The nature of instabilities in blocked media and seismological law of Gutenberg-Richter

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Abstract: - This paper studies properties of a continuum with structure. The characteristic size of the structure governs the fact that difference relations do not automatically transform into differential ones [1]. It is impossible to consider an infinitesimal volume of a body, to which we could apply the major conservation laws, because the minimal representative volume of the body must contain at least a few elementary microstructures. The corresponding equations of motions are the equations of infinite order, solutions of which include, along with sound waves, the unusual waves propagating with abnormal low velocities, not bounded below. It is shown that in such media weak perturbations can increase or decrease outside the limits. The variance of structure sizes plays a double role. The intensity of instabilities decreases due to dispersion, thereby stabilizing the media, while the frequency range of unstable solutions expands, and disasters can occur at very low frequencies. The equation of equilibrium is not satisfied at any point in the medium. It is true only at an average. Hence there is a possibility to have a lot of micro-dynamic acts, in spite of static macroscopic state in average. This paper describes some of the conditions under which the possible occurrence of usual wave motion in media in the presence of certain dynamic phenomena. The number of complex roots of the corresponding dispersion equation, which can be interpreted as the number of unstable solutions, depends on the specific surface cracks and is an almost linear dependence on a logarithmic scale, as in the seismological law of Gutenberg-Richter.

Key-words: - Specific surface, Operator of continuity, Equation of motion, Catastrophes, Structured media, Gutenberg–Richter law

1 Introduction

The characteristic size of structure leads to the fact that the average distance between one of the crack to another one or one of the pore to another one is given by specific surface of sample. Fig.1 shows an element of the volume of a structured body, where l_0 is the average distance between one pore to another. There is a theorem of integral geometry, which relates the specific surface σ_0 and l_0 with porosity f [2]

$$\sigma_0 l_0 = 4(1-f) \tag{1}$$

Therefore, if there is a specific surface of sample, the average range of microstructure l_0 is automatically defined.

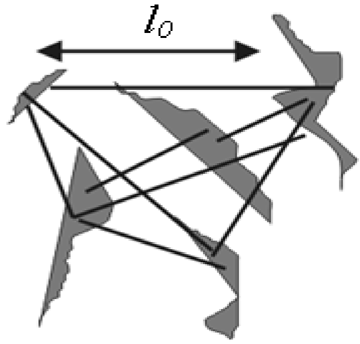

Fig.1 The element of a structured body with an average distance l_0 between pores.

It is evident that the minimal distance, which gives us a structure, cannot be less than the distance from one particle to its nearest neighbor for granular medium.

The same we can tell about average distance from any crack to its nearest neighbor for cracked medium. Actually, the representative size of structure is related with statistical characteristics of pore space.

The distinction of the classic and the structured continuum is clear from Fig.2. In the volume bounded by surface *C* there is equilibrium as a result of compensation of all internal volume forces.

There is no equilibrium in the volume bounded by surface *D* because all forces are concentrated on one side of a surface of grain while another side is free from action of forces.

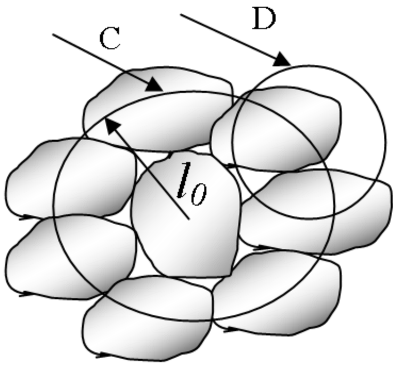

Fig.2 The problem of creation of equilibrium equation into arbitrary element of discrete medium

We can construct new model of a medium as follows. Let's consider finite element of volume of the structured body bounded by the sphere of radius l_0. Surface forces will act on the surface of the element, while the forces of inertia will be attached to its center and in our case, there is no possibility to turn the elementary volume to zero and to combine points of the application of surface and inertial forces, as in classical a continuum. So we must consider namely a finite volume as representative volume of the body.

Physically based equations of motion of such elementary volumes will be possible if we apply the operator of translation of the surface forces to the center of this structure.

So there appears a possibility to use usual laws of conservation for the forces transferred to the center of structure. In other words, operators of translation will transform the real medium to its continuous image, where all space is filled by a field of forces.

Some results of a new model of the structured continuum was published earlier [3], but now we repeat some formulas in order to present an absolutely new idea about the intermediate state between statics and dynamics

The one-dimensional operator of translation of the field from the point x into the point $x \pm l_0$ is specified by the formula the symbolic formula [4]

$$u(x \pm l_0) = u(x)e^{\pm l_0 D_x} \tag{2}$$

This formula contains both spatial variables x, l_0 and the symbolic variable D_x

$$D_x = \frac{\partial}{\partial x} \tag{3}$$

The formal Taylor series expansion in (2) by using (3) gives the finite increment of the field as a series of the infinite number of all order derivatives with different powers of l_0. The difference of the first order is given by

$$\begin{aligned} \Delta_1 &= u(x)\frac{1}{l_0}\left(e^{\frac{l_0 D_x}{2}} - e^{-\frac{l_0 D_x}{2}} \right) = \\ &= u(x)\frac{\sinh\left(\frac{l_0 D_x}{2}\right)}{\frac{l_0}{2}} \end{aligned} \tag{4}$$

