Ahead prediction of kinematics of vehicles under various collision circumstances by application of ARMAX autoregressive model

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Abstract: In this paper we present the application of regressive models to simulation of a full-scale vehicle-to-pole impact as well as virtual vehicle-to-barrier collision. The capability of an ARMAX model to reproduce vehicle kinematics was examined. Regressive model parameters were established by minimizing a weighted sum of squares of prediction errors. The prediction horizon was assigned to evaluate model's robustness and verify its time series data forecasting performance. It was found that the ARMAX model does not only reproduce the signal which was used for its establishment (i.e. real vehicle's acceleration) but it predicts another signal as well (i.e. virtual vehicle's acceleration). Moreover, such estimation technique preserves all characteristic information relevant for a given collision, since integration of the estimated acceleration pulse yields plots of velocity and displacement which closely follow the reference ones.

Key–Words: ARMAX model, prediction horizon, vehicle crash, vehicle kinematics

1 Introduction

Transport plays a crucial role in an economy, transferring goods between the place of production and consumption, as well as transporting passengers for work or pleasure. However, transport problems such as congestion, quality of services (such as punctuality and connectivity), affordability and environmental impact put general economic developments at risk [1]. When it comes to Europe, road traffic related accidents are considered to be the major threat to life in the European Union. In 2000 40 761 people were killed in road accidents in the 15 member EU countries. The amount of people being injured is 1.7 million. European Commision has set up an ambitious goal to reduce the number of deaths on roads by 50% by 2010. What is significant is the fact that 52% of all road fatalities were passenger car occupants. This statistics unequivocally shows that there is still a lot to be done when it comes to protection not only of the vulnerable road users (like cyclists or pedestrians) but also of the drivers and their passengers as well ([2]).

One of the tools which is the most frequently used and helpful in evaluation of vehicle safety is a crash test. There are different vehicle safety programs and organizations (e.g. NHTSA or EuroNCAP) which specify how such tests should be performed, what factors should be investigated and how the car vehicle safety should be assessed. However, they do agree that the most important ratings are: occupant protection, pedestrian protection and safety assist technologies. Crash tests are not only performed when the car design is completed and a prototype is ready but also throughout the whole vehicle development and validation. It is a well known fact that those experiments are complex ones. Vehicle crash test standards and procedures designate detailed test procedures and requirements. Considerable resources are required to successfully conduct vehicle crash tests. These involve skilled and trained personnel along with a large variety and quantity of sophisticated monitoring and measurement equipment and post-crash data analysis software. Apart from that, information about the measuring equipment, visual inspection, data acquisition process or cameras layout are included in those documents. What is more, e.g. a runaway itself needs a lot of space therefore the appropriate facilities are required (huge hall or open-air yard). In the case of an outside testing site, one needs to be sure that the experimental procedure is conducted in the appropriate weather conditions. Of course we cannot forget about the qualified staff responsible for the whole crash test realization. They need to be well trained and have the adequate knowledge concerning the number of standards to execute such an experiment in a correct way and, what is of the same importance, to collect the representative data and measurements which can be further analyzed. Those issues make a crash test extremely complex and complicated project.

Virtual crash experiments, using mathematical modeling, can help reduce the number of full-scale vehicle tests. Results from these experiments can be used to predict real vehicle behavior, interactions between a vehicle and its occupants, or deformation during a collision - all of that in the virtual environment, without a need of full-scale tests - and providing high degree of models fidelity. In the same time the results from such analysis can be used by the specialists who can e.g. redesign particular car components so that their safety ratings can be raised. Therefore, even if almost all the research is done on a computer, this approach has a huge influence on vehicle safety.

