Effective Optimal Power Flow Solution with Transient Stability Constraints based on Functional Transformation Technique

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Abstract: - Optimal power flow with transient stability constraints (OTS) can coordinate the economical efficiency, reliability and quality of the electric power at the same time. However, OTS is a nonlinear infinite optimization problem, which is difficult to be solved precisely even for power systems of small dimension. In this paper, we propose a new transformation method, which exactly converts the infinite-dimension OTS into a finite-dimension optimization problem. The transformed OTS problem has the same variables of the associated OPF problem, and is tractable even for the large-scale power systems with a large number of transient stability constraints. In addition, this paper uses a modified SQP (sequential quadratic programming) approach to solve the formulated nonlinear optimization problem. By using this approach, the optimal operation points can be determined in short calculation time.

Key-Words: - Transient stability, Optimal power flow, Sequential quadratic programming, Power system, Functional equality, Available transfer capability.

1 Introduction

OTS (optimal power flow with transient stability constraints) can be defined as an optimization problem that produces a solution which can let the power system to perform in the best way under any kinds of contingencies, when structure parameters and loads are given. Along with growth of the power system, not only reliability but also economical efficiency is considered to be very important. The OTS is very powerful tool that allows obtaining the economical efficiency, reliability and quality of the electric power at the same time completely. However, according to the mathematical model, OTS is a nonlinear infinite optimization problem with a large number of constraints, which is difficult to be solved precisely even for a small power system. One of the major difficulties in the OTS is how to deal with infinite-dimension variables, which are constrained by infinite algebraic inequalities and equalities [1].

Traditionally, the OTS problem is solved mainly by the approximate discretization methods [2-6]. However, discretization methods may not only suffer from the inaccuracy in computation by the approximation, but also cause convergence difficulties due to the introduction of a large number of variables and equations at each time step of the original OPF. Because of the long computing time in this approach, the calculation itself will become extremely complicated for power systems with a large number of contingencies. To deal with this kind of problem, this paper proposes a functional transformation method, which converts the infinite-dimensional OTS into a finite-dimensional optimization problem. The transformed OTS problem has the same variables of the OPF and almost has no limitation on the models of generators, controls and transmission networks.

In general, the fast convergence algorithm such as the Newton method is useful to calculate the OPF problems [7]. However, the Newton method requires the LU factorization or inverse of the Hessian matrices. This involves a large number of time-consuming integrations of the differential equations for the OTS due to the functional constraints [8]. To overcome this problem, a modified SQP approach has been applied in this paper. This approach has faster convergence rate than the conventional Newton method and does not require exact evaluations of the Hessian matrices.

This paper is organized as follows. In the next section, several mathematical definitions used in this paper as well as steady and transient stability are briefly described. In section 3, OTS is formulated as a nonlinear optimization problem in functional space or infinite optimization problem with the constraints of both algebraic and differential equations, and then transformed into an initial values problem with functional equalities. In section 4, the modified SQP approach is described. In section 5, in order to show the validity of the proposed method, the performances are compared with the conventional method, and some numerical examples are described. Finally we summarize the conclusions in section 6.

2 Dynamic Model of Power System

Power system operations are generally described with a set of differential and algebraic equations [9]

$$\frac{dx(t)}{dt} = F\left(x(t), y(t), \bar{z}\right) \tag{1}$$

$$G\left(x(t), y(t), \bar{z}\right) = 0 \tag{2}$$

$$H\left(x(t), y(t), \bar{z}\right) \le 0 \tag{3}$$

where vectors $x(t) \in R^{n_x}$ and $y(t) \in R^{n_y}$ are the state variables such as power outputs of generators, voltage values and angles, while $\bar{z} \in R^{n_z}$ are the control variables such as transformer tap positions, phase shifter angle positions and shunt capacitor/reactors. The control variables are generally independent of time t in transient stability analysis and can be treated as parameters in (1)-(3). $F : R^{n_x+n_y+n_z} \mapsto R^{n_x}$ represent the dynamics of the generators and the loads as well as the controllers, while $G : R^{n_x+n_y+n_z} \mapsto R^{n_y}$ are the current balance equations of the transmission network (the Kirchhoff's laws have to be hold at each bus) and internal static behaviors of passive devices (e.g shunt capacitors and static loads). $H : R^{n_x+n_y+n_z} \mapsto R^{n}$ express the stability and operating conditions, such as lower and upper bounds of generators and voltages.