Expression Δ_1 in (4) tends to the first derivative at $l_0 \to 0$. Analogously the second difference may be written as

$$\begin{aligned} \Delta_2 &= u(x)\frac{1}{l_0^2}\left(e^{\frac{l_0 D_x}{2}} - e^{-\frac{l_0 D_x}{2}} \right)^2 = \\ &= u(x)\frac{\sinh^2\left(\frac{l_0 D_x}{2}\right)}{\left(\frac{l_0}{2}\right)^2} \end{aligned} \tag{5}$$

Expression Δ_2 in (5) tends to the second derivative at $l_0 \to 0$. The similar operator of translation in three-dimension space for some sphere is given by expression

$$P(D_x, D_y, D_z, l_0) = \\ = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\exp[l_0(D_x\sin\theta\cos\varphi + \\ + D_y\sin\theta\sin\varphi + D_z\cos\theta)]\sin\theta d\theta d\varphi = \\ = \frac{\sinh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} = E + \frac{l_0^2}{3!}\Delta + \frac{l_0^4}{5!}\Delta\Delta + ... \tag{6}$$

According to Poisson formula [3] we have

$$\int_0^{2\pi}\int_0^{\pi} f(\alpha\cos\theta + \beta\sin\theta Cos\varphi + \\ + \gamma\sin\theta\sin\varphi)\sin\theta d\theta d\varphi = \\ = 2\pi\int_0^{\pi} f(\sqrt{\alpha^2+\beta^2+\gamma^2}\cos p)\sin p dp \tag{7}$$

so the operator P in (6) may be rewritten as follows

$$P(D_x, D_y, D_z) = \frac{1}{2}\int_{-1}^{1}\exp(l_0\sqrt{\Delta}\, t)dt = \\ = \int_0^1\cosh(l_0\sqrt{\Delta}\, t)dt = \frac{\sinh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} = \\ = E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + ... \tag{8}$$

It is interesting that a P operator in (8) as a function of symbolic variables $p_1 = \frac{\partial}{\partial x}$, $p_2 = \frac{\partial}{\partial y}$, $p_3 = \frac{\partial}{\partial z}$ satisfies to Helmholtz equation with pure image frequency

$$\frac{\partial^2 P}{\partial p_1^2} + \frac{\partial^2 P}{\partial p_2^2} + \frac{\partial^2 P}{\partial p_3^2} = l_0{}^2 P \tag{9}$$

One of fundamental solution of (9) which tends to unit at $l_0 \to 0$ takes a form

$$P = \frac{\sinh(l_0\sqrt{p_1^2+p_2^2+p_3^2})}{l_0\sqrt{p_1^2+p_2^2+p_3^2}} = \frac{\sinh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} \tag{10}$$

2 The equation of motion in blocked media

By using operator P we can write the equation of motion of microinhomogeneous body because for an average stresses in structure the law of impulse conservation takes a usual form, namely

$$\frac{\partial}{\partial x_k}[P(\sigma_{ik})] = \rho\ddot{u}_i \tag{11}$$

By using (10) the equation (11) may be rewritten in more detailed form as follows

$$\frac{\partial}{\partial x_k}\left(E + \frac{l_0^2}{3!}\Delta + \frac{l_0^4}{5!}\Delta\Delta + ...\right)\sigma_{ik} = \rho\ddot{u}_i \tag{12}$$

In particular, if we take into account only the first and second terms in this equation, we obtain the fourth-order equilibrium equation. For one dimensional case equation (12) takes more simple expression

$$u''(E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + ...) + k_S^2 u = 0 \tag{13}$$

This equation by substitution of $u = Aexp(ikx)$ into (13) gives us the dispersion equation for unknown wave number k

$$\frac{\sin(kl_0)}{kl_0} - \frac{k_S^2}{k^2} = 0 \tag{14}$$

Or, for unknown wave velocity, which depends on range l_0 of structure or specific surface σ_0 of sample, according to the formula (1). In (14) k_S is usual wave number for shear or longitude waves.
In case of $l_0 \to 0$, the wave number $k \to k_S$, i.e. the wave velocity is equal to V_P or V_S elastic wave velocities. However, if l_0 is not very small value, the wave velocity decreases up to a zero when $kl_0 \to m\pi$, where m is integer number.
Hence this model describes along with usual seismic waves a lot of waves of very small velocities, which unbounded from below. Such effect is more substantial for the P waves than for the S ones. Thus, if the Poisson's ratio is measured on the samples using V_P and V_S velocities, we have the growth of ratio V_S / V_P with growth of l_0, and this effect can produce abnormally small Poisson's ratio up to its negative values. Set of velocities, described by (14) closely relates with infinite degrees of freedom of structured media.

The same approach for case of infinite degrees of freedom was recently published in the work [3].
Another feature of equation (14) is the existence of complex roots representing the unstable solutions, describing phenomenon of so-called parametric resonance.

