Optimal Size and Location of Capacitors Placed on a Distribution System

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***Abstract*：** –This paper proposes a study of reactive power compensation for electric distribution systems to find out the optimal capacitive compensation location and quantity. The most commonly used reactive power sources for compensation in distribution systems are capacitor banks. The compensation problem under study is an optimization problem. A new method-evolutionary simulated annealing, with evolution strategies embedded in the simulation annealing, is employed to solve a 36-bus distribution system composed of radial-type and loop-type circuits, to find out the optimal capacitor placement solutions. Lastly, according to the results of simulation, it validates the proposed approaches and provides the way to promote the power-supply quality of distribution systems.

Keywords： –Capacitive compensation, Radial-type, Loop-type, Evolutionary simulated annealing.,Power loss, Bus voltage

1. Introduction

The radial type system is the most commonly used one for power distribution[1-6], yet when something does go wrong in the system, an instantaneous blackout may occur. In order to satisfy the high expectations of the high-tech industry for a reliable power supply system, we pay greater attention to the use of a loop distribution system. This system not only improves reliability and service quality of the power supply, but also entails less power loss than occurs with the radial type.

At present, Taiwan Power Company has been actively promoting the use of the loop type distribution system to satisfy the demand of users in general, and those in high-tech industry, in particular, for reliable power supply, and to prevent the occurrence of instantaneous blackouts caused by maintenance, switching operation, and single incidents of the distribution system. This paper aims to identify the best position setting (fixed or switched capacitor) and the capacity of capacitors in the distribution system by adding the loop type to that of the radial type, to minimize investment cost and prevent power loss. The ultimate goal is to achieve the lowest total cost, improve voltage drops and reliability of power supply, and simultaneously reduce line loss.

This study proposes a large-scale distribution system by using evolutionary simulated annealing which has been effectively applied to dispatch in var compensator to examine whether the same effectiveness can be acquired. This method adopts competition procedures in evolution strategy and multi-path searching skills of individuals to concentrate the limited computing resources on the “possible” area of ‘best-so-far’ for improving the effectiveness of simulated annealing and the determination of the ‘best-so-far’.

2. Feeder Power Flow Equation

For a balanced three-phase system with single feeder as shown in Fig. 1. Let P_i and Q_i refer to the real power and reactive power of bus i, respectively; $|V_i|$ shows the voltage, δ_i the phase angle; also let $R_{i,i+1}$ and $X_{i,i+1}$ indicate the resistance and reactance for the feeder section from bus i to bus $i+1$, respectively. Some results are acquired from equations (1) ~ (6) [1,2,7]：

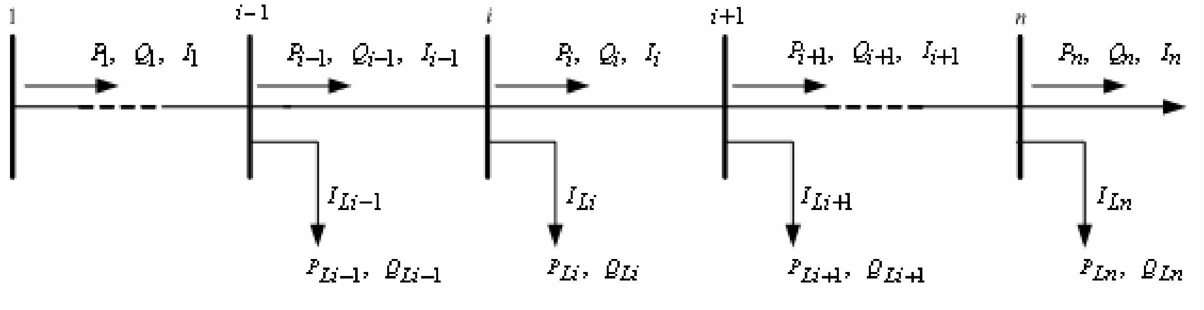

Fig. 1 A single feeder of distribution system.