Let define $F_x = \partial F\left(x, y, \bar{z}\right)/\partial x$ and $\left|F_x\right|$ the determinant of F_x. A^T is the transpose of a vector or matrix A. Vectors $\left(\bar{x}, \bar{y}, \bar{z}\right)$ are called an equilibrium of (1) and (2) if $F\left(\bar{x}, \bar{y}, \bar{z}\right) = 0$ and $G\left(\bar{x}, \bar{y}, \bar{z}\right) = 0$. Let m be the number of disturbances under study. Then, for any specified large disturbance, e.g the k-th disturbance, power system (1)-(3) can be viewed as going through changes in configuration in three stages:

I. Pre-disturbance system (t=0)

$$0 = F\left(\bar{x}, \bar{y}, \bar{z}\right) \tag{4}$$

$$0 = G\left(\bar{x}, \bar{y}, \bar{z}\right) \tag{5}$$

$$0 \ge H\left(\bar{x}, \bar{y}, \bar{z}\right) \tag{6}$$

II. During-disturbance system $\left(0 < t \le t^k\right)$

$$\frac{dx^k(t)}{dt} = F_1^k\left(x^k(t), y^k(t), \bar{z}\right) \text{ with initial value } \bar{x} \tag{7}$$

$$0 = G_1^k\left(x^k(t), y^k(t), \bar{z}\right) \tag{8}$$

$$0 \ge H_1^k\left(x^k(t), y^k(t), \bar{z}\right) \tag{9}$$

III. Post-disturbance system $\left(t^k < t \le \bar{t}\right)$

$$\frac{dx^l(t)}{dt} = F_2^k\left(x^k(t), y^k(t), \bar{z}\right)$$

$$\text{with initial value } x\left(t^k\right) \tag{10}$$

$$0 = G_2^k\left(x^k(t), y^k(t), \bar{z}\right) \tag{11}$$

$$0 \ge H_2^k\left(x^k(t), y^k(t), \bar{z}\right) \tag{12}$$

where t^k is the clearing time of the k-th disturbance and $\bar{t}\left(> t^k\right)$ is the study period. Actually, the second stage, i.e. during disturbance stage can be further divided into several sub-stages corresponding to the actions of the relays and circuit breakers. For the sake of simplicity, this paper omits the detail descriptions of these sub-stages although there is no difficulty to incorporate them into the analysis. A solution satisfying all (4)-(12) is transiently stable for the k-th disturbance. Since the trajectories of a disturbance during the transient period is uniquely determined by the equilibrium $\left(\bar{x}, \bar{y}, \bar{z}\right)$ according to the three stages, a transient stability problem consists in finding an equilibrium, which satisfies (4)-(12) for all disturbances $k = 1,\ldots,m$.

3 OTS problem

3.1 Formulation of OTS

To emphasize the initial values $\left(\bar{x},\bar{y},\bar{z}\right)$ of (1), next we express *F*, *G*, *H* of (1)-(3) in more general forms of $F\left(x(t),y(t),\bar{x},\bar{y},\bar{z}\right)$, $G\left(x(t),y(t),\bar{x},\bar{y},\bar{z}\right)$, $H\left(x(t),y(t),\bar{x},\bar{y},\bar{z}\right)$.