3 Seismological law of Gutenberg–Richter

Equation (14) gives us an infinite number of roots both real and complex ones. The real roots correspond to stable solutions, while complex ones correspond to instabilities. The number of complex roots is growing at decreasing of dimensionless specific surface of cracks, which represents by expression

$$\frac{1}{\varepsilon} = \frac{\sigma_0 \lambda_S}{8\pi(1-f)} \qquad (15)$$

The symbol λ_S in (15) is the wavelength of a usual shear wave, while $\varepsilon = k_s l_0$. Theoretical dependence of the number of complex roots versus specific surface on the log-log scale plot (Fig.3) represents something closed to linear dependence. It is evident that the energy of cracked body is proportional to surface of cracks. The deficit of potential energy owing to occurrence of cracks is equal to product of stresses before cracking process and displacement of cracks surface, namely

$$\bar{E} = \int_S P_i u_i dS \qquad (16)$$

Value $\bar{E}$ in (16) is equal to kinetic energy of waves due to law of energy conservation. It means that we can compare the experimental seismological relation (number of earthquakes versus energy) with the theoretical relationship (number of instable solutions versus specific surface). On the Fig.3-4 there are theoretical and experimental diagrams.
The Fig.3 shows series of straight horizontal segments, representing complex roots, corresponding to detached number of events
The experimental diagram was made Riznichenko [5]. On the theoretical plot (Fig.3) this tangent is closed to 0.5. It is interesting that for large energy there is set of non-uniqueness. One line segment of the vertical scale corresponds a lot of surfaces (energy) on the horizontal exes.

The tangent of the angle with vertical axes is changing on the experimental plot from 0.5 to 0.52 and depends on different data processing (Fig.4).

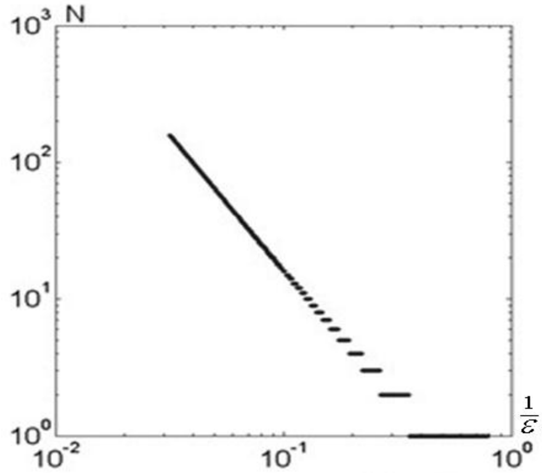

Fig.3 Theoretical dependence of instable events number from cracks specific surface. Tangent of the angle γ = 0.5 It is clearly visible non-uniqueness of solutions with the great energies

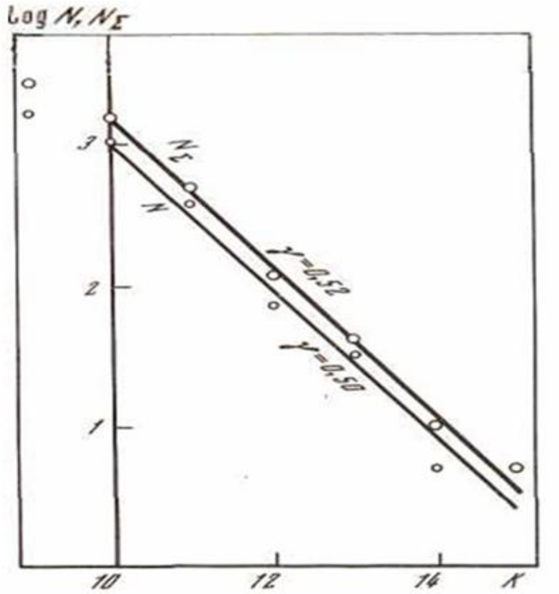

Fig.4 Experimental dependence seismic events numbers versus energy, which is proportional to cracks specific surface. Tangent of the angle γ = 0.5 – 0.52

Such non-uniqueness in experimental data usually interpreted as a deficiency of number of great events and bad statistic estimations of such events.
Proposed theoretical approach explains a principal non-uniqueness of such processes. Besides of it, these processes have shear character without of sufficient crack opening (Fig.5).

At sufficient crack opening the potential energy would be proportional not to surface, but to volume of cracks.
Hence the theoretical plot on Fig.3 shows, that law Gutenberg–Richter is obliged to extremely shift mechanisms of earthquakes.

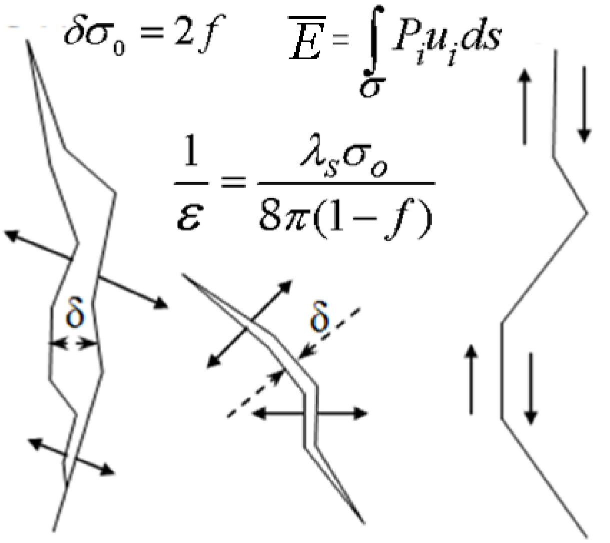

Fig.5 Real shear cracking process (right hand) without of sufficient crack opening corresponds to Richter-Gutenberg law

