# Hysteresis Identification methodology for the SAS rotational MR damper

Y. Iskandarani, G.A. Moslatt, E. Myklebust, J.P. Kolberg, L.L. Solberg, H.R. Karimi
Department of Engineering, Faculty of Engineering and Science
University of Agder, 4879 Grimstad, Norway
Hamid.r.karimi@uia.no

*Abstract:* vibrations is an extremely important issue to consider when designing various systems. It may lead to discomfort and malfunction or in some cases collapse of structures. To compensate for these vibrations different types of damping devices can be introduced and applied.
The main scope which the following work addresses has been to look at standard methodology which enables determining the hysteresis from predefined steps for range of measurements which will be applied later on specific system integrated with Magnetorheological damper.The mathematical equations that lie behind the Bingham, Dahl, Lugre and Bouc-Wen have been studied to describe the behavior of the MR damper.
The hysteresis equations of Bouc-Wen, Lugre, and Dahl have been modeled and simulated in Matlab/Simulink. The different parameters in the models have been manipulated and analyzed to determine the effects on the outcome. The hysteresis models of Bouc-Wen, Dahl and LuGre have been analyzed and compared analytically to show the difference in the models. At last the Bouc-Wen model was implemented together with the SAS(Semi Active Suspension) experimental system. The model parameters were tuned manually to fit the response of the system.
This paper introduces the methodology flowchart which can be implemented and generalized for any kind of dampers, in the following work, the methodology was used to find hysteric behavior of MR damper with different mathematical models.

*Key–Words:* *Bouc-Wen model, Dahl model, Hysteresis methodology, Lugre model, Magnetorheological damper,Semi-Active Suspension System*

## 1 Damper types

### 1.1 Passive dampers

Passive dampers are dampers with constant damper parameters. It does not depend on an external controller and requires no input power to operate . Since the passive damper is not controlled we are not able to adjust parameters dynamically which is a great disadvantage. This type of damper is the most common type. A typical passive rotary damper consists of a rotor surrounded by an outer housing. Viscous fluid is interposed between these two, causing a velocity dependent force between the rotating rotor and the fixed outer housing.[1]

### 1.2 Active dampers

Active damping involves an active controller that continuously regulate the damper properties. Active dampers only work when controlled. The active damper uses external power supply unlike for the passive dampers and the loss of power supply will disable the damping. This type of damping is commonly used in cars to improve handling and comfort[2] Loss of power could have serious consequences and, when used in buildings, earthquakes are a real threat to these systems.

### 1.3 Semi-active dampers

Semi-active dampers are a combination of features of passive and active dampers. Earlier semi-active damping was regulated by changing the orifice area, which changed the resistance. The MR damper is a type of semi-active damper where the flow of MR fluid is controlled by varying the amount of current supplied and thus change the level of damping. Like other semi-active control devices the MR damper is able to damp even if the current goes to zero[3]. The damper is capable of producing large control forces by changing parameters such as damping coefficient and stiffness trough changing the magnetic field and thereby control the response of large scaled structures[4]. The average use of energy in semi active dampers is considerably smaller than in active and unlike active dampers the semi-active does not shut down when the power is cut off it just behaves like the passive would. Semi-Active dampers also has the advantage of changing the natural frequency of the system thus avoiding resonance as can be found when the static and dynamic vibration analysis performed.

## 2 The Magnetorheological Damper

The MR damper looks like a normal damper, but has surrounding coils. These coils makes it possible to set up a magnetic field through the fluid. The magnetic field can be controlled by a active/semi-active or passive controlled current.
Passive means that the current applied is constant. Active controlled system is always ”active” and often demands more power than semi-active. Semi active can be both passive and active, it all depends on the situation. In situations like earthquakes, there exists power sources that activates because of the vibrations, and thereby generates energy to the controller and then starts the active controller.

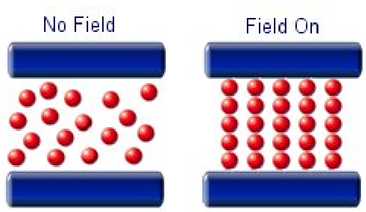


Figure 1: The Magnetorheological fluid[5]

In the MR damper there are magnetorheological fluids. These fluids can vary in viscosity. This is done by applying a magnetic field to the oil/fluid. The fluid contains iron particles(micron sized) that align with the magnetic field[6]. This alignment makes the oil stiffer and more rigid. The fluid respond very quickly and this alignment is done within 6.1ms[2]. Because of this quick response the damper is much used in active and semi-active controlled systems. The MR damper also demands a small amount of current and a voltage source of 2-2.25V[2].
By changing the current the MR damper change properties as a result of the changed fluid viscosity. This change is approximately linear with the control current. MR fluids have a wide operation temp, from -40 to 150 degrees Celsius and its not sensitive to additives and impurities.
The MR damper is seen as a safe damper, because of its action when power loss occur. With loss of power it reverts to a passive damper[2].

Figure 2: The MR damper components[5]

## 3 Hysteresis properties

The hysteresis phenomena in the MR damper can be modeled with different types of mathematical models. Each model describes different aspects of friction and/or dynamic properties of the MR damper. In this section we will present these friction aspects together with the models we later will simulate, compare and analyze.

Hysteresis is a dynamic friction phenomena which represents the history dependence of physical systems.[7] We get a model for the systems nonlinear behavior at low velocities. Figure 3 show an example of hysteresis. The force versus velocity curve is not coincide for increasing and decreasing velocities.

