Dynamic Analysis of Electric Arc: From the Single-Phase State Approach to the Three-Phase Clarke Approach

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Abstract: - In the present paper the analysis based on state equations method is applied to the transient study of interrupted single- and three-phase networks. The proposed approach is independent on the mathematical model adopted to represent the electric arc.
In the three-phase case, the Clarke transformation allows us to perform the energy analysis of the three-phase electric arc during any balanced or unbalanced transient conditions. The Clarke energy balance related to the imaginary power concept permits to design the breaker under study without the use of empirical coefficients normally adopted in literature.

Key-Words: - Circuit breakers, Switching transients, Transient analysis, Clarke Approach.

1 Introduction

The analysis based on the use of state-equations and graph theory is the standard approach to the network dynamic analysis [1]. In fact, it allows a direct formulation of the network mathematical model, not only in case of linear systems, but also in case of non-linear ones, i.e. magnetic circuits, electronic devices and interrupted networks.

Clarke transformation [2] allows the application, in instantaneous form, of the symmetrical components algebra and the sequence networks to study the three-phase symmetrical network. Also, under the energy point of view, it brings to a formulation of the energy balance suitable for generic waveform and related to the imaginary power concept [3].

Moreover, the circuital analysis of electric arc − that, in the past, was very simplified because of the very modest available computation power [4] − requires some corrective factors that have to be applied to the obtained results [5]. The more useful methods [6] are based on numeric simulations where the arc models are advanced respect to the Mayr [7] and Cassie [8] classic models. Assigning the network structure and choosing some basic parameters, the solution are deduced with a procedure, useful from an applicative point of view, that does not allow to analyze the system and the topological aspects of the interruption phenomena [9]-[12], [16], [17].

On the other hands, the three-phase interrupted networks are studied, by a circuital and energy point of view, with the method of equivalent single-phase circuit. As a matter of fact, in this way it is ignored the fact that the electric arc imposed an instantaneous unbalanced condition that invalidates the use of this type of algorithm. This influences also the energy balance: in fact, the obtained results are normally corrected with empirical coefficients deduced by experimental methods.

In the present paper an innovative approach to the transient study of interrupted single- and three-phase networks is proposed. It is based on state-equations and it is independent on the mathematical model adopted to represent the electric arc in the breaker. Concerning the three-phase networks, it is also proposed an energy analysis based on the use of Clarke transformation: the obtained results do not need the application of corrective factors.

The paper is organized as follows. In Section 2 the systemic, topological and energy approach to the single-phase electric arc are recalled. In Section 3 a new equivalent three-phase bipole, the "Clarke breaker", is introduced and used for obtain more information from the energy balance, neglecting the use of empirical coefficients. Finally, in Section 4, some numeric examples show the validity of the proposed approach.

2 Single-Phase Case

2.1 Arc empirical models

Even if some evolutes models of electric arc have been proposed in the present paper, as an example, the classic model of Mayr [7] and Cassie [8] are adopted. This assumption, justified by the independence of the proposed approach on the considered arc model, permits to referring to well-known used models. The further extension of the developed approach to other types of arc models is always possible.

Generally, the electric arc is described by one of the following equivalent representation:

$$\mathrm{p}\, r_{ar} = F_1\left(v_{ar}, i_{ar}\right), \qquad \mathrm{p}\, g_{ar} = G_1\left(v_{ar}, i_{ar}\right) \qquad (1)$$

where r_{ar}, g_{ar}, v_{ar} and i_{ar} are, respectively, the resistance, the conductance, the voltage and the current of the arc. F_1 and G_1 are two functionals and p=d/dt is the Heaviside operator.

The Cassie's model postulates that the arc has a constant current density and that the cross-sectional area varied directly with the current [8]. It also has a constant resistivity and stored energy per unit volume. In this case, the (1) becomes:

$$\frac{1}{r_{ar}}\mathrm{p}\, r_{ar} = \frac{1}{\theta}\left(1 - \frac{v_{ar}^2}{V_{ar0}^2}\right) \qquad (2)$$

where q is the arc time constant, coinciding to the energy stored per unit volume and V_{ar0} is the constant steady-state arc voltage.

As anyway it results $v_{ar} = r_{ar} \cdot i_{ar}$, it is possible to rewrite the (2) as:

$$\mathrm{p}\, r_{ar} = \frac{1}{\theta}\left(r_{ar} - \frac{r_{ar}^3 \cdot i_{ar}^2}{V_{ar0}^2}\right) \qquad (3)$$

In Mayr's model, the heat loss is assumed to occur from the periphery of the arc only, and the conductance of the arc varied with the energy stored in it [7]. So, in this case, the (1) becomes:

$$\frac{1}{r_{ar}}\mathrm{p}\, r_{ar} = \frac{1}{\theta}\left(1 - \frac{v_{ar} \cdot i_{ar}}{P_{ar0}}\right) \qquad (4)$$

or:

$$\mathrm{p} r_{ar} = \frac{1}{\theta}\left(r_{ar} - \frac{r_{ar}^2 \cdot i_{ar}^2}{P_{ar0}}\right) \qquad (5)$$

where P_{ar0} is the steady-state constant power loss.

2.2 The system interpretation

The electric arc is formally considered as any other one port of the network, a priori non-linear and time-varying, of which is known the input-output relation. During the opening phases, the analysis adopt as electrical state variables the flux ψ or the current i for the inductors and the charge q or the voltage v for capacitors. In addition, a suitable state variable r_{ar} that represents the component circuit breaker now seen as a dynamic element is added up (see Fig. 1).

