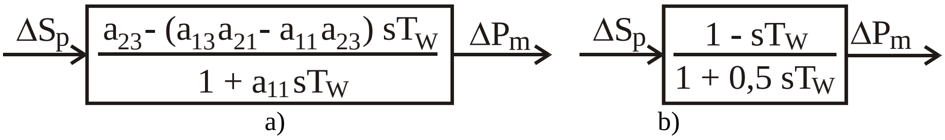



Fig.1. Simplified mathematical models of hydromechanical equipment of a hydro-unit.
a) Block scheme of a linear mathematical model of hydraulic turbine (*Sp*-deviation of valve position; *Pm* – variation of turbine mechanical power. *Tw* – time constant of water; $s = d/dt$ time operator), $a_{11}=0.5$, $a_{12}=0$, $a_{13}=1$, $a_{21}=1.5$, $a_{22}=-1$, $a_{23}=1$ for an ideal turbine at small oscillations; b) Ideal model of the hydraulic turbine.

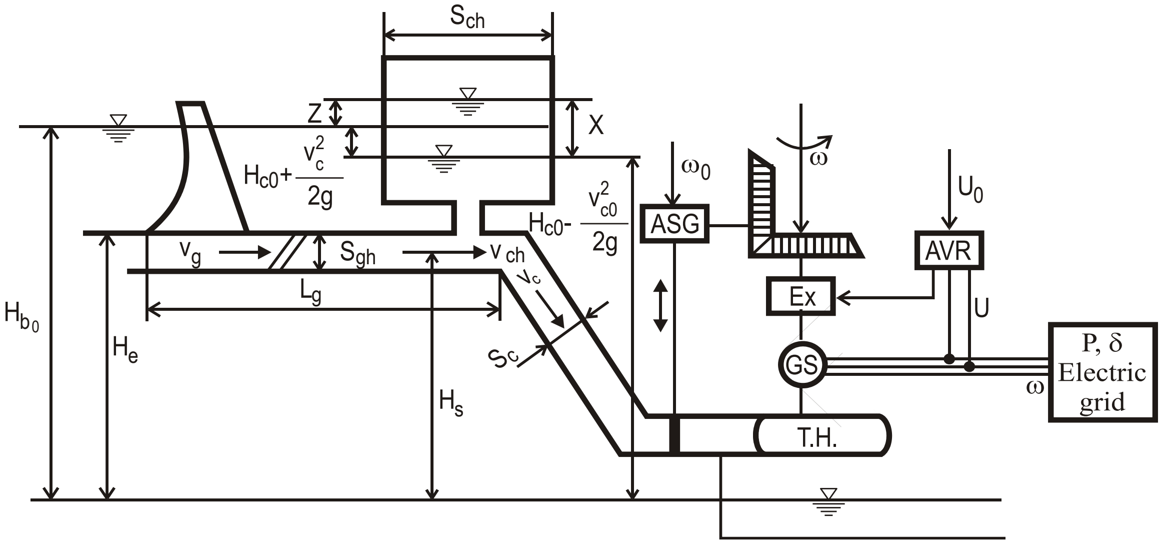



Fig.2. General diagram of a hydro-unit, detailing the main elements of the hydraulic system.

In order to get such a complex mathematical method, the following steps have been followed:

a) All the values have been expressed in per units (p.u) referred to the basic values represented by the corresponding absolute values in the working point of the stationary regime, previous to perturbation.

b) There have been neglected the small infinities of a degree bigger than two, retaining only the first two terms of the series development around the point corresponding to the permanent ante-perturbations regime, thus realizing the linearity of equations.

c) The following basic values have been established for the water tower and feed pipe:

$Z^* = v_{g0}\sqrt{\dfrac{L_g S_g}{g S_{ch}}}$ – the maximum water level in the water tower when the valves are totally closed and the load loss is zero;

$T_g = \sqrt{\dfrac{L_g S_{ch}}{g S_g}}$ – the time constant of the feed pipe.

d) The mathematical modelling of the hydraulic turbine has been realized according to the following algorithm:

$d_1$) There have been defined four adimensional values for the hydraulic turbine, depending on its mechanical, hydraulic and geometrical parameters.