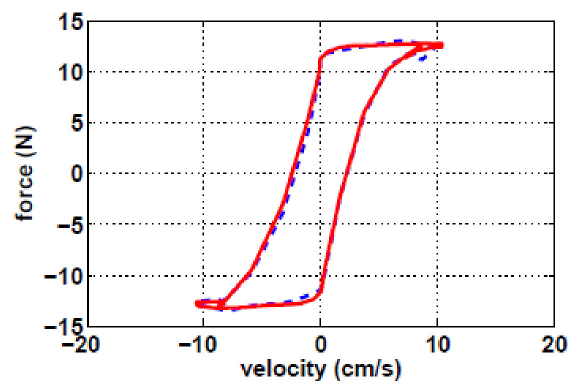


Figure 3: Example of hysteric behavior[5]

## 4 The MR damper hysteresis models

### 4.1 Dahl model

Using the Dahl model of the MR damper [8]

$$\begin{aligned} F_{mr} &= k\dot{x} + (k_{wa} + k_{wb}v)w, \\ \dot{w} &= \rho(\dot{x} - |\dot{x}|w) \end{aligned} \tag{1}$$

, we obtain the expression:

$$\begin{aligned} T_{mr} &= k\dot{\theta} + (k_{wa} + k_{wb}v)w, \\ \dot{w} &= \rho(\dot{\theta} - |\dot{\theta}|w) \end{aligned} \tag{2}$$

, with new parameter values. $T_{mr}$ is the exerted torque, $\theta$ is the angle, $v$ is the control voltage, w is a dynamic hysteresis coefficient, and $k$,$k_{wa}$, $k_{wb}$, and $\rho$ are parameters that control the shape of the hysteresis loop.

Dahl's first paper states: "The origin of friction is in quasi static bonds that are continuously formed and subsequently broken" [9]. This can be seen in hysteresis loops for torque vs velocity; when the velocity changes sign, the torque does not change instantly. This does not happen until a certain change in displacement allows these "bonds to be broken".

### 4.2 Dahl model: Parameters effect

These plots are used for further reference when analysing the parameters. Notice that the "knees" of the hysteresis are at points $(0, \pm 80)$ in the velocity graph. This value is decided by v, and $k_w$, which is now represented by $T(\dot{\theta}) = T_0 \approx k_{wa} + v \cdot k_{wb}$. Now, when increasing the value of k, with the result showing in figure 4. The knees are still in the area of $(0, \pm 80)$. This establish that the hysteresis does not change much, but the linear part of the graph has a greater slope. This results in higher torque with respect to velocity.

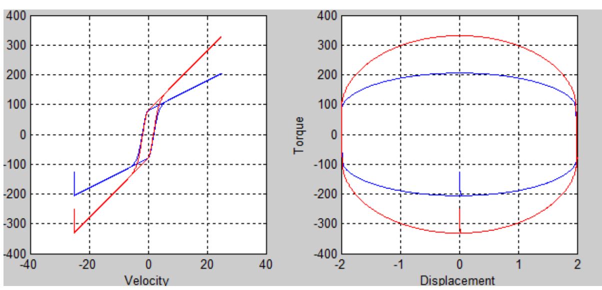


Figure 4: Plot of the Dahl model with k=5(blue) and 15(red), $k_{wa}$=80 = $k_{wb}$, $v$=0,$\rho$=15

From equation 2 it can be shown that increasing either voltage, v, or $k_w$ will have the same impact on the shape. An increase in either one will result in an increase of the torque in the hysteresis loop. This is shown in figure 5.

Inreasing $\rho$ will change the width of the hysteresis loop, giving a fast change in torque. This is illustrated in figure 6 where $\rho$ is increased from 2 to 15.

### 4.3 Lugre model

In *Modeling of MR damper with hysteresis for adaptive vibration control*[10] an MR damper model based on the earlier mentioned LuGre model is described. This model expresses the dynamic friction characteristics and the hysteresis effect. The equation looks like this:

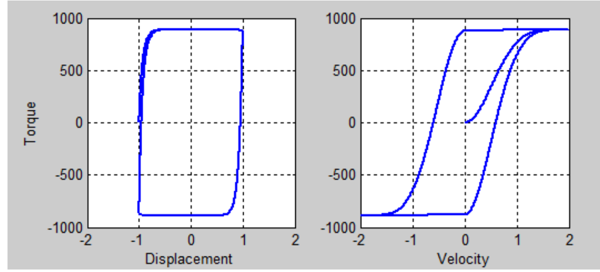


Figure 5: Plot of the Dahl model with k=5, $k_{wa}$=80 = $k_{wb}$, v=10,$\rho$=15

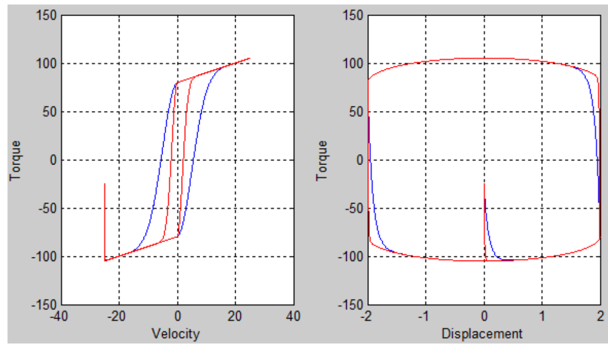


Figure 6: Plot of the Dahl model with k=5, $k_{wa}$=80 = $k_{wb}$, v=0,$\rho$=2(blue) and 15(red)

$$T = \sigma_a z + \sigma_0 z v + \sigma_1 \dot{z} + \sigma_2 \dot{x} + \sigma_b \dot{x} v \quad (3)$$
$$\dot{z} = \dot{x} - \sigma_0 a_0 |\dot{x}| z \quad (4)$$