Then, OTS can be defined as a nonlinear optimization problem in functional space with both algebraic and differential constraints as formulated in (13)-(18).

$$\min \quad f\left(\bar{x},\bar{y},\bar{z}\right) \quad for\ \bar{x},\bar{y},\bar{z},x^k(t),y^k(t) \tag{13}$$

$$s.t. \quad G^0\left(\bar{x},\bar{y},\bar{z}\right)=0 \tag{14}$$

$$H^0\left(\bar{x},\bar{y},\bar{z}\right)\le 0 \tag{15}$$

$$\frac{dx^k(t)}{dt}=F^k\left(x^k(t),y^k(t),\bar{z}\right);\quad x^k(0)=\bar{x} \tag{16}$$

$$G^k\left(x^k(t),y^k(t),\bar{z}\right)=0 \tag{17}$$

$$H^k\left(x^k(t),y^k(t),\bar{z}\right)\le 0 \tag{18}$$

$$k=1,\ldots,m;\quad 0\le t\le \bar{t}$$

where F^k, G^k and H^k are piecewise functions as indicated in (4)-(12). In particular, (18) is the state inequality constraints including transient stability constraints, such as rotor angle limits, voltage stability limits and tie-line stability limits.

Since the rotor angles with respect to a center of inertia reference frame are usually used to evaluate the transient stability in power industry, they can be adopted to construct the constraints of (18). Besides, other constraints such as voltage dips or rises, line flow limits and power oscillations can also be included in (18) during transient period.

The problem (13)-(18) is an OTS problem because all of the trajectories must satisfy constraints (17) and (18) before, during and after the transient period. OTS is not easy to solve because there are infinite-dimensional variables and infinite algebraic inequalities (18) and equalities (17) for all $0\le t\le \bar{t}$, not to mention a large number of the specified contingencies.

3.2 Transformation of OTS

In this section, the original OTS of (13)-(18) is transformed from an infinite-dimensions problem to a finite-dimensions one.

Let $h_i^k\left(x^k(t),y^k(t),\bar{z}\right)\in R$ the i-th element of $H^k\left(x^k(t),y^k(t),\bar{z}\right)$. Then, it is possible to convert (18) into the following functional equality

$$\bar{h}_i^k\left(\bar{x},\bar{y},\bar{z}\right)=\int_0^{\bar{t}}\left[\max\left\{0,h_i^k\left(x^k(t),y^k(t),\bar{z}.\right)\right\}\right]^2 dt \tag{19}$$

where for $i=1,\ldots,n$

$$\left(\bar{h}_1^k\left(\bar{x},\bar{y},\bar{z}\right),\ldots,\bar{h}_n^k\left(\bar{x},\bar{y},\bar{z}\right)\right)^T=0 \tag{20}$$

which are also differentiable functions with respect to time-invariant $\bar{x},\bar{y}$ and $\bar{z}$. Therefore, OTS of (13)-(18) can be rewritten as an optimization problem of the Euclidean space, i.e. to find an operation point $\left(\bar{x},\bar{y},\bar{z}\right)$ such that

$$\min \quad f\left(\bar{x},\bar{y},\bar{z}\right) \quad for\ \bar{x},\bar{y},\bar{z} \tag{21}$$

$$s.t. \quad G^0\left(\bar{x},\bar{y},\bar{z}\right)=0 \tag{22}$$

$$H^0\left(\bar{x},\bar{y},\bar{z}\right)\le 0 \tag{23}$$

$$\bar{H}^k\left(\bar{x},\bar{y},\bar{z}\right)\le 0 \quad k=1,\ldots,m \tag{24}$$

where $\bar{H}^k\left(\bar{x},\bar{y},\bar{z}\right)=\left(\bar{h}_1^k\left(\bar{x},\bar{y},\bar{z}\right)-\bar{\rho},\ldots,\bar{h}_n^k\left(\bar{x},\bar{y},\bar{z}\right)-\bar{\rho}\right)^T$ is defined by (19). $\bar{\rho}$ is a positive constant which is introduced to satisfy the constraint qualification of the Kuhn-Tucker conditions.

Obviously, the reformulated OTS of (21)-(24) has only variables $\left(\bar{x},\bar{y},\bar{z}\right)$ which are the same as those of the original OPF, with finite constraints. (21)-(24) can actually be viewed as a problem seeking an optimal initial value $\left(\bar{x},\bar{y},\bar{z}\right)$ for all specified disturbances. As far as the Jacobian matrices of (24) can be computed, the modified SQP approach can be applied to solve this problem.