The arc black box models bring to the introduction of the following sets of equations:

$$\begin{cases} \mathrm{p}\, r_{ar}(t) = F_1\left(r_{ar}(t), i_{ar}(t)\right) \\ v_{ar}(t) = r_{ar}(t) \cdot i_{ar}(t) \end{cases} \qquad (6)$$

where the functional F_1 depends on the used arc models.

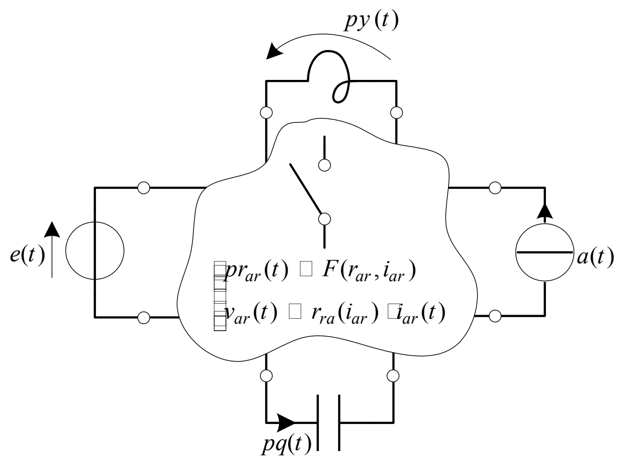


Fig. 1. The systemic interpretation of a single-phase network in which an electric arc is present.

2.3 The topological approach

In case of non-degenerating network, the dynamic one port breaker can be insert on the tree or co-tree branches: its position does not depend on any preliminary rule and it is carried out time by time according to the topology of the network in which is located. This comes from the algebraic characteristic of the voltage-current relation express by the second of (6), that matches formally the dynamic component arc to any other resistance of the network.

In Fig. 2a, as an example, the elementary case of a series RLC circuit during the opening process is presented. In this case, the breaker is positioned on the tree.

The following state model is correspondingly deduced:

$$\begin{cases} p\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{R+r_{ar}(t)}{L} & -\dfrac{1}{L} \\ \dfrac{1}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \cdot e(t) \\ p\, r_{ar}(t) = F_1\left(v_{ar}(t), i_{ar}(t)\right) \end{cases} \quad (7)$$

The network in Fig. 2b, of the fifth order, is instead referred to an electrical line during the opening stage. In such a case, given the presence of the two capacitors on the tree, the breaker is necessarily located on the co-tree. Its model is the following:

$$\begin{cases} p\begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} = = \begin{bmatrix} -R_1/L_1 & 0 & -1/L_1 & 0 \\ 0 & -R_2/L_2 & 1/L_2 & 0 \\ 1/C_1 & -1/C_1 & 0 & 0 \\ 0 & -1/C_2 & 1/\left(r_{ar}C_2\right) & -1/\left(r_{ar}C_2\right) \end{bmatrix} \\ \cdot \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot e(t) \\ p\, r_{ar}(t) = F_1\left(v_{ar}(t), i_{ar}(t)\right) \end{cases} \quad (8)$$

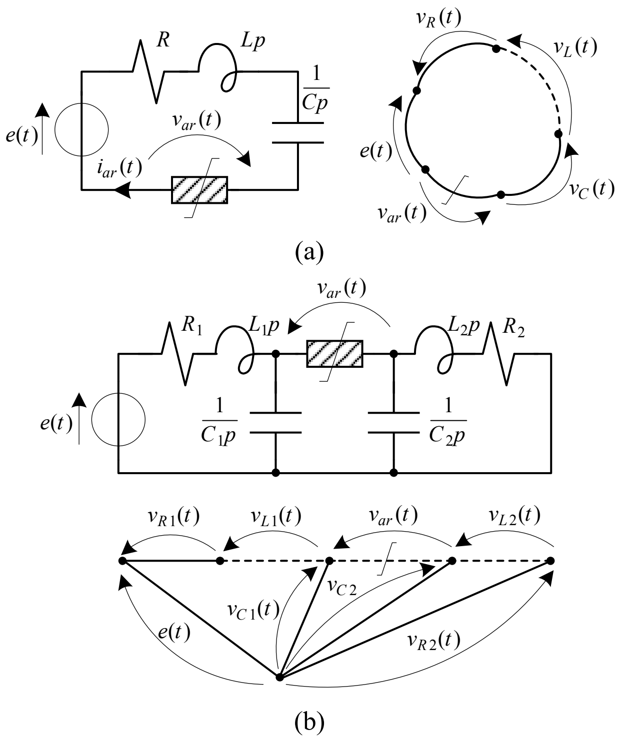


Fig. 2. Interrupted single-phase network. Case in which the electric arc must be put on the tree (a) and on the co-tree (b).

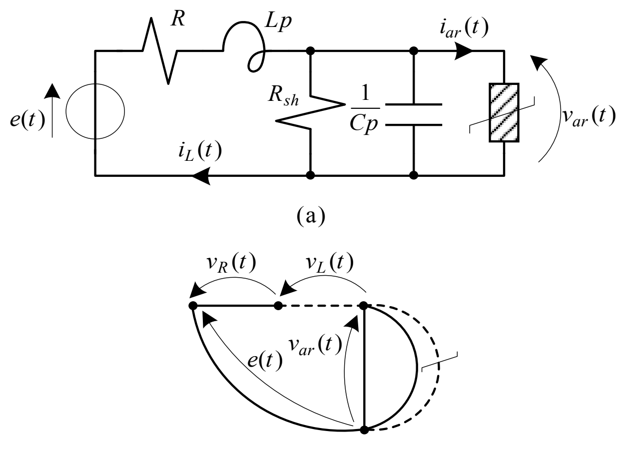


Fig. 3. Considered single-phase example.

