Dynamic analysis of cracks running at a constant velocity in a strip

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Abstract: - The scattering of a time-harmonic anti-plane shear (SH) wave by finite length cracks with constant velocity in an elastic medium is considered. In the first case, cracks are assumed to propagate in an infinite elastic space. In the second case, the problems of propagation of subsurface crack and edge crack propagating with uniform velocity in a half-space are treated. In the third case, based on the extension of the dislocation model and images method for the free surface, an analysis of the scattering of SH waves by cracks moving in a strip is carried out. The effects of the wave number, crack velocity and relative position of the cracks are presented. The dynamic stress intensity factors (SIFs) are numerically computed and the results are shown graphically.

Key-Words: - Finite length moving cracks, Antiplane shear waves, Galerkin numerical method, Colinear cracks, Half-space, Strip

1 Introduction

The study of elastodynamic problems of moving or extending cracks has received considerable attention in the field of fracture mechanics. Considerable progress has been made toward the understanding of the problem of diffraction of SH waves by a stationary line crack [1-3]. However, the angular distribution of the local stress will be distorted if the crack moves in the elastic solid. One of the first studies of a traveling crack was made by Yoffé [4] who introduced the model of a finite width crack extending uniformly through an elastic medium. This model was improved by Broberg [5] who considers the crack tips to move in opposite directions with constant velocities. Sih [6] made use of the Riemann-Hilbert problem in complex function theory and obtained the solutions of general loading conditions and various crack geometries. Based on the extension of the integral transforms method, the dynamic problem of a running crack opened out by anti-plane shear waves has been solved by Sih and Loeber [7]. It is found that the dynamic stresses can be higher than the static ones depending upon the frequency of the incoming waves and speed of crack propagation. Since similar topics are of great interest to both the academic and engineering communities, it has received lasting attention from the researchers [8]. Singh *et al.*[9] used Fourier transform method to solve the problem of diffraction of SH waves by a running crack of finite length. The problem of diffraction of SH waves by two co-planar Griffith cracks of finite length was investigated by Dhaliwal *et al.*[10]. Using the finite Hilbert transform technique, a solution of the pair of triple integral equations is obtained for the small wave number.

Scattering of elastic waves by near surface cracks in a homogeneous, isotropic medium has important applications in seismology and nondestructive testing. The concerns with mechanical failures or earthquake disaster initiating largely at the one or more cracks have led to extensive studies for the purpose of understanding the interaction between flaws that may exit in these regions and applied loads and other environmental factors [11]. Most of the existing solutions mentioned above invoke the simplifying assumption that the crack travels in an infinite medium. The propagation characteristics of the waves depend on structural boundaries such as those in plates, tubes, rods, and embedded layers. The solution of the problems involving moving cracks in a bounded medium poses difficulties and sometimes it is impossible to obtain in analytical treatment by the presence of the free surface. By using the Riemann-Hilbert formulation together with Schwartz-Christoffel transformation, Sih and Chen [12] have obtained the solution of the problem of a running crack with semi-infinite length in a strip under anti-plane shear. The assumption of a constant length crack is expected to be reasonable for long cracks in a narrow strip where the reflected stress waves from the external boundaries are more dominant than the waves generated by the trailing end of the crack. With the help of conformal mapping method, the exact solution for a propagating semi-infinite crack in a strip was solved by Fan [13]. Wang and Wang [14] established a moving dislocation model of a propagating self-similar interface crack and obtain Cauchy singular equations from which the dynamic SIFs were calculated. By the use of Fourier transform and finite Hilbert transform techniques, the distribution of stress and displacement due to propagation of three co-planar Griffith cracks with constant velocity under normally incident SH wave at the interface of two dissimilar elastic media were investigated by Das [15].