Autoregressive models like NAR (nonlinear autoregressive), NARX (nonlinear autoregressive with exogenous input) or NARMAX (nonlinear autoregressive moving average with exogenous input) are commonly used to predict the time-series data. Particular parameters of those models can be successfully estimated by neural networks - see [3]-[5]. The proper selection of a network's activation function has a significant effect on the accuracy of the network's output. Machine diagnostics and failure detection is a frequent field of application of this approach. Vibration data was processed in [6] to predict machine state in the future. [7] describes time series data analysis by using ARMA model. On the other hand, in [8] a black box model created as a NARX model was capable of representing a gas turbine operating in isolated and nonisolated mode. [9] presents examination of recurrent NARX model's output according to the various configurations of a network's structure. Regressive models are used not only in the engineering applications. Topics such as financial and economic processes forecasting, e.g. stock market data, medical care, e.g. cardiovascular diseases prevention, or natural sciences, e.g. phytoplankton dynamics, can also be modeled by this method, see [10]-[14]. In [15] lumped parameters (stiffness, damping, and mass) of physical systems were estimated by autoregressive moving average (ARMA) model. The advantage of this approach is that those parameters were nonlinear, therefore the accuracy of models' simulation was high.

Recently we can distinguish two main approaches of vehicle crash modeling: FEM (Finite Element Method) simulations and mathematical LPM (Lumped Parameter Modeling). Creating a three dimensional car or obstacle model and its further simulation in an appropriate software is a common way of describing the car collision without performing a real test. After an item model is created in a CAD (Computer Aided Design) program, the mesh is applied to it and structural parameters are assigned. With a complete model and knowledge of the initial conditions one can proceed to its validation and responses observation. On the other hand, LPM is an analytical method of formulating a model which can be further used for simulation of a real event. It allows us to establish dynamic equations of the system - differential equations - which give the complete description of the models behavior - see [16]-[26]. In the most up-to-date scope of research concerning crashworthiness, parameters that change according to the changeable input (e.g. initial impact velocity) are to be defined in a dynamic vehicle crash model. One of such trials is presented in [27] - a nonlinear occupant model is established and scheduling variable is defined to formulate LPV (linear parametrically varying) model.

Up-to-date technologies are currently being utilized in the area of vehicle crash modeling: wavelets, neural networks, and fuzzy logic - see [28]-[31]. In [32] and [33] Radial Basis Function (RBF) neural network was used to obtain the parameters of vehicle crash viscoelastic models. Prediction of the average speed on highways or injury severity of an occupant using statistical data and artificial neural networks (ANNs) is shown in [34] and [35]. Intelligent approach for deformation energy assessment, consisting of vision systems joined together with fuzzy logic is presented in [36].

This paper contains detailed description of two experiments which results are used to evaluate ARMAX model performance. The major modeling principles and formulas needed to establish such a regressive model are provided. Subsequently, models are simulated for the prediction horizon being equal to $K = 0$ and $K = 50$, respectively. Finally, the obtained estimated acceleration plots are integrated and compared to the original vehicles' kinematics in order to evaluate ARMAX model's performance. The major contribution of this paper is the verification of the simulation results with the full-scale crash test data. Furthermore, it is proved that ARMAX model created for one set of time series data can be successfully applied to simulate and predict another data set.

2 Full-scale experiment description

The experimental data which we deal with come from the typical vehicle-to-pole collision ([37]) - see Fig. 1. A test vehicle was subjected to impact with a vertical, rigid cylinder. The acceleration field was 100 meter long and had two anchored parallel pipelines. The vehicle was steered using those pipelines that were bolted to the concrete runaway.

The initial velocity of the car was $35\ km/h$, and

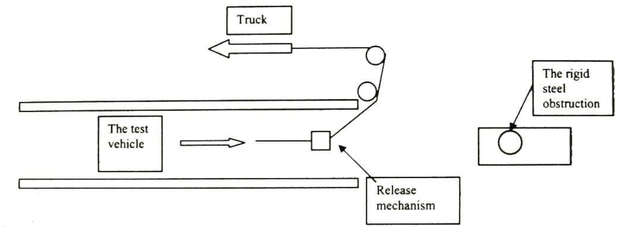

Figure 1: Experiment's scheme.