4 The long wave approach. Equations kind of Korteweg-de-Vries and Boussinesq

For small values l_0 in comparison with wavelength there is a possibility to reduce the equation of motion of an infinite order to the equation of the fourth order, neglecting the members containing values l_0^4 and above.
In this case, we can consider some nonlinear relations between stress and strain.
Let's assume that we have a nonlinear loading and linear unloading.
For rocks and grounds the reduction of stresses takes place at the increase in deformations.
It means, that in such media, where shock waves are absent while nonlinear waves are represented by Riemann waves.
The reduced equation of motion (11) takes a form

$$\frac{\partial}{\partial x_k}\left(E+\frac{l_0^2}{3!}\Delta\right)\sigma_{ik}=\rho\ddot{u}_i \qquad (17)$$

On the Fig.6 is shown quadratic relations between stress and strain of loading process in the form $\sigma_{xx}=(\lambda+2\mu)(u_x-bu_x^2)$ and the linear behavior of unloading process. The area of the hysteresis loop represents the energy dissipated.
Under a long wavelength approximation, when a wavelength much more than the linear size of microstructure, the equation of motion by neglecting third and higher order terms, takes a form

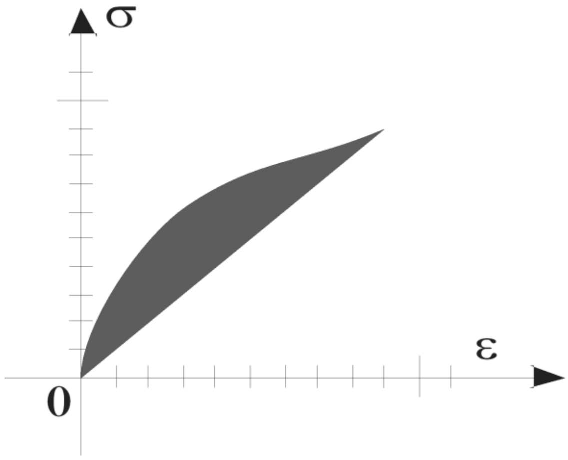

Fig.6 There is relation between stress and strain. The loading part has a positive curvature. The downloading part is a straight line. The area between them is dissipated energy.

$$u_{xx}(1-2b^2u_x)+\frac{l_0^2}{3!}u_{xxxx}=\frac{1}{c^2}u_{tt} \qquad (18)$$

By means of change of variables $\xi=ct-x;\ \eta=ct+x$ an equation of motion (4) reduces to similar to Korteweg- de -Vries equation.

$$u_\eta-b^2uu_\xi+\frac{l_0^2}{3!}u_{\xi\xi\xi}=0 \qquad (19)$$

The classical *KdV*-equation has another sign of nonlinear term. Due to this fact equation (18) has no solutions type of solitons, and the role of nonlinear term will be represent below.
If the nonlinear term is absent, this equation is similar to the equation of Boussinesq type, i.e.

$$u_{xx}+\frac{l_0^2}{3!}u_{xxxx}=\frac{1}{c^2}u_{tt} \qquad (20)$$

In (20) second summand $\frac{l_0^2}{3!}u_{xxxx}$ is the dispersion term. Let's find the solution of (17) in the wave form

$$u = cTF\left(\frac{t-\alpha x/c}{T}\right) \qquad (21)$$

In (21) T is a characteristic time of pulse, while α is the some constant value, greater unit. Assuming $F' = \varphi(\xi)$, we can write the ordinary nonlinear equation

$$\varphi'' + \frac{3!(\alpha-1)}{l_0^2\alpha^4}\varphi = -\frac{3!b^2}{l_0^2\alpha}\varphi^2 \qquad (22)$$

`Let's assume, that in (22)

$$\frac{3!(\alpha^2-1)(cT)^2}{l_0^2\alpha^4} = 1,\ \varepsilon = \frac{l_o}{cT},\ \alpha = 1 + \frac{1}{2}\varepsilon^2$$

The value of φ is the product $\varphi = \varphi_0\bar{\varphi}$, where the constant φ_0 is equal to characteristic value of strain, for example, to the elastic limit of shear deformation. At the assumptions entered above it is received more simple equation for a variable $\bar{\varphi}$. Ignoring the bar over the variable $\bar{\varphi}$, we have the nonlinear equation in the form

$$\varphi'' + \varphi + \beta\varphi_0\varphi^2 = 0 \qquad (23)$$

In equation (23) $\beta = \frac{3!\,b^2}{\alpha\varepsilon^2}$. Thus, in spite of φ_0 is very small value, the product of it on large value β is not very small one, due to $\alpha \approx 1$, while ε is a very small value for small size of structure compare to the wavelength. Hence, the dispersion phenomena in microinhomogeneous media increase the nonlinear effects. The exact solution of (23) in implicit form is

$$\frac{t-\alpha x/c}{T} = \int_0^{\varphi} \frac{dp}{\sqrt{1-p^2-\beta\varphi_0 p^3}} \qquad (24)$$

This solution at $\beta\varphi_0 \to 0$ tends to the usual sinusoidal function, while the more common exact solution of (23) takes a form

$$\frac{t-\alpha x/c}{T} = \int_0^{\varphi} \frac{dp}{\sqrt{C_1 + Cp - p^2 - \beta\varphi_0 p^3}} \qquad (25)$$

Here C_1 and C are arbitrary constants. The integral (25) describes wider class of phenomena, than the integral (24).