Solutions of many of the problems considering the free surface effect were obtained by various methods, and these studies lay a foundation for knowledge of signal interpretations of nondestructive testing. The effect of free surface on dynamic stress intensity factor of a moving center-crack in a finite solid is investigated by Agrawal [16]. It is predicted that for high speeds of crack propagation, crack front maintains its straight profile. A modified zigzag approach was proposed by Liu et al. [17] to track the shape changes of crack front during crack propagation based on virtual crack closure-integral technique. Analytical solutions for an anti-plane Griffith moving crack inside an infinite magnetoelectroelastic medium under the conditions of permeable crack faces are formulated using integral transform method by Hu and Li [18]. Several studies regarding the dynamic crack propagating problem of a moving crack in a piezoelectric ceramic strip have been reported [19]. Singular stresses and electric fields around an eccentric crack moving at constant speed in a piezoelectric ceramic strip which is sandwiched between two elastic half planes are determined by Soon et al. [20] by the integral transform approach. Application of dislocation methods has in recent years provided some new insights. In dealing with the scattering field around damage area, especially when multiple cracks and boundaries are in existence, the dislocation model is one of the powerful tools for the solution. The effects of SH waves on the SIFs of two cracks at arbitrary positions in an infinite space and half-space had been analyzed by using a dislocation model developed by Huang [21]. In the present study, the dislocation model is extended to determine the problem of wave scattering by multiple moving cracks in various geometries. Numerical results are presented for SIFs for various values of wave numbers, crack velocities, crack positions and orientations.

To show the versatility of the dislocation method and to approach the complicate problem step by step, three kinds of elastodynamic problems are solved consecutively in this paper. First, considering the running cracks subjected to a SH wave in an infinite space. Secondly, the problem of running cracks in a half-space subjected to SH wave are analyzed. Finally, the dynamic SIFs of running cracks in a strip engulfed in SH waves are calculated. All the cracks considered in this paper are assumed to posses a finite length with a=1 while it propagates at a uniform velocity v.

2 A running crack in an infinite medium stretched by a horizontal shear stress

This example is given in the first place to describe the concept of dislocation method and the related numerical scheme. By employing the same procedure, the following three kinds of problems studied in this paper can then be solved. For analyzing the problem of a crack extending at a constant velocity, it is convenient to use the Galilean transformation, i.e., by introducing a set of moving axis (x,y,z) attached to the middle of the crack plane such that it is at a distance vt measured form the fixed axis (X,Y,Z) as follows

$$x = X - vt,\ \ y = Y,\ \ z = Z \tag{1}$$

Referring to Fig.1, the crack occupies the region y=0, $|x| \leqq a$, and the uniform motion of the crack is maintained by a anti-plane shear stress of magnitude τ_o

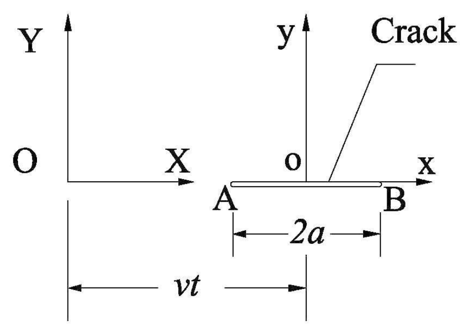

Fig.1. Moving crack and coordinate system.

Based on the fact that the stress distribution close to one end of the crack is not influenced by its distance from the other end [4], it is assumed that the crack reseals itself spontaneously and its length remains constant at all times. Consider the plane strain deformation of an infinite body, i.e., the particles in the elastic body move only in the *X* and *Y* directions. Then the boundary conditions satisfied by this problem for a crack loaded in anti-plane shear are:

$$\tau_{ZY}(X,0,t) = -\tau_0, \qquad |x| \le a \tag{2}$$

Considering the static condition, that is, when the moving velocity of the crack v=0. These conditions are all readily satisfied by representing the stationary crack as an array of screw dislocations with density function $D(x)$. Applying the traction boundary condition, we have the equilibrium equation

$$\frac{\mu\gamma b}{2\pi}\int_{-a}^{a}\frac{D(s)}{x-s}ds + \tau_0 = 0 \tag{3}$$

where

$$\gamma = \sqrt{1-\frac{v^2}{c^2}} = \sqrt{1-M^2}$$

The shear wave velocity is given by $c=(\mu/\rho)^{1/2}$, ρ is the density of material, μ is the shear modulus and b is the Burgers vector. The ratio $M=v/c$ is referred to as the Mach number, which is smaller than 1. The Cauchy type singular integral equations have received considerable attention from a number of researchers over the last seventy years [22]. The solution of the dislocation density can readily be obtained analytically as

$$D(x) = \frac{2x\tau_o}{\mu\gamma b\sqrt{a^2 - x^2}} \tag{4}$$

Then, the value of the SIFs can be obtained as

$$K_3 = \tau_0\sqrt{a} \tag{5}$$

In the case of uniform motion, where v=constant, the displacement field surrounding the dislocation must appear to be constant with time in the moving coordinate system. The solution of the stress can be expressed as [23]

$$\sigma_{YZ} = \mu\frac{\partial u_z}{\partial y} = -\frac{\mu b}{2\pi}\frac{\gamma(x-vt)}{(x-ct)^2+\gamma^2 y^2} \tag{6}$$

