HARMONIC DISTORTION AND REACTIVE POWER COMPENSATION IN SINGLE PHASE POWER SYSTEMS USING ORTHOGONAL TRANSFORMATION STRATEGY

W HOSNY[*], B DOBRUCKY[**]
* School of Computing and Technology
University of East London
4-6 University Way, Docklands E16 2RD
United Kingdom
wada@uel.ac.uk
** Department of Power Systems and Electrical Engineering
University of Zilina
Velky Diel ND 119, Zilina
Slovak Republic
dobrucky@fel.utc.sk

Abstract: - The orthogonal transformation technique which was instigated by Akagi et al in 1984 for the investigation of the instantaneous power in a three phase power system is extended in this paper in order to derive novel compensating current expressions for a single phase, shunt active power filter. This filter is used for a single phase power supply feeding a non-linear load in order to compensate for the instantaneous reactive power or the harmonic current distortion or for compensating of both. The configuration of the active power filter utilised in this investigation is included. In addition, the filter compensating current control technique and methods implemented for the evaluation of the filter ratings and size are included. An expression for the distortion power equation for a single phase power supply feeding a non-linear load is derived in the Appendix. This expression is extremely useful as an aid to design active power filters which compensate for both of the reactive power and the current higher harmonics. Experimental results demonstrated the effectiveness of the novel expressions for the active power filter compensating currents derived in this paper.

Key-Words: - Single phase power systems, orthogonal transformation technique, harmonic distortion and reactive power compensation

1 Introduction

In this section the orthogonal transformation technique applied to a single phase power system instigated by Akagi et al; (Akagi Kanazawa and Nabae, 1983) is described. By adopting this technique expressions for the reference currents used in an active power filter for the compensation of harmonic distortion or reactive power or both, are derived.

Consider a single phase power system which is defined by its input voltage and input current as follows:

$$
\begin{aligned}
v_{Re}(t) &= V \cos \omega t \\
i_{Re}(t) &= I \cos (\omega t - \Phi)
\end{aligned}
\tag{1}
$$

Where V and I respectively are the peak values of the voltage and current, ω is the angular frequency of the power supply and Φ is the phase shift between voltage and current.

The power system described by Eq.(1) is termed as the real part in a complex power system and is complemented by a fictitious/imaginary phase defined as follows:

$$
\begin{aligned}
V_{im}(t) &= V \sin \omega t \\
I_{im}(t) &= I \sin (\omega t - \Phi)
\end{aligned}
\tag{2}
$$

Comparing Eqs (1) and (2), it is obvious that the imaginary or fictitious phase of the voltage or current in a single phase power supply can be created in the time domain by shifting the real component on the time axis to the right by an equivalent phase shift of л/2.

According to Eqs(1) and (2), the α-β orthogonal co-ordinate systems for both of the voltage and current are defined as follows:

$$v_\alpha = v_{Re}(t) \text{ and } v_\beta = v_{im}(t)$$
$$i_\alpha = i_{Re}(t) \text{ and } i_\beta = i_{im}(t) \quad (3)$$

According to (Akagi, Kanazawa and Nabae, 1983) , each of the α and β components of the voltage and current are combined to form a vector, x(t).
This vector can be represented by the following equation:

$$x(t) = x_\alpha + x_\beta\, e^{j\pi/2} = x_{Re}(t) + x_{im}(t)\, e^{j\pi/2} \quad (4)$$

This vector is represented in the Gaussian complex domain as a four sided symmetrical trajectory, Fig.1.

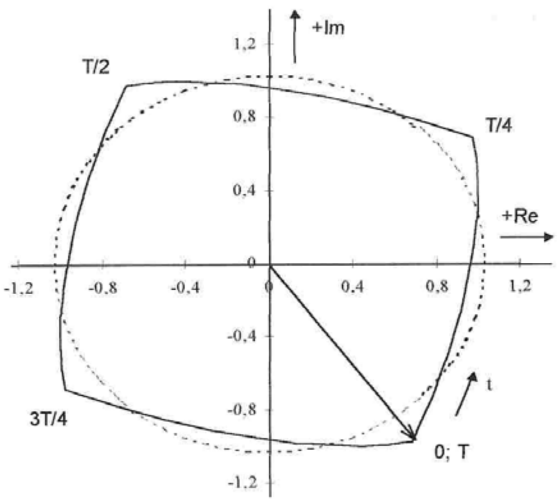

Fig1. Trajectory of vector x(t)

Because of the symmetry of the x(t) trajectory shown in Fig.1, it is evident that the voltage and current investigation for the complex power system (including both of real and imaginary voltage and current components), could be carried out within quarter of the periodic time of the voltage and current waveforms (T/4). Thus, Fourier transforms applied for the harmonic analysis of non-sinusoidal waveforms could be carried out during this time interval only as it will be shown later. Fig.2 shows the arrangement of the real and fictitious/imaginary circuits of the complex single phase power system under investigation. As it is shown on this figure, the real and fictitious circuits should be synchronised by the so called "SYNC" signal. This implies that the x (0) is a priori zero.