Let the Jacobian matrices of (24) be $\bar{H}_{\bar{x}}^k\left(\bar{x},\bar{y},\bar{z}\right)=\left(\bar{h}_{1\bar{x}}^k,\ldots,\bar{h}_{n\bar{x}}^k\right)^T$, $\bar{H}_{\bar{y}}^k\left(\bar{x},\bar{y},\bar{z}\right)=\left(\bar{h}_{1\bar{y}}^k,\ldots,\bar{h}_{n\bar{y}}^k\right)^T$ and $\bar{H}_{\bar{z}}^k\left(\bar{x},\bar{y},\bar{z}\right)=\left(\bar{h}_{1\bar{z}}^k,\ldots,\bar{h}_{n\bar{z}}^k\right)^T$. Assume $\left|G_y^k\right|\neq 0$ for $0\le t\le \bar{t}$. Then, the Jacobian matrices can actually be derived as follows

$$\bar{h}_{i\bar{x}}^k=2\int_0^{\bar{t}}\max\left\{0,h_i^k\right\}h_{i\bar{x}}^k dt \tag{25}$$

$$\bar{h}_{i\bar{y}}^k=2\int_0^{\bar{t}}\max\left\{0,h_i^k\right\}h_{i\bar{y}}^k dt \tag{26}$$

$$\bar{h}_{i\bar{z}}^k=2\int_0^{\bar{t}}\max\left\{0,h_i^k\right\}h_{i\bar{z}}^k dt \tag{27}$$

where $h_i^k=h_i^k\left(x^k(t),y^k(t),\bar{z}\right)$ at *t*.

$\left(h_{i\bar{x}}^k,\ldots h_{n\bar{x}}^k\right)=\frac{\partial H^k}{\partial \bar{x}}$, $\left(h_{i\bar{y}}^k,\ldots h_{n\bar{y}}^k\right)=\frac{\partial H^k}{\partial \bar{y}}$ and $\left(h_{i\bar{z}}^k,\ldots h_{n\bar{z}}^k\right)=\frac{\partial H^k}{\partial \bar{z}}$ at $\left(x^k(t),y^k(t),\bar{z}\right)$ can be written as follows

$$\frac{\partial H^k}{\partial \bar{x}} = \left[H_x^k - H_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{x}} \tag{28}$$

$$\frac{\partial H^k}{\partial \bar{y}} = \left[H_x^k - H_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{y}} \tag{29}$$

$$\frac{\partial H^k}{\partial \bar{z}} = \left[H_x^k - H_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{z}} - H_y^k \left(G_y^k \right)^{-1} G_{\bar{z}}^k + H_{\bar{z}}^k \tag{30}$$

where $\frac{\partial \phi_t^k}{\partial \bar{x}} \in R^{n_x \times n_x}, \frac{\partial \phi_t^k}{\partial \bar{y}} \in R^{n_x \times n_y}, \frac{\partial \phi_t^k}{\partial \bar{z}} \in R^{n_x \times n_z}$ follow linear differential equations at t.

$$\frac{d}{dt} \frac{\partial \phi_t^k}{\partial \bar{x}} = \left[F_x^k - F_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{x}} + F_{\bar{x}}^k \tag{31}$$

$$\frac{d}{dt} \frac{\partial \phi_t^k}{\partial \bar{y}} = \left[F_x^k - F_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{y}} + F_{\bar{y}}^k \tag{32}$$

$$\frac{d}{dt} \frac{\partial \phi_t^k}{\partial \bar{z}} = \left[F_x^k - F_y^k \left(G_y^k \right)^{-1} G_x^k \right] \frac{\partial \phi_t^k}{\partial \bar{z}} - F_y^k \left(G_y^k \right)^{-1} G_{\bar{z}}^k + F_{\bar{z}}^k \tag{33}$$

with $\frac{\partial \phi_0^k}{\partial \bar{x}} = I, \frac{\partial \phi_0^k}{\partial \bar{y}} = 0, \frac{\partial \phi_0^k}{\partial \bar{z}} = 0$

4 Modified SQP approach

Generally, to solve problems similar to (21)-(24), algorithms such as the quasi-Newton method or successive linear programming are utilized. However, the convergence rates are expected only to be linear or superlinear. In other words, it may require a large number of iterations to obtain a solution, thereby impeding the possible online applications.