We can linearize the equation (23) using instead of function φ^2 expression $\varphi^2 = \varphi\cos\xi$ close to it in which the term $\cos\xi$ is the solution of the linear equation arising at $\beta\varphi_0 \to 0$. Other words, we can write $\varphi^2 \approx \varphi\cos\xi$, and $\xi = (t-\alpha x/c)/T$. So there is a possibility to linearize equation into form of Mathieu's equation

$$\varphi'' + \varphi\left(1 + \frac{\varphi_0}{\varepsilon^2}\cos\xi\right) = 0 \qquad (26)$$

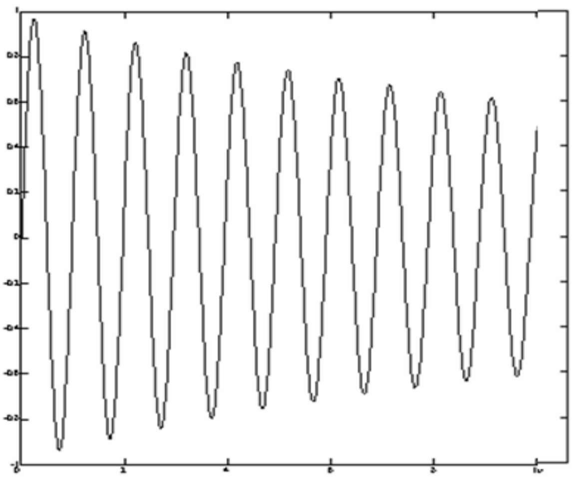

Fig.7 The attenuation of sinusoidal impulse depends on distance. The nonlinear parameter is 0.1. The logarithmic decrement is almost constant value.

This equation contains both stable solutions and instabilities. Instabilities contain attenuation of vibrations and growing of them i.e. catastrophes.

The role of parameter plays not small value φ_0, but the more significant value, namely, the ratio φ_0/ε^2. Attenuation of sinusoidal pulse with distance due to nonlinearity is displayed on Fig.7-8 The parameter of nonlinearity is not very small due to large factor $1/\varepsilon^2$.

On these figures is shown, how to appear the attenuation for sinusoidal impulse. When the nonlinear parameter equal to zero, attenuation is absent.

If it is equal to 0.1 there is some small attenuation with almost constant logarithmic decrement. For larger nonlinear parameter equal to 0.5 there is a sufficient attenuation without of constant decrement. Equation (26) also contains growing solutions. But in this section, we shall deal only with damping vibrations.

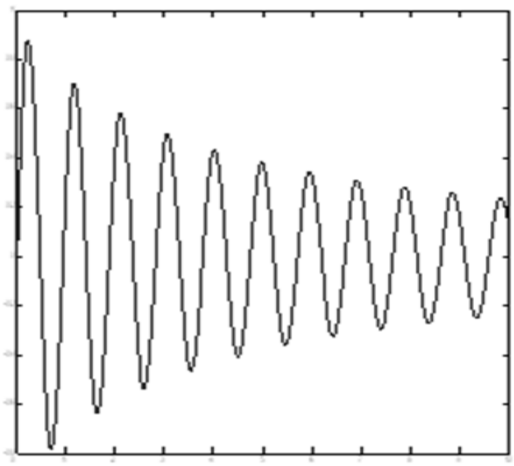

Fig.8 The attenuation of sinusoidal impulse depends on distance. The nonlinear parameter is 0.5.

According to experimental observations the sample of artificial sandstone (length-1m, diameter-0.76m, porosity-0.3 and density 2g/cm^3) undergoes of excitation simultaneously by two vibrators with frequencies 6100Hz and 7720 Hz.

The spectra of signals on the plane of cylindrical sample, where the source is located, are given on the Fig.9.