the mass of the vehicle (together with the measuring equipment and dummy) was $873\ kg$. During the test, the acceleration at the center of gravity in three dimensions (x - longitudinal, y - lateral and z - vertical) was recorded. The yaw rate was also measured with a gyro meter. The obstruction and car themselves are shown in Fig. 2 and Fig. 3, respectively. The obstruction was constructed with two steel components - a pipe filled with concrete and a baseplate mounted with bolts on a foundation. Using normal-speed and high-speed video cameras (the layout of the visual data acquisition system is illustrated in Fig. 4), the behavior of the test vehicle during the collision was recorded - see Fig. 5.

Figure 2: Obstruction.

Figure 3: Car's deformation.

2.1 Real vehicle's crash pulse analysis

Having at our disposal the acceleration measurements from the collision, we are able to describe in details motion of the car. Since it is a central impact, we analyze only the pulse recorded in the longitudinal direction (x-axis). By integrating car's deceleration we obtain plots of velocity and displacement, respectively - see Fig. 6. At the time when the relative approach velocity is zero (t_m), the maximum dynamic crush (d_c) occurs. The relative velocity in the rebound

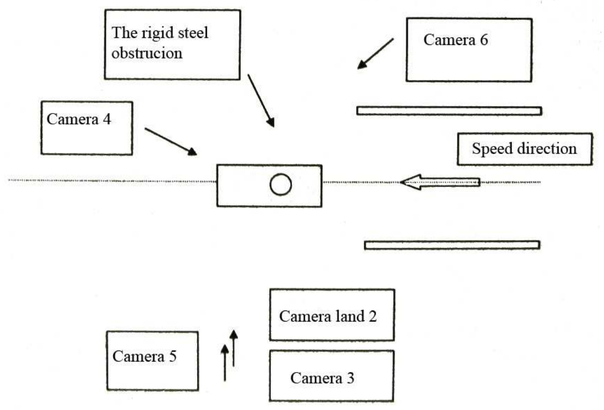

Figure 4: Cameras layout.

Figure 5: Subsequent steps of the crash test.

phase then increases negatively up to the final separation (or rebound) velocity, at which time a vehicle rebounds from an obstacle. The contact duration of the two masses includes both contact times in deformation and restitution phases. When the relative acceleration becomes zero and relative separation velocity reaches its maximum recoverable value we have the separation of the two masses. From the crash pulse analysis we obtain the data listed in Table 1.

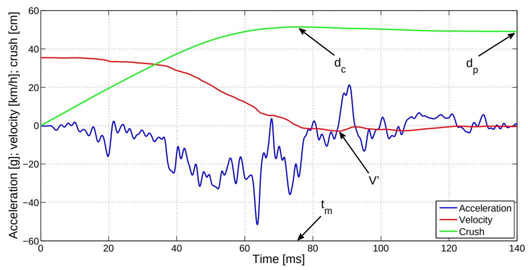

Figure 6: Real car's kinematics.

3 Virtual experiment description

Because of the fact that the data from only one full-scale crash test was available, it was necessary to perform a virtual experiment in order to acquire measurements needed to better assess model's suitability. That

Table 1: Relevant parameters characterizing the real collision

Parameter	Value
Initial impact velocity V $[km/h]$	35
Rebound velocity V' $[km/h]$	3
Maximum dynamic crush d_c $[cm]$	52
Time when it occurs t_m $[ms]$	76
Permanent deformation d_p $[cm]$	50

is the reason to perform it - not to simulate the analyzed full-scale crash test, but to obtain a new set of measurements for model's validation.

3.1 Experiment's overview

From the number of experiments analyzed in [38] on purpose we have chosen to reproduce a low-speed collision (22 km/h compared to 35 km/h from the real crash test presented in Section 2) and similar car's type as well as its dimensions to check if it is possible to establish a model applicable to two different collision types and two different initial impact velocities (however, to the similar car's type). Virtual experimental setup is presented in Fig. 7.

