Effect of transition stresses in a disc having variable thickness and Poisson's ratio subjected to internal pressure

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Abstract: - Elastic-plastic transitional stresses in an annular disc having variable thickness and variable passion's ratio subjected to internal pressure has been derived by using Seth's transition theory. It is seen that thickness and passion's ratio variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully plastic state. It is seem for fully plastic state that circumferential stresses is maximum at the outer surface.

Key-Words: - disc, thickness, pressure, stresses, transitional, elastic, plastic.

1 Introduction

Elastic – plastic circular disks under the action of internal pressures have been investigated by several workers. It is well known that disc with variable thickness are frequently found in mechanical engineering. A literature survey indicates that several workers have analyzed circular discs with constant material properties under various conditions. Durban [1] found an exact solution for the internally pressurized elastic - plastic, strain hardening, annular plate and Chaudhuri [2] obtained stresses in a non-homogeneous rotating annulus by varying Poisson's ratio of the material while keeping Young's modulus constant. The problem of a rotating disc with varying thickness and in homogeneity subjected to steady inhomogeneous temperature field has been solved by Yeh and Han, and Tutuncu [3] investigated effect of anisotropy on stress in rotating disc. . Güven [4,5] studied the plane state of stress in elastic-plastic annular discs with variable thickness subjected to external and internal pressures, assuming Tresca's yield condition, its associated flow rule and strain hardening. In analyzing the problem, these authors used some simplifying assumptions. First, the deformation is small enough to make infinitesimal strain theory applicable. Second, simplifications were made regarding the constitutive equations of the material like incompressibility of the material and an yield criterion. Incompressibility of the material is one of the most important assumptions which simplifies the problem. In fact, in most of the cases, it is not possible to find a solution in closed from without this assumption. Seth's transition does not require these assumptions and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. Seth's transition theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deforming field and has been successfully applied to a large number of the problems .

Seth [8] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \quad (i = 1,2,3) \qquad (1)$$

where 'n' is the measure and $\overset{A}{e}_{ii}$ is the Almansi finite strain components[9]. For n = -2, -1, 0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures respectively. In this paper, we investigate the problem of elastic-plastic transition in a disc having variable thickness and passion's ratio subjected to internal pressure. Non-homogeneity in the disc is taken due to variation of Poisson's ratio of the material. The thickness 'h' and Poisson's ratio 'ν' are assumed to vary in the radial direction as:

$$h = h_0 (r/b)^{-k}, \nu = \nu_0 (r/b)^m, \tag{2}$$

where h_0, ν_0, k and m are real constants and $0 < \nu \le 1/2$.

2 Governing Equation

We consider a thin disc of non-homogeneous material having variable thickness with internal radius a and external radius b subjected to internal pressure p as shown in figure 1. The disc is thin and it is effectively in a state of plane stress that is, the axial stress T_{zz} is zero. The displacement components in cylindrical polar co- ordinate are given by [6]:

$$u = r(1-\beta), v = 0, w = dz, \tag{3}$$

where β is function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.v_0

The finite strain components are given by [7]:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1 - (r\beta' + \beta)^2\right], \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1 - \beta^2\right],$$

$$\overset{A}{e}_{zz} = \frac{1}{2}\left[1 - (1-d)^2\right], \overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0 \tag{4}$$

where $\beta' = d\beta/dr$ and meaning of superscripts "A" is Almansi.

Using equation (4) in equation (1), the generalized components of strain are:

$$e_{rr} = \frac{1}{n}\left[1 - (r\beta' + \beta)^n\right], e_{\theta\theta} = \frac{1}{n}\left[1 - \beta^n\right],$$

$$e_{zz} = \frac{1}{n}\left[1 - (1-d)^n\right], e_{r\theta} = e_{\theta z} = e_{zr} = 0, \tag{5}$$

where $\beta' = d\beta/dr$.

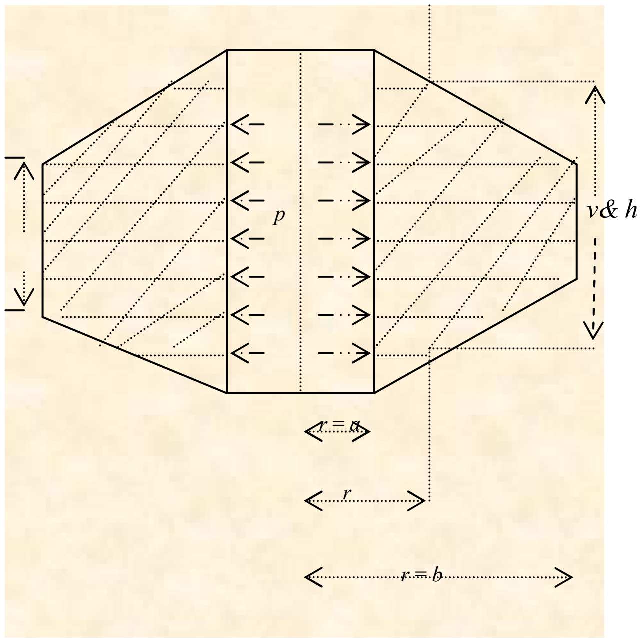

Figure 1. Geometry of Disc Having variable thickness and Passion's ratios.

The stress –strain relations for isotropic media is given by [21]:

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij}, (i,j = 1,2,3) \tag{6}$$

where $I_1 = e_{kk}$ (k =1,2,3).

Equation (6) for this problem becomes:

$$T_{rr} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{rr},$$

$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda + 2\mu}\left[e_{rr} + e_{\theta\theta}\right] + 2\mu e_{\theta\theta},$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0. \tag{7}$$

Substituting equation (5) in equation (7), the stresses are obtained as:

$$T_{rr} = \frac{2\mu}{n}\left[3 - 2c - \beta^n\left\{1 - c + (2-c)(P+1)^n\right\}\right],$$

$$T_{\theta\theta} = \frac{2\mu}{n}\left[3 - 2c - \beta^n\left\{2 - c + (1-c)(P+1)^n\right\}\right],$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0, \tag{8}$$

where $r\beta' = \beta P$ and $c = 2\mu/(\lambda + 2\mu)$.