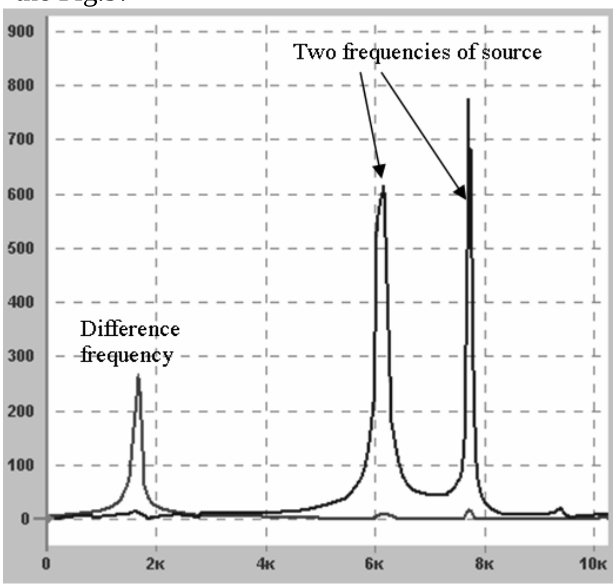

Fig.9 Spectra of quasi-sinusoidal signals. The horizontal axis shows the frequency with step equal 2000Hz. The vertical axis shows the amplitudes. (Egorov G.V. [6])

The receiver registers different frequency 1620 Hz on the distance 75 cm from the source. It is interesting that the amplitude of difference frequency extremely high, i.e. reached the order of a several percent of initial signal [6].

Classic approach, connected with second-order equations of motion predicts the effect, which is proportional to quadrate of strains.

The dispersion phenomena in porous media sharply strengthen nonlinear processes so even weak fluctuations are accompanied by appreciable nonlinear effects.

5 Gamma-distribution of sizes of random structures. The role of average size deviation of structures in vibrations

One of the most close to the reality size's distribution of particles is two-parameter gamma distribution with probability density function

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \qquad (27)$$

At usual conditions about norm of (27) $F(\infty) = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{\alpha-1} e^{-\beta x} dx = 1$ an average value Mx is given by expression

$$Mx = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^\alpha e^{-\beta x} dx = \\ = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+1)}{\beta^{\alpha+1}} = \frac{\alpha}{\beta} \qquad (28)$$

Random value $l(\xi)$ is a real distance between blocks. The random value is ξ. The average value of $l(\xi)$ is equal to $< l(\xi) >= l_0$.

If we require, that average value in (28) was equal to the unit i.e. average distance between blocks is l_0, we need put $\beta = \alpha$ in (27) and the variance takes a form

$$\sigma^2 = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{\alpha-1} (x-1)^2 e^{-\beta x} dx = \frac{\alpha}{\beta^2} = \frac{1}{\alpha} \qquad (29)$$

It is evident that the average value of random variable $e^{\xi\omega}$ is equal

$$< e^{\xi\omega} >= \frac{\alpha^{\alpha}}{\Gamma(\alpha)}\int_0^{\infty} e^{\xi\omega}\xi^{\alpha-1}e^{-\alpha\xi}d\xi = \left(\frac{\alpha}{\alpha-\omega}\right)^{\alpha} \quad (30)$$

with $\omega = l_0(D_x n_x + D_y n_y + D_z n_z)$. Formula (30) gives a possibility to write operator P for gamma distribution of cracks. Using a Poisson formula (7) we have

$$P(D_x, D_y, D_z, \alpha) = \frac{1}{2}\int_{-1}^{1}\left(\frac{\alpha}{\alpha - l_0\sqrt{\Delta}t}\right)^{\alpha} dt \quad (31)$$

Value α determines from (29). In turn by using (31) we have the corresponding dispersion equation for random structures with Gamma distribution of $l(\xi)$ and $< l(\xi) >= l_0$

$$\frac{1}{2}\int_{-1}^{1}\left(\frac{\alpha}{\alpha - ikl_0 t}\right)^{\alpha} dt = \frac{k_S^2}{k^2} \quad (32)$$

At $\alpha \to \infty$ equation (32) is reducing to equation (14), namely $\frac{\sin(kl_0)}{kl_0} = \frac{k_S^2}{k^2}$ or in another form $z\sin(z) = \varepsilon^2$ with $z = kl_0$ and $\varepsilon = k_S l_0$.

In case of $|\mathrm{Im}(z)| = |y| < \alpha$ we can use the another form for the integral (31), namely [7]

$$\int_0^{\infty} x^{\alpha-2}e^{-\alpha x}\sin(zx)dx = \frac{\Gamma(\alpha-1)}{\left(\alpha^2+z^2\right)^{(\alpha-1)/2}}\sin\left((\alpha-1)\arctan\frac{z}{\alpha}\right) \quad (33)$$

Equation (33) may be written as

$$\frac{z\alpha\sin\left((\alpha-1)\arctan\frac{z}{\alpha}\right)}{\left(1+z^2/\alpha^2\right)^{(\alpha-1)/2}(\alpha-1)} = \varepsilon^2 \ \left(\alpha > |\mathrm{Im}(z)|\right) \quad (34)$$

For large values α we get an equation (14) with replacement of random value ξ per unit.