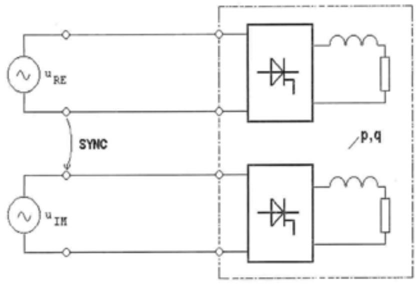

Fig.2 Real and fictitious circuits of the complex single phase power system

2 Instantaneous Reactive Power

In this section the use of the p-q-r instantaneous reactive power method, described in references (Dobrucky, 1985), (Kim and Akagi, 1999) and Akagi, Kanazawa and Nabee, 1984), for compensation of the reactive power and harmonic filtering is explained.

Consider a single phase power system with a co-sinusoidal voltage supplying a solid state controlled rectifier, thus yielding a non-sinusoidal supply current waveform. The supply current is assumed to have a square waveform. Thus, the supply voltage and the fundamental component of the supply current could be written as:

$$v_{Re}(t) = V \cos \omega t$$
$$i_{1Re}(t) = I_1 \cos(\omega t - \pi/3) \quad (5)$$

The supply voltage and current waveforms as well as the fundamental current waveform are depicted in Fig.3.

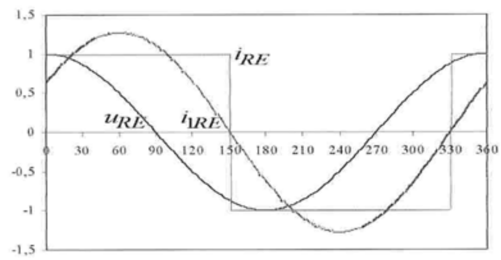

Fig.3 Supply voltage, current waveforms and the fundamental current component waveform for a single phase power circuit consisting of a co-sinusoidal voltage supply feeding a solid state controlled rectifier.

The imaginary/fictitious components of the waveforms shown in Fig.3 are depicted in Fig.4.

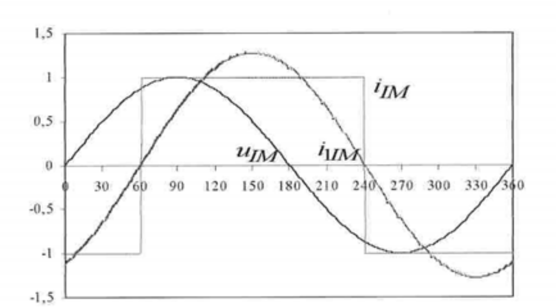

Fig.4: Imaginary/fictitious components of the supply voltage, current waveforms and the fundamental current component waveform for a single phase power circuit consisting of a co-sinusoidal voltage supply feeding a solid state controlled rectifier

The voltage and fundamental current components are given as:

$$v_{im}(t) = V \sin \omega t$$
$$i_{im}(t) = I_1 \sin (\omega t - \pi/3) \qquad (6)$$

The instantaneous active and reactive power equations for the complex power system under consideration are given in the α-β domain, as described in references (Akagi, Kanazawa and Nabae, 1983),(Kim and Akagi, 1999) and (Akagi, Kanazawa and Nabae, 1984), as follows:

$$p = v_\alpha i_\alpha + v_\beta i_\beta$$
$$q = v_\alpha i_\beta - v_\beta i_\alpha \qquad (7)$$

Fig.5 depicts the time variation of p and q for the complex single phase power system under consideration. In this figure P_{AV} and Q_{AV} respectively are the average values of the active and reactive power.

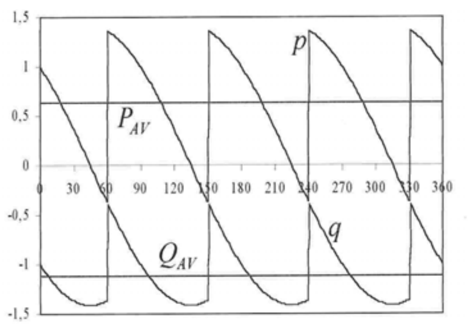

Fig.5 Instantaneous and average values for p and q for a complex single phase, the real component of which consists of a co sinusoidal voltage supply feeding a solid state controlled rectifier

The instantaneous power factor, Φ, is defined as:

$$\Phi = \tan^{-1} (q/p) \qquad (8)$$

It is important to point out that the values of p, q and Φ in Eqs (7) and (8) are instantaneous values.