In addition, adopting the Newton method requires the computation of the Hessian matrices [10]. This involves a large number of time-consuming integrations of the differential equations for the OTS due to the functional constraints. To overcome this problem, this paper applies a modified SQP approach for solving the OTS problem; this method has the important advantage that it does not require the computation of the Hessian matrices [11].

4.1 SQP approach

In general, an optimization problem can be defined as follows:

$$\min \quad f(x) \tag{34}$$

$$s.t. \quad c_i(x) = 0, \qquad i = 1, \ldots, m_e \tag{35}$$

$$c_i(x) \geq 0, \qquad i = m_e + 1, \ldots, m \tag{36}$$

For the OPF problem, the Lagrangian function can be defined as follows:

$$L(x,u) = f(x) - \sum_{i=1}^{m} u_i c_i(x) \tag{37}$$

The key to implement the OPF is to solve the following equations

$$\begin{bmatrix} \nabla_x^2 L\left(x^{(k)}, u^{(k)}\right) & -\nabla c\left(x^{(k)}\right) \\ \nabla c\left(x^{(k)}\right)^T & 0 \end{bmatrix} \begin{bmatrix} \Delta x^{(k)} \\ \Delta u^{(k)} \end{bmatrix} = -\begin{bmatrix} \nabla_x L\left(x^{(k)}, u^{(k)}\right) \\ c\left(x^{(k)}\right) \end{bmatrix} \tag{38}$$

Instead of solving (38), a QP subproblem to compute descent direction d^k at each iteration is solved. This is the SQP approach. It can be verified that the following QP problem defined by quadratic approximation of Lagrangian function solves the (38).

$$\min \quad \nabla f\left(x^{(k)}\right)^T d + \frac{1}{2} d^T B^{(k)} d \tag{39}$$

$$s.t. \quad c_i\left(x^{(k)}\right) + \nabla c_i\left(x^{(k)}\right)^T d = 0, \quad i = 1, \ldots, m_e \tag{40}$$

$$c_i\left(x^{(k)}\right) + \nabla c_i\left(x^{(k)}\right)^T d \geq 0, \qquad i = m_e + 1, \ldots, m \tag{41}$$

where vector d expresses the displacement from the $x^{(k)}$ and $B^{(k)}$ is a matrix which approximates the Hessian matrix in the Lagrangian function L.

4.2 Modified SQP approach

The method described in the previous section has only a local convergence theoretically. Then, we proposed a method, which guarantees the global convergence by introducing an exact penalty function.

The constrained problem expressed by (39)-(41) can be rewritten as a non-constrained minimization problem expressed by (42). By using (42), the global convergence is guaranteed.

$$F_r(x) = f(x) + r\left[\sum_{i=1}^{m_e} \left|c_i(x)\right| + \sum_{i=m_e+1}^{m} \left|\min\{0, c_i(x)\}\right|\right] \tag{42}$$

Next, the convergence rate is evaluated.

In each step of the quadratic problem, the next iteration point $x^{(k+1)}$ is calculated by (43).

$$x^{(k+1)} = x^{(k)} + t^{(k)} d^{(k)} \tag{43}$$

where, for guaranteeing the global convergence, $t^{(k)}$ has to fulfill the (44)

$$F_r\left(x^{(k)} + t d^{(k)}\right) < F_r\left(x^{(k)}\right) + \alpha t\left[\bar{F}_r\left(x^{(k)}, d^{(k)}\right) - F_r\left(x^{(k)}\right)\right] \tag{44}$$

where α is a parameter between 0 and 1 and $\bar{F}_r\left(x^{(k)}, d^{(k)}\right)$ is expressed as follows