The equations of equilibrium are all satisfied except:

$$\frac{d}{dr}\left(rT_{rr}h\right)-hT_{\theta\theta}=0. \tag{9}$$

Using equation (8) in equation (9), one get a non- linear differential equation in β as:

$$(2-c)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta}=\left[\begin{array}{l} r\left(\frac{\mu'}{\mu}+\frac{h'}{h}\right) \\ \left[\begin{array}{l} 3-2c \\ -\beta^{n}\left\{\begin{array}{l}1-c+\\(2-c)(P+1)^{n}\end{array}\right\} \\ +\beta^{n}\left[1-(P+1)^{n}\right]+ \\ r\beta^{n}c'\left[1+(P+1)^{n}\right]- \\ nP\beta^{n}\left[1-c+(2-c)(P+1)^{n}\right] \\ -2rc' \end{array}\right] \end{array}\right] \tag{10}$$

where $r\beta'=\beta P$ (P is function of β and β is function of r), $h'=dh/dr, \mu'=d\mu/dr$ and $c'=dc/dr$.

Transition points of β in equation (10) are $P=-1$ and $P=\pm\infty$.The boundary condition are:

$$T_{rr}=-p \text{ at } r=a, \; T_{rr}=0 \text{ at } r=b. \tag{11}$$

2.1 Solution Through The Principal Stress

It has been shown [6-20] that the asymptotic solution through the principal stress lead from elastic state to plastic state at the transition point $P\to\pm\infty$. We define the transition function *'R'* as

$$R=T_{\theta\theta}=\left(\frac{2\mu}{n}\right)\left[(3-2c)-\beta^{n}\left\{\begin{array}{l}2-c+\\(1-c)(P+1)^{n}\end{array}\right\}\right]. \tag{12}$$

Taking the logarithmic differentiation of equation (12) with respect to r and using equation (10), we get

$$\frac{d(\log R)}{dr}=\frac{\mu'}{\mu}-\frac{1}{\left[\begin{array}{l}3-2C-\\ r\left[\beta^{n}\left\{\begin{array}{l}2-C\\+(1-C)\\(P+1)^{n}\end{array}\right\}\right]\end{array}\right]}\left[\begin{array}{l}\frac{2rc'}{(2-c)}-\frac{\beta^{n}rc'}{(2-c)}\left\{\begin{array}{l}1+\\(P+1)^{n}\end{array}\right\}\\ +\frac{n\beta^{n}P}{(2-c)}(3-2c)\\ +r\left(\frac{1-c}{2-c}\right)\left(\frac{\mu'}{\mu}+\frac{h'}{h}\right)\left[\begin{array}{l}3-2c-\beta^{n}\left\{\begin{array}{l}1-c\\+(2-c)\\(P+1)^{n}\end{array}\right\}\end{array}\right]\\ +\beta^{n}\left(\frac{1-c}{2-c}\right)\left\{1-\{P+1\}^{n}\right\}\end{array}\right] \tag{13}$$

Taking the asymptotic value $P\to\pm\infty$ in equation (13), we get:

$$\frac{d(\log R)}{dr}=\left(\frac{\nu-1}{r}\right)-\frac{c'}{(1-c)}+\frac{c'}{(2-c)}-\frac{h'}{h} \tag{14}$$

Integrating (14) both side with respect to r, one get:

$$R=\frac{A_1\nu}{rh}\exp\left[\frac{\nu_0}{m}\left(\frac{r}{b}\right)^{m}\right] \tag{15}$$

where A_1 is a constant of integration.
From equation (12) and (15), we have

$$R=\frac{A_1\nu}{rh}\exp\left[\frac{\nu_0}{m}\left(\frac{r}{b}\right)^{m}\right]=T_{\theta\theta} \tag{16}$$

where A_1 is a constant of integration.
Substituting equation (16) in equation (9), we get after integration:

$$rhT_{rr}=A_2+A_1\exp\left[\frac{\nu_0}{m}\left(\frac{r}{b}\right)^{m}\right] \tag{17}$$

Substituting equation (11) in equation (17), we get:

$$A_1=\frac{pah(a)}{\left[e^{\nu_o/m}-e^{\left[\frac{\nu_o}{m}\left(\frac{a}{b}\right)^{m}\right]}\right]}$$

and $A_2 = \dfrac{pah(a)e^{\nu_o/m}}{\left[e^{\left[\frac{\nu_o}{m}\left(\frac{a}{b}\right)^m\right]} - e^{\nu_o/m}\right]}$,

where A_1 and A_2 is a constants of integration. Substituting the value of A_1 and A_2, using (21) from equations (16) and (17), we get transitional stresses

$$T_{rr} = \frac{pR_0^{1-k}R^{k-1}\left\{\begin{array}{l}\exp(\nu_0 R^m/m)\\ -\exp(\nu_0/m)\end{array}\right\}}{\left[e^{\nu_0/m} - e^{\nu_0 R_o^m/m}\right]} \quad (18)$$

$$T_{\theta\theta} = \frac{pR_0^{1-k}R^{m+k-1}\nu_0\exp(\nu_0 R^m/m)}{\left[e^{\nu_0/m} - e^{\nu_0 R_o^m/m}\right]} \quad (19)$$

where $R = r/b$ and $R_0 = a/b$.