2.4 The energy aspect

The numerical integration of the complete network model brings to the descriptive port quantities $\{v_{ar}(t), i_{ar}(t)\}$ associated to the breaker. In such conditions, naming t_i and t_f, respectively, the starting and the ending instants of the interrupting process, the maximum arc current $I_{ar,M}$, related to the requested interruption power, and the recovery voltage $v_{ar,r}(t)$, from which depends the potential subsequent restart of a new arc [5], can be deduced as follows:

$$\begin{cases} I_{\mathrm{ar,M}} = \max\left\{i_{ar}(t)\right\} & \mathrm{t_i} \le \forall t \le t_f \\ v_{ar,r}(t) = v_{ar}(t) & \forall t \ge t_i \end{cases} \quad (9)$$

With reference to the electrical energy of the interruption phenomenon, the Joule integral is then defined [5]:

$$\int_{ti}^{t_f} i_{ar}^2(\xi) \cdot d\xi = I_{ar,RMS}^2 \cdot \left(t_f - t_i\right) = I_{ar,RMS}^2 \cdot T \quad (10)$$

linked to the rms arc current $I_{ar,RMS}$ and to the total time $T = t_i - t_f$ of the interrupting process.

The electric work elaborated by the breaker can be adopted as a measure of the thermal stress of the component:

$$L = \int_{t_i}^{t_f} v_{ar}(t) \cdot i_{ar}(t) dt = P_{ar} \cdot \left(t_f - t_i\right) = P_{ar} \cdot T \quad (11)$$

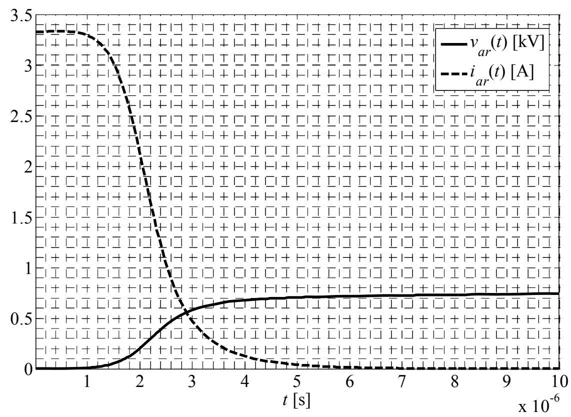
where P_{ar} is the average power elaborated by the arc.

In Fig. 3 is reported, as an example, a simple single-phase network in presence of a breaker, represented by the Cassie model [8]. This network, which numerical parameters are deduced by [14], can be represented by the following state equations:

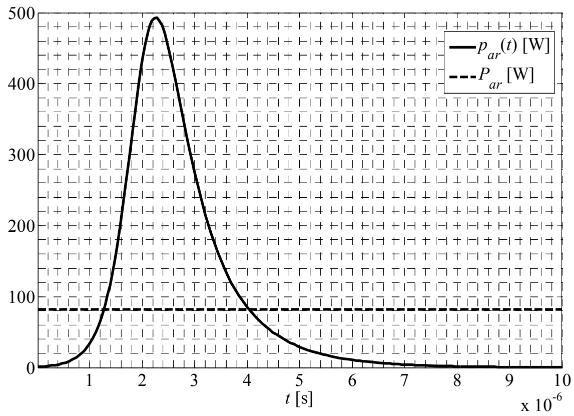
$$\begin{cases} \mathrm{p}\begin{bmatrix} v_{ar} \\ i_L \end{bmatrix} = \begin{bmatrix} -\dfrac{R_{sh}+r_{ar}}{R_{sh}r_{ar}C} & \dfrac{1}{C} \\ -\dfrac{1}{L} & -\dfrac{R}{L} \end{bmatrix} \cdot \begin{bmatrix} v_{ar} \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} \cdot \begin{bmatrix} e \\ i_{ar} \end{bmatrix} \\ \mathrm{p}\, r_{ar} = \dfrac{1}{\theta}\left(r_{ar} - \dfrac{r_{ar}^3 \cdot i_{ar}^2}{V_{ar0}^2} \right) \end{cases} \quad (12)$$

The results reported in Fig. 4 are obtained integrating these equations by numeric techniques. In particular, Fig. 4a shows the arc voltage and current, Fig. 4b shows the instantaneous and average arc power, and Table 1 summarized some obtained results. All these data are necessary for the breaker design.

The obtained results are in agreement with those presented in [14].



(a)



(b)

Fig. 4. (a) Arc voltage and current and (b) instantaneous power for the breaker in the single-phase network of Fig. 2.

Table 1. Numeric Results Obtained for the Example in Fig. 3.

Quantities	Values
RMS value of arc voltage ($V_{ar,RMS}$)	555.61 V
RMS value of current voltage ($I_{ar,RMS}$)	1.7355 A
Arc active power (P_{ar})	82.150 W
Sizing arc power (S_{ar})	963.13 VA
Electric work (*L*)	0.8215 mJ

3 Three-Phase Case

3.1 Preliminary considerations

The considerations presented in Section 2 are valid in three-phase case too: the most important case from the applications point of view.