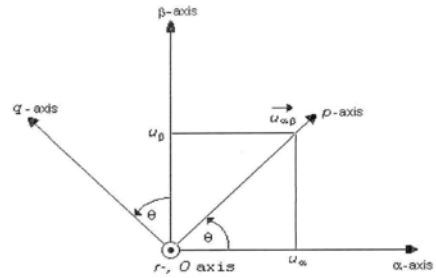

Fig.6 Voltage in a complex single phase power system represented in fixed and rotating frames of reference

The p-q-r theory is introduced in references (Kim and Akagi, 1999) and (Kim, Blaabjerg, Bak-Jensen and Choi, 2001), where the current, voltage and power equations are projected in p-q-r rotating frame of reference. Fig.6 shows the voltage components in both of the fixed α-β and rotating p-q frame of reference for a single phase power system.

In Fig.6, $v_{\alpha\beta}$ is defined as:

$$v_{\alpha\beta} = \sqrt{v_\alpha^{\ 2} + v_\beta^{\ 2}} \qquad (9)$$

Angle, θ is defined as:

$$\Theta = \tan^{-1} (v_\alpha/v_\beta) \qquad (10)$$

The r-axis is considered to be identical to the zero axis, hence the voltage transformation equation from the fixed frame of reference α-β to the rotating frame of reference p-q-r, can be written as:

$$\begin{bmatrix} v_p \\ v_q \\ v_r \end{bmatrix} = (1/v_{\alpha\beta}) \begin{bmatrix} v_\alpha & v_\beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_{\alpha\beta} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \\ 0 \end{bmatrix} = \begin{bmatrix} v_{\alpha\beta} \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$

The currents in the rotating frame of reference, i_p, i_q and i_r are related to the currents in the stationary frame of reference, i_α and i_β by similar equations as the voltage equations in Eq.11.

Moreover, the following relations can be derived in the p-q-r rotating frame of reference:

$$\begin{aligned} &p_{\alpha\beta} = v_{\alpha\beta}\, i_{\alpha\beta} = v_\alpha\, i_\alpha + v_\beta\, i_\beta = p \\ &v_p = v_{\alpha\beta} \\ &v_q = v_r = i_q = i_r = 0 \\ &i_p = i_{\alpha\beta} \\ &p = v_p\, i_p \end{aligned} \tag{12}$$

3 Derivations of Reference Current Expressions for the Active Filter

In this section instantaneous expressions for the reference currents for an active power filter to compensate for the harmonic distortion or reactive power or both in the single phase power system under investigation are derived.

Because of the symmetry of the complex voltage and current vectors trajectories, Fig.1, the average value of the active and reactive powers for both of the real and imaginary/fictitious phases can be evaluated from Eq.7 as follows:

$$\begin{aligned} P_{RE\,AV} &= P_{AV}/2 = 2/T \int_0^{T/4} (v_\alpha i_\alpha + v_\beta i_\beta)\,dt \\ Q_{RE\,AV} &= Q_{AV}/2 = 2/T \int_0^{T/4} (v_\alpha i_\alpha - v_\beta i_\beta)\,dt \end{aligned} \tag{13}$$

According to (Akagi, Kanazawa and Nabae, 1983), the instantaneous expressions for the active and reactive power in the real phase of the single phase power system under analysis are given as:

$$\begin{aligned} P_{RE} &= (v^2_\alpha / v^2_{\alpha\beta})p \\ q_{RE} &= (- v^2_\beta / v^2_{\alpha\beta})q \end{aligned} \tag{14}$$

The real phase average value, fundamental and ripple components of the active and reactive power are extracted from Eqs (13) and (14) and are depicted in Fig. 7 and Fig.8 respectively.

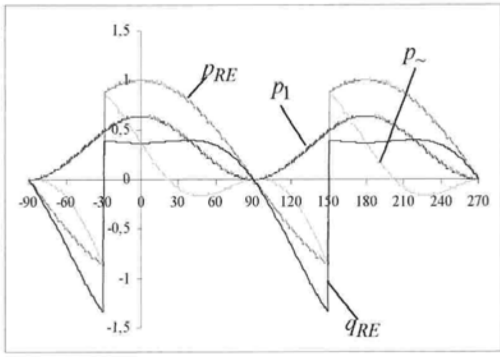

Fig.7 Components of the active power for the real phase of the complex single phase power system

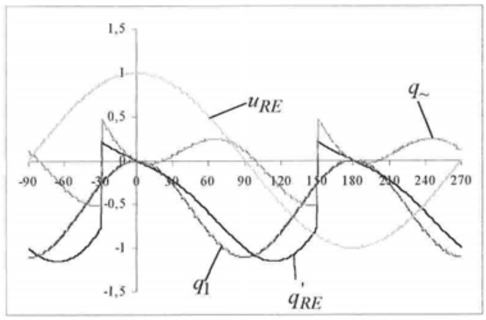

Fig.8 Components of the reactive power for the real phase of the complex single phase power system