Initial Yielding

The maximum value of $T_{\theta\theta}$ occurs at radius $R = \left(\frac{1-m-k}{\nu_0}\right)^{\frac{1}{m}} = R_1$ (say) for $m < 0$. For example, if we take m = -1, -1.2, -1.5 and k =1.34, 1.44, 1.566 and $\nu_0 = 0.333$ yielding starts at the internal surface and for k = 1.67, 1.87, 2.17 yielding starts at the external surface .For yielding at $R = R_1$,equation (19) becomes:

$$\left|T_{\theta\theta}\right|_{R=R_1} = \left|\frac{p\nu_0 R_0^{1-k}\exp\left(\frac{1-m-k}{m}\right)}{\left(e^{\nu_o/m} - e^{\nu_o R_o^m/m}\right)}\left(\frac{1-m-k}{\nu_0}\right)^{\frac{m+k-1}{m}}\right| \equiv Y(say). \quad (20)$$

and the required pressure is

$$p_1 = \frac{p}{Y} = \left|\frac{\left[e^{\nu_o/m} - e^{\nu_o R_o^m/m}\right]}{\nu_0 R_0^{1-k}\left[(1-m-k)/\nu_0\right]^{\frac{m+k-1}{m}}\exp[(1-m-k)/m]}\right| \quad (21)$$

Using equation (21) in equations (18) and (19), one gets the transitional stresses as:

$$\sigma_r = \frac{T_{rr}}{Y} = \frac{p_1 R_0^{1-k}R^{k-1}\left[e^{\nu_o R^m/m} - e^{\nu_o/m}\right]}{\left\{e^{\nu_o/m} - e^{\nu_o R_o^m/m}\right\}}$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y} = \frac{p_1 R_0^{1-k}R^{m+k-1}\nu_0 e^{\left[\nu_o R^m/m\right]}}{\left\{e^{\nu_o/m} - e^{\nu_o R_o^m/m}\right\}} \quad (22)$$

Stresses for fully-plastic state are obtained from equation (18) and (19) by taking $\nu_0 \to 1/2$.

There are two plastic zones:

(1) Inner-plastic zone: $R_0 \le R \le R_1$

(2) Outer-plastic zone: $R_1 \le R \le 1$

For Inner-plastic zone, equation (19) become:

$$\left|T_{\theta\theta}\right|_{R=R_0} = \left|\frac{pR_0^m e^{R_o^m/2m}}{2\left[e^{1/2m} - e^{R_o^m/2m}\right]}\right| \equiv Y^*(say) \quad (23)$$

and the required pressure is

$$p_1^* = \frac{p}{Y^*} = \left\{\frac{2\left[e^{1/2m} - e^{R_o^m/2m}\right]}{R_0^m e^{R_o^m/2m}}\right\} \quad (24)$$

Using equation (24) in equations (18) and (3.8), one gets:

$$\sigma_r^* = \frac{T_{rr}}{Y^*} = \frac{p_1^* R_0^{1-k}R^{k-1}\left[e^{\left[\frac{R^m}{2m}\right]} - e^{\frac{1}{2m}}\right]}{\left(e^{1/2m} - e^{R_o^m/2m}\right)} \quad (25)$$

$$\sigma_\theta^* = \frac{T_{\theta\theta}}{Y^*} = \frac{p_1^* R_0^{1-k}R^{m+k-1}e^{\frac{R^m}{2m}}}{2\left(e^{1/2m} - e^{R_o^m/2m}\right)} \quad (26)$$

For Outer-plastic zone, equation (19) become:

$$\left|T_{\theta\theta}\right|_{R=1} = \left|\frac{pR_0^{1-k}e^{1/2m}}{2\left(e^{1/2m} - e^{R_o^m/2m}\right)}\right| \equiv Y^{**}(say) \quad (27)$$

and the required pressure is

$$p_1^{**} = \frac{p}{Y^{**}} = \left(\frac{2\left(e^{1/2m} - e^{R_o^m/2m}\right)}{R_0^{1-k}e^{1/2m}}\right) \quad (28)$$

Using equation (28) in equations (18) and (19) , one gets:

$$\sigma_r^{**} = \frac{T_{rr}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{k-1}\left[e^{\left[\frac{R^m}{2m}\right]} - e^{\frac{1}{2m}} \right]}{\left(e^{1/2m} - e^{R_o^m/2m}\right)},$$

$$\sigma_\theta^{**} = \frac{T_{rr}}{Y^{**}} = \frac{p_1^{**} R_0^{1-k} R^{m+k-1} e^{\frac{R^m}{2m}}}{2\left(e^{1/2m} - e^{R_o^m/2m}\right)}. \quad (29)$$

Particular Cases:

1) For a flat disc (k = 0) ,elastic-plastic transitional stresses (18) and (19) become

$$T_{rr} = \frac{pR_0\left(\exp\left[\frac{\nu_0}{m}R^m\right] - \exp\left[\frac{\nu_0}{m}\right]\right)}{R\left[e^{\nu_o/m} - e^{\left[\frac{\nu_o}{m}R_o^m\right]}\right]} \quad (30)$$

$$T_{\theta\theta} = \frac{pR_0 R^{m-1}\nu_0 \exp\left[\frac{\nu_0}{m}R^m\right]}{\left[e^{\frac{\nu_o}{m}} - e^{\left[\frac{\nu_o}{m}R_o^m\right]}\right]} \quad (31)$$

It has been seen that $T_{\theta\theta}$ is maximum at radius $R = \left(\frac{1-m}{\nu_0}\right)^{\frac{1}{m}} = R_2$ (say) for $m < 0$ and $\nu_0 = 0.333$,therefore yielding starts at $R = R_2$ equation (31) becomes:

$$\left|T_{\theta\theta}\right|_{R=R_2} = \left| \frac{p\nu_0 R_0 \exp\left(\frac{1-m}{m}\right)}{\left[e^{\frac{\nu_o}{m}} - e^{\left[\frac{\nu_o}{m}R_o^m\right]}\right]} \left(\frac{1-m}{\nu_0}\right)^{\frac{m-1}{m}} \right| \equiv Y_1(\text{say})$$