$\sigma_0$: stiffness of z(t) influenced bu v(t), $(N/(m \cdot V))$
$\sigma_1$: damping coefficient of z(t),$(N \cdot s/m)$
$\sigma_2$: viscous damping coefficient,$(N \cdot s/m)$
$\sigma_a$: stiffness of z(t), $(N/m)$
$\sigma_b$: v(t) dependent viscous damping,$(N \cdot s/(m \cdot V))$
$a_0$: constant value, $(V/N)$

In simulink, the model looks like this:

A sine wave and its derivative is used as an input to the model:

There are outputs connected to the different terms of the equation. These values are plotted with respect to the speed. The model parameters used are from table I in [10] Using this model, we can find out what the different parts of the model adds to the result. This helps us to understand how the model works and how we can adjust the model. The plots is viewed starting with output 1 from the left

Out1 and Out5 and captures the nonlinear effect. Out1 is the active part of the damping, and Out5 is

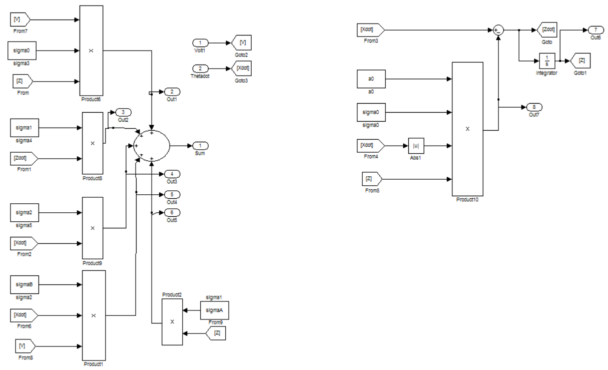

Figure 7: MR-damper subsystem

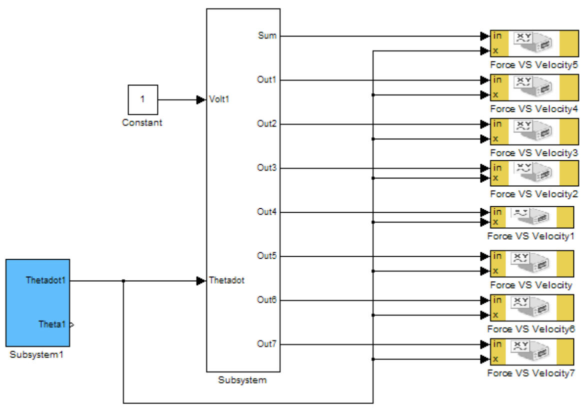


Figure 8: Lugre system

the passive part.
Out2 gives the stribeck effect (without the viscous friction), Out3 gives the passive linear damping and Out4 gives the active linear damping.
Out6 is the state variable Z and Out7 is the state variable Z multiplied with $\sigma_0$, $a_0$ and $|\dot{x}|$

Sigma0 affects both the gain of the state variable Z and the active nonlinear damping
Sigma1 affects the gain of the "stribeck"-part
Sigma2 affects the gain of the passive linear damping
SigmaA affects the gain of the passive nonlinear damping
SigmaB affects the gain of the active linear damping
a0 affects the gain of the state variable Z
The voltage affects the gain of all the active parts

The same procedure was done with respect to the displacement. Initialy, the parameters from [10] where used. But to clearly show the caracteristics of the different parts of the equation, these parameters where modified. The parameters used where:

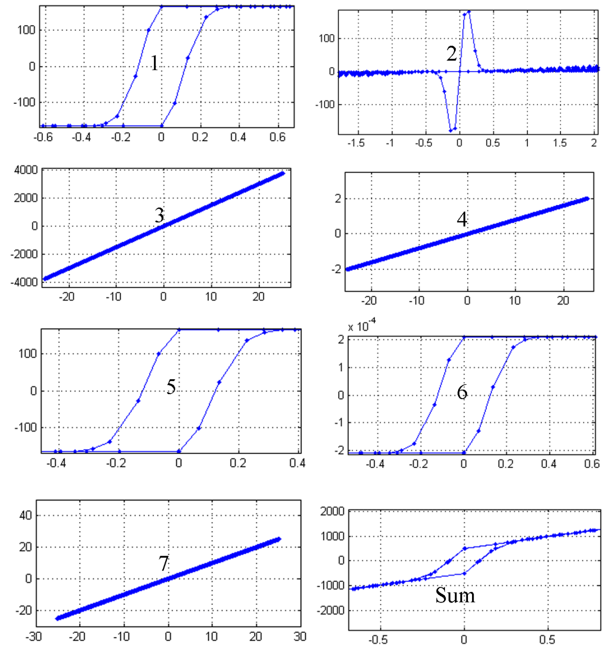


Figure 9: LuGre equation speed

$\sigma_0$: 800
$\sigma_1$: 1600
$\sigma_2$: 150
$\sigma_a$: 800000
$\sigma_b$: 0.0080
$a_0$: 0.0030

Here, in Out2, we can clearly see how the force is rapidly increasing and then decreasing as the direction changes.(Stribeck)

As earlier, the following pictures shows how changes in the parameters will affect the output force of the model. The blue graph is the initial value

## 4.4 Bouc-Wen model

The Bouc-Wen model is used to describe a hysteric effect. By applying the hysteric effect of Bouc-Wen, we can establish a good model of the MR damper. In this section we model the Bouc-Wen and test the effect of changing the implemented parameters.