$$P_i = P_{i+1} + P_{Li+1} + R_{i,i+1}\frac{P_i^2 + Q_i^2}{|V_i|^2} \qquad (1)$$

$$Q_i = Q_{i+1} + Q_{Li+1} + X_{i,i+1}\frac{P_i^2 + Q_i^2}{|V_i|^2} \qquad (2)$$

$$|V_{i+1}|^2 = |V_i|^2 - 2(R_{i,i+1}P_i + X_{i,i+1}Q_i) + (R_{i,i+1}^2 + X_{i,i+1}^2)\frac{P_i^2 + Q_i^2}{|V_i|^2} \qquad (3)$$

$$\delta_{i+1} = \delta_i - \tan^{-1}\frac{P_i X_{i,i+1} - Q_i R_{i,i+1}}{|V_i|^2 - (P_i R_{i,i+1} + Q_i X_{i,i+1})} \qquad (4)$$

$$I_i = I_{i+1} + I_{Li+1} \qquad (5)$$

$$I_{Li+1} = \frac{\sqrt{P_{Li+1}^2 + Q_{Li+1}^2}}{\sqrt{3}\ V_{i+1}} \qquad (6)$$

In general, the two values of $R_{i,i+1}\frac{P_i^2 + Q_i^2}{V_i^2}$ and $X_{i,i+1}\frac{P_i^2 + Q_i^2}{V_i^2}$ are far smaller than the transmission power of feeders, P_i and Q_i; it is reasonable to make the following assumptions to speed up the computation :

1) The initial value of bus voltage is assumed as 1pu; i.e. $|V_i|$=1pu. T

2) The power flow of the feeders is equivalent to:

$$P_i \cong P_{i+1} + P_{Li+1}$$
$$Q_i \cong Q_{i+1} + Q_{Li+1}$$

Above equations (1) and (2) are amended to the following equations of (7) and (8)：

$$P_i = P_{i+1} + P_{Li+1} + R_{i,i+1} \bullet$$

$$\frac{(P_{i+1}+P_{Li+1})^2+(Q_{i+1}+Q_{Li+1})^2}{|V_i|^2} \quad (7)$$

$$Q_i = Q_{i+1}+Q_{Li+1}+X_{i,i+1}\bullet$$

$$\frac{(P_{i+1}+P_{Li+1})^2+(Q_{i+1}+Q_{Li+1})^2}{|V_i|^2} \quad (8)$$

The power of buses is derived from the end of the feeder to the starting bus of the feeder. Bus voltages can be computed from the starting bus to that of the end of the feeder.

3. Mathematical Model

The compensating capacitor in the distribution system is used to provide reactive power, in order to reduce power loss of the feeder and improve the stability of bus voltage within the specified range. The installation aims to define the most appropriate position, size, and type of the compensating capacitor to minimize power loss and investment cost at the same time. By doing so, the system will work to minimize the total cost goal, and the objective function is demonstrated in the following equation :

$$F = K_P P_{loss} + K_c \sum_{i=1}^{m} \Delta Q_i^c \quad (9)$$

Where

K_P ： Cost factor of real power loss (dollar/kW-year)

K_c ： Annual cost of capacitor per kVAr (dollar/kVAr-year)

P_{loss} ：Total power loss of the system during peak load (kW)

ΔQ_i^c ：Capacitance added at bus i (kVAr); and an integral time of the unit capacitance ΔQ_0^c is assumed.

Equation (10) shows the cost of power loss [8]：

$$E = C \times P_{loss} \times FLS \times 365 \times 24 = K_P \times P_{loss} \quad (10)$$

Where

Q_i^c ：Capacitor being equipped to bus i

Q_o ：Unit capacitor available

S_i ：Positive integer

m: system bus number

Based on the above objective function and limitations, the problem is modeled as equation (14)：

Minimize：

$$F = K_P P_{loss} + K_c \sum_{i=1}^{m} \Delta Q_i^c \quad (14)$$

$$\text{Subject to} \quad \begin{cases} V_{min} \le V_i \le V_{max} \\ \sum_{i=1}^{m} \left| Q_i^c - S_i Q_0^c \right| = 0 \\ Q_i^C > 0 \end{cases}$$

4.Evolutionary Simulated Annealing

P.C.Yip and Y.H Pao combined simulated annealing with another powerful optimum tool, evolutionary strategy algorithm, to solve the famous traveling salesperson problems in 1995 and this new method is referred to as evolutionary simulated annealing [9-12].