Although the stress singularity is the same as in static loading, the angular dynamic stress pattern will change in accordance with crack speed. Considering an array of moving dislocations which representing a crack running with a constant velocity, there is no relative velocity between the dislocations located at *x'* and *s'*. Therefore, through the coordinate transformation, the equilibrium equations take the same form as that of stationary one shown in Eq.(3). Again, we have the dynamic SIFs $K_3 = \tau_0\sqrt{a}$, where K_3 is independent of the crack velocity and is the same as in static loading. Note that although the parameter γ is included Eq.(6), the SIFs is not influenced by this parameter.

The unknown dislocation density function in Eq. (3) can also be obtained by the numerical method proposed in reference [21]. Approximating the density function by their first kind of Chebyshev polynomial in the following form

$$D(s) \approx \sum_{n=0}^{N}\frac{a_n T_n(s)}{\sqrt{1-s^2}}, \quad |s| \le 1 \tag{7}$$

Let a=1, the unknown coefficients a_n can be obtained by changing Eq.(3) into the form

$$\int_{-1}^{1}\frac{U_m(x)}{\sqrt{1-x^2}}\left\{\frac{\mu\gamma b}{2\pi}\left[\int_{-1}^{1}\sum_{n=1}^{N}\frac{a_n}{\sqrt{1-s^2}}\frac{T_n(s)}{s-x}\right]ds + \tau_o\right\}dx = 0 \tag{8}$$

where U_m is the second kind of Chebyshev polynomial. The divergent integration can be obtained by

$$\int_{-1}^{1}\frac{1}{x-s}\frac{T_n(s)}{\sqrt{1-s^2}}ds = \begin{cases} -\pi U_{n-1}(x) & n \ge 1 \\ 0 & n = 0 \end{cases}$$

$$|x| \le 1 \tag{9}$$

Following the integration procedures depicted in reference [21], the unknowns a_n can be determined. In this case, the only nonzero unknown coefficients left is $a_1 = 2\tau_0 s/\gamma\mu b$. Thus, we have the same dislocation density function as the closed form solution shown in Eq.(4). Therefore, the same SIFs can be derived. Since the expansion of the Chebyshev polynomial converges quickly, the numerical method is effective in solving more complicated problems easily.

3 Crack propagating at a constant velocity in an infinite space subjected to SH waves

Consider a finite length crack running at a constant velocity *v* along the *x*-axis and it is engulfed in a harmonic SH waves arriving from infinity at an incident angle *θ*. The displacement field of the undisturbed region can be expressed mathematically as

$$u_z^{(i)} = u_o \exp\{-i[\alpha(x\cos\theta + y\sin\theta) - \omega t]\} \quad (10)$$

Here $\alpha(=\omega/c)$ is the free-space wave number; ω is the circular frequency. If the disturbance due to the crack motion is confined within the *X*-*Y* plane of a rectangular coordinate system (*X*,*Y*, *Z*), then only plane elastic wave arise. Let the disturbance in the body be such that the particles are displaced only in the *Z*-direction. In this case, only two of the six stress components are nonzero, σ_{XZ} and σ_{YZ}. The theoretical analyses of this problem were made by Sih [8]. By applying the coordinate transformation to the wave equation, the displacement function $u_z(x,y,t)$ will be degenerated into the following form

$$c^2 s^2 (u_{z,xx} + u_{z,yy}) + 2Mc(u_{z,x} - u_{z,t}) = 0 \quad (11)$$

In the moving coordinate system, Eq. (10) can be rewritten as

$$u_z^{(i)} = u_o \exp\{i\beta[(Mx\alpha^* - \omega^* t) - \alpha^*(x\cos\theta + y\sin\theta)]\} \quad (12)$$

where

$$\alpha^* = \alpha\beta/\gamma^2,$$
$$\beta = 1 + M\cos\theta,$$
$$\omega^* = \beta\omega$$