The real phase current, i_α, can be derived from Eq.7 as follows:

$$\begin{aligned} i_\alpha &= 1/v_{\alpha\beta}^2 (v_\alpha\, p - v_\beta\, q) \\ &= 1/v_{\alpha\beta} (v_\alpha (P_{AV} + p_\sim) - v_\beta (Q_{AV} + q_\sim)) \end{aligned} \tag{15}$$

In Eq.15, $p_\sim$ and $q_\sim$ respectively are the ripple active and reactive power components.

Reference current for the active filter of the single phase system under consideration can assume different expressions depending on the special requirements of compensating for the reactive power or filtering the distortion harmonics or both. Three special cases are listed below:

i) Reference current for distortion harmonic filtering and reactive power compensation

$$\begin{aligned} i_{ref} &= 1/v_{\alpha\beta}^2 (v_\alpha\, p_\sim - v_\beta\, q) \\ &= 1/v_{\alpha\beta}^2 (v_\alpha (p-P_{AV}) - v_\beta\, q) \end{aligned} \tag{16}$$

ii) Reference current for average reactive power compensation

$$i_{ref} = 1/v_{\alpha\beta}^2 (-v_\beta\, Q_{AV}) \tag{17}$$

iii) Reference current harmonic distortion compensation

$$i_{ref} = 1/v_{\alpha\beta}^2 \, (v_\alpha \, p_\sim - v_\beta \, q_\sim) \qquad (18)$$

4 Active Power Filter Configuration, Rating and Sizing

4.1 Active Power Filter configuration

The active power filter used in this investigation is a shunt type. This implies that it is connected in parallel with the single phase power supply and the load, Fig.9. If the load is solid state full-wave rectifier bridge supplying inductive impedance, it is a non-linear load and the current supplied by this load requires compensation in order to cancel the higher harmonics and reactive components currents. Since the active power filter is a shunt type, the following relationship is valid between the supply current, load current and the active power filter type:

Supply Current = Load Current – Filter Current

The filter current is controlled to follow one of the reference currents delineated to above in order to compensate for the higher harmonic load currents or the reactive load currents or both. The active power filter implemented is a pulse-width modulated (PWM) voltage source inverter. Thus, it comprises an H-bridge connected, four IGBT solid state electronic devices G_1,G_2,G_3 and G_4, Fig.9. Each electronic device is connected anti-parallel to a free-wheeling diode. The active power filter inverter circuit is connected in parallel with a dc capacitor. The latter acts as the dc power supply to this circuit. It should be noted that the inverter circuit is not connected directly in parallel with the single phase power supply and the load. An isolating inductor,L_f , Fig.9, is connected in series with the inverter terminal which is connected to the high voltage side of the power supply. The purpose of this inductor is to convert the inverter output voltage into the active power filter compensating current.

4.2 Active power Filter Current Control

The desired compensating current waveform is obtained by controlling the PWM switching of the four IGBTs constituting the inverter. This is accomplished by comparing the current taken by the active power filter to the reference current and applying the difference between these two currents (ΔI) to a hysteresis comparator. If the magnitude of the ΔI exceeds the upper boundary of the hysteresis band of the comparator, the comparator output goes high, thus firing the inverter two IGBT devices G_2 and G_3 that make the active power filter current decreases. However, if the magnitude of ΔI is less than the lower boundary of the hysteresis band of the comparator, the comparator output goes low, thus firing the inverter two IGBT devices G_1 and G_4 that make the active power filter current increases.

4.3 Factors Affecting the Active power Filter Ratings

The frequency of switching of the IGBT devices, constituting the active power filter, is inversely related to the hysteresis controller bandwidth. The higher the value of the switching frequency, the lower the value of the active power filter isolating inductor value and vice versa. A low inductor value implies compactness and reduced cost. For a constant inverter dc voltage low inductor values implies higher rates of current rise. This is a good condition for cancellation of higher order harmonic components of the load current. But a larger inductor is better for isolation of the active power filter from the single phase power system and protection from transient disturbances. Therefore, there is a trade-off involved in sizing the isolating inductor. It should also be noticed that higher switching frequencies result in increasing the inverter IGBTs switching losses. This is another factor to be noticed in sizing the isolator inductor. The voltage ratings of the inverter switching devices and the dc capacitor are determined from the inverter dc supply voltage.