Evolutionary simulated annealing integrates evolutionary strategy algorithm with simulated annealing to counter the drawbacks of random searching as well as the vulnerability of being influenced by initial searching. Evolutionary strategy algorithm comes from the Darwinian notion of “survival of the fittest,” emphasizing the elements of competition and elimination among various groups. Based on natural competition, those performing well, expand in number, and those doing poorly decrease or are even eliminated. Through evolutionary simulated annealing and this inspirational evolutionary strategy, we are able to use limited computing resources on “the more likely” best-so-far area to avoid random searches, and the influence of initial value to improve solution effectiveness. We assume that there is a need to determine the minimum value of the objective function $f(x_1, x_2)$ on the developed surface of x_1 horizontal axis and x_2 vertical axis; that is, to evenly distribute N on the space for search before computing, and then treat N points as the parents of N families. In this example, each of the parents and the offspring constitute the individual of (x_1, x_2), and the value of $f(x_1, x_2)$ refers to the quality of each individual used as the elimination base. There are two competition levels in the evolutionary simulated annealing method.

Flow chart of Fig. 2 demonstrates the process of the application of evolutionary simulated annealing to capacitive compensation in distribution systems.

5. Illustrative Application Example

This application system adopts the structure of the distribution system, as shown in Fig. 3 [13,14]. Fig. 3 is composed of one main feeder with 18 buses and three branching lines (among them, one branching wire contains 4 loop system buses) for a total of 36 buses. The load capacity of each bus is indicated in Table 1 and the information on the resistance and reactance of the feeder is shown in Table 2.

Table 1：The three phase load power of the distribution system

Bus no.	Real power load (kW)	Reactive Power load (kVAr)	Bus No.	Real power load (kW)	Reactive power Load (kVAr)
1	0	0	20	90	50
2	100	60	21	90	50
3	90	40	22	90	50
4	120	80	23	90	50
5	60	30	24	400	300
6	60	20	25	400	300
7	200	300	26	60	40
8	200	300	27	60	40
9	60	20	28	60	30
10	60	20	29	120	70
11	50	30	30	200	100
12	60	40	31	125	90
13	60	40	32	200	300
14	120	80	33	60	40
15	60	20	34	125	90
16	60	20	35	125	90
17	60	20	36	125	90
18	90	50			
19	90	50	Total	4020	3000

Table 2：The impedance parameters for feeders of the distribution system

From Bus i	To Bus $i+1$	$R_{i,i+1}$ (Ω)	$X_{i,i+1}$ (Ω)
1	2	0.099	0.2189
2	3	0.0786	0.2124
3	4	0.0655	0.1770
4	5	0.1048	0.2832
5	6	0.1179	0.3186
6	7	0.1048	0.2832
7	8	0.0917	0.2478
8	9	0.1572	0.4248
9	10	0.1441	0.3894
10	11	0.0786	0.2124
11	12	0.1834	0.4956
12	13	0.1179	0.3186
13	14	0.0655	0.1770
14	15	0.1179	0.3186
15	16	0.1703	0.4602
16	17	0.1048	0.2832

17	18	0.1572	0.4248
2	19	0.4725	0.2505
19	20	0.7560	0.4008
21	22	0.9450	0.5010
3	23	0.5670	0.3006
23	24	0.8505	0.4509
24	25	0.6615	0.3507
6	26	0.5670	0.3006
26	27	0.4725	0.2505
27	28	0.6615	0.3507
28	29	1.0395	0.5511
29	30	0.2835	0.1503
30	31	0.4725	0.2505
31	32	0.7560	0.4008
32	33	0.5670	0.3006
33	34	0.9450	0.5010
33	35	0.9450	0.5010
33	36	0.9450	0.5010
34	36	0.9450	0.5010
35	36	0.9450	0.5010

The nominal voltage of the distribution system is 11 kV, and the base power is 5 MVA. Before compensation, buses 16 through 18 and 28 through 36, with a total of 12 buses have voltages violating the voltage limitation range of 0.95 p.u. ~ 1.05 p.u. as shown in Table 3, and the bus voltage profile is shown in Fig. 4.