$$\bar{F}_r\left(x^{(k)}, d^{(k)}\right) = f\left(x^{(k)}\right) + \nabla f\left(x^{(k)}\right)^T d^{(k)} + \frac{1}{2}\left(d^{(k)}\right)^T B^{(k)} d^{(k)}$$
$$+ r\left[\sum_{i=1}^{m_e} \left|c_i\left(x^{(k)}\right) + \nabla c_i\left(x^{(k)}\right)^T d^{(k)}\right| + \sum_{i=m_e+1}^{m} \left|\min\left\{0, c_i\left(x^{(k)}\right) + \nabla c_i\left(x^{(k)}\right)^T d^{(k)}\right\}\right|\right] \tag{45}$$

Here, if (44) is fulfilled only when value of $t^{(k)}$ is very small, generated points converge but its convergence rate deteriorates extremely.

This problem is solvable by working out the quadratic problem expressed by (48)-(50) instead of the quadratic problem expressed by (39)-(41), while fulfilling the conditions expressed by (46) and (47).

$$J_\varepsilon\left(x^{(k)}\right)=\left\{i\left\|c_i\left(x^{(k)}\right)\right|<\varepsilon,\ i=1,2,\cdots,m\right\}\neq\phi \tag{46}$$

$$\begin{aligned}&\max\left\{\left|c_i\left(x^{(k)}\right)+\left(a_i^{(k)}\right)^T d^{(k)}-c_i\left(x^{(k)}+d^{(k)}\right)\right\| i\in J_\varepsilon\left(x^{(k)}\right)\right\}\\&<\sigma\max\left\{\left|c_i\left(x^{(k)}\right)+\nabla c_i\left(x^{(k)}\right)^T d^{(k)}-c_i\left(x^{(k)}+d^{(k)}\right)\right\| i\in J_\varepsilon\left(x^{(k)}\right)\right\}\end{aligned} \tag{47}$$

$$\min \quad \left(p^{(k)}\right)^T d+\frac{1}{2}d^T B^{(k)} d \tag{48}$$

$$s.t. \quad c_i\left(x^{(k)}\right)+\left(a_i^{(k)}\right)^T d=0,\ i=1,\ldots,m_e \tag{49}$$

$$c_i\left(x^{(k)}\right)+\left(a_i^{(k)}\right)^T d\geq 0,\ i=m_e+1,\ldots,m \tag{50}$$

where ε is a sufficiently small positive number and σ is a constant between 0 and 1.

$p^{(k)}, a_i^{(k)}$ in (46)-(50) are expressed as follows.

$$p^{(k)}=\nabla f\left(x^{(k)}\right)+\frac{1}{2}\sum_{i=1}^{m}u_i^{(k+1)}\left(\nabla c_i\left(x^{(k)}+d^{(k)}\right)-\nabla c_i\left(x^{(k)}\right)\right) \tag{51}$$

$$a_i^{(k)}=\frac{1}{2}\left(\nabla c_i\left(x^{(k)}+d^{(k)}\right)+\nabla c_i\left(x^{(k)}\right)\right),\quad i=1,\ldots,m \tag{52}$$

As mentioned above, the algorithm of the modified SQP method converges to the global minimum rapidly. The algorithm sequence is as follow:

1) Select an initial point and set the penalty parameter *r*.
2) Solve the quadratic problem expressed by (39)-(41), and calculate the $\left(d^{(k)},u^{(k+1)}\right)$.
3) Update the penalty parameter *r* as follow.

$$r=\max\left\{\left|u_i^{(k+1)}\right\| i=1,2,\cdots,m\right\}+\rho$$

$$\text{if } r<\max\left\{\left|u_i^{(k+1)}\right\| i=1,2,\cdots,m\right\} \tag{53}$$

4) If (47) is fulfilled, go to step (6).
5) Carry out the line search toward a direction $d^{(k)}$ and calculate the step width $t^{(k)}$ which satisfies (44). Set $x^{(k+1)}=x^{(k)}+t^{(k)}d^{(k)}$ and go to step (8).
6) Solve the quadratic problem expressed by (48)-(50).
7) Carry out the one-dimensional search on the curve defined by (54) and calculate the step width $t^{(k)}$ which fulfills (55).