- turbine energy value: $\varepsilon = \dfrac{2gH_n}{R^2 n^2}$
- turbine flow capacity value:

$$\gamma = \frac{Q_n}{SR^3 n} \;; \qquad (1)$$

- turbine power value: $\psi = \dfrac{2Pm_n}{\rho S R^5 n^3}$
- turbine output: $\eta = \dfrac{P_{m_n}}{\rho g H_n Q_n}$

The relation among the four values is, the following:

$$\psi = \varepsilon \cdot \gamma \cdot \eta$$

$d_2$) There has been defined the reference section of the turbine:

$$s = \frac{S}{R^2} \;\; [u \cdot r] \qquad (2)$$

where: *S* is the turbine section and *R* is the radius of the bladed turbine.

For Pelton turbine $S = \pi R_e^2$, where: $R_e$ = radius of distributor jet.

For Francis, Kaplan and helicoid turbine, $S = \pi\left(R^2 - R_n^2\right)$, where: $R_n$ - axis radius.

d$_3$) For the hydraulic turbine, taking into account the output curve, η = invariable and the position of the directing apparatus, $A$ = invariable, represented in plan as shown in Fig.3, any stable working point is contained in the two plan, tangent to the surfaces described by the following functions $F(\varepsilon,\gamma,\eta) = 0$ and $G(\varepsilon,\gamma,A) = 0$.

Since the position of each plan is determined by two angle coefficients, there have been defined for angle coefficients to allow the treatment of any stability problem of the turbine around the point corresponding to the permanent regime. Expressed in relative units, these angle coefficients are:

$$t_1 = \left.\frac{\partial\gamma}{\partial\varepsilon}\right|_A ;\ t_2 = \left.\frac{\partial\gamma}{\partial A}\right|_\varepsilon ;\ t_3 = \left.\frac{\partial\eta}{\partial\varepsilon}\right|_A ;\ t_4 = \left.\frac{\partial\eta}{\partial A}\right|_\varepsilon . \qquad (3)$$

The four angle coefficients are the fundamental hydraulic turbine parameters and their values change according to the values in Fig.4, depending on the speed value expressed as follows

$$v = n\sqrt{\frac{Q}{s\left(gH_n\right)^{3/2}}} \qquad (4)$$

where: $Q$ = flow capacity; $n$ = number of revolutions; $s$ = reference section; $H_n$ = net fall; $g$ = gravitation acceleration.

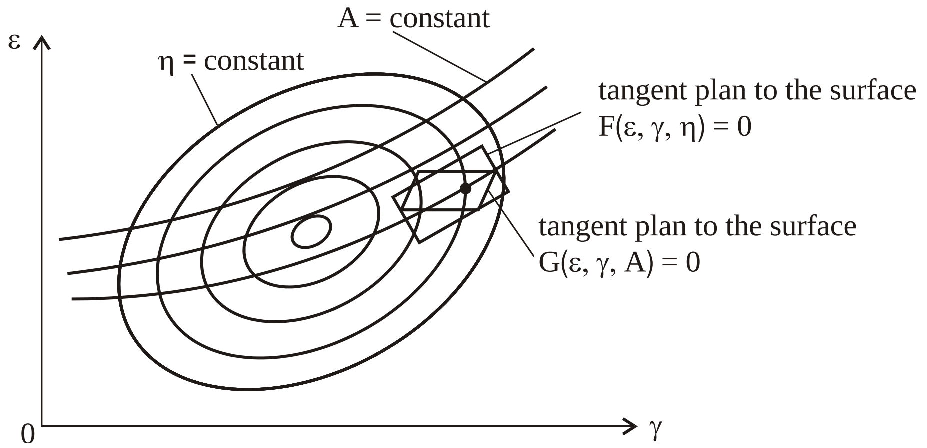



Fig.3. Representations of the working conditions of the hydraulic turbine.

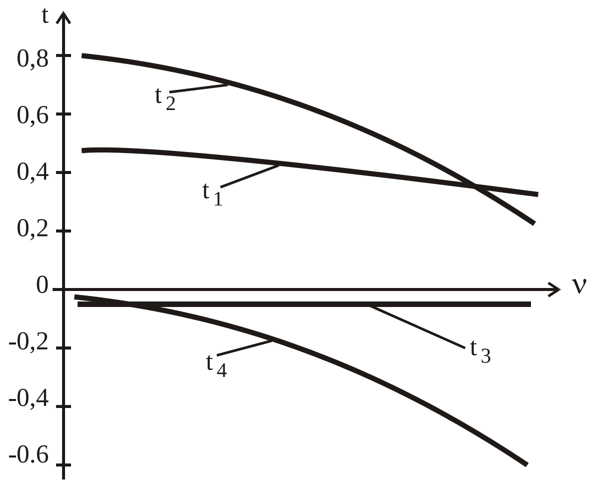



Fig.4. Relations of statistic nature among the fundamental hydraulic parameters and value of the hydraulic turbine.