In the case of three-phase networks with electric arc (Fig. 5a), an approach based on the instantaneous symmetrical components cannot be used. This comes from the fact that, even although the three distinct arcs may be considered (for construction symmetry) with the same constitutive relation $r_{ar,k}=F_2(v_{ar,k},i_{ar,k})$ (with $k=a,b,c$), as a matter of fact the non-linearity of the functional F_2 introduces in the three arc resistances a condition of instantaneous asymmetry of the following type:

$$\begin{cases} r_{ar,a}(t) = F_2\left(v_{ar,a}, i_{ar,a}\right) \\ r_{ar,b}(t) = F_2\left(v_{ar,b}, i_{ar,b}\right) \Rightarrow \\ r_{ar,c}(t) = F_2\left(v_{ar,c}, i_{ar,c}\right) \end{cases} \quad (13)$$
$$\Rightarrow r_{ar,a}(t) \neq r_{ar,b}(t) \neq r_{ar,c}(t)$$

for which is not possible to define the equivalent single-phase sequence circuits. In fact, expressing the relations between arc voltages and currents as (see Fig. 5b):

$$\begin{bmatrix} v_{ar,a}(t) \\ v_{ar,b}(t) \\ v_{ar,c}(t) \end{bmatrix} = \begin{bmatrix} r_{ar,a}(t) & 0 & 0 \\ 0 & r_{ar,b}(t) & 0 \\ 0 & 0 & r_{ar,c}(t) \end{bmatrix} \cdot \begin{bmatrix} i_{ar,a}(t) \\ i_{ar,b}(t) \\ i_{ar,c}(t) \end{bmatrix} \quad (14)$$

this implies that is not possible a reformulation of (14) in the following terms:

$$\begin{bmatrix} v_{ar,\alpha}(t) \\ v_{ar,\beta}(t) \\ v_{ar,0}(t) \end{bmatrix} = \begin{bmatrix} r_{ar,\alpha}(t) & 0 & 0 \\ 0 & r_{ar,\beta}(t) & 0 \\ 0 & 0 & r_{ar,0}(t) \end{bmatrix} \cdot \begin{bmatrix} i_{ar,\alpha}(t) \\ i_{ar,\beta}(t) \\ i_{ar,0}(t) \end{bmatrix} \quad (15)$$

related to $\{\alpha,\beta,o\}$ variables, that has to be write as:

$$\begin{aligned}\bar{v}_{ar,\alpha\beta}(t) &= v_{ar,\alpha}(t) + jv_{ar,\beta}(t) = \\ &= r_{ar,\alpha}(t)\cdot i_{ar,\alpha}(t) + jr_{ar,\beta}(t)\cdot i_{ar,\beta}(t) \\ &= r_{ar,\alpha\beta}(t)\cdot \bar{i}_{ar,\alpha\beta}(t)\end{aligned} \tag{16}$$

The dynamical analysis, not anymore single-phase as in the case of symmetric networks, must then be exclusively referred to the arc three-phase port variables. The application of the Clarke transformation to these quantities permits, as it is shown in the following, to take into account some fundamental aspects of the energetic balance of the three-phase arc.

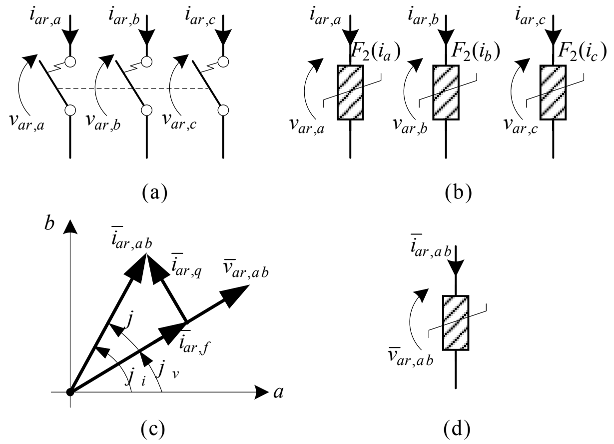


Fig. 5. Genesis of the "Clarke breaker".

3.2 The "Clarke breaker"

Once integrated the model of the interrupted three-phase network, the matrix functions $[i_{arc,abc}(t)]$ and $[v_{arc,abc}(t)]$ are obtained as port variables of the three-phase arc (see Fig. 5b). Eq. (9) can be read in the following three-phase form:

$$\begin{cases} I_{ar,M} = \max\left\{i_{ar,a,M}, i_{ar,b,M}, i_{ar,c,M}\right\} & t_i \le \forall t \le t_f \\ \left[v_{ar,r}(t)\right] = \left[v_{ar,a,b,c}(t)\right] & \forall t \ge t_i \end{cases} \tag{17}$$

where:

$$i_{ar,k,M}(t) = \max\left\{i_{ar,k}(t)\right\} \qquad k = a,b,c \tag{18}$$

The transformation to the Clarke variables $\{\alpha,\beta,o\}$ can be achieved only after the integration process and in an exclusive energy range.

The three arc currents $[i_{a,b,c}]$ can be matched to the corresponding three-phase Clarke arc current [2] (see Appendix A):

$$\begin{aligned}\bar{i}_{ar,\alpha\beta}(t) &= \sqrt{\frac{2}{3}}\left\{i_{ar,a}(t) + \bar{\alpha}\cdot i_{ar,b}(t) + \bar{\alpha}^2\cdot i_{ar,c}(t)\right\} = \\ &= i_{ar,\alpha\beta}(t)e^{j\vartheta_i(t)}\end{aligned} \tag{19}$$

Also to the three arc voltages $[v_{arc,abc}(t)]$ corresponds the three-phase Clarke voltage:

$$\begin{aligned}\bar{v}_{ar,\alpha\beta}(t) &= \sqrt{\frac{2}{3}}\Big\{F_2\left(v_{ar,a}, i_{ar,a}\right)\cdot i_{ar,a} + \\ &+ \bar{\alpha}\cdot F_2\left(v_{ar,b}, i_{ar,b}\right)\cdot i_{ar,b} + \\ &+ \bar{\alpha}^2\cdot F_2\left(v_{ar,c}, i_{ar,c}\right)\cdot i_{ar,c}\Big\} = \\ &= v_{ar,\alpha\beta}(t)\cdot e^{j\vartheta_v(t)}\end{aligned} \tag{20}$$

As can be observed, even although it is a resistive one-port, the Clarke arc voltage results to be instantaneously shifted of the angle $\vartheta_i(t) - \vartheta_v(t) = \varphi(t)$ respect to the Clarke arc current. Even although it is not related to any process of energy storage, it is consequence of the instantaneous arc asymmetry introduced by the non-linearity.