To model the Bouc-Wen we used Matlab Simulink. To make sure that the model was correct, the model was set up in a system identically to the one in file "Characterization of a commercial magnetorheological brake/damper in oscillatory motion" [11]. By doing this we could use the same parameters and thereby confirm that the model was correct. The system used was a simple system with Bouc-Wen and a linear damper. In translational dampers the model

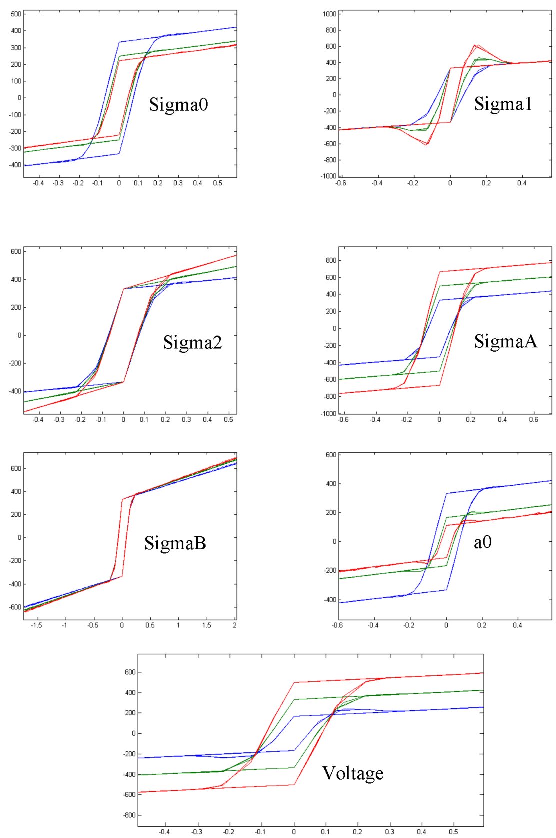


Figure 10: Parameter change speed

also contains a spring. In the rotational MR damper it can be neglected.

Formulas used to model the mr-damper[11]:

$$T = \alpha(i)z + c(i)\dot{\theta} \tag{5}$$

$$\dot{z} = -\gamma|\dot{\theta}|z|z|^n - \beta\dot{\theta}|z|^n + \delta\dot{\theta} \tag{6}$$

i: The current applied to the mr-damper
z: The hysteretic parameter from Bouc-Wen

The constants $\alpha$ and c are linear to the current [2]:

$$c(i) = c_1 + c_2 \cdot i \tag{7}$$

$$\alpha(i) = \alpha_1 + \alpha_2 \cdot i \tag{8}$$

The $\alpha_1$ and $c_1$ are constants for the passive damping. $\alpha_2$ and $c_2$ are parameters for the active damping.

To show the hysteresis and Bouc-Wen, the model was implemented in simulink.

The simulink model in fig13 corresponds to the equations 5,6,7 and 8. In fig26 compressed into the subsystem "Bouc-Wen MR damper".

## 4.5 Bouc-Wen model: The current effect

By changing the current from 0 to 1, a significant change in the two hysteresis appears. In this test a sinusoidal velocity profile i used.

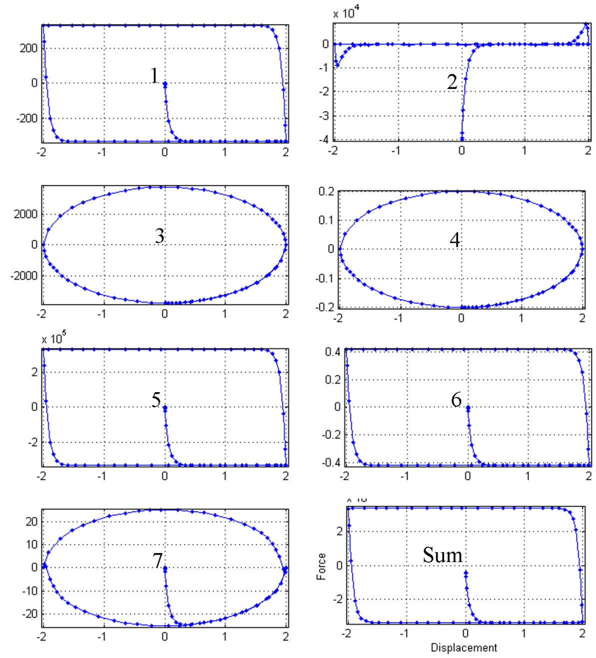


Figure 11: LuGre equation displacement

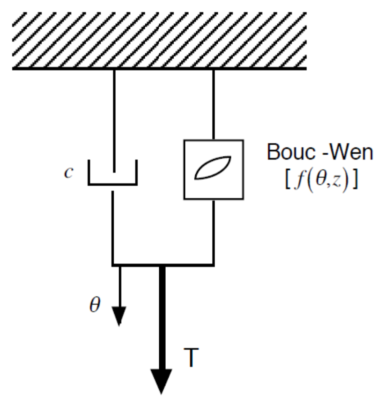


Figure 12: Bouc-Wen model of MR-damper[11]

In Figure 15 we see that the increased current increases the maximum torque. It also increases the slope of the linear damping outside the hysteresis. When looking at the torque vs displacement, we see that the shape of the curve is kept the same. Compared with the linear damper we see a noticeable difference(17).

From eq.8 we should have a linear dependency of the parameter $\alpha_1$ and the applied torque from the hysteresis effect.