d$_4$) By means of the fundamental parameters, the following auxiliary hydraulic parameters have been defined:

$$t_5 = 1 + t_1 + t_3 ;\ t_6 = t_2 + t_4 ;\ t_7 = 1 - 2t_1 ;\ t_8 = t_2 t_5 ;$$

$$t_9 = 1 - 2t_1 - 2t_3 ;\ t_{11} = -2t_3 . \qquad (5)$$

d$_5$) Applying the differentials for the adimensional values of the hydraulic turbine, they have resulted in:

$$d\psi = d\varepsilon + d\gamma + d\eta \qquad (6)$$

and marking the position of the directing apparatus, with, $A$, in p.u and the differentials of the adimensional values with $a$, expressed in accordance with hydraulic parameters $t_i$, there have been established:

$$d\gamma = t_1 d\varepsilon + t_2\, da ;$$

$$d\eta = t_3 d\varepsilon + t_4\, da;\ d\psi = t_5 d\varepsilon + t_6\, da. \qquad (7)$$

e) In writing the working equations of the composing elements of the hydraulic system, we have taken into account the laws of hydro-static pressure, of hydro-dynamic pressure, of position pressure in the gravitational field as well as the principle of continuity at the inter-face of the pipes with the water dam and with the water tower. The load losses in the forced pipe and the "ram blow" in the forced pipes were also taken into account. Therefore, the working equations of the hydraulic system will be:

$e_1$) The feed pipe losses equation:

Considering the perfectly rigid feed pipe, the load losses may be considered proportional to the square of the water flow, $Q_v$. If the losses are represented as the head, $H_g$, of the feed pipe, we may write:

$$H_g = \frac{K_g}{g} Q_{V_g}^2 ,$$

where $K_g$ is the load losses coefficient. In the stationary regime operating point, we obtain:

$$H_{g0} = \frac{K_g}{g} Q_{V_{g0}}^2$$

and, thus, in p.u., in relation to the quantities in the operating point, we obtain:

$$h_g = q_{V_g}^2$$

Losses variation is obtained via differentiation:

$$dh_g = 2 \cdot q_{V_g} \cdot dq_{V_g}$$

or, by linearization and accepting that $q_{V_g} = 1$, results:

$$\Delta h_g = 2\Delta \cdot q_{V_g}^2 \tag{8}$$

$e_2$) The kinetic energy equation in the insertion point of the water tower:

Accepting that this kinetic energy is determined by the flow of the water that courses through the feed pipe, hypothesis that is obvious in stationary regime and, although debatable in transitory regime, sufficient for small oscillations of the water plane within the water tower, we have:

$$E_{ch} = \frac{mv_{ch}^2}{2} ,$$

or, expressing the kinetic energy relative to the fluid-mass, or the so-called specific kinetic energy, we may write, in p.u.:

$$e_{ch} = \frac{v_{ch}^2}{2} = K_c \cdot q_{V_g}^2 .$$

Subsequently to differentiating and linearization, we find:

$$\Delta e_{ch} = 2 \cdot \Delta q_{V_g} \tag{9}$$

$e_3$) The water tower equation:

The water tower is considered cylindrical, rigid and frictionless. Its minimal surface (for a stable behaviour, except for small flow variations) is stated in the Thoma condition:

$$S_{ch} > S_g \frac{L_g}{k_g \cdot 2g \cdot H_{bo}} \tag{10}$$

The water tower filling equation is:

$$S_{ch} \frac{dX}{dt} = Q_{ch} \tag{11}$$

If the time constant of the water tower is defined as:

$$T_{ch} = \frac{S_{ch}\left(H_{bo} - H_{go}\right)}{Q_{V_{go}}} \tag{12}$$

and the following notations are taken under consideration:

$$q_{ch}^* = \frac{Q_{ch}}{Q_{V_{go}}} \text{ and } X = \frac{x}{H_{bo} - H_{go}}$$

the equation becomes:

$$T_{ch} \frac{dX}{dt} = q_{ch}^* \tag{13}$$

$e_4$) The flow equation:

The flow rates in the feed pipe, $Q_{vg}$, the forced pipe, $Q_c$, and the water tower, $Q_{ch}$, are linked through:

$$Q_{V_g} = Q_c + Q_{ch} \tag{14}$$

$e_5$) The feed pipe equation:

Considering that there are present only small oscillations of the water plane around the stationary regime position, and that the water flows in the feed pipe only downstream, the theorem of momentum, relative to the liquid tube formed by the water in the feed pipe leads (after substituting all the forces with their expressions) to:

$$S_g L_g \frac{dv_g}{dt} = -S_g \left( gZ + gH_g + \frac{v_{ch}^2}{2} - g\Delta H_b \right) \cdot$$

Since, by definition, $gX = gZ + gH_{go} + \dfrac{v_{ch}^2}{2}$,

the equation becomes:

$$L_g \frac{dv_g}{dt} = -gX - g\Delta H_g - \Delta \frac{v_{ch}^2}{2} + g\Delta H_b \tag{15}$$

If the time constant of the feed pipe is defined as:

$$T_{g_i} = \frac{T_g}{h_0} , \tag{16}$$

where: $h_0 = \frac{H_{bo} - H_{go}}{Z^*}$, is a non-dimensional constant which expresses the relative pressure of the water in the feed pipe, in stationary regime through $p_0 = \frac{H_{go}}{Z^*}$, we obtain the load losses in the feed pipe, in stationary regime, as:

$$c_2 = \frac{p_0}{h_0} \tag{17}$$

and the kinetic energy in the insertion point of the water tower, in stationary regime as:

$$c_4 = \frac{v_{ch}^2}{2g\left(H_{bo} - H_{go}\right)} \tag{18}$$

Nevertheless, the feed pipe may be written, in p.u., as:

$$T_{gi} \frac{d\Delta q_{V_g}}{dt} = -X - c_2 g \Delta h_g - -c_4 \Delta e_{ch} + \left(1 + c_2\right) g \Delta h_b \tag{19}$$

or, combining the previous relations, as:

$$T_{ch} \cdot T_{gi} \frac{d^2 X}{dt} + 2\left(c_2 + c_4\right) T_{ch} \frac{dX}{dt} + X = = -T_{gi} \frac{dq_c}{dt} - 2\left(c_2 + c_4\right) \Delta q_c + \left(1 + c_2\right) g \Delta h_b \tag{20}$$

e6) The specific energy relative to mass equation in the insertion point of the water tower:

Considering the hypothesis that the kinetic energy in the insertion point of the water tower is, at any given time, determined by the flow rate of the water the courses through the feed pipe, for small oscillations of the water plane, the energy is obtained via the following balance:

$$E_a = mg\left(H_b - H_g\right) = mg\left(H_{bo} - H_{go}\right) - -\frac{mv_{ch0}^2}{2} + mgX + \frac{mv_{ch}^2}{2}$$

According to this equation, the water tower behaves as a piezometric tube, meaning that its level measures the static pressure in the insertion point.

The energy relative to mass yields the specific energy whose variation may be expressed as:

$$\Delta e_a^* = gX + \Delta e_{ch}^*$$

or, in p.u. obtained by division with $e_{a0}^*$, leads to:

$$\Delta e_a = X + c_4 \Delta e_{ch} \tag{21}$$

If the (9) equation is also taken into consideration, then:

$$\Delta e_a = X + 2c_4 \Delta q_{vg}$$

and, by combining the (8), (9), (19) and (21) equations, we obtain:

$$T_{gi} \frac{d\Delta e_a}{dt} + 2\left(c_2 + c_4\right)\Delta e_a = T_{gi} \frac{dX}{dt} + 2c_2 X + +2c_4\left(1 + c_2\right) g h_b \tag{22}$$

e7) The load losses in the forced pipe equation:

The specific energy relative to mass of the load losses in the forced pipe is, in p.u., proportional to the square of the water flow through the pipe:

$$e_c = q_c^2$$

while its variation around the stationary regime operating point is:

$$de_c = 2dq_c$$

which, after linearization, becomes:

$$\Delta e_c = 2\Delta q_c \tag{23}$$

e8) The ram blow in the forced pipe:

In writing this equation, the elasticity of water and of the forced pipe is considered negligible. Also, the load losses in the pipe are accepted as null and the pressure is constant at the top of the pipe as a result of the presence of water in the water tower.

Applying the theorem of momentum on an elementary liquid section $S_c$, of section and length $dL_c$, we have:

$$\frac{d}{dt}\left(\rho \cdot S_c \cdot v_c \cdot dL_c\right) = -S_c dp,$$

where $dp$ is the pressure difference generated by the variation of the water speed between the upstream and the downstream surfaces of the elementary liquid section involved.