4.4 Determination of the Active Power Filter Size

A shunt connected active power filter, such as the one used in this investigation, should be rated in terms of its rms current. This current is evaluated as the rms current required to compensate for the non-linear loads harmonic and reactive currents. One merit of this shunt active power filter as compared to passive filters is that it is self-limiting in terms of the harmonic and the reactive currents cancellation provided. There is no concern of overloading the filter due to harmonics from the single phase power supply or under-rating the filter for the loads involved. The reason for this is due to the fact that if the filter is under-rated, it would not completely compensate for all the nonlinear load current harmonics. In practice, it is not necessary to compensate for all the harmonics generated from a non-linear load. The size can be selected to achieve any desired level of harmonic current compensation. In this investigation the guidelines for harmonic generation specified in IEEE 519-1992 are adopted in order to determine the required level of the harmonic current compensation and the size of the active power filter.

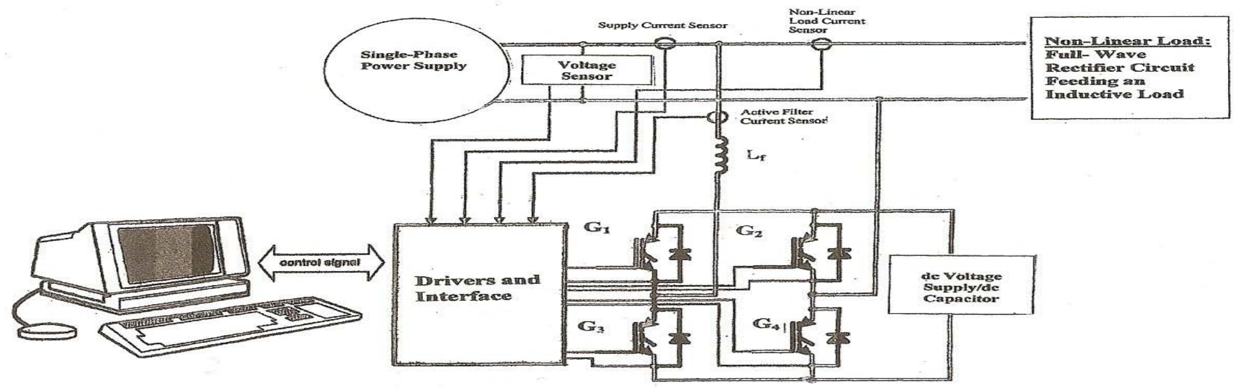

Fig.9 Schematic Diagram Showing the Structure and Connection of the Active Power Filter

5 Modular Active Power Filters

5.1 Structure of the Modular Active Power Filter

Modular active power filters, Fig.10, are capable of compensating for the higher harmonics load currents at the vicinity of their generation. Hence they ensure that these harmonics are not transmitted throughout the single phase electrical power system. However, these filters can not eliminate the higher switching frequency harmonics introduced by the power electronic converters constituting these filters. These switching frequencies are kept constant as determined by the bandwidth of the hysteresis current controllers embedded in the control system of these filters. Therefore, these higher switching frequencies currents can be eliminated by using specially designed passive filters. Modular active power filters are based on the separate computation and implementation of control strategies in order to evaluate the reference currents, Eqs 16-18, for individual modules of the modular structure active power filter. These modules will operate at the fraction of the rating of a singular active power filter and will compensate for a specific harmonic frequency and carry their own individual computational facilities. Therefore, modular active power filters are capable of compensating for higher order harmonic currents than the singular structure ones. Additional advantages of modular filters are their enhanced reliability and efficiency of operation. The reason for their enhanced reliability is due to the fact that the failure of one module in these modular filters will neither adversely affect the operation of the modular filter as a whole, nor will it bring it to a complete halt. The enhanced efficiency of the modular filters is due to the smaller switching loss in the solid state converter included in each module because of the converter reduced power handling requirement. This leads to a reduction in the total switching loss in a modular filter in comparison to a singular one handling an equivalent power rating. One additional advantage of modular filters is their improved transient response in comparison to singular filters due to the capability of each module in the modular filters of handling rapid current changes than singular filters. Obviously, this improvement is highly dependent on the selected control strategies to evaluate the required harmonic reference currents for each module.