$$x(t)=x^{(k)}+td^{(k)}+t^2\left(\hat{d}^{(k)}-d^{(k)}\right) \tag{54}$$

where $\hat{d}^{(k)}$ is optimal solution of quadratic problem expressed by (48)-(50).

$$F_r(x(t))<F_r\left(x^{(k)}\right)+\alpha t\left[\overline{F}_r\left(x^{(k)},d^{(k)}\right)-F_r\left(x^{(k)}\right)\right] \tag{55}$$

8) Update the matrix *B* and set *k*=*k*+1 and then, go back step (2).

By use of this algorithm, it becomes possible in a short time to determine the optimal operation point.

5 Applications to test systems

In the following section, firstly, the performances of the proposed method are compared with OPF without stability constraints and OPF in which stability constraints are incorporated directly. In addition, a numerical example is described.

5.1 Comparison of calculation performances

There are various techniques for the solution of nonlinear optimization problems such as sequential quadratic programming, quasi-Newton method, and interior point method. According to the applied method, there may be large differences regarding the calculation amount. Additionally, irrespective of applied solution method, it can say that if the number of constraints increases, the amount of calculations will increase. So, in this section, the amount of calculations is investigated by comparing number of equality constraints and inequality constraints. In this session, the proposed method is expressed as PM, the conventional method (discretization method) and the OPF without stability constraints as CM and OPF respectively.

The number of equality and inequality constrains in a system, which has N_g -generators, N_b -buses and N_l -lines are shown in Table 1. Here, N_c -contingencies are considered and simulation time is *t*-sec. In the conventional method, integration step-width is 0.01sec. N_t is the number of integration time intervals. In Table 2, the numbers of equality and inequality equations indispensable to solve the problem are compared for simulations of 2 and 5 seconds. The assumed power system models are the IEEJ East 10 and 30. 3 and 5 contingencies are considered.

Table 1 The number of constraints

PM	equality	$2N_b$
	inequality	$N_c*N_g+N_b+2N_g+N_l$
CM	equality	$2N_b+2*N_c*N_g*N_t+2N_g$
	inequality	$N_c*N_g*N_t+N_b+2N_g+N_l$
OPF	equality	$2N_b$
	inequality	$N_b+2N_g+N_l$

Table 2 The number of constraints on EAST10,30 model system

			EAST 10		EAST 30	
			t=2	t=5	t=2	t=5
PM	3-c	equality	94	94	216	216
		inequality	136	136	326	326
	5-c	equality	94	94	216	216
		inequality	156	156	386	386
CM	3-c	equality	12114	30114	36276	90276
		inequality	6106	15106	18236	45236
	5-c	equality	20114	50114	60276	150276
		inequality	10106	25106	30236	75236
OPF	-	Equality	94	94	216	216
		inequality	106	106	236	236

*c:contingency

From Table 1,2, we can found that the number of equality and inequality constraints in the proposed method is about $1/2N_t$, $1/N_t$ times respectively compared with the conventional method. Moreover, by comparing the OPF without stability constraints, we can find that there is no increase in constraints except for the increase of inequality constraints by only $N_c * N_g$. Also, Table 2 shows that the proposed method does not bring the increase in the number of constraints by the increase of simulation time. This is the remarkable advantage compared with the conventional method.

5.2 Applications to test systems

In this section, in order to illustrate the potentiality of the proposed algorithm, we have applied a prototype program to IEEJ EAST10 system [12]. Fig. 1 shows the one-line diagram of the IEEJ EAST10 test system, where the node and line numbers are indicated with ( ) and < >, respectively. The IEEJ EAST10 test system, which has 10 generators, 47 buses, 53 lines, 22 transformers and no capacitor/shunt element, is a simplified model of the 50Hz power system in Japan. It has 500 kV AC loops and 500/275 kV different voltage level AC loops. In this simulation, the load at each node has been decreased to 70%, since the original loading lead almost full output for all generators. It is assumed that all the transmission lines are operated with double circuit before the fault. The system data including impedance map, generator data and loading conditions can be obtained from the web site [12].