The Clarke approach confirms anyhow its conceptual and practical importance under the energy aspect. Once associated to the three-phase arc functions $[i_{arc,abc}(t)]$ and $[v_{arc,abc}(t)]$, the corresponding Clarke quantities (19), (20), it is possible to introduce a new circuital component, the "Clarke breaker" (see Fig.5d). Basing on the previous considerations, this component can be exclusively used for the external effects studies.

3.3 The energy aspect

Confirming the formalism introduced in Section 2 for the single-phase case, the "Clarke breaker" allows to derive the three-phase Joule integral as:

$$\begin{aligned}\int_{t_i}^{t_f} \bar{i}_{ar,\alpha\beta}(\xi)\cdot \bar{i}^*_{ar,\alpha\beta}(\xi)\cdot d\xi &= I^2_{ar,\alpha\beta,RMS}\cdot\left(t_f - t_i\right) = \\ &= I^2_{ar,\alpha\beta,RMS}\cdot T\end{aligned} \tag{21}$$

where the three-phase rms value $I_{ar,\alpha\beta,RMS}$ of the arc current is considered (see Appendix A).

Concerning on the energy aspects of three-phase arc, the "Clarke breaker" model allows the calculation of the following instantaneous complex power:

$$\bar{a}_{ar}(t) = \bar{v}_{ar,\alpha\beta}(t)\cdot \bar{i}^*_{ar,\alpha\beta}(t) = p_{ar}(t) + jq_{ar}(t) \tag{22}$$

The real component of the power can be deduced as:

$$\begin{cases} p_{ar}(t) = \mathrm{Re}\left\{\overline{a}_{ar}(t)\right\} = \left|\overline{v}_{ar,\alpha\beta}(t)\right| \cdot \left|\overline{i}_{ar,f}(t)\right| \\ P_{ar} = \mathrm{Re}\left\{\dfrac{1}{T}\displaystyle\int_{t_i}^{t_f} \overline{a}_{ar}(t) \cdot dt\right\} \end{cases} \quad (23)$$

where the on-phase component $\overline{i}_{ar,f}(t)$ of Clarke arc current is introduced (Fig. 4c).

The imaginary power component − linked both to the distortion of the waveforms and the asymmetry introduced by the non-linearity of the three-phase electric arc − can be also introduced as follows:

$$\begin{cases} q_{ar}(t) = \mathrm{Im}\left\{\overline{a}_{ar}(t)\right\} = \left|\overline{v}_{ar,\alpha\beta}(t)\right| \cdot \left|\overline{i}_{ar,q}(t)\right| \\ Q_{ar} = \mathrm{Im}\left\{\dfrac{1}{T}\displaystyle\int_{t_i}^{t_f} \overline{a}_{ar}(t) \cdot dt\right\} \end{cases} \quad (24)$$

where $\overline{i}_{ar,q}(t)$ is the quadrature component of Clarke arc current (Fig. 5c).

The imaginary power affects, as index of the "three-phase unbalance" instantaneously caused by the arc, the three-phase Joule integral, as results from the following expression:

$$\int_{t_i}^{t_f} \overline{i}_{ar,\alpha\beta}(\xi) \cdot \overline{i}^*_{ar,\alpha\beta}(\xi) \cdot d\xi = \int_{t_i}^{t_f} \frac{p^2_{ar}(\xi) + q^2_{ar}(\xi)}{v^2_{ar,\alpha\beta}(\xi)} d\xi$$
$$= \left\{I^2_{ar,f,RMS} + I^2_{ar,q,RMS}\right\} \cdot T > I^2_{ar,f,RMS} \cdot T \quad (25)$$

The three-phase electric work, due to the presence of the quadrature component $\overline{i}_{ar,q}(t)$, becomes complex:

$$\overline{L}_{\alpha\beta} = \int_0^T \overline{v}_{ar,\alpha\beta}(t) \cdot \overline{i}^*_{ar,\alpha\beta}(t)dt = \int_0^T p_{ar}(\mathrm{t})dt +$$
$$+ j\int_0^T q_{ar}(\mathrm{t})dt = P_{ar} \cdot T + jQ_{ar} \cdot T = L_f + jL_q \quad (26)$$

which module:

$$L_{\alpha\beta} = \left|\int_0^T \overline{v}_{ar,\alpha\beta}(t) \cdot \overline{i}^*_{ar,\alpha\beta}(t)dt\right| = \sqrt{L_f^2 + L_q^2} =$$
$$= \sqrt{P^2_{ar} + Q^2_{ar}} \cdot T > P_{ar} \cdot T = L \quad (27)$$

results bigger than the single-phase electric work that can be obtained considering a single-phase equivalent representation. Together with the sizing classical three-phase power:

$$S_{ar} = V_{ar,\alpha\beta,RMS} \cdot I_{ar,\alpha\beta,RMS} =$$
$$= \sqrt{\frac{1}{T}\int_0^T \overline{v}_{ar,\alpha\beta}(t) \cdot \overline{v}^*_{ar,\alpha\beta}(t)dt} \quad (28)$$
$$\cdot \sqrt{\frac{1}{T}\int_0^T \overline{i}_{ar,\alpha\beta}(t) \cdot \overline{i}^*_{ar,\alpha\beta}(t)dt}$$

it could be a useful element for the breaker design.