. (32)

and the required pressure is

$$p_2 = \frac{p}{Y_1} = \left| \frac{\left[e^{\frac{\nu_o}{m}} - e^{\left[\frac{\nu_o}{m}R_o^m\right]}\right]}{\nu_0 R_0^{1-k}\left(\frac{1-m}{m}\right)^{\frac{m-1}{m}} \exp\left(\frac{1-m}{\nu_0}\right)} \right| \equiv Y_1(say)$$

. (33)

Using equation (33) in equations (30) and (31), one get the transitional stresses as:

$$\sigma_r = \frac{T_{rr}}{Y_1} = \frac{p_2 R_0\left[e^{\left[\frac{\nu_o}{m}R^m\right]} - e^{\frac{\nu_o}{m}}\right]}{R\left[e^{\frac{\nu_o}{m}} - e^{\left[\frac{\nu_o}{m}R_o^m\right]}\right]} \quad (34)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y_1} = \frac{p_2 R_0 R^{m-1}\nu_0 e^{\left[\frac{\nu_o}{m}R^m\right]}}{\left[e^{\nu_0/m} - e^{\nu_0 R_0^m/m}\right]} \quad (35)$$

Stresses for fully-plastic state are obtained from equations (30) and (31) by taking ($\nu_0 \to 1/2$).
There are two plastic zones:

(1) Inner-plastic zone: $R_0 \le R \le R_1$

(2) Outer-plastic zone: $R_1 \le R \le 1$.

For Inner-plastic zone, equation (31) become:

$$\left|T_{\theta\theta}\right|_{R=R_0} = \left| \frac{pR_0^m e^{R_0^m/2m}}{2\left[e^{1/2m} - e^{R_0^m/2m}\right]} \right| \equiv Y_1^*(\text{say}) \quad (36)$$

and the required pressure is

$$p_2^* = \frac{p}{Y_1^*} = \left\{ \frac{2e\left[e^{1/2m} - e^{R_0^m/2m}\right]}{R_0^m e^{\left[\frac{R_o^m}{2m}\right]}} \right\} \quad (37)$$

Using equation (37) in equations (30) and (31), one gets:

$$\sigma_r^* = \frac{T_{rr}}{Y_1^*} = \frac{p_3^* R_0\left[e^{R^m/2m} - e^{1/2m}\right]}{R\left[e^{1/2m} - e^{R_0^m/2m}\right]} \quad (38)$$

$$\sigma_\theta^* = \frac{T_{\theta\theta}}{Y_1^*} = \frac{p_3^* R_0^{1-k} R^{m-1} e^{\frac{R^m}{2m}}}{2\left[e^{1/2m} - e^{R_0^m/2m}\right]} \quad (39)$$

For Outer-plastic zone, equation (31) become:

$$\left|T_{\theta\theta}\right|_{R=1} = \left| \frac{pR_0 e^{1/2m}}{2\left[e^{1/2m} - e^{R_0^m/2m}\right]} \right| \equiv Y_1^{**}(\text{say}) \quad (40)$$

and the required pressure is:

$$p_2^{**} = \frac{p}{Y_1^{**}} = \left(\frac{2\left[e^{1/2m} - e^{R_0^m/2m}\right]}{R_0 e^{1/2m}} \right) \quad (41)$$

Using equation (41) in equations (30) and (31), one gets:

$$\sigma_r^{**} = \frac{T_{rr}}{Y_1^{**}} = \frac{p_3^{**} R_0 \left(e^{R^m/2m} - e^{1/2m} \right)}{R\left(e^{1/2m} - e^{R_0^m/2m} \right)}$$

$$\sigma_\theta^{**} = \frac{T_{rr}}{Y_1^{**}} = \frac{p_1^{**} R_0 R^{m-1} e^{\frac{R^m}{2m}}}{2\left(e^{1/2m} - e^{R_0^m/2m} \right)} \quad (42)$$

2) For (m = 0) ,equation (3.1) taking logarithmic differentiation and taking $P \to \pm\infty$, one get:

$$\frac{d(\log R)}{dr} = -\frac{1}{r(2-c)} - \frac{h'}{h}. \quad (43)$$

Integrating equation (13)with respect to r, one get:

$$R_2 = \frac{A_3 r^{-1/(2-c)}}{h} = T_{\theta\theta} \quad (44)$$

where A_3 is a constant of integration.

Substituting equation (44) from equation (9), we get after integration

$$rT_{rr}h = \frac{A_3 r^{\frac{1-c}{2-c}}(2-c)}{(1-c)} + A_4 \quad (45)$$

where A_4 is a constant of integration.

Using boundary condition (2.9) in equation (4.16), one gets:

$$A_3 = \frac{aph(a)(1-c)}{(2-c)\left[b^{\frac{1-c}{2-c}} - a^{\frac{1-c}{2-c}} \right]}$$

and $$A_4 = \frac{-aph(a) b^{\frac{1-c}{2-c}}}{\left[b^{\frac{1-c}{2-c}} - a^{\frac{1-c}{2-c}} \right]}.$$

Substituting A_3 and A_4 in equations (44) and (27) and using (2), one gets:

$$T_{rr} = \frac{pR_0^{1-k}\left(R^{\nu_0} - 1\right)R^{k-1}}{\left(1 - R_0^{\nu_0}\right)} \quad (46)$$

$$T_{\theta\theta} = \frac{p\nu_0 R_0^{1-k} R^{\nu_0+k-1}}{\left(1 - R_0^{\nu_0}\right)} \quad (47)$$