Table 3：Bus voltages before compensation

Bus no	Voltage (p.u.)	Degree	Bus no.	Voltage (p.u.)	Degree
1	1.0000	0.0000	19	0.9719	-0.0503
2	0.9915	-0.2631	20	0.9697	-0.0477
3	0.9843	-0.5181	21	0.9676	-0.0453
4	0.9797	-0.6756	22	0.9666	-0.0443
5	0.9727	-0.9180	23	0.9666	-0.3097
6	0.9650	-1.1870	24	0.9584	-0.2333
7	0.9617	-1.2928	25	0.9552	-0.2032
8	0.9596	-1.3746	26	0.9573	-1.0769
9	0.9575	-1.4959	27	0.9501	-1.0020
10	0.9556	-1.5970	28	0.9406	-0.8981
11	0.9546	-1.6465	29	0.9348	-0.8588
12	0.9527	-1.7523	30	0.9313	-0.8140
13	0.9516	-1.8130	31	0.9265	-0.7369
14	0.9511	-1.8426	32	0.9200	-0.6221
15	0.9505	-1.8809	33	0.9169	-0.5969
16	0.9498	-1.9236	34	0.9142	-0.5803
17	0.9495	-1.9420	35	0.9142	-0.5803
18	0.9492	-1.9580	36	0.9153	-0.6030

Table 4 shows the real and reactive power losses of each feeder section before compensation for the illustrative system. Table 5 shows those capacitors available for addition.

Table 4：The real and reactive power losses of every feeder section before compensation

From Bus i	To Bus $i+1$	Real power loss (kW)	Reactive power loss (kVAr)
1	2	19.43	42.96
2	3	13.17	35.59
3	4	6.26	16.92
4	5	9.18	24.80
5	6	9.91	26.78
6	7	1.94	5.25
7	8	0.99	2.66
8	9	0.83	2.24
9	10	0.64	1.73
10	11	0.29	0.78
11	12	0.56	1.51
12	13	0.28	0.74
13	14	0.11	0.30
14	15	0.09	0.25
15	16	0.08	0.22
16	17	0.03	0.07
17	18	0.02	0.04
2	19	79.17	41.97
19	20	0.64	0.34
20	21	0.39	0.21
21	22	0.09	0.05
3	23	54.20	28.73
23	24	7.67	4.06
24	25	1.50	0.79
6	26	10.51	5.57
26	27	10.53	5.58
27	28	13.44	7.12
28	29	7.34	4.65
29	30	4.30	2.28
30	31	4.92	2.61
31	32	5.74	3.04
32	33	1.59	0.84
33	34	0.74	0.39
33	35	0.74	0.39
33	36	0.28	0.15
34	36	0.12	0.07
35	36	0.12	0.07
Total		267.80	271.75

We assume the unit capacitor Q_0 as 100 kVAr. Limit the capacitive compensation of each bus as $Q_i^c \le Q_{L,total}$, where $Q_{L,total}$ is the

Table 5：Available capacitors for compensation andtheir associated annual costs

i	0	1	2	3	4	5	6	7
Q_i^c (kVAr)	0	100	200	300	400	500	600	700
(Dollar /kVAr-year)	0	8	7	7.33	7	7.2	7	7.14
i	8	9	10	11	12	13	14	15
Q_i^c (kVAr)	800	900	1000	1100	1200	1300	1400	1500
(Dollar /kVAr-year)	7	7.11	7	7.09	7	7.07	7	7.066
i	16	17	18	19	20	21	22	23
Q_i^c (kVAr)	1600	1700	1800	1900	2000	2100	2200	2300
(Dollar/kVAr-year)	7	7.058	7	7.053	7	7.047	7	7.043
i	24	25	26	27	28	29	30	
Q_i^c (kVAr)	2400	2500	2600	2700	2800	2900	3000	
(Dollar /kVAr-year)	7	7.04	7	7.037	7	7.034	7	

reactive power loaded on the feeders of the 36-bus system and is assumed as 3000 kVAr. Based on the above information, there are up to 30 types ($\frac{Q_{L,total}}{Q_i^c} = \frac{3000}{100} = 30$) of unit prices of capacitor available for bus reactive compensation.

Other parameters for the proposed method are as follows: the annual cost of power loss per kW, $K_P = 6307 = 1.2 \times 0.6 \times 365 \times 24$ (dollar/kW-year); and its maximum and minimum limitations of bus voltage are $V_{\max} = 1.05$ p.u. and $V_{\min} = 0.95$ p.u., respectively.

Parameters specified for applying the simulated annealing

Simulated annealing is used to determine the compensation of the capacitor, and the parameters are set as initial temperature, $T_0 = 100$, final temperature, $T_f = 25$, temperature dropping co-efficient, $\alpha = 0.95$, and iteration times, $K_{\max}$ (perturbations) = 1000.

Parameters specified for applying the evolutionary simulated annealing

The family of N parents in evolutionary annealing is set as 10 and 100; initial temperature, $T_o = 100$, final temperature, $T_f = 25$; and temperature dropping co-efficient, $\alpha = 0.95$.