## 4.6 Bouc-Wen model: Parameters effect

In Figure18 the torque is plotted for $\alpha$=[0,0.5,1.5,2]. We see that the hysteresis torque increases linearly. The dark blue line represents the $\alpha$=0, and from the

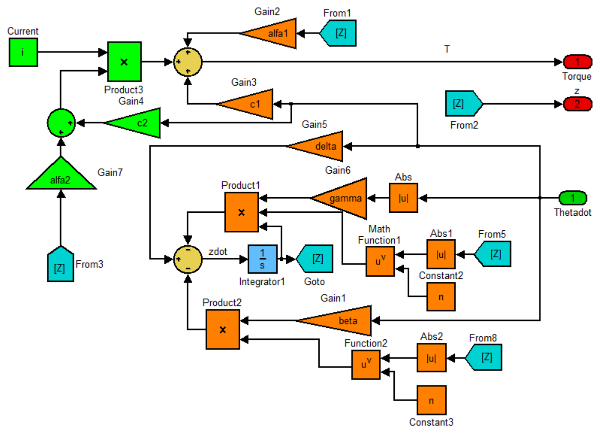


Figure 13: MR-damper model in simulink

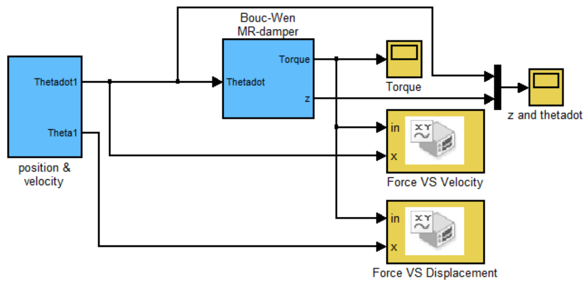


Figure 14: System design in simulink

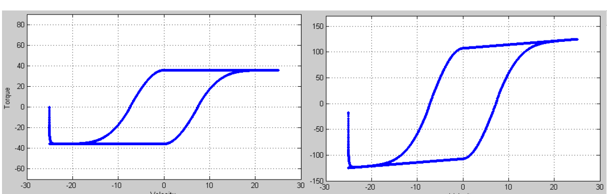


Figure 15: Torque vs Velocity. Hysteresis for 0 and 1 ampere.

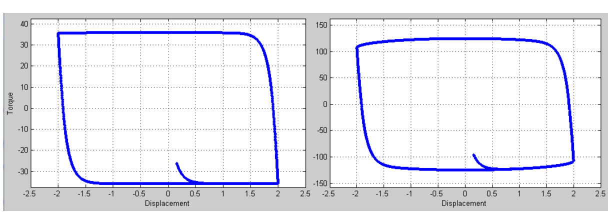


Figure 16: Torque vs displacement. Hysteresis for 0 and 1 ampere.

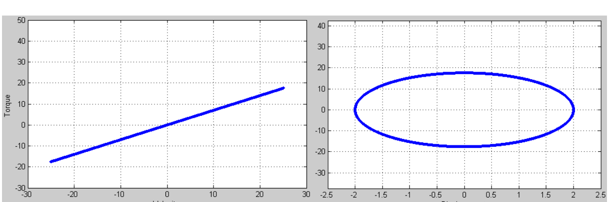


Figure 17: Linear damper

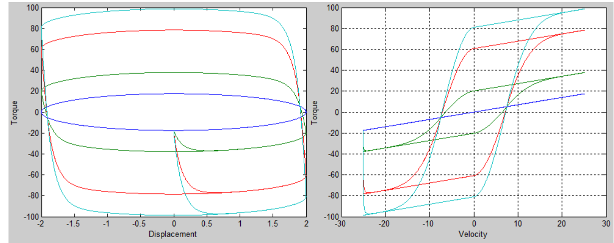


Figure 18: Change of the parameter alpha=[0,0.5,1.5,2]

eq.5 we easily see that this should be linear .

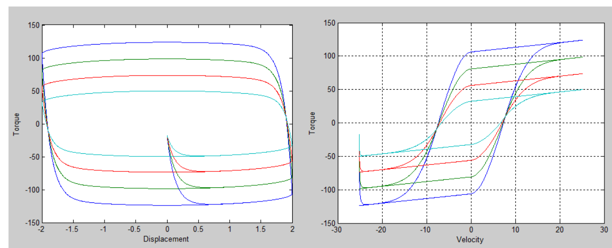


Figure 19: Change of the parameter beta=[740,760,780,800]

When we increase the beta the absorbed torque decreases(light blue: $\beta = 800$,dark blue: $\beta = 740$).

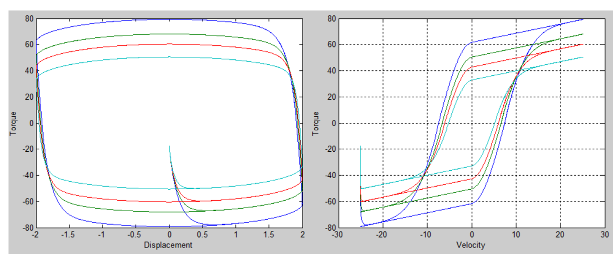


Figure 20: Change of the parameter gamma=[1,1.25,1.5,2]

The hysteresis get narrower with increase of the parameter gamma. The absorbed torque decreases. We also see a tendency of a steeper slope in the hysteresis area.

The increase in n shows a nonlinear development of the absorbed torque and the velocity hysteresis gets narrower and shorter.

The change of delta increases the hysteresis with increased delta.
The increase in the constants, c1 and c2 will give a steeper slope in the linear areas and give the same effect as the current gave in Figure (15). c1 controls how steep the curve should be when there is no current and c2 when current is applied.