It has been seen that $T_{\theta\theta}$ has maximum value at the internal surface $R = R_0$ (say) for k < 1, therefore yielding start at inner surface, then equation (47) becomes:

$$\left|T_{\theta\theta}\right|_{R=R_0} = \left| \frac{\nu_0 p R_0^{\nu_0}}{1 - R_0^{\nu_0}} \right| = Y_2 \quad (48)$$

and required pressure for initial yielding is

$$p_3 = \frac{p}{Y_2} = \frac{1 - R_0^{\nu_0}}{\nu_0 R_0^{\nu_0}} \quad (49)$$

Stresses for fully-plastic state are obtained from equation (47) by taking ($\nu_0 \to 1/2$) and R = 1, one gets:

$$\left|T_{\theta\theta}\right|_{R=1} = \left| \frac{pR_0^{1-k}}{2\left(1 - \sqrt{R_0}\right)} \right| = Y_2^*. \quad (50)$$

and the required pressure for fully-plastic state is given by:

$$p_3^* = \frac{p}{Y_2^*} = \frac{2\left(1 - \sqrt{R_0}\right)}{R_0^{1-k}}. \quad (51)$$

Substituting equation (51) in equations (46) and (47), one get stresses:

$$\sigma_r = \frac{T_{rr}}{Y_2^*} = \frac{p_3 R_0^{1-k}\left(R^{\nu_0} - 1\right)R^{k-1}}{\left(1 - R_0^{\nu_0}\right)} = \frac{\left(R^{\nu_0} - 1\right)R_0^{1-k-\nu_0} R^{k-1}}{\nu_0} \quad (52)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y_2^*} = \frac{\nu_0 p_3 R_0^{1-k} R^{k+\nu_0-1}}{\left(1 - R_0^{\nu_0}\right)} = \frac{R_0^{1-k-\nu_0} R^{k+\nu_0-1}}{R_0^{\nu_0}}. \quad (53)$$

For fully-plastic state ($\nu_0 \to 1/2$) using equation (51) in equations (52) and (53), one get:

$$\sigma_r = \frac{T_{rr}}{Y_1^*} = \frac{p_2^* R_0^{1-k}\left(\sqrt{R} - 1\right)R^{k-1}}{\left(1 - \sqrt{R_0}\right)} = 2\left(\sqrt{R} - 1\right)R^{k-1} \quad (54)$$

$$\sigma_\theta = \frac{T_{\theta\theta}}{Y_1} = \frac{p_2 R_0^{1-k} R^{k-\frac{1}{2}}}{2\left(1 - \sqrt{R_0}\right)} = R^{k-\frac{1}{2}} \quad (55)$$

It can be seen that equation (54) and (55) for homogeneous material are same for fully plastic state as given by and Pooja Kumari [20].

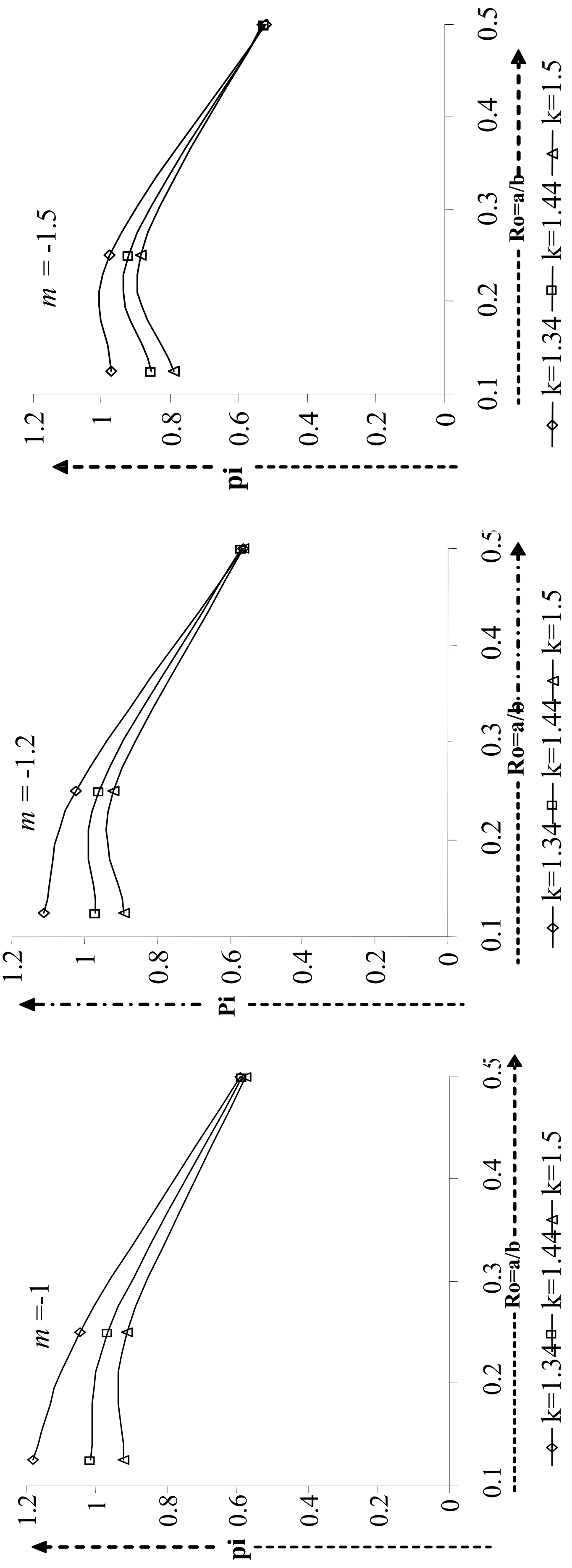

Figure 2 pressure required for initial yielding of non-homogeneous disc having variable thickness and radii ratio a/b