4 Numeric Examples

Without lack of generality, the case reported in Fig. 6 is considered. When the breaker is closed, the network is linear and symmetric, so the voltage between the two star centers is nil. In these conditions the network admits a representation based on Clarke equivalent networks.

The breaker opening introduces the three non-linearities due to the electric arc, and, together with them, the asymmetry. In such a case the three arc resistors must be placed onto the tree. By the systemic point of view, the inductive cut set introduce degeneracy and makes two the order of the network, which is three without the opening process. For the topological aspect, this implies that one of the three inductances must be placed on the three. With reference to the graph reported in Fig. 6b, it is obtained:

$$\begin{bmatrix} 2L & L \\ L & 2L \end{bmatrix} \mathrm{p} \begin{bmatrix} i_{ar,a} \\ i_{ar,b} \end{bmatrix} =$$
$$= \begin{bmatrix} 2R + r_{ar,a} + r_{ar,c} & R \\ R & 2R + r_{ar,b} + r_{ar,c} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \quad (29)$$
$$+ \begin{bmatrix} e_a - e_c \\ e_b - e_c \end{bmatrix}$$

The three-phase degeneracy of the network impose that this model is not in normal form. But the inverse of the inductances network always exist and the model can be resolved.

It can be also investigated in the phase domain with a numerical algorithm. As an example, we have used the "SimPower System" Toolbox of MatLab-Simulink.

Once performed the numeric integration of the model and obtained the phase arc voltages and currents (see Fig. 7), applying the Clarke transformation the {α,β} Clarke voltages and currents are obtained. By the application of the procedure (23)−(28) and of what is reported in Appendix A, the numerical results summarized in Table 2 and the imaginary power shown in Fig. 8 can be obtained.

From these data is clear that the proposed approach allows more sophisticated and correctly breakers' design. In particular, the possibility to derive the value of the three-phase electric work and of the sizing three-phase power without the use of empirical coefficients permits a more rigorous approach to the technical problem of the project of the component.

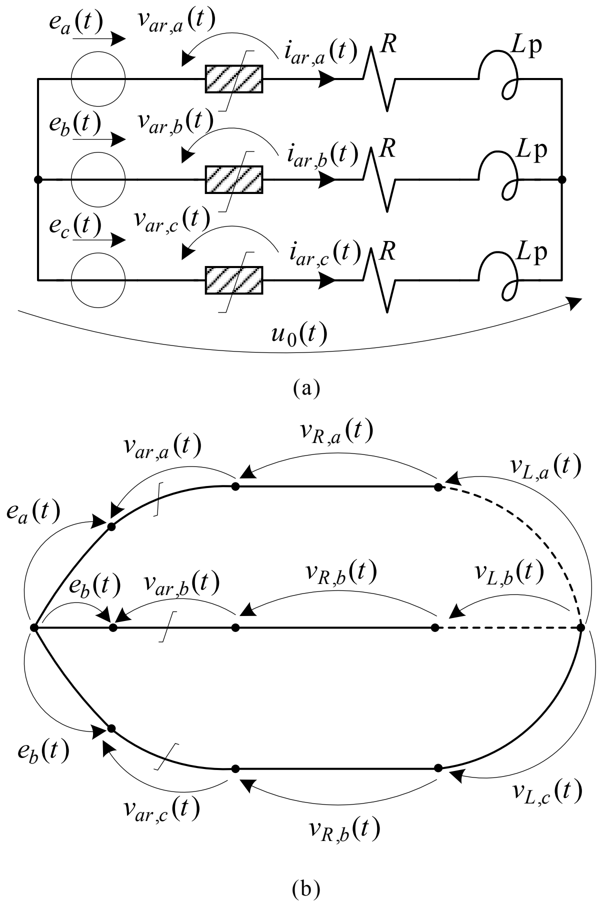


Fig. 6. Considered three-phase interrupted network. (a) Three-phase network and (b) adopted graph.

Table 2. Numeric Results Obtained for the Network in Fig. 5.

Quantities	Values
RMS value of arc voltage ($V_{ar,\alpha\beta,RMS}$)	2.6495 kV
RMS value of arc current ($I_{ar,\alpha\beta,RMS}$)	6.0245 A
Arc active power (P_{ar})	14.6355 kW
Average arc imaginary power (Q_{ar})	6.3707 kVAr
Arc sizing power (S_{ar})	15.9619 kVA
Time length of interruption process	0.005 s
Electric work ($L_{\alpha\beta}$)	79.8095 J

In Fig. 9 the voltage between the two star centers is reported. It confirms the invalidity of the approach based on the equivalent single-phase network during the interrupting process: in fact, the voltage assumes values different from zero during the opening stage, even if it is nil before and after the process.