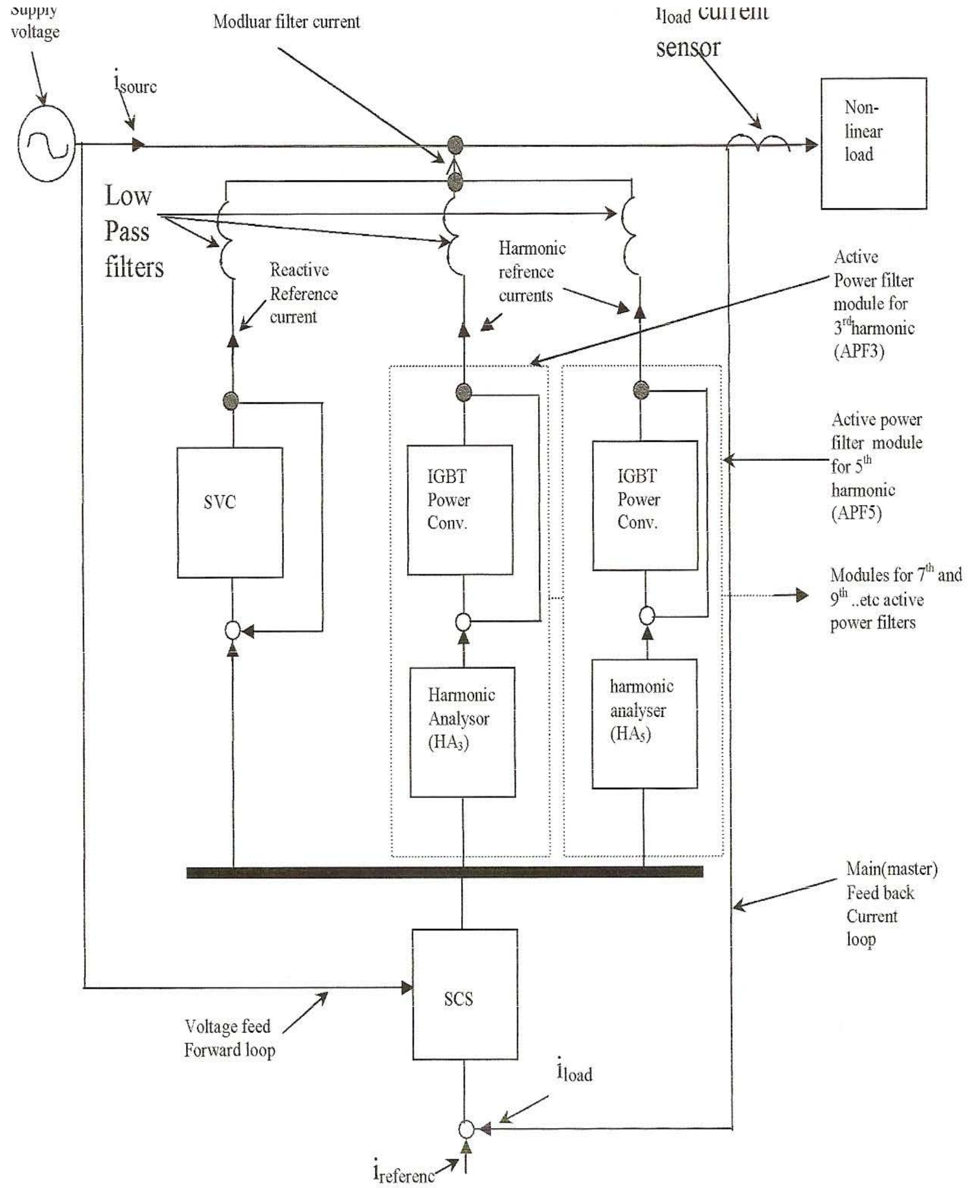

Fig.10 Block Diagram of a Modular Active Power Filter

In Fig.10, SVC is the static Var Compensator module; it generates the reactive reference current component for the modular active power filter. This reference current is controlled so as to follow Eq 17 delineated to earlier. APF3, APF5, APF7 and APF9 are the modular active power filter modules respectively for the compensation of the third, fifth, seventh and ninth harmonics. The reference currents for these modules are controlled so as to follow the corresponding harmonic component of Eq 18. Each of these modules comprises an IGBT converter of the type explained in Section 4.1. In addition each modules must include a harmonic analyser to extract the required current harmonic and a passive low pass filter. Each harmonic analyser mainly contain an analogue to digital converter, a digital to analogue converter and a digital signal processor in order to carry out the necessary computations to evaluate the harmonic reference currents. The low pass filter compensates for the switching frequency signal introduced by the switching action of the IGBT converter. The SCS is the Supervisory Control System; it oversees the operation of all of the above modules and takes appropriate actions to compensate for the failure of any of them.