The parameter n has the most significance on the

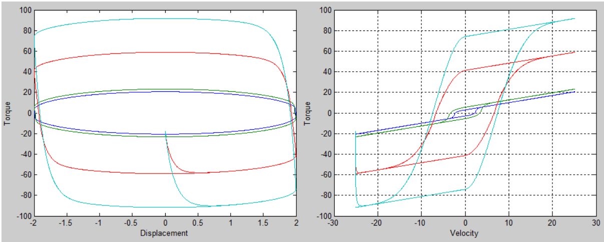


Figure 21: Change of the parameter n=[0.1,0.05,0.01,0.001]

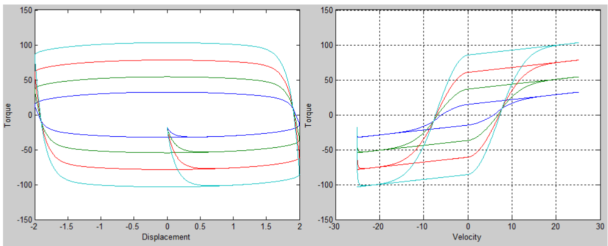


Figure 22: Change of the parameter delta=[800,820,840,860]

hysteresis and is very sensitive of change. Rough adjustment of the hysteresis would be done by changing c1 and c2.

## 5 Hysteresis models: Benchmarking

Dahl equation for MR damper[8]:

$$T_{mr} = k_x \cdot \dot{\theta} + (k_{wa} + k_{wb} \cdot v) \cdot w \quad (9)$$
$$\dot{w} = \rho \cdot (\dot{\theta} - |\dot{\theta}|w) \quad (10)$$

$v$: current or voltage
Bouc-Wen equation for MR damper 5:

$$T = \alpha_1 z + \alpha_2 z \cdot i + c_1\dot{\theta} + c_2\dot{\theta} \cdot i \quad (11)$$
$$\dot{z} = -\gamma|\dot{\theta}|z|z|^n - \beta\dot{\theta}|z|^n + \delta\dot{\theta} \quad (12)$$

Lugre equations for MR damper 3, 4

$$T = \sigma_a z + \sigma_0 zv + \sigma_1 \dot{z} + \sigma_2 \dot{x} + \sigma_b \dot{x} v \quad (13)$$
$$\dot{z} = \dot{x} - \sigma_0 a_0 |\dot{x}| z \quad (14)$$

Comparing the three equations for torque:

| Dahl | | Bouc-Wen | | Lugre |
|---|---|---|---|---|
| $k_x \cdot \dot{\theta}$ | $\leftrightarrow$ | $c_1\dot{\theta}$ | $\leftrightarrow$ | $\sigma_2\dot{x}$ |
| $k_{wa} \cdot w$ | $\leftrightarrow$ | $\alpha_1 z$ | $\leftrightarrow$ | $\sigma_a z$ |
| $k_{wb} \cdot v \cdot w$ | $\leftrightarrow$ | $\alpha_2 z \cdot i$ | $\leftrightarrow$ | $\sigma_0 z \cdot v$ |
| 0 | $\leftrightarrow$ | $c_2\dot{\theta} \cdot i$ | $\leftrightarrow$ | $\sigma_b \dot{x} \cdot v$ |
| 0 | $\leftrightarrow$ | 0 | $\leftrightarrow$ | $\sigma_1 \dot{z}$ |

Comparing the two equations for torque we see the bouc-wen has an additional term compared to the Dahl model($c_2\dot{\theta} \cdot i$). This expression increases the linear damping force, due to increased current. Lugre is much similar to the Bouc-Wen, but has an additional term dependent on the slope of the hysteresis-value, z,($\sigma_1 \dot{z}$).

Comparing the two equations for the hysteresis parameter:

| Dahl | | Bouc-Wen | | Lugre |
|---|---|---|---|---|
| $\dot{w}$ | $\leftrightarrow$ | $\dot{z}$ | $\leftrightarrow$ | $\dot{z}$ |
| $\rho\dot{\theta}$ | $\leftrightarrow$ | $\delta\dot{\theta}$ | $\leftrightarrow$ | $\dot{x}$ |
| $-|\dot{\theta}|w$ | $\leftrightarrow$ | 0 | $\leftrightarrow$ | $-\sigma_0 a_0 |\dot{x}| z$ |
| 0 | $\leftrightarrow$ | $-\beta\dot{\theta}|z|^n$ | $\leftrightarrow$ | 0 |
| 0 | $\leftrightarrow$ | $-\gamma|\dot{\theta}|z|z|^n$ | $\leftrightarrow$ | 0 |

Comparing the three equations for the hysteresis value we see that Dahl and Lugre is pretty similar, but the Bouc-Wen stands out. The main difference is that the Bouc-Wen has exponential terms and more tuning parameters. This makes the model more advanced.

To summarize the benchmarking of the studied models, it is found that Dahl and Bouc-Wen model have pretty much the same equations for torque. The Lugre torque equation stands out with an extra term dependent on $\dot{z}$, but disregard that, Lugre is very similar to Bouc-Wen. When comparing the hysteresis equations we see that the Dahl and Lugre are allmost identical. The Bouc-Wen stands out with some nonlinear exponential terms,$-\beta\dot{\theta}|z|^n$and$-\gamma|\dot{\theta}|z|z|^n$ and by changing some of these parameters someone can smoothly create a hysteresis similar to the the one found from dahl model.

Comparing the three hysteresis:

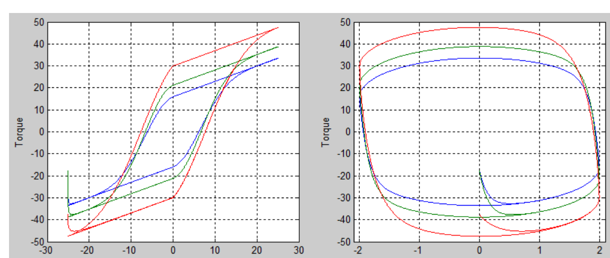


Figure 23: Dahl model (blue), Bouc-Wen(green), and Lugre(red)

From the magnified figure i the middle in fig 24, it can be shown that the Lugre model is a bit more straight than Bouc-Wen and Dahl.