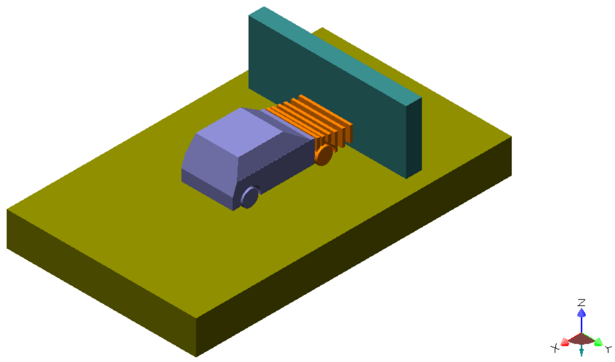

Figure 7: Virtual experimental setup.

3.2 Methodology and assumptions

A multi-body car model has been built (its dimensions were selected to fulfill general requirements concerning mid-size vehicles - see Fig. 8 for front, side, and top views of the vehicle). We have divided the front part of the vehicle into 6 undeformable components as we can assume that in such a type of collision only this car's section undergoes the deformation. To simulate elasto-plastic properties of the car's body, its particular components were connected with springs and dampers - see Fig. 9. To make the vehicle follow the reference car's behavior from [38], their values were assigned in the trial and error method - see Table 2. The most relevant dimensions of the car are shown in Fig. 10. They were assigned to fulfill the overall mid-size vehicle geometric requirements. Mass of the whole vehicle is equal to 1000 kg. Mass distribution was also taken into account (e.g. the front part of the hood is not heavy, on the other hand, the axle together with wheels and engine weigh more). Furthermore, since we investigate a central collision, the whole model is constrained in such a way that its motion is possible only in one direction - longitudinal. By doing this we analyze only its longitudinal acceleration component - the same as we did in Section 2.

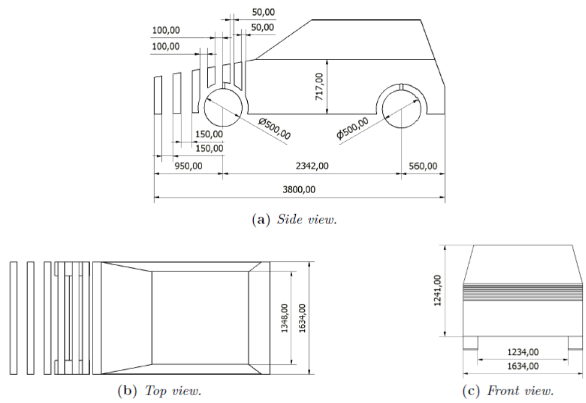

Figure 8: The most relevant dimensions of the virtual vehicle $[mm]$.

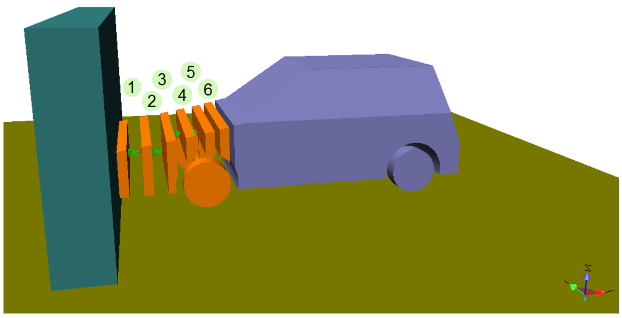

Figure 9: Virtual experiment's overview.