The previous relation may be written, alternatively, in p.u.:

$$de_p = \frac{dp}{\rho} = -\frac{v_{c0} \cdot dL_c}{e_{k0}^*} \cdot \frac{d\Delta q_c}{dt},$$

which, after integration, yields:

$$e_p = -T_c \frac{d\Delta q_c}{dt}, \tag{24}$$

where, $e_p = \frac{e_p^*}{e_{k0}^*}$ is the specific energy relative to mass and, in p.u., due to the ram blow, $e_{k0}^* = gH_{n0}$ is the specific energy corresponding to the net fall in stationary regime, while $T_c$ is the hydraulic inertia time constant of the forced pipe and is calculated via:

$$T_c = \frac{\int_0^{L_c} v_{c0} dL_c}{gH_{no}} \quad (25)$$

$e_9$) The equation of the net specific energy relative to mass:

The net specific energy relative to mass, meaning the specific energy as far as the upstream of the turbine, is:

$$e_k^* = e_a^* - e_c^* + e_p^*,$$

while the stationary regime point is:

$$e_{k0}^* = e_{a0}^* - e_{c0}^*.$$

Considering the ratio:

$$h_2 = \frac{e_{c0}^*}{e_{k0}^*}, \quad (26)$$

which represents the load loss in the forced pipe in statio-nary regime, in p.u, it follows:

$$e_k = (1 + h_2)e_a - h_2 e_c + e_p.$$

If we take into account that $e_{p0} = 0$, i.e. $\Delta e_p = e_p$, differentiation and linearization yields:

$$\Delta e_k = (1 + h_2)\Delta e_a - h_2 \Delta e_c + e_p \quad (27)$$

or, by means of (23) and (24), we obtain:

$$\Delta e_k = (1 + h_2)\Delta e_a - 2h_2 \Delta q_c - T_c \frac{d\Delta q_c}{dt} \quad (28)$$

$e_{10}$) The turbine flow rate equation:

In the stationary regime operating point, the energy figure, $\varepsilon$, and the flow rate figure, $\gamma$, of the turbine given by (1) may be written:

$$\varepsilon_0 = \frac{2gH_{n0}}{R^2 n_0^2} \text{ and } \gamma_0 = \frac{2Q_0}{sR^3 n_0}$$

Since the net specific energy is $E_k = g \cdot H_n$, by dividing the 2 equations member by member and working in p.u., we obtain:

$$\varepsilon_r = \frac{e_k}{n_r^2}; \; \gamma_r = \frac{q}{n}$$

and differentiation yields:

$$d\varepsilon_r = \frac{1}{n_r^2} de_e - \frac{2e_k}{n_r^3} dn_r,$$

$$d\gamma_r = \frac{1}{n_r} dq - \frac{q}{n_r^2} dn_r$$

or, around the stationary regime point:

$$d\varepsilon_r = de_k - 2dn_r, \; d\gamma_r = dq - dn_r \quad (29)$$

By introducing these in the expression of $d\gamma_r$, given by (7), we reach:

$$dq = (1 - 2t_1)dn_r + t_1 de_k + t_2 da$$

which, after linearization, becomes:

$$\Delta q = t_7 \Delta n_r + t_1 \Delta e_k + t_2 \Delta a \quad (30)$$

We note that, in order to respect the law of conti-nuity, we always have $\Delta q_c = \Delta q$.

$e_{11}$) The hydraulic turbine output equation:

If in the output relation presented in (7), we substitute the expression of $d\varepsilon_r$ from (29), linearization yields:

$$\Delta \eta_r = t_{11} \Delta n_r + t_3 \Delta e_k + t_4 \Delta a. \quad (31)$$

$e_{12}$) The hydraulic turbine mechanical power equation:

Based on (1), for a given turbine, we have, in p.u.:

$$\varepsilon_r = \frac{e_k}{n_r^2} \text{ and } \psi_r = \frac{p_m}{n_r^3},$$

and around the stationary regime operating point:

$$d\varepsilon_r = de_k - 2dn_r \text{ and } d\psi_r = dp_m - 3dn_r.$$

Substituting these in the expression of $d\psi_r$ from (7), we obtain, after linearization:

$$\Delta p_m = t_9 \Delta n_r + t_5 \Delta e_k + t_6 \Delta a \quad (32)$$

Based on all the statements mentioned previously, we may synthesise a 5 differential and algebraic equation system that encompasses the mathematical models of the various elements of the hydraulic system that comprises a feed pipe and a water tower. This equation system, together with the motion equation of the rotors (turbine + generator) and with the speed control system equations for the power system, is able to fully characterise the behaviour of a hydro-electrical system in what concerns the dynamic stability problems.