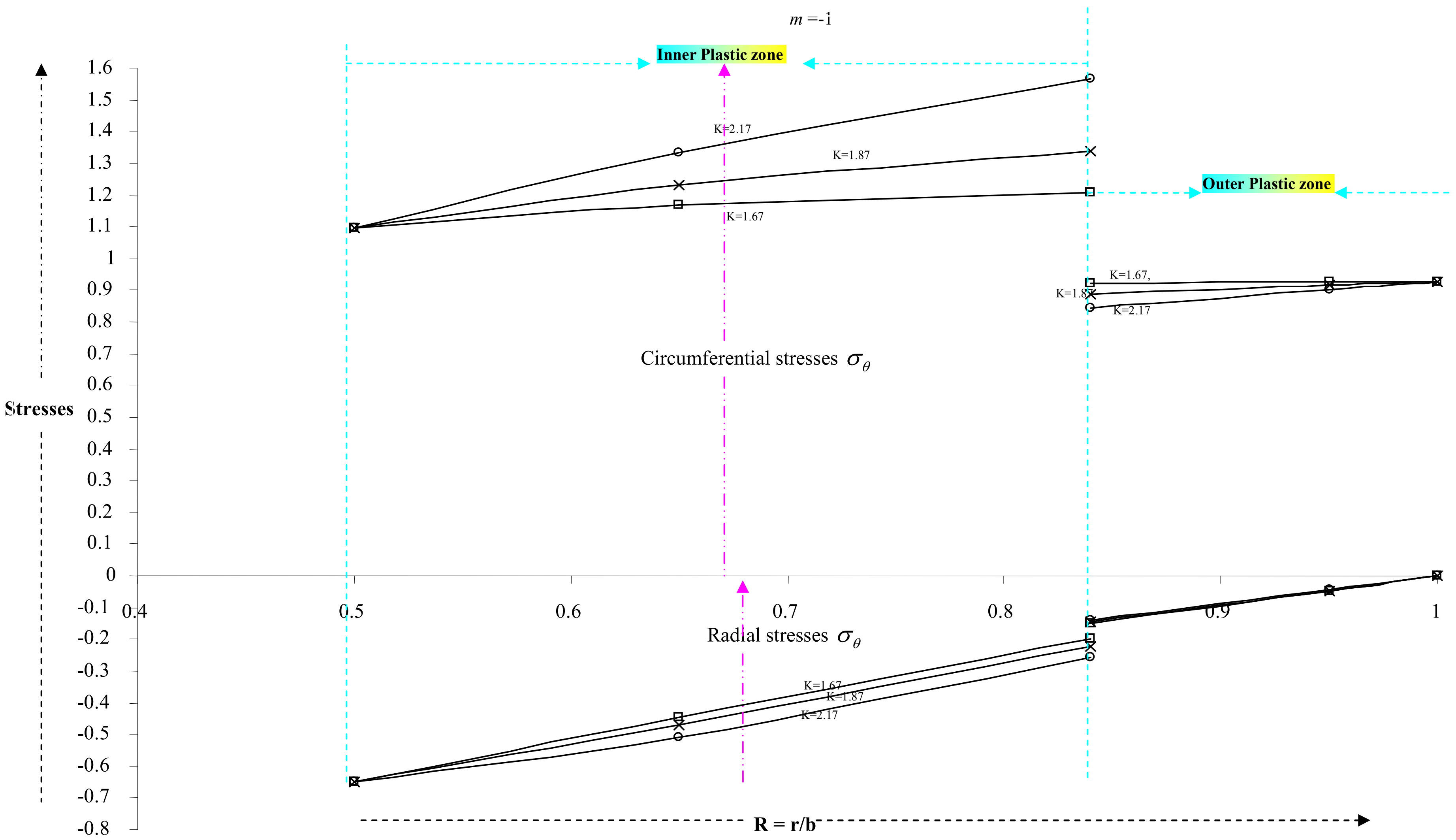

Figure 3 Stresses distribution in a non homogeneous disc with variable thickness at the transitional state for different values of k with respect to radii ratio R = r/b and m= -1.