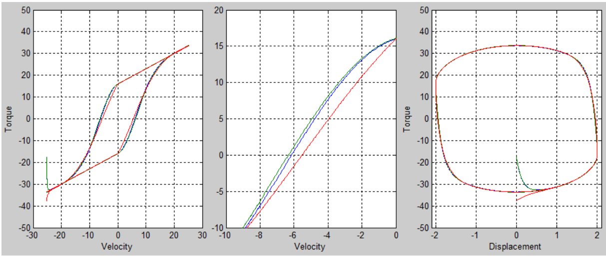


Figure 24: Dahl model (blue), Bouc-Wen(green), and Lugre(red)

# 6 Identification methodology

The methodology of identifying the hysteresis parameters using different models is crucial, hereby, this work should provide an insight into determining a systematic which enable and facilitate the identification problem.

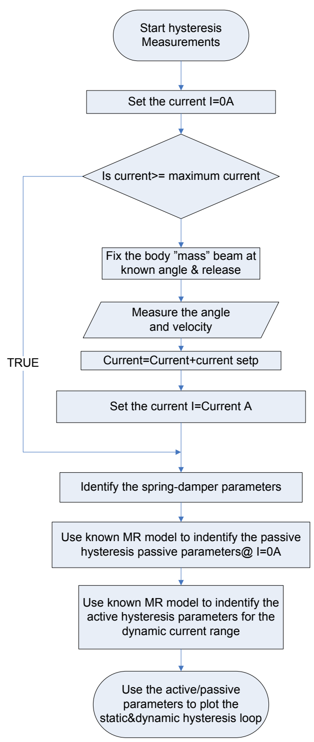


Figure 25: Hysteresis Identification flowchart

Figure (25) shows the systematic approach in a step wise manner in order to determine the hysteresis in any kind of damper, the approach is quiet simple, it starts with identifying the passive parameters of the hysteresis at a current I=0A using known model, afterwards the current is stepped up with known steps, , in between few steps need to be done as follows: (i) The beam which represent the mass of the body which can reassemble for example a mass of car,plane or any other vehicle is lifted to a known angle, (ii) the body is released from the known angle whereby the data for the angles/angular velocity is registered, (iii) the angles/angular velocities is feed into the damper model.

The data for the angles/angular velocities enables determining the passive parameters of the damper, on the other hand, the parameters whereby the current is above zero can be used to define the most accurate parameters which regulate the hysteresis loop over the predefined range of current.

# 7 Semi-Active Suspension system

The SAS (Semi active suspension system) test rig, which is a mass-spring-damper mechanical system with an MR-damper. This system allows someone to run tests and observe reactions of different inputs as traveling speed and currents, as it has a different instruments mounted on to it to allow measurements at different sections of the setup. The SAS used rig is like the one seen in Figure (26), only with an angular damper instead of a linear one.

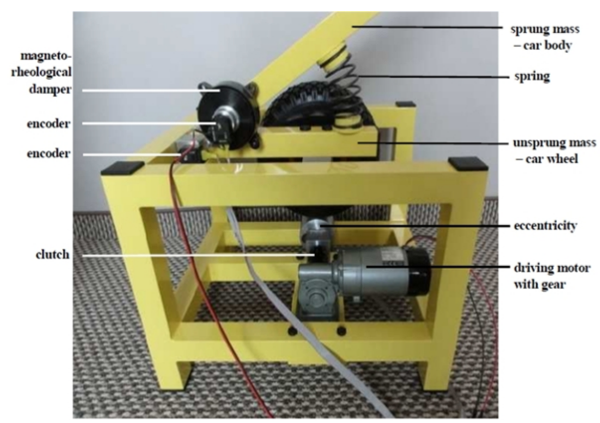


Figure 26: System design in simulink

Since a static vibration analysis with no motion of the wheel in sufficient for hysteresis studies, it is assumed that the wheel is stiff and with no damping.

The spring represents the physical spring on the model. The damper represents the constant damping in the MR damper plus other external things as friction and so on. The last part is representing the force from the nonlinear damping in the MR damper. $\theta_1$ and $\theta(\theta = \theta_2 - \theta_1)$ is measured trough a angle sensor on the model and plotted on the computer.

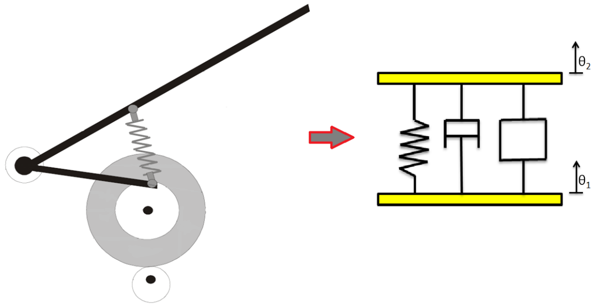


Figure 27: Model of system

## 8 Equations for system with Bouc-Wen and Dahl

Model-equations with Bouc-Wen:

$$T = J\ddot{\theta} = -k\theta - c(i)\dot{\theta} - \alpha(i)z \qquad (15)$$

J is redundant:

$$\ddot{\theta} = -k_1\theta - c(i)\dot{\theta} - \alpha(i)z \qquad (16)$$

$\alpha(i)$ and c(i) is defined in equation 7 and 8.
The Bouc-Wen non linearity is a function of z.
Z can be calculated from the differential equation:

$$\dot{z} = -\gamma|\dot{\theta}|z|z|^n - \beta\dot{\theta}|z|^n + \delta\dot{\theta} \qquad (17)$$

This equation was modeled in matlab/simulink and then merged together with the rest of the system.