6 Dry friction in microinhomogeneous bodies

The surface friction force is the product of normal stress on friction coefficient p, namely

$$F_i = p\sigma_{jl}n_j n_l\left|\sin(n, w_i)\right| \quad (35)$$

In (35) n is the normal vector to contact of grains and w_i is the displacement vector. We define the volume friction force as a product of surface one on specific surface of contacts, i.e.

$$F_i = p\sigma_0\sigma_{jl}n_j n_l\left|\sin(n, w_i)\right| \quad (36)$$

The average value of $\left|sin(n, w_i)\right| = \frac{1}{2}$, so

$$F_i = \frac{1}{2}p\sigma_0(\sigma_{xx}\bar{n}_x^2 + \sigma_{yy}\bar{n}_y^2 + \sigma_{zz}\bar{n}_z^2)e_i \quad (37)$$

In (37) e_i is i-component of unit vector. For one dimensional case the equation of motion takes a form

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{1}{2}p\sigma_0\sigma_{xx} = \frac{1}{V^2}\ddot{u}_x \quad (38)$$

Thus, the phenomenon of friction initiates an appearance of diffusion term $\frac{1}{2}p\sigma_0 u'$ in equation of motion which is follow from (38) in the form

$$u'' + \frac{1}{2}p\sigma_0 u' = \frac{1}{V^2}\ddot{u} \quad (39)$$

For microinhomogeneous bodies the equation of motion (39) takes a form

$$Pu'' + \frac{1}{2}p\sigma_0(P-E)u' = \frac{1}{V^2}\ddot{u} \quad (40)$$

In (40) operator P for one dimensional case is given by expression (8).

7 Viscose friction in microinhomogeneous bodies

At a flow through granular collector with average radius r_0 local speed of a particle can be presented

$$v_r = u\cos\theta\left(1 - \frac{3r_0}{2r} + \frac{r_0^3}{2r^3}\right)$$
$$v_\theta = -u\sin\theta\left(1 - \frac{3r_0}{4r} - \frac{r_0^3}{4r^3}\right) \quad (41)$$

In (41) u means the fluid velocity in infinity. The divergence of this field is equal to zero, and this field itself satisfies to Navier-Stokes equation. Besides of it, on the interface grain-fluid, at $r = r_0$, the relative velocity is also equal to zero. The viscose stresses may be written as

$$\sigma_{rr} = 2\eta \frac{\partial v_r}{\partial r} = -2u\cos\theta\left(\frac{3r_0}{2r^2} - \frac{3r_0^3}{2r^4}\right)\Big|_{r=r_0} = 0$$
$$\sigma_{r\theta} = \eta\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right) = \qquad (42)$$
$$= -\frac{3}{2}u\sin\theta\frac{r_0^3}{r^4}\Big|_{r=r_0} = -\frac{3}{2r_0}\eta u\sin\theta$$

In (42) first and second equations are normal and tangent stresses in solid. Therefore, the viscose resistivity is equal to tangent stress in (36). Force of friction does not change the sign at variation only speed so it is necessary to use the module of $\sin\theta$. Taking into account identity $<|Sin\theta|>=\frac{1}{2}$ we can rewrite the volume friction force in the form

$$F_f = \frac{3\sigma_0}{4r_0}u = \frac{3\sigma_0}{4r_0}\dot{w} \qquad (43)$$

In (43) w is a field of displacement. Equation of motion for classic continuum is the telegraph equation, namely

$$(\lambda+2\mu)w_{xx} = \rho\ddot{w} + \frac{3\eta\sigma_0}{4r_0}\dot{w} \qquad (44)$$

Equation of motion for structured continuum is

$$(\lambda+2\mu)Pw_{xx} = \rho\ddot{w} + P\frac{3\eta\sigma_0}{4r_0}\dot{w} \qquad (45)$$

Diffusion terms in (44) and (45) means that resistivity is proportional to viscosity and specific surface of grains
.

8 Waves due to static loading in blocked media

Let's return to the Fig.2. In the volume, which bounded by surface *C* there is an equation of equilibrium, because all surface forces compensate each other, while in the volume bounded by surface *D*, there is no equation of equilibrium, because all surface forces concentrate on the one part of grain, and another its part has no forces. In order to describe, how the total statics can contain micro-dynamics, we should use operator $P-E$ for inertial forces. In this assumption, the equation of balance takes the form

$$\frac{\partial}{\partial x_k}[P(\sigma_{ik})] = \rho[P-E]\ddot{u}_i \qquad (46)$$

For classic continuum model $P \to E$ and inertial forces tend to zero. Actually, they are equal to zero only in the average, but not in each point. In blocked media, there is a possibility to have a lot of inertial forces with random orientation. It is an interesting question, when these events of micro-dynamics may create real, usual waves, which was initiated by static loading. In order to answer this question, it is necessary to solve the equation (46).
Assuming $f(z,\alpha) =$

$$= \frac{z\alpha}{(\alpha-1)\left(1+\frac{z^2}{\alpha^2}\right)^{(\alpha-1)/2}}\sin\left[(\alpha-1)\arctan\frac{z}{\alpha}\right]$$

and using formulas (46), (34) and (10), we can write the dispersion equation for quasi-static process in the form

$$f(z,\alpha) = \varepsilon^2\left[\frac{f(z,\alpha)}{z^2} - 1\right] \qquad (47)$$

In (47) $\frac{1}{\alpha} = \sigma^2$ is variance of gamma distribution.