5.2 Modular Active Power Filter Control Strategy

Fig.11 illustrates the control strategy adopted for the modular active power filter of Fig.10. As it can be seen, there are two levels of control systems. The reference current algorithms in these control systems are executed simultaneously with the aid of parallel computation algorithms. Level 1 control system carries out the following tasks: computing the first harmonic of the load current for the power factor compensation, performing the main (master) control loop algorithms, and supervising the successful operation of the filter modules for higher harmonics and taking appropriate action in case of their failures. Level 2 control system computes the higher load current harmonics and provides local slave reference current control. It is important to mention that both of level 1 and level 2 control systems operate synchronously. In order to compute the fundamental and higher harmonics load currents, Fourier harmonic analysis is carried out in the stationary frame of reference (α, β). As it was mentioned in section 1, the mathematical integration performed for Fourier analysis could be carried out within quarter of the periodic time of the voltage and current waveforms due to the four sided symmetrical trajectory of both of the voltage and current when represented in the complex domain (Fig.1). Methodology for computing of the reference current harmonics for modular single phase active power filters is included in (Roch, Dobrucky and Hosny, 2003).

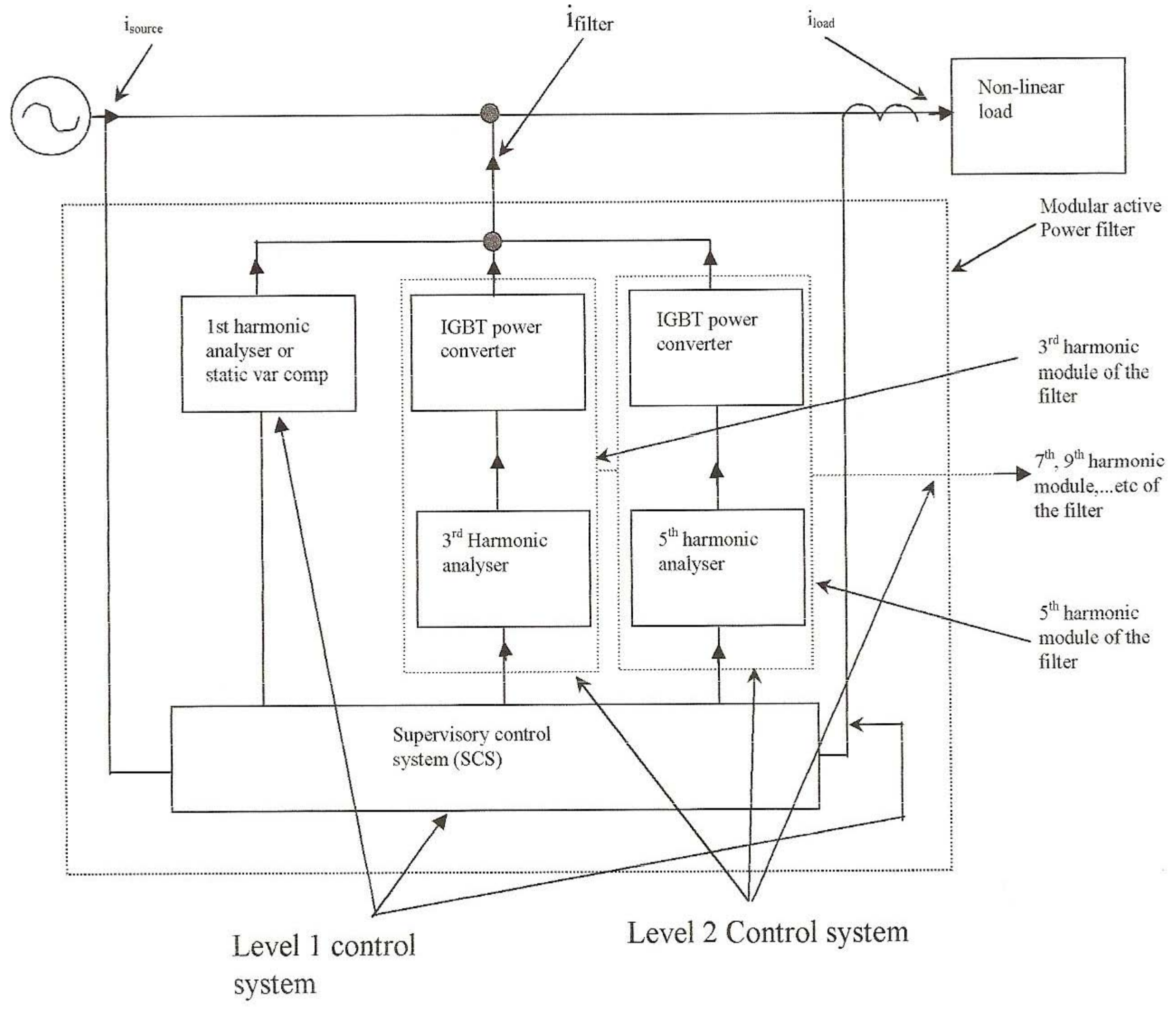

Fig.11 Modular Active Power Filter Control System

6 Experimental Results

A test rig was set up to verify the theoretical derivations above for a singular active power filter. The active power filter is implemented with the current reference of Eq.(15) used as an input to the filter and the digital signal processing of the voltages and currents is implemented using a 32 bit floating point DSP,TMS320C31.The configuration of the experimental setting is shown in Fig.12.