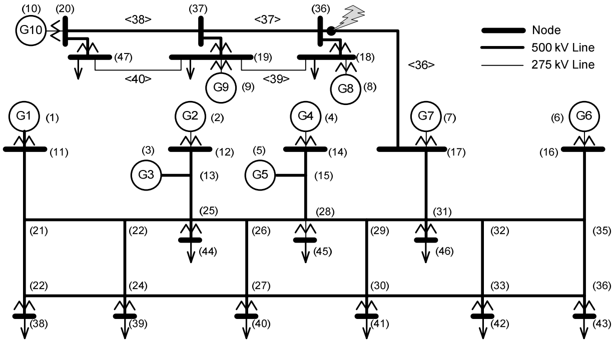

Fig.1 IEEJ EAST10 Test System

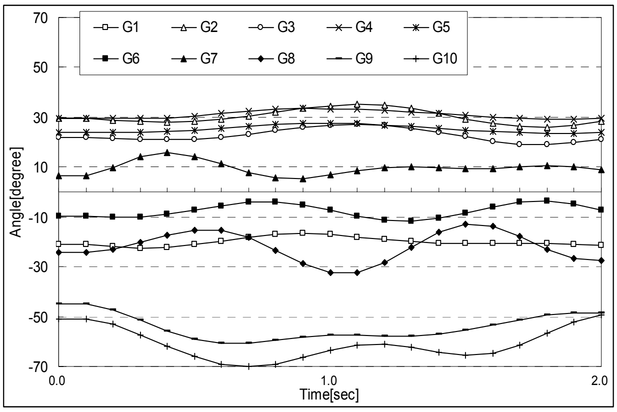

Fig.2 Rotor angle for each generators

Table 3 Generators' outputs at 70 degree limit

	P_G	P_{min}	P_{max}
G1	14,000	14,000	70,000
G2	110,500	109,500	110,500
G3	59,996	15,000	60,000
G4	109,986	33,000	110,000
G5	59,991	15,000	60,000
G6	51,253	33,000	110,000
G7	110,500	109,500	110,500
G8	26,545	17,500	70,000
G9	17,500	17,500	70,000
G10	10,000	10,000	50,000

In the conducted simulation, we maximized the real power transfer from areas, which have cheaper generators (G8~G10) to those areas where high price generator are used through the transfer path <36>. Fig. 2 shows the TTC with the limit of rotor angle with respect to COI at 70 degree. In this case, we considered a 3LG-O fault at the end of line <36> near node (36). The fault occurred at 0.1 sec and cleared at 0.15 sec. After the clearing of the fault, the line is operated as single line.

In Fig. 2, it can be seen that G10 reaches the rotor angle limit at 0.8 sec. The total transfer power from node (36) to node (17) is 3142 MW. Table 3 shows the generators' outputs for this simulation result. The real power output of G9 and G10 are close to their lower limit, while G8 operated above the lower limit to maximize the real power transfer.

6 Conclusion

In this paper, the infinite-dimensions OTS was transformed into a finite-dimensions optimization problem in Euclidean space. By comparison of computational performances among the proposed method, conventional method, and optimal power flow without stability constraints, we found that the number of stability constraints can be reduced remarkably, and transformed OTS problem is tractable even for the large-scale power systems with a large number of transient stability constraints. In addition, we apply the modified SQP approach to the optimization of OTS. This approach not only has the convergence rate as fast as q-super linear but also does not require exact evaluations of the Hessian matrices; thereby significant reduction of the computation time can be attained. The application of the modified SQP permits the OTS to be tractable even for large-scale power systems with a large number of transient stability constraints. The approach proposed here, can be used not only for the solution of OPF with practical stability constraints but also for the evaluations of electricity or ancillary prices, voltage dip problems, available transfer capability (ATC) [13] and other important task of the power system operation.

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