Computational results

The compensated results from applying the two methods — the simulated annealing and the evolutionary simulated annealing — together with the results before compensation are shown in Table 6 and Fig. 5 for convenient comparison.

The computational results reveal that the objective function value has been greatly reduced by reactive power compensation. Moreover, the evolutionary simulated annealing gives a better objective function value of 1,068,090 comparing with that from the simulated annealing of 1,080,540.

6. Conclusion

Previous literature has shown that the study of reactive power compensation in the distribution system mostly focuses on radial structure and this paper takes another perspective to address the effectiveness issue based on circuit with loop-type structure. At present, power systems use capacitors to reduce feeder loss, to improve bus voltage, and to achieve the ultimate goal of safety and quality of power supply. In addition to system loss, economic factors such as the installation cost of the capacitor, are also taken into consideration and would be beneficial to future studies of the distribution system.

The comparison results in Table 6 indicate that the evolutionary simulated annealing works better than the simulated annealing in searching for the optimum value, and both methods have significantly reduced the power loss and improved the system voltage profile.

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Table 6：Computation results obtained by applying the simulated annealing and the evolutionary simulated annealing

Bus no.	Voltage before compensation (p.u.)	Simulated annealing		Evolutionary simulated annealing	
		number of perturbations=1000		N = 100 parents	
		Compensated capacitor (kVAr)	Voltage after compensation (p.u.)	Compensated capacitor (kVAr)	Voltage after compensation (p.u.)
1	1.0000	0	1.0000	0	1.0000
2	0.9915	100	0.9961	0	0.9964
3	0.9843	100	0.9931	300	0.9940
4	0.9797	0	0.9911	0	0.9920
5	0.9727	0	0.9883	0	0.9892
6	0.9650	300	0.9853	300	0.9862
7	0.9617	100	0.9820	100	0.9830
8	0.9596	100	0.9800	0	0.9809
9	0.9575	0	0.9779	0	0.9788
10	0.9556	0	0.9760	0	0.9770
11	0.9546	0	0.9751	0	0.9760
12	0.9527	200	0.9732	0	0.9741
13	0.9516	100	0.9721	0	0.9730
14	0.9511	0	0.9716	0	0.9725
15	0.9505	0	0.9710	0	0.9719
16	0.9498	0	0.9704	100	0.9713
17	0.9495	0	0.9701	100	0.9710
18	0.9492	0	0.9698	0	0.9707
19	0.9719	0	0.9816	100	0.9826
20	0.9697	100	0.9797	0	0.9807
21	0.9676	0	0.9781	0	0.9786
22	0.9666	100	0.9772	0	0.9777
23	0.9666	300	0.9800	300	0.9809
24	0.9584	200	0.9738	300	0.9751
25	0.9552	0	0.9712	0	0.9728
26	0.9573	200	0.9777	300	0.9786
27	0.9501	100	0.9738	200	0.9747
28	0.9406	0	0.9681	0	0.9687
29	0.9348	200	0.9649	0	0.9651
30	0.9313	300	0.9629	200	0.9629
31	0.9265	200	0.9603	300	0.9602
32	0.9200	100	0.9561	100	0.9563
33	0.9169	100	0.9541	100	0.9544
34	0.9142	0	0.9523	0	0.9526
35	0.9142	0	0.9523	0	0.9526
36	0.9153	0	0.9523	0	0.9526
Total real power loss (kW)	267.80	167.90		166.05	
Objective function value (Dollar/year)	1,689,030	1,080,540		1,068,090	