The 5 differential and algebraic system of equations may be represented as follows:

- water level equation in the water tower;
- specific energy equation in the insertion point of the water tower;
- net specific energy (net fall);
- turbine flow capacity equation;
- turbine mechanical power equation.

The differential equations have been set up in a form that allows the application of Runge-Kutta's type integration numerical methods, resulting in the following system:

$$\frac{dB}{dt}=-\frac{2(c_2+c_4)}{T_{gi}}\cdot B-\frac{1}{T_{gi}T_{ch}}X-\frac{2(c_2+c_4)}{T_{gi}T_{ch}}Q-\frac{1}{T_{ch}}\frac{dQ}{dt}+\frac{1+c_2}{T_{gi}T_{ch}}\cdot gH_b;\quad \frac{dX}{dt}=B;$$
$$\frac{dE_a}{dt}=B+\frac{2c_2}{T_{gi}}X-\frac{2(c_2+c_4)}{T_{gi}}E_a+\frac{2c_4(1+c_2)}{T_{gi}}gH_b;\quad \frac{dQ}{dt}=\frac{1+h_2}{T_c}E_a-\frac{2h_2}{T_c}Q-\frac{1}{T_c}E_k; \tag{33}$$
$$E_k=\frac{1}{t_1}(Q-t_7\omega-t_2a);\ P_m=t_5E_k+t_9\cdot\omega+t_6a.$$

where $c_2$ = load losses in the feed pipe; $c_4$ = kinetical energy at the insertion point of the water tower; $h_2$ = load loss in th forced-pipe.

To this set of equations there have been added the movement equation, through which the values of ω are obtained, that is:

$$\frac{d\omega}{dt}=\frac{1}{T_l}(P_m-P) \tag{34}$$

where: $T_1$ = launching time of the hydroelectric unit, as well as the equations of the mathematical model of the speed governing system which determines value positions, $a$.

For describing speed governing system behaviour, we have proposed a mathematical model which allows its use for all types of hydraulic turbines, trough simply assigning values in accordance with the amplifying factor of the time constant. The diagram and the corresponding transfer function are represented in Fig.5.

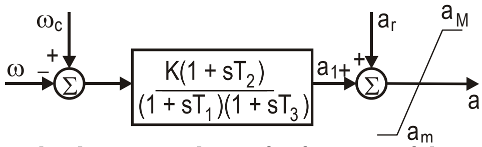



Fig.5. The diagram and transfer function of the model generated for speed governing system with hydraulic turbines.

The equations describing speed governorning system behaviour are:

$$\frac{dz}{dt}=\frac{1}{T_1}\left(\omega_c-\omega-z-T_2\frac{d\omega}{dt}\right);$$
$$\frac{da_1}{dt}=\frac{1}{T_3}\left(K\cdot z-a_1+KT_2\frac{dz}{dt}\right);\ a=a_r+a_1; \tag{35}$$
$$a_m\le a<a_M$$

The values of speed governing system coefficients may take the following values: *K = (10; 15; 25)*; *$T_1$ = (0,2÷ 2,8)s*; *$T_2$ = (0÷1)s*; *$T_3$ = (0,025 ÷ 0,15)s*; *$a_m$ = 0,1 p.u.*; *$a_M$ = 1,1 p.u.*.

Taking into account the above mentioned equations, we have drawn the diagram of the working conditions of the hydromechanical equipment of a hydro-unit described in Fig.6, containing, synthetically, the transfer functions corresponding to its main composing elements.

Using the mathematical models of the composing elements of the hydromechanical equipment and their proper interconnections, the calculating "HYDRO" programme has been written in DELPHI. It aimed at studying the way in which the system of equations satisfies the initial conditions corresponding to an ante-perturbation stationary regime and they allow the calculus of the initial values of variables. We have studied the way in which the model responds to a given perturbation, the adjustment of the model according to the response to perturbations as well as checking the stability of the mathematical model having in view the possibility of linking them to the mathematical models of the synchronous generators and electric networks.