3.3 Virtual crash pulse analysis

Sampling rate for the virtual experiment is exactly the same as the one for the real collision elaborated in Section 2 - i.e. 10 kHz (according to [39]). Similarly, the acceleration was measured in the car's center of gravity - COG of a virtual vehicle is illustrated in Fig. 11. Sequence of the virtual crash is shown

Table 2: Values of stiffness and damping for each spring

Spring number	Stiffness k $[kN/m]$	Damping c $[kNs/m]$
1	90	70
2	500	80
3	100	10
4	800	6
5	600	10
6	30	70

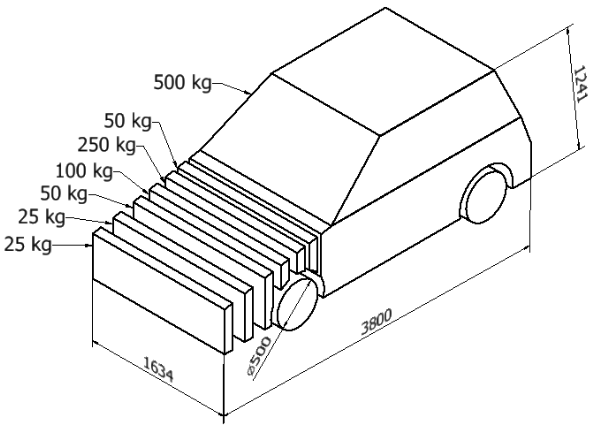

Figure 10: 3D model of a virtual vehicle - dimensions in $[mm]$.

in Fig. 12. In Fig. 13 there is shown virtual experiment's outcome. Keep in mind that the response obtained from the virtual crash test should be treated as an approximated crash pulse since it is not possible to get such rapidly changing acceleration plot (as it is in the real experiment) in this kind of simulation. However, the results are satisfactory because the obtained virtual car's deformation closely follows the reference one from [38] as well as the overall shapes of velocity and acceleration do. As we see, the crush curve in Fig. 13 does not achieve a steady value. However, the behavior of this model in the crush time interval (up to the moment when the acceleration plot reaches the zero value) is satisfactory. See Table 3 for the most relevant crash pulse characteristic parameters. The value of permanent deformation d_p for the virtual experiment should be treated as an approximated one, since the whole model is represented as a multi-body.

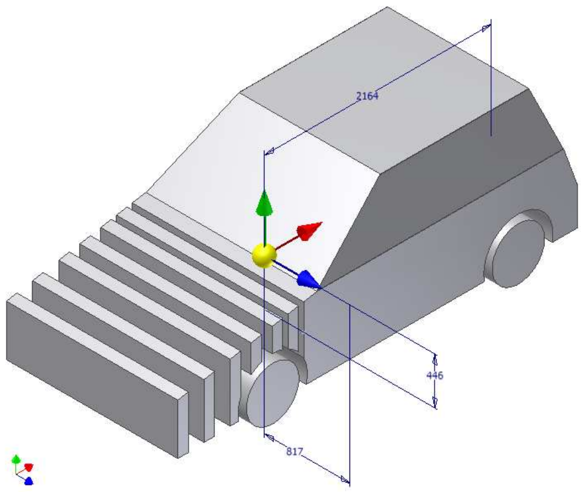

Figure 11: Center of gravity (COG) of a virtual vehicle.

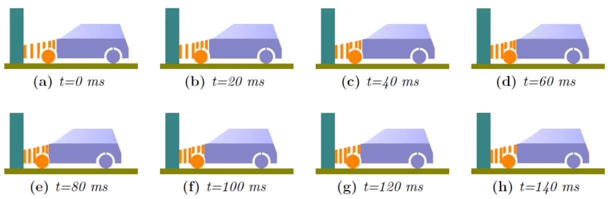

Figure 12: Sequence of the virtual crash.

4 ARMAX model analysis

Two types of mathematical modeling of real world systems are commonly used [40]:

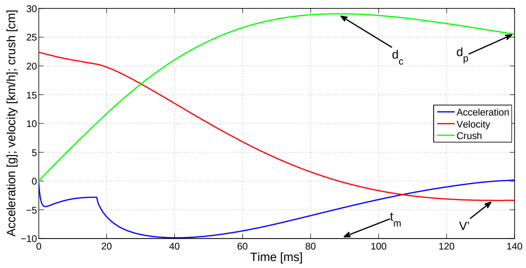

Figure 13: Virtual car's kinematics.