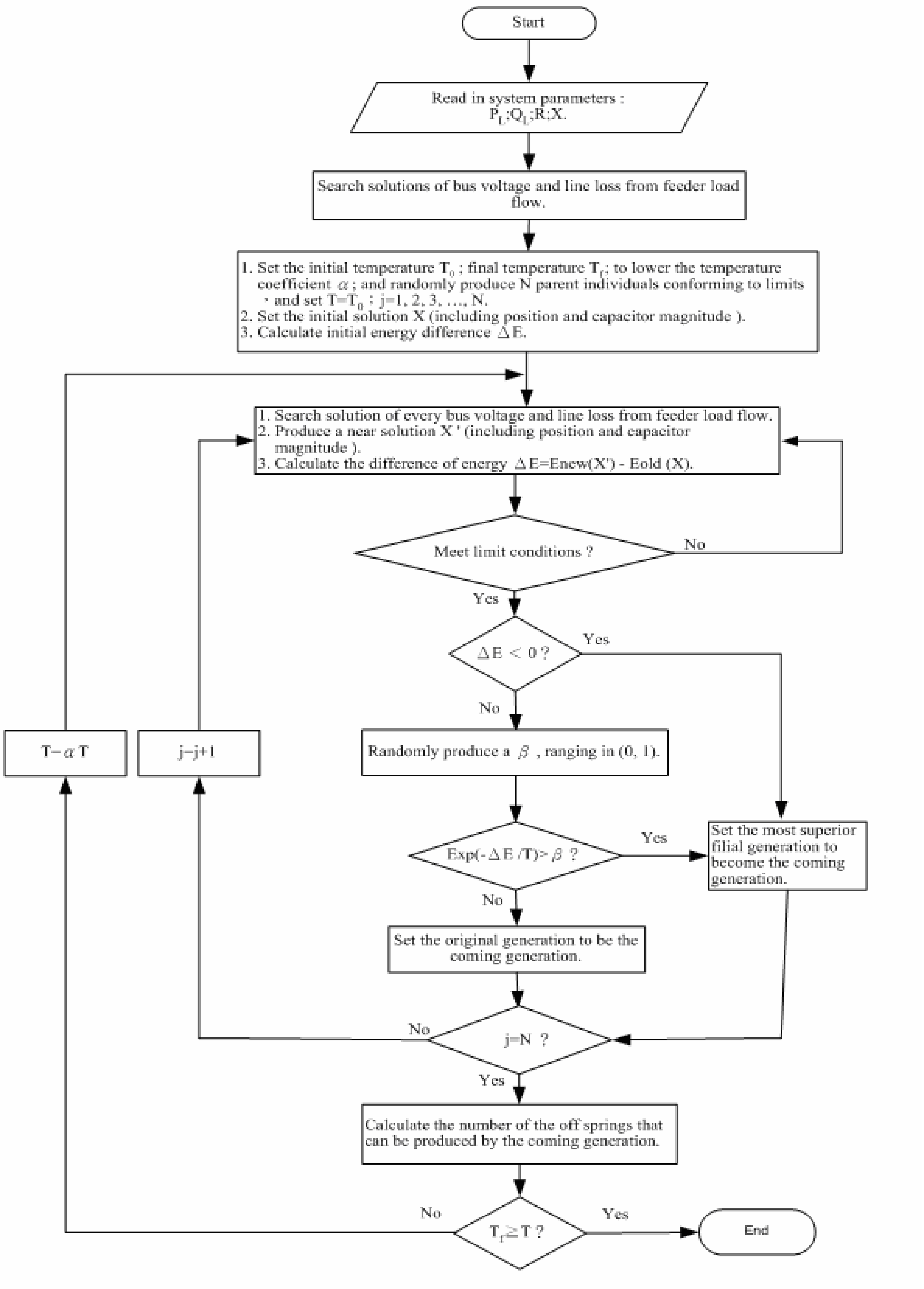

Fig. 2 Flow chart of application of evolutionary simulated annealing algorithm to capacitor VAr compensation.