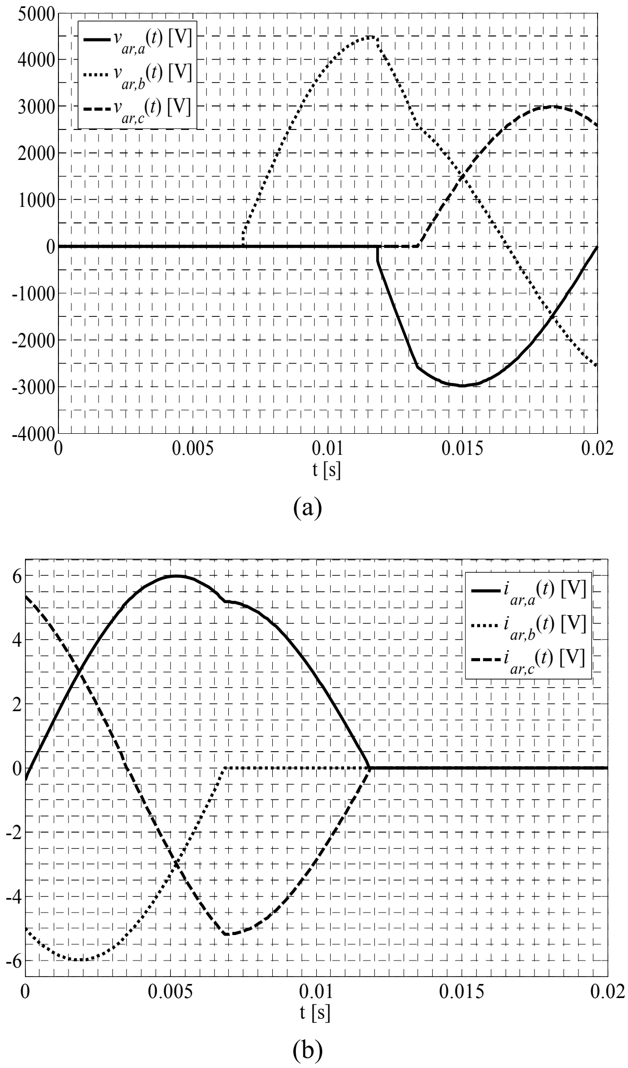


Fig. 7. Arc voltages (a) and arc currents (b) obtained for the opening process of the network in Fig. 6.

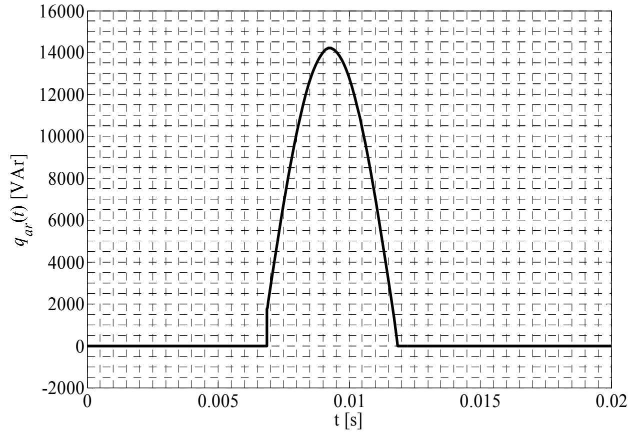


Fig. 8. Imaginary power obtained for the opening process of the network in Fig. 6.

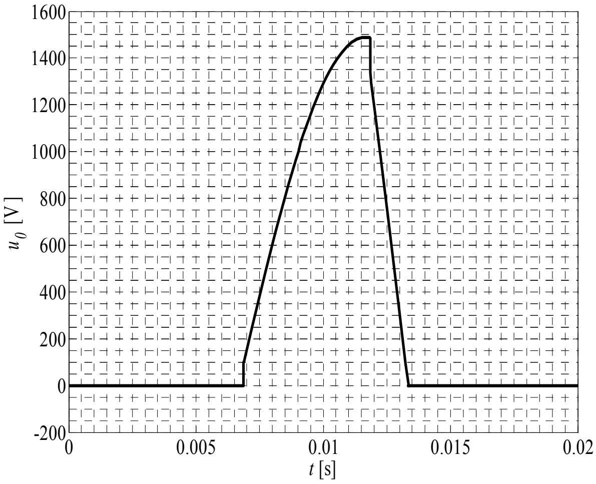


Fig. 9. Voltage between the star centers during the three-pole interruption of the network in Fig. 6.

The same three-phase network is analyzed in case of single-pole opening. This case cannot be studied by means of the equivalent single-phase network, so the three-phase state equations approach is mandatory. Assuming that the opening happen on phase a, it is sufficient to impose that the values of arc equivalent resistances on phase b and c remain to their initial values. The equivalent arc resistance on phase a follows the trend represented by empirical Mayr or Cassie models.

In Figs. 10, 11 and 12 the obtained diagrams are reported; in Table 3 the corresponding energetic values are summarized.

The comparison between the results shown in Table 2 and Table 3 confirms that the three-phase opening is more onerous than the single-phase opening.

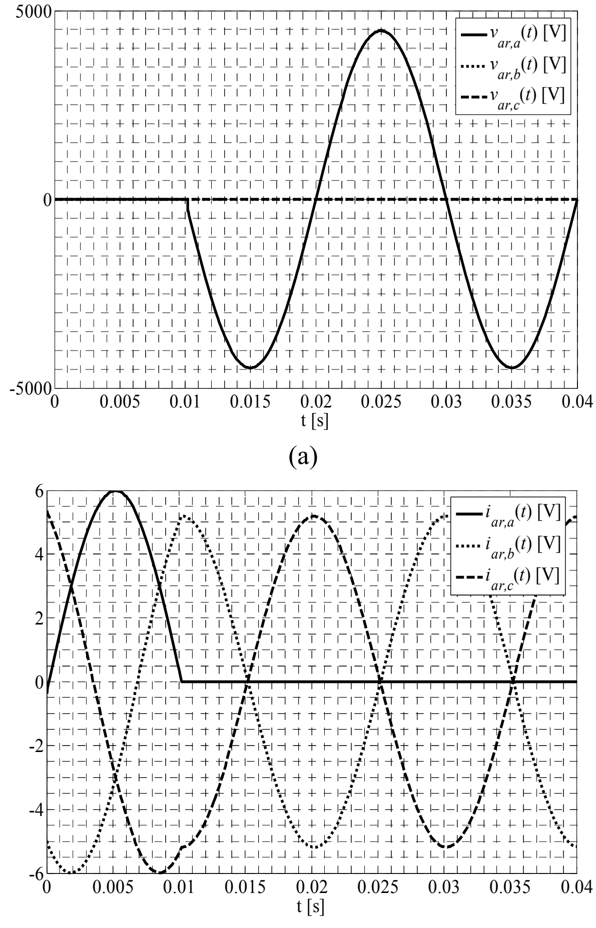


Fig. 10. Arc voltages (a) and arc currents (b) in term of phase variables for the single-phase interruption of the network in Fig. 6.