Table 3: Relevant parameters characterizing the virtual collision

Parameter	Value
Initial impact velocity V $[km/h]$	22
Rebound velocity V' $[km/h]$	3.2
Maximum dynamic crush d_c $[cm]$	29
Time when it occurs t_m $[ms]$	88
Permanent deformation d_p $[cm]$	25

1. Mathematical approach - dynamics of a phenomenon or system is derived from the fundamental law of physics (e.g. Newton's Laws or conservation principle).

2. System identification - experimental approach. After examination of the system by performing on it experiments, model parameters are selected in such a way, that model's behavior fits to the experimental data.

In this paper, the second approach is followed.

4.1 Model's description

Analysis of the autoregressive model with moving average and exogenous input (ARMAX) was done according to [41]. ARMAX model is defined as:

$$y(t) + a_1 y(t-1) + \ldots + a_{na} y(t-n_a) = b_1 u(t-n_k) + \ldots + b_{nb} u(t-n_k-n_b+1) + c_1 e(t-1) + \ldots + c_{nc} e(t-n_c) + e(t) \quad (1)$$

where:

- t - time
- $y(t)$ - system's output
- $a_1, \ldots, a_{na};\ b_1, \ldots, b_{nb};\ c_1, \ldots, c_{nc}$ - model's parameters
- n_a - number of model's poles
- n_b - number of model's zeros + 1
- n_c - number of model's parameters in C vector
- n_k - order of delay
- $y(t-1), \ldots, y(t-n_a)$ - system's output in the previous moment
- $u(t-n_k), \ldots, u(t-n_k-n_b+1)$ - system's input in the previous moment
- $e(t-1), \ldots, e(t-n_c)$ - white noise.

ARMAX model can be also formulated as:

$$A(q)y(t) = B(q)u(t-n_k) + C(q)e(t) \quad (2)$$

where A, B, and C are expressed as functions of q:

$$A(q) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \quad (3)$$

$$B(q) = b_1 + b_2 q^{-1} + \ldots + b_{nb} q^{-nb+1} \quad (4)$$

$$C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}. \quad (5)$$

4.2 Model's establishment

ARMAX model was created for the acceleration pulse recorded during a full-scale crash test. Since this type of data is a regular time series, the order of ARMAX model is specified by the parameters n_a and n_c. We assumed that $n_a = n_c = 3$. The obtained ARMAX model is as follows:

$A(q) = 1 - 2.907 \cdot q^{-1} + 2.865 \cdot q^{-2} - 0.9582 \cdot q^{-3}$
$C(q) = 1 + 1.08 \cdot q^{-1} + 0.9191 \cdot q^{-2} + 0.4136 \cdot q^{-3}$.

This is a particular case of an ARMAX model in which $n_b = 0$. The model's output is predicted acceleration. To obtain the complete vehicle's kinematics (apart from acceleration also its velocity and displacement) during a collision, model's response is integrated. Results of simultaneous model's simulation (no ahead prediction) for both: full-scale crash test and virtual one are shown in Fig. 14 and Fig. 15, respectively. Please note that ARMAX model established for the real crash data set (mid-speed collision) is also suitable to simulate the virtual crash test (low-speed collision).

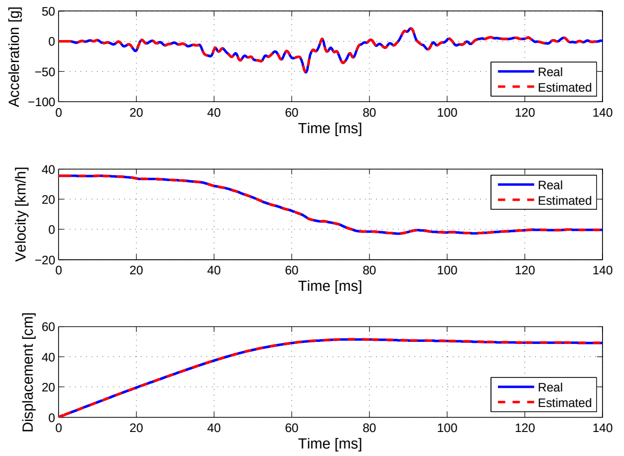

Figure 14: ARMAX model's simultaneous prediction of the real crash pulse.