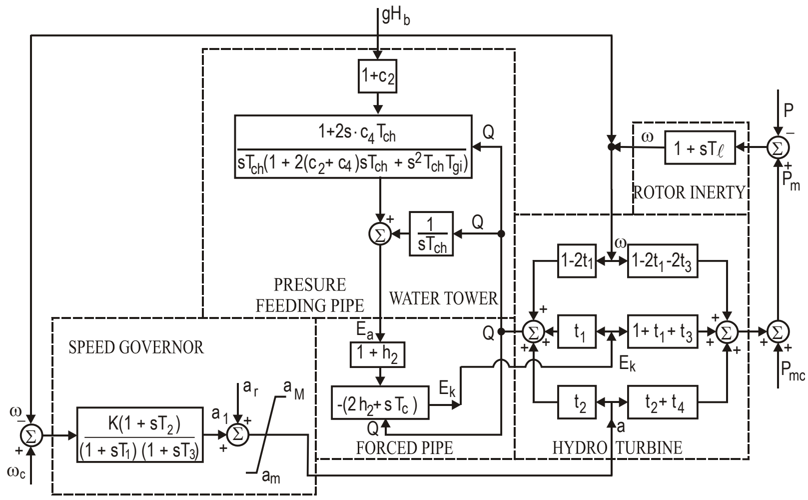



Fig.6. The diagram of the working conditions and transfer functions for the hydromechanical equipment of a hydro-power plant.

## 3 Application of “Hydro” mathematical model

In order to apply the mathematical model “HYDRO”, we have chosen the hydro-power plant Raul Mare-Retezat which has the advantage of being monitored in SCADA, (monitoring in real time the main hydroelectric values). This has allowed to compare the results obtained by mathematical modelling with those obtained by direct measurements (and registrations) by means of transducers.

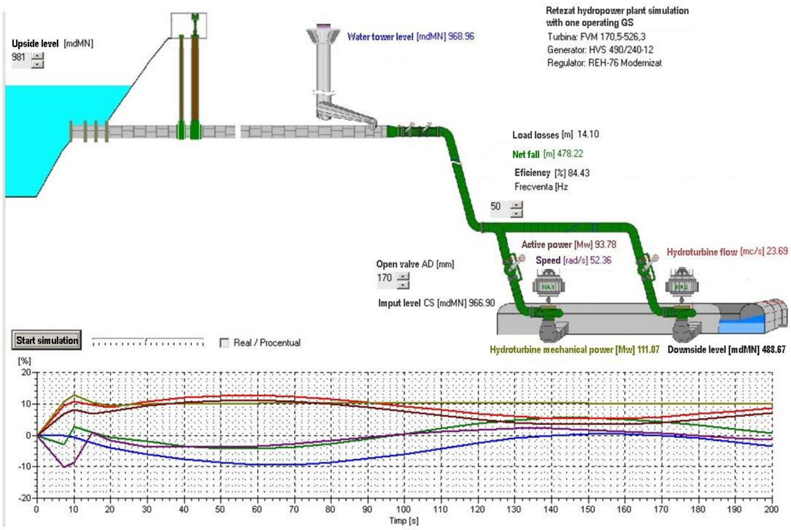
Hydro power plant Raul Mare-Retezat is an underground power plant of derivation under pressure type, having installed power of *335 MW*, concentrated in two power units equipped with Francis turbines, with installed power of *157,5 MW* each. The power plant is fed with water from the water accumulation Gura Apei realized trough building a stone dam downstream the confluence of the rivers Ses, Lapusnicul Mare and Lapusnicul Mic. The total water volume of Gura Apei water accumulation is 220 million $m^3$, of which 200 million $m^3$ is the useful water volume.

The flow capacities in the limitrophe water accumulations are brought to Gura Apei accumulation trough 21 collectors meaning *29,3 km* of galleries of secondary feed pipes.

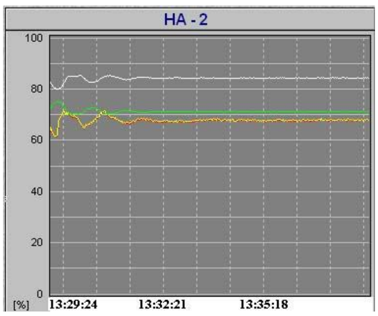
The water of Gura Apei is led to hydro power plant Raul Mare – Retezat through a pressure feed pipe of *18,4 km* having an interior diameter of *4,9 m*.

The underground power plant has got a total sheer fall of *582,5 m* and an installed flow capacity of $70\ m^3/s$. The Francis turbines are directly coupled to synchronous generators having an apparent power of *186 MVA* each and nominal revolution of *500 r/min*, having a nominal power factor $\cos\varphi_n = 0{,}9$. Each turbine is protected by a spherical valve with a diameter of *2,2 m*. The evacuation of the sewage is done through an expansion room followed by an escape pipe of *789 m* having a three step section continued by a canal with a free loose level which is the feed pipe of Clopotiva power plant placed downstream to hydro power plant Raul Mare-Retezat, Romania.