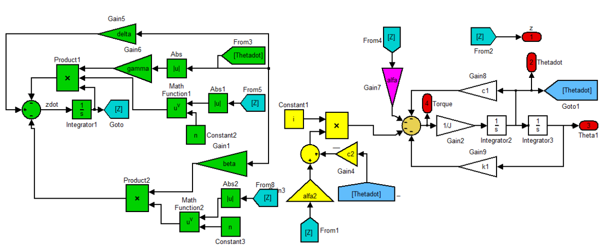


Figure 28: Total system with Bouc-Wen

On the left side, the bouc-wen model(green system) for calculating the parameter z. The yellow system is the contribution from the active damping, dependent on the current. The white blocks illustrates the physical system with a linear spring and damper. The purple block multiplied by z is the passive non-linear damping.

Model-equations with Dahl:

$$T = J\ddot{\theta} = -k\theta - k_x\cdot\dot{\theta} + (k_{wa} + k_{wb}\cdot v)\cdot w \qquad (18)$$

J is redundant:

$$\ddot{\theta} = -k\theta - k_x\cdot\dot{\theta} + (k_{wa} + k_{wb}\cdot v)\cdot w \qquad (19)$$

$$\dot{w} = \rho\cdot(\dot{\theta} - |\dot{\theta}|w) \qquad (20)$$

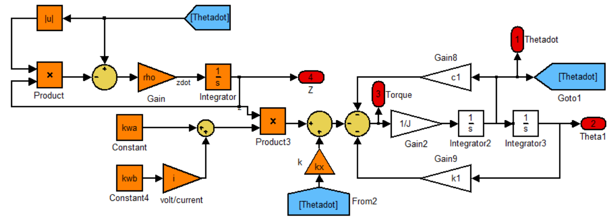


Figure 29: Dahl and SAS system in matlab/simulink

The orange blocks represents the dahl-model for MR-damper and the white blocks represents the SAS-model.

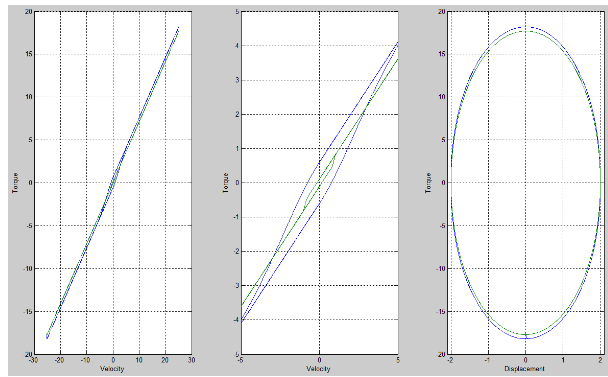


Figure 30: comparing hysteresis with 0A (Bouc-Wen:[green],Dahl:[blue])

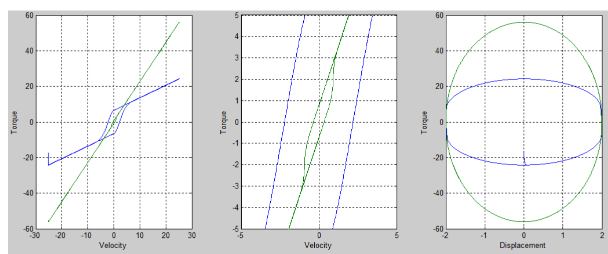


Figure 31: comparing hysteresis with 1A (Bouc-Wen:[green],Dahl:[blue])

## 9 Results

These experiments show that Bouc-Wen is more adaptable to the SAS system. Even though the Dahl hysteresis is quite similar to the Bouc-Wen, it could not fit the system as good. With no current on the damper the differences were quite small, and could probably been even smaller if advanced algorithm like Particle Swarm Optimization had used as the method to estimate the parameters. When the current was set to one, the Dahl model could not manage to increase the linear slope of the hysteresis. This problem could easily be solved by adding a term for the linear damping dependent on current. Like, $k_i \cdot i$

A source of error in addition to inaccurate parameters could be that the neglect of the tire damping makes our simplified model too inaccurate.

# 10 Conclusion

This work addresses the conceptualization, implementation and verification of the hysteresis identification methodology as a generalized concept of determining the hysteresis for all kind of dampers, however, the methodology has been implemented and assessed for the Magnetorheological damper of the Semi Active Suspension setup as the case study.

In order to build up the knowledge and the understanding of the methodology, some steps are investigated and the following has been concluded:

- The MR damper is a highly nonlinear device, and is commonly used in many occasions like damping of automobiles and buildings. The semi-active properties of the MR damper has several advantages including being very adaptive.
- The Bingham model is linear and has no hysteresis. This makes it less adaptive for the MR damper.
- The Dahl, Lugre and Bouc-Wen have all an adaptable hysteresis, but it is found that the Bouc-Wen model to be the best model for illustrating the MR damper. The Dahl is also pretty good in the passive part but it needs to be modified to fit the active properties. This may be done by adding a voltage dependent damping coefficient similar to the Bouc-Wen and Lugre models.
- The simulation of the SAS with the implemented models of Dahl and Bouc-Wen gave us the opportunity to compare the models and select the best one. The Lugre model was not tested into the SAS system like Bouc-Wen and Dahl, but since the Lugre is a modified version of the Dahl, it is assumed that Lugre would give a better fit, but still not give better hysteresis results than the Bouc-Wen model.

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