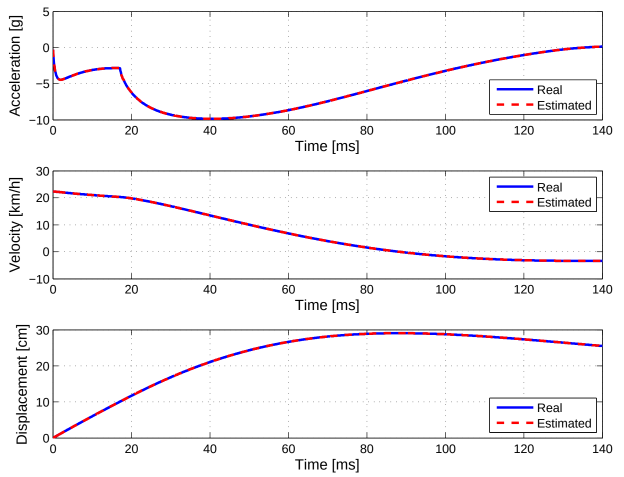

Figure 15: ARMAX model's simultaneous prediction of the virtual crash pulse.

5 ARMAX model ahead prediction

Ahead prediction of time series data is based on the prediction horizon concept. This method takes the old model's output up to time $t - K$ and uses it to predict the output at time t. Thanks to that it is possible to forecast the behavior of a system according to its past output. In Section 4.2 it was presented a particular case of prediction in which $K = 0$. Estimation process was performed simultaneously therefore the model's orders were low. Let us verify the performance of an ARMAX model if the prediction horizon is equal to $K = 50$. Because of the fact that the data sampling frequency used in the experiment is $10\ kHz$, this corresponds to the situation in which the current model's output is created based on the reference system's behavior from $5\ ms$ in the past. Since the ahead prediction methodology demands more complex model's structure, its order has been increased in the trial and error process to $n_a = 25$ and $n_c = 20$. Similarly to Section 4.2, ARMAX model was created from the acceleration pulse recorded during the full-scale crash test. Obtained system has the following form:

$A(q) = 1 - 8.181 \cdot q^{-1} + 33.37 \cdot q^{-2} - 92.03 \cdot q^{-3} + 195.8 \cdot q^{-4} - 345.3 \cdot q^{-5} + 526.7 \cdot q^{-6} - 713.5 \cdot q^{-7} + 872.1 \cdot q^{-8} - 970.5 \cdot q^{-9} + 987.6 \cdot q^{-10} - 920 \cdot q^{-11} + 783.9 \cdot q^{-12} - 609.6 \cdot q^{-13} + 431 \cdot q^{-14} - 275.1 \cdot q^{-15} + 157 \cdot q^{-16} - 79.85 \cdot q^{-17} + 37.63 \cdot q^{-18} - 19.31 \cdot q^{-19} + 13.07 \cdot q^{-20} - 10.23 \cdot q^{-21} + 7 \cdot q^{-22} - 3.52 \cdot q^{-23} + 1.12 \cdot q^{-24} - 0.167 \cdot q^{-25}$

$C(q) = 1 - 0.285 \cdot q^{-1} - 0.41 \cdot q^{-2} + 0.7149 \cdot q^{-3} - 0.7731 \cdot q^{-4} + 0.7477 \cdot q^{-5} - 0.3222 \cdot q^{-6} - 0.2005 \cdot q^{-7} + 0.1955 \cdot q^{-8} - 0.7391 \cdot q^{-9} + 1.139 \cdot q^{-10} - 0.5949 \cdot q^{-11} - 0.1957 \cdot q^{-12} + 0.2036 q^{-13} - 0.4528 \cdot q^{-14} + 0.1967 \cdot q^{-15} - 0.07959 \cdot q^{-16} - 0.05506 \cdot q^{-17} - 0.3742 \cdot q^{-18} + 0.01382 \cdot q^{-19} + 0.275 \cdot q^{-20}$.