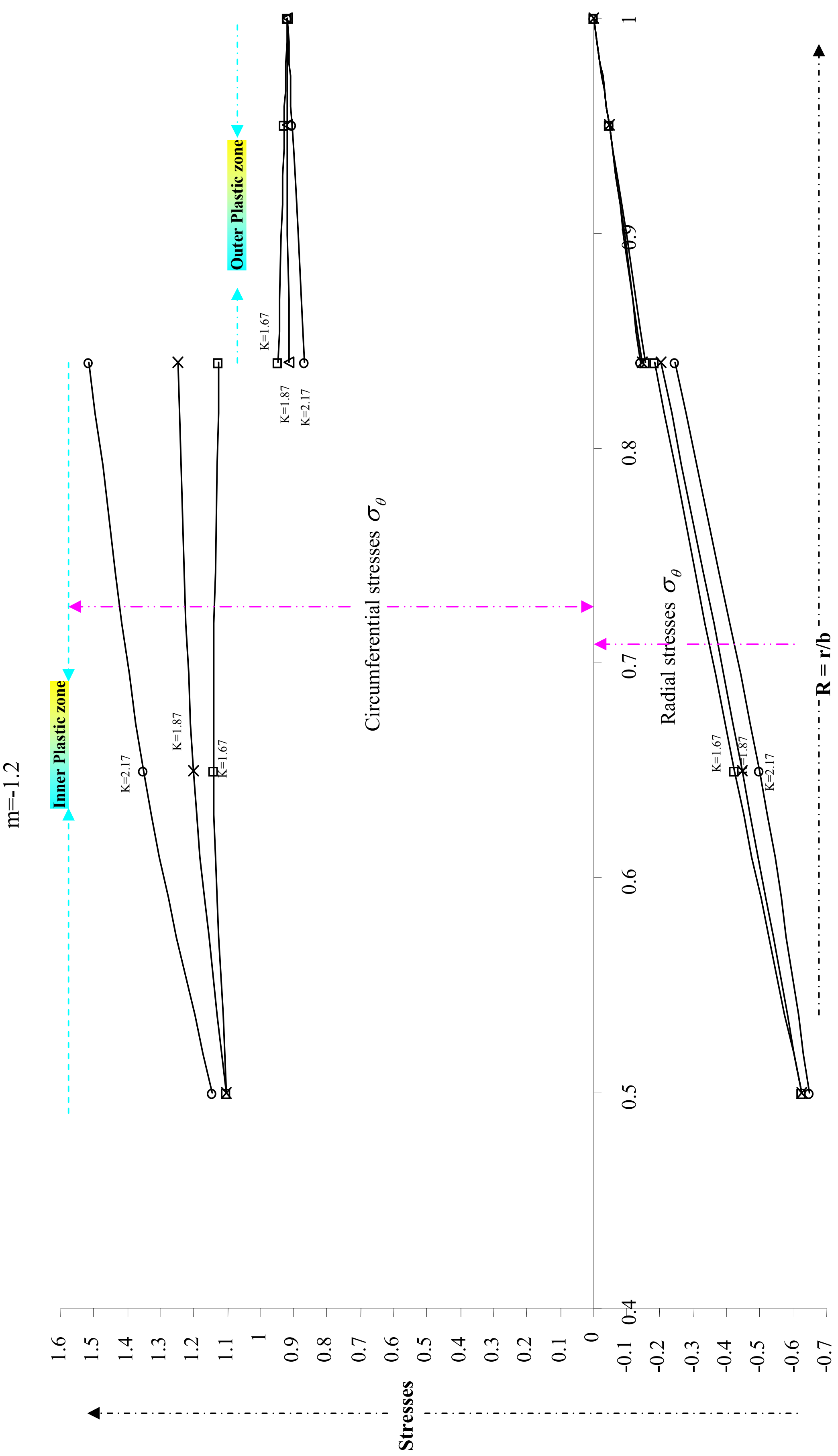

Figure 4 Stresses distribution in a non homogeneous disc with variable thickness at the transitional state for different values of k with respect to radii ratio R = r/b. and m= -1.2.

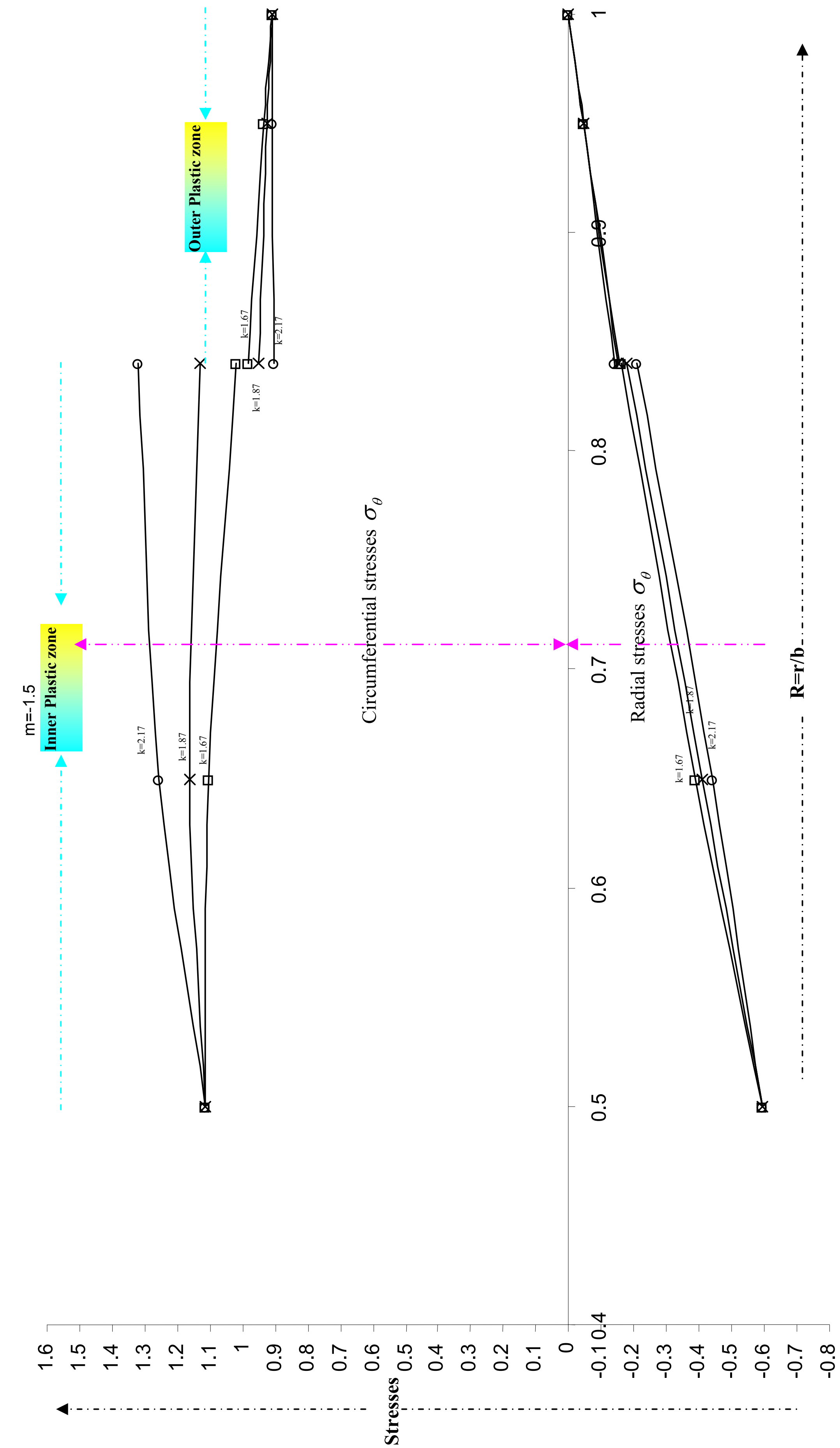

Figure 5 Stresses distribution in a non homogeneous disc with variable thickness at the transitional state for different values of k with respect to radii ratio R = r/b and m= -1.5.