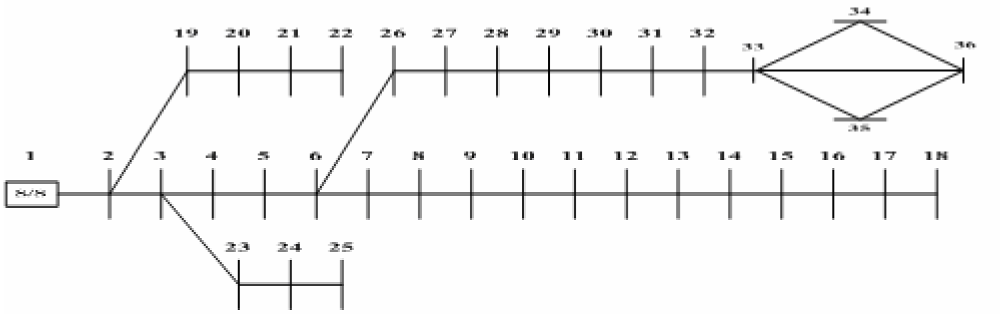

Fig. 3 A 36-bus distribution system

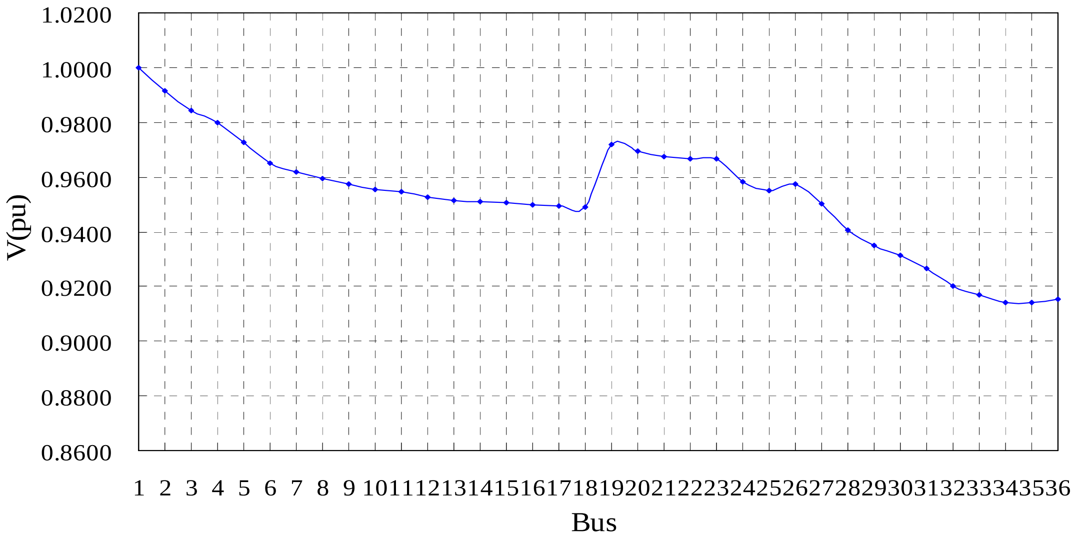

Fig. 4 Bus voltage profile before compensation.

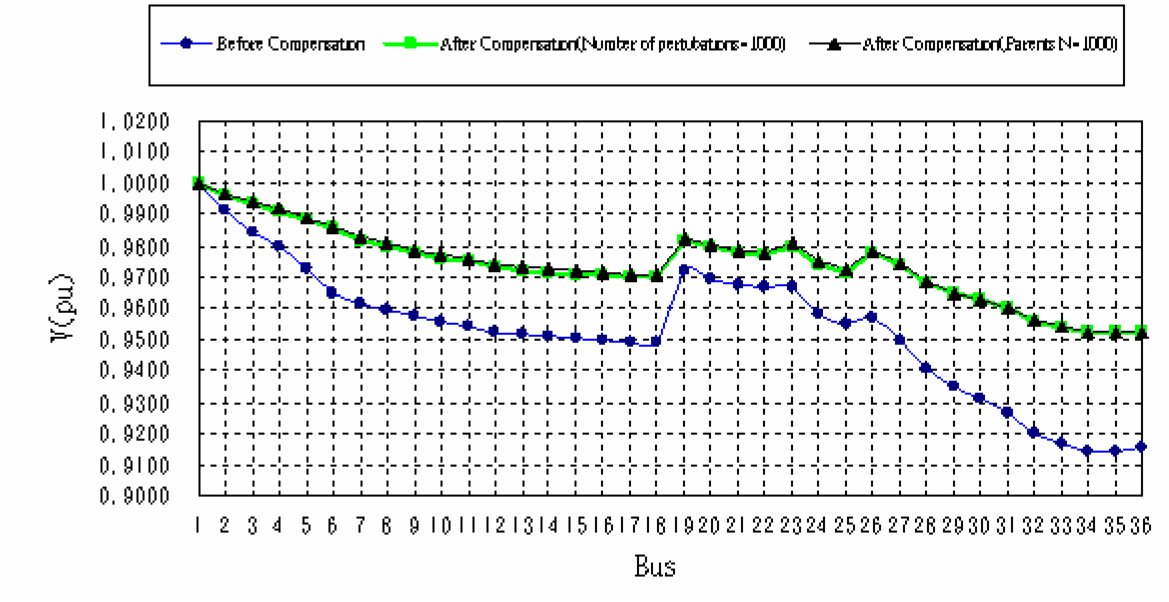

Fig. 5 Bus voltage magnitude (p.u.) before and after compensation.