Parameters of this regressive model are estimated by minimizing a weighted sum of squares of prediction errors. The criterion applied is interpreted as estimation of the covariance matrix of the noise source and use of the inverse of that matrix as the weighting [42]. Simulation results for this system are illustrated in Fig. 16 and Fig. 17.

The accuracy of the proposed method is satisfactory. It is important that the kinematics of both vehicles: the real and virtual one, derived from the estimated acceleration pulse closely reproduces the reference plots of velocity and displacement. Hence it is proved, that ARMAX model allows us to obtain vehicles' acceleration plots which preserve the overall characteristics and information concerning the collision circumstances and its nature. Therefore integration of the estimated signals yields the kinematic plots which closely resemble the original ones.

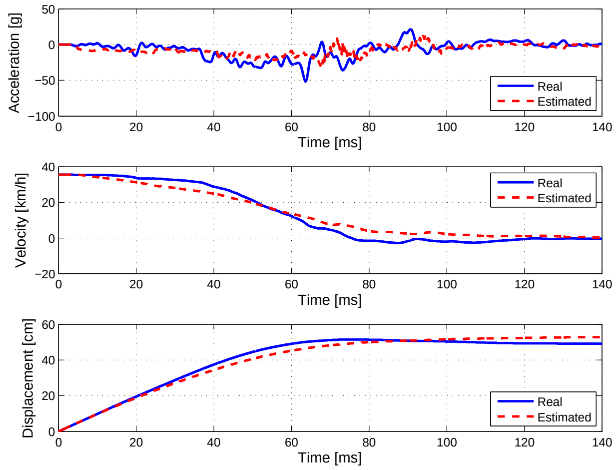

Figure 16: ARMAX model's 50 samples ahead prediction of the real crash pulse.

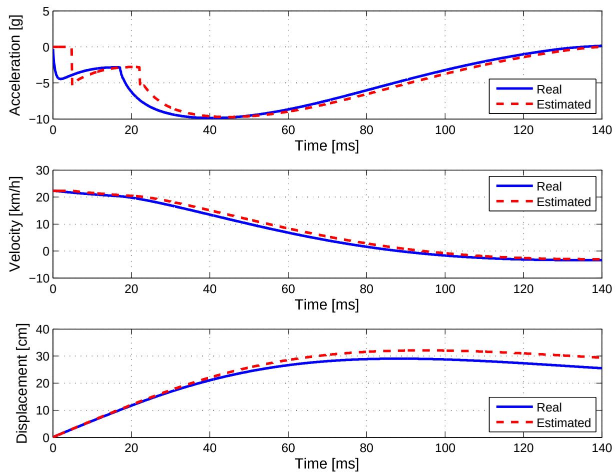

Figure 17: ARMAX model's 50 samples ahead prediction of the virtual crash pulse.

6 Conclusions

Two ARMAX models have been established. One for the simultaneous estimation of the vehicle's acceleration signal, whereas another one for the signal ahead prediction, where prediction horizon was equal to 50 samples. As expected, the higher the prediction horizon is demanded, the higher the orders of the model should be applied. However, at some point the increase of model's orders does not improve the results obtained - this is associated with the loss of model's stability. Moreover, creating complex, large models greatly extends the computational time needed to estimate their parameters and obtain their output.

Advantage of the approach presented in this paper is that a model created for one set of data can be successfully used to reproduce another data set. This reduces the time needed for model's creation and enhances the simulation stage as well. Future work in this area may cover performance investigation of models established for another time series data and analysis of the estimated signals in the frequency domain. It is also advisable to conduct simulations of the regressive models for the higher prediction horizon to evaluate in details robustness of this modeling approach.

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