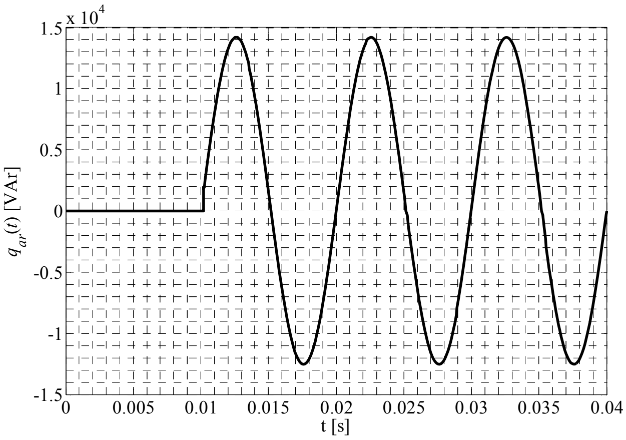


Fig. 11. Imaginary power obtained for the single-phase interruption process of the network in Fig. 6.

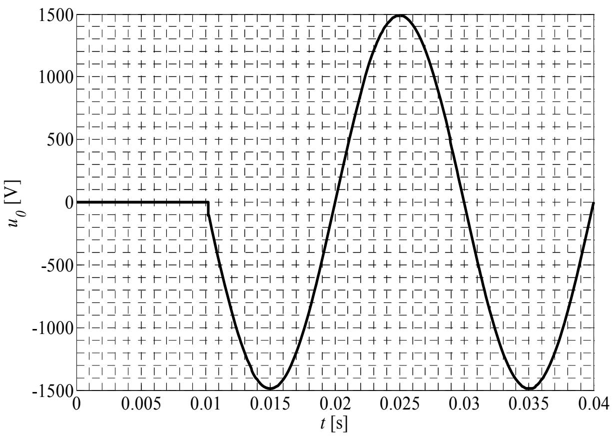


Fig. 12. Voltage between the star centers for the single-phase interruption of the network in Fig. 6.

Table 3. Numeric Results Obtained for the *a*-Phase Opening of Three-phase Network in Fig. 6.

Quantities	Values
RMS value of arc voltage ($V_{ar,\alpha\beta,RMS}$)	823.6657 V
RMS value of arc current ($I_{ar,\alpha\beta,RMS}$)	7.2492 A
Arc active power (P_{ar})	4.5692 kW
Average arc imaginary power (Q_{ar})	3.8437 kVAr
Arc sizing power (S_{ar})	5.9709 kVA
Time length of interruption process	0.0015 s
Electric work ($L_{\alpha\beta}$)	8.9563 J

5 Conclusion

The arc phenomena analysis has been included in the state equations approach to study the dynamic of single-phase and three-phase interrupted electric networks. The obtained results show new perspectives of investigation. In particular, given the flexibility and computational power of the method, there is no limitation for the arc model and the network complexity.

Under the energy point of view, the three-phase energy balance is analyzed by Clarke transformation. As a matter of fact, it shows the role of the three-phase arc as an "imaginary power source". Accordingly, the sizing power and electric work are greater than those calculated by the equivalent single-phase model usually adopted. By means of these energy results, it is possible to design the breaker under study without the use of empirical coefficients.

The Clarke-based energy analysis presented in the paper can be applied independently on the considered three-phase arc model and on the software used to perform the simulation of the network.

Appendix A. The Clarke Transformation

The information associated to a three-phase set $\{y_a(t),\ y_b(t),\ y_c(t)\}$ can be retrieved in the combination of a three-phase vector and a zero-sequence scalar component expressed as [2], [18]:

$$\begin{cases} \bar{y}_{\alpha\beta}(t) = \sqrt{\dfrac{2}{3}}\left\{y_a(t) + \bar{\alpha}\cdot y_b(t) + \bar{\alpha}^2\cdot y_c(t)\right\} \\ y_o(t) = \dfrac{1}{\sqrt{3}}\left\{y_a(t) + y_b(t) + y_c(t)\right\} \end{cases}$$

where $\bar{\alpha} = \exp(j2\pi/3)$ is the Fortescue operator.

Their knowledge in the time domain allows to derive, by applying the inverse Clarke transformation, the phase quantities [2]:

$$\begin{bmatrix} y_a(t) \\ y_b(t) \\ y_c(t) \end{bmatrix} = \frac{1}{\sqrt{3}}\mathrm{Re}\left\{\begin{bmatrix} 1 \\ \bar{\alpha}^2 \\ \bar{\alpha} \end{bmatrix}\cdot \bar{y}_{\alpha\beta}(t)\right\} + \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\cdot y_o(t)$$

Basing on the Clarke transformation, it is possible to introduce the RMS three-phase value as:

$$W_{RMS} = \sqrt{\frac{1}{T}\int_0^T \bar{w}(t)\cdot \bar{w}(t)^*\,dt + \frac{1}{T}\int_0^T w_o^2(t)\,dt}$$

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