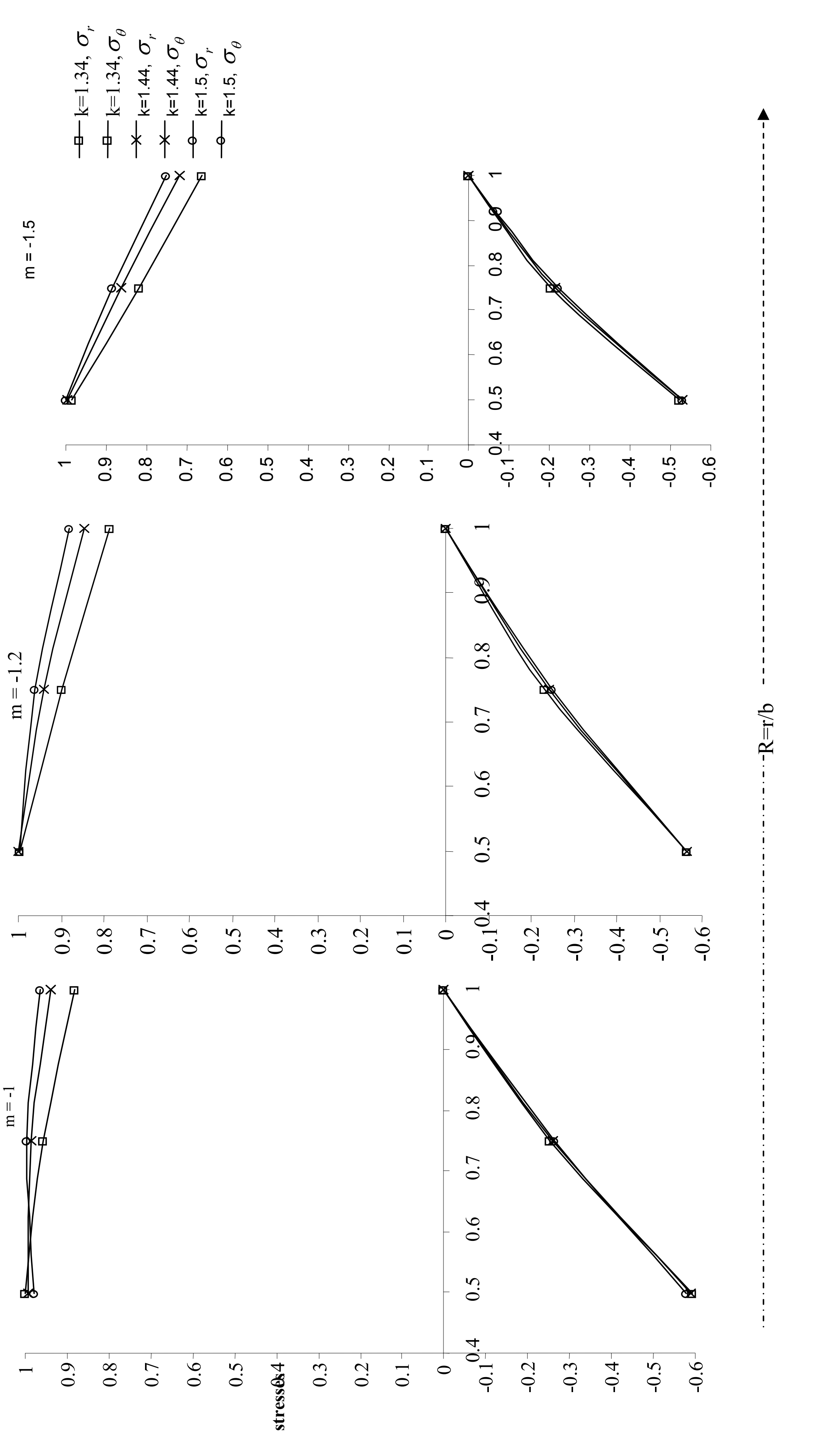

Figure 6 Stresses distribution in a non homogeneous disc with variable thickness for fully plastic state for different values of *k* with respect to radii ratio R = r/b.

Numerical Illustration and Discussion

To study the combined effects of non-homogeneous and thickness variation as given in equation (1.1) for disc with radii ratio $R_0 = 0.5$ and $\nu_0 = 0.333$ has been consider. As a numerical example, if we take $m = -1,-1.2$ we seen from that yielding of disc ($R_0 = a/b = 1/2$) at different pressure and different values of k has been given. It can be seen that yielding occurs at radius $R = R_1$ for m < 0 and depending upon different values of k. For $m = -1, -1.2$ and $k = 1.34, 1.441859$ and $\nu_0 = 0.333$ yielding starts at the internal surface of the disc at pressures 1.184873 P, 0.973439 P and for $k = 1.67, 1.87$ it occurs at the yielding starts at the outer surface at pressures 0.752243 P, 0.627435 P i.e., as the values of $k = 1.34$ to 1.87, yielding of disc shifted from internal surface towards the outer surface at a lesser pressure. It is also seen that the non-homogeneous disc having variable thickness requires high percentage increase in pressure to become fully plastic from initial yielding as compared to isotropic material (Brass).

In fig. 2, curves have been drawn between pressure required for initial yielding of non-homogeneous disc having variable thickness and radii ratio a/b. It is seen that, less pressure is required for yielding as the thickness ratio a/b of the disc increases. In Figs. 3, 4 and 5, curve have been drawn between stresses distribution in a non homogeneous disc with variable thickness at the transitional state for different values of k and m with respect to radii ratio R = r/b. It is seen that circumferential stress is maximum at the internal surface of the flat disc whereas it is maximum at the outer surface of the disc having variable thickness.

In fig.6, curve have been drawn Stresses distribution In a non homogeneous disc with variable thickness for fully plastic state for different values of k with respect to radii ratio R = r/b, It is seen that the circumferential stresses is maximum at the outer surface.

Conclusion

It is seen that thickness and passion’s ratio variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully plastic state. It is seem for fully plastic state that circumferential stresses is maximum at the outer surface.

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