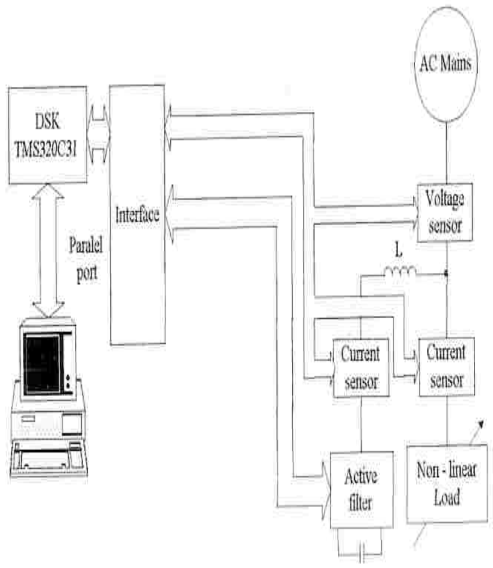

Fig.12: Block Diagram of the Experimental Setting

The single phase power system under experimentation is a diode bridge rectifier with an RL load connected to the dc side. The ac to dc converter is rated at 25 kVA. An inductor, L, with a value of 1.2 mH and a capacitor with a value of 10,000 μF are used as dc output filter. The output current of the active power filter is controlled by a hysteresis comparator to confine the switching frequency to 15 kHz. Fig.11 shows the waveforms of the load current, the compensating current of the active power filter and the supply current. It is clear that active power filter performed its task of compensating for the harmonic distortion as the supply current is converted to a pseudo-sinusoidal waveform from its original square shape waveform. The top waveform in Fig.13 shows the original supply current waveform and the bottom waveform shows the supply current wave form after the implementation of the active power filter. The middle wave form is the compensating current of the active power filter.

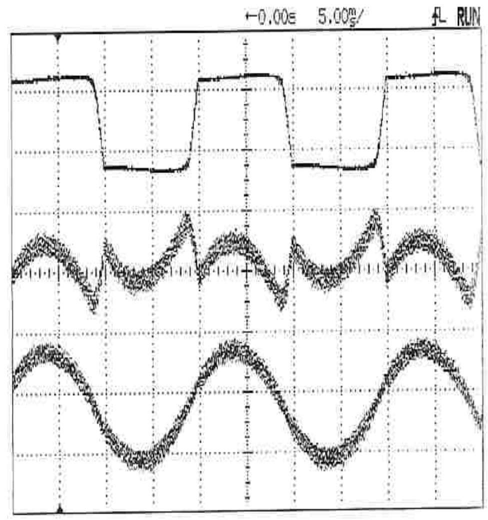

Fig.13 Supply current to the single phase power system under consideration before (top) and after (bottom) the implementation of the active power filter

7 Conclusions

A novel strategy, orthogonal transformation technique, is used to yield reference current expressions for the active power filter of a single phase power supply feeding a solid state power converter, in terms of the supply voltage and current. The power active filter control strategy could compensate for either the harmonic distortion of the supply current or the reactive power or both. The configuration for the both of singular and modular single phase active power filters and their control strategies are included. Experimental results for a singular active power filter demonstrated the effectiveness of the novel active power filter control strategy reported in this paper.

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Appendix

In this Appendix the distortion power for a single phase power supply feeding a non-linear is evaluated. Distortion power includes the total reactive power and the higher harmonics active power. Accordingly, the rating of the active power filter, which compensates for the total reactive power and the higher harmonics of the active power, can be determined.

Assuming a single phase sinusoidaly time varying power supply feeding a non-linear load which draws a current of a square waveform, the distortion power, D, is given as follows:

$$D = \sqrt{S^2 - P^2} \qquad \text{(A1)}$$

In Eq.A1, S is the apparent power and P is the active power associated with the fundamental current harmonic, thus:

$$S = VI \quad \text{and} \qquad \text{(A2)}$$

$$P = V \times \left(\frac{4I}{\sqrt{2}\pi} \right) \times \cos\Phi \qquad \text{(A3)}$$

In Eqs. A2 and A3, V is the rms voltage of the single phase power supply, I is the amplitude of the non-linear load current square waveform and cos Φ is the inductive load power factor.

Inserting Eqs (A2) and (A3) into Eq (A1), yields the final expression for the distortion power:

$$D = VI \sqrt{1 - \frac{8}{\pi^2}\cos^2\Phi} \qquad \text{(A4)}$$

For a unity power factor load, Eq (A4) can be written as:

$$D = VI \sqrt{1 - \frac{8}{\pi^2}} \qquad \text{(A5)}$$