The protection for hydro power plant Raul Mare-Retezat is realized by means of a water tower of superior room type, diaphragm, inferior room and overflow (spillway) the height of the well being *162 m* and its diameter *5,9 m*. The forced-pipe leading the water from the water tower is *812 metres* long and its diameter is *6,3 m*.




a)




b)

Fig.7. The variation of hydromechanical values caused by a power sprind of $\Delta P = +10\%$; a) mathematical modelling; b) measurements in real time.

For the mathematical simulation of the dynamic behaviour, hydro power plant Raul Mare-Retezat has been considered as working with a single hydroelectric unit, the other being a spare one, thus reproducing the most frequent real working situations. The main initial data for the mathematical simulation have been considered as follows: the nominal power of Francis turbine (mechanical power) $P_{nm}=167,5\ MW$ with nominal rotation $n_0=500\ r\ /\ min$ at a frequency of $f = 50\ Hz$ ($52,36\ rad\ /\ s$), turbine reference radius $R=1,425\ m$ and unit launching time $T_1=7,5\ s$. For the nominal flowing capacity $Q_k=35\ m^3/s$, the speed figure has resulted in $v_0 = 0,408$ and the angle turbine parameters in proper nominal working are $t_1 = 0.47$; $t_2 = 0.6$; $t_3 = -0.03$; $t_4 = -0.23$. The hydroelectric system is made up of feed pipe of $L_g = 18400\ m$, section $S_g = 18,8\ m^2$, water tower with $S_{ch}=109\ m^2$ and forced-pipe of $L_c = 812\ m$ and section $S_c=31,2\ m^2$. The sheer fall of system is $H_{b0} = 526\ m$ and fall in the feed pipe $H_{g0} = 15\ m$, resulting in a net fall $H_{k0} = 516\ m$. The speed of the water in the feed pipe, in nominal working conditions has been $v_{g0} = 3,52\ m\ /\ s$ and in the forced-pipe, $v_{c0} = 4,07\ m\ /\ s$. The inertial time constants for the feed pipe, water tower and forced-pipe have been considered: $T_{gi} = 6.7s$; $T_{ch} = 99s$ and $T_c = 1.15s$.

The numerical simulation of the dynamic behaviour of the hydromechanical equipment of hydro power plant Raul Mare Retezat has been analysed for an application consisting in a sudden change of power at the clamps of the synchronous generator through an increase with 10 % as compared with the value of the stationary regime. We have observed the evolution in time of the mechanical and hydraulic values in a period of 200 seconds and the results have been compared with the SCADA monitored ones for a similar situation.

Fig. 7, a and b show, comparatively, the results obtained through mathematical modelling and the ones obtained through direct measurements with SCADA translators.

## 4 Results and conclusion

Fig.7 shows that in both mathematical modelling and real monitored situation, the dynamic process has got a similar development, thus when the power at the clamps of the synchronous generator is suddenly increased, due to the appearance of a strong braking couple, the frequency (rotation) rapidly decreases. The speed governing system feels the decrease of frequency and orders the opening of the admission valves (the control of the directing apparatus). The directing apparatus first opens very rapidly and the mechanical power by the turbine increases, surpassing the required power value that leading to an accelerating couple and an increased frequency. The opening of the valves makes the water flow capacity increase into the hydraulic turbine, but this water flow cannot be compensated by the water which drains through the feed pipe.

This necessary water excess is taken over by the water tower and the water level in it starts becoming lower leading to a decreasing of the net fall, respectively of the pressure in the turbine. This dynamic process, due to the big inertia of the hydroelectric system, is much slower than the process of dynamic regulation given by the speed governing system, which feeling the increase in frequency orders the closing of the valves and thus diminishes the mechanical power under the value of electrical power. The frequency decreases again and it restarts opening the valves but, since the water level in the water tower is more reduced and the net fall smaller, in order to get the necessary mechanical power, a bigger flow capacity is needed in a larger period of time. The oscillatory process continues until the powers are balanced, when the closing of the valves is ordered, that leading to an increase in the water level in the water tower and in the corresponding net energy.

The similar evolution of the dynamic processes of the hydroelectric unit described by the mathematical model and the real monitored situation has proved the validity of the HYDRO mathematical model. It may be considered a useful instrument for the theoretical study of the dynamic behaviour of hydroelectric units in the case of a multitude of preestablished situations, providing a range of corresponding prophylactic solutions.

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