9 Quasi-statics at whole and micro-dynamics in microstructure

In case of absence of macro-dynamics equation (11) for the micro-dynamic processes can be written as

$$\frac{\partial}{\partial x_k}[P(\sigma_{ik})] = [P-E]\rho\ddot{u}_i \qquad (48)$$

The presence of the operator $P-E$ in this equation means that in a classical continuum ($P=E$), dynamic phenomena are absent
This equation in one dimensional case for internal dry friction with coefficient p may be rewritten like

$$Pu'' - (P-E)u'\sigma_0 p = \frac{1}{c^2}(P-E)\ddot{u} \qquad (49)$$

Equation (48) describes intermediate (statics-dynamics) processes. Equation (49) describes the same processes with friction. The blocked medium has equilibrium state as a whole, but not in any of its points

$$P\frac{\partial \sigma_{ik}}{\partial x_k} = (P-E)\rho \ddot{u}_i \qquad (50)$$

Equation (50) derives from (46) due to commutativity of differential operator and, also P-E and P operators.
For internal dry friction with friction coefficient *p*, the dispersion equation for blocked medium becomes

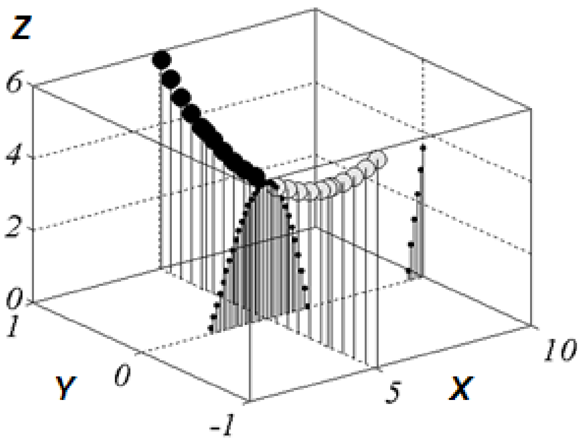

Fig.10 Waves in statics with α=300, i.e. at small variance of blocks sizes (z=x+iy).

$$\mathrm{Re}\frac{f(z,\alpha)z^2 + ip\delta\left(f(z,\alpha)-z^2\right)}{f(z,\alpha)-z^2} = \varepsilon^2 \qquad (51)$$

$$\mathrm{Im}\frac{f(z,\alpha)z^2 + ip\delta\left(f(z,\alpha)-z^2\right)}{f(z,\alpha)-z^2} = 0 \qquad (52)$$

In formulas (51)-(52) $\varepsilon = k_S l_0$, $z = kl_0$, p is friction coefficient, δ is specific surface of contacts. On the Fig.10 are shown the roots of dispersion equation corresponding to equation (48), representing waves in statics with α=300. It corresponds to small variation of the sizes of blocks. At the vicinity of origin of coordinates waves are absent (pure statics).Waves are beginning with very small velocities of them (Z<<X). Small black points mean stable solutions. Catastrophes (large black balls) are beginning when wave velocities tend to shear waves ones ($Z \approx X$). Large white balls mean damping solutions.

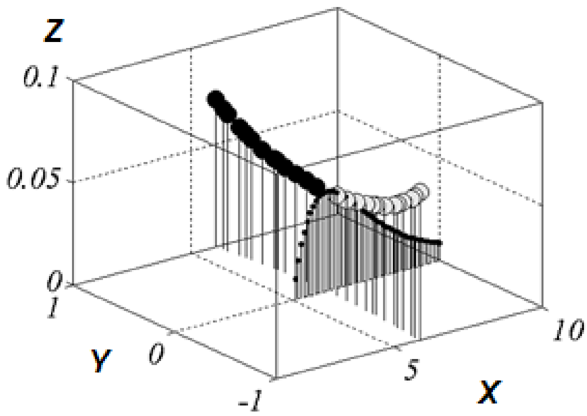

Fig.11 Waves in statics with α=5, i.e. at large variance of blocks sizes (z=x+iy).

On the Fig.11 are shown waves in statics with α=5. It corresponds to large variation of the sizes of blocks. At the vicinity of origin of coordinates waves are absent (pure statics). In this case waves also are beginning at very small velocities (*Z*<<*X*). Small black points mean stable solutions. In that case catastrophes (large black balls) are beginning at very small wave velocities (*Z*<<*X* too) with compare to shear wave velocities.

10 Conclusions

1. The new model of the structured continuum with an account of specific surface of blocked medium or average size of structure, gives us the differential equations of motion of the infinite order. This model predicts besides of usual elastic waves a lot of unusual waves with very small velocities. The reason of these unusual waves consists in the infinite degrees of freedom for such media.
2. In blocked media the representative volume element has finite size. Hence the equilibrium state is possible on some minimal volume, but not on arbitrary small volume. It means that macro-statics may contain micro-dynamics. It is shown, that separate dynamic acts also may produce waves, namely a slow and a fast usual waves, and can generate instabilities due to static loads.
3. The dry friction can block catastrophes. For almost periodic structures this effect is not very strong, while for structures with high variation of sizes such blocking effect is much more than for periodic medium. Geometrical chaos stabilizes the

blocked medium. However, the decreasing of a friction coefficient due to fluids or to temperature is the instability factor, especially for almost periodic structures.

4. The number of instable solutions relates to energy of them like in seismological law of Gutenberg–Richter

5. For usual continuous medium nonlinear effects in weak waves are negligible small. But in structured media this effect is not small, it is real phenomenon.

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