Based on the appearance of Eq.(12), Sih [8] proposed the incident displacement potential be written in terms of x and y as

$$u_z^{(i)}(x,y,t) = u_o \exp\{-i[\alpha^*(x\cos\theta^* + y\sin\theta^*)]\} \exp\{i\beta[M\alpha^* x - \omega^* t]\} \quad (13)$$

where

$$U_z^{(i)}(x,y) = u_o \exp\{-i[\alpha^*(x\cos\theta^* + y\sin\theta^*)]\}$$

Here the apparent parameters are given by

$$\sin\theta^* = \frac{\gamma\sin\theta}{\beta}, \cos\theta^* = \frac{M + \cos\theta}{\beta} \quad (14)$$

Substituting Eq.(12) into the transformed wave equation, $U_z^{(i)}(x,y)$ is found to satisfy a Helmholtz equation, which is in the same form as derived from the problem of diffraction of SH waves by a stationary crack. Thus, by introducing the apparent parameters, the total stress wave released from the moving crack can be obtained by replacing *ω, α, θ* with the apparent frequency ω^*, apparent wave number α^* and apparent incidence angle θ^*, respectively.

The class of problems involving cracks running in elastic medium with various boundaries can also be solved by employing the dislocation method together with the introduction of the apparent parameters. To fulfil the requirements of the equation of motion of each dislocation on the plane of *y*=0, the dislocation density function must satisfy the following equation for all points in the range of $-a \leqq x \leqq a$,

$$\int_{-a}^{a} D(s) A\alpha^{*2} \{[J_o(\alpha^*|x-s|) + J_2(\alpha^*|x-s|)] \cos[\omega^* t + p(s)] + [Y_o(\alpha^*|x-s|) + Y_2(\alpha^*|x-s|)] \cos[\omega^* t + p(s)]\} ds = \sigma_o \frac{\beta}{\gamma^2} \sin\theta^* \cos[\alpha^* \cos\theta^* - \omega^* t] \quad (15)$$

where $A = -bA_o\mu/8$, A_o is the amplitude of the screw dislocation, μ is the shear modulus, $p(s)$ is the phase lag compared with incident wave. $\sigma_o(=\mu u_o \alpha)$ denotes the value of the shear stress at the incident wave front and is taken to remain finite as ω approaches zero. J_o and J_2 are the Bessel functions of the first kind of order zero and two, respectively. Y_0 and Y_2 are the Bessel functions of the second kind of order zero and two, respectively. The left hand side of the Eq. (15) is the same as the governing equation for single stationary crack subjected to SH wave as stated in reference [3], while the right hand side of the equation is modified from the original incident wave due to the crack movement. Applying the same approach as stated in reference [3], the singular integral equation can be solved and the dynamic SIFs are obtained. The values of SIFs vary with wave numbers for different values of crack velocities as illustrated in Fig. 2. For small wave numbers, it is observed from the graphs that the SIFs increase in magnitude with velocity and the values are in accordance with the results obtained by Singh [9]. While for larger wave numbers, refer to the curves shown in Fig.2, the SIFs decreases in magnitude with the crack velocity as it has been stated by Sih [8].

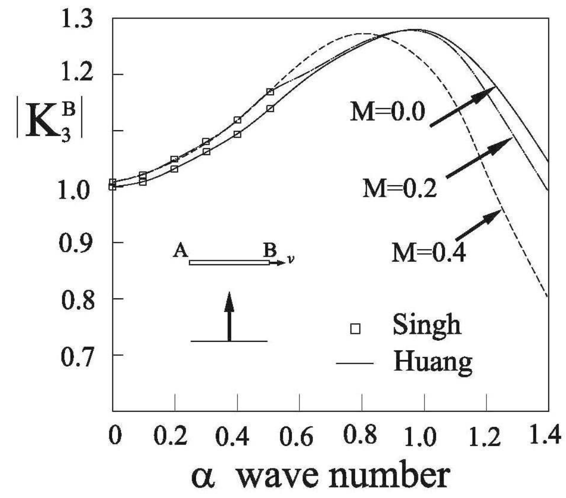

Fig.2. Variation of SIFs vs. wave numbers for incident angle of 90° and M=0, 0.2, 0.4.

The dislocation method can be extended to solve the problem of scattering of SH wave by two cracks without difficulty [25]. To show the interaction between two cracks, the SIFs of two moving co-planar crack subjected to normally incident wave are displayed in Fig.3. The analysis data agree with the results obtained by Itou [24] for a zero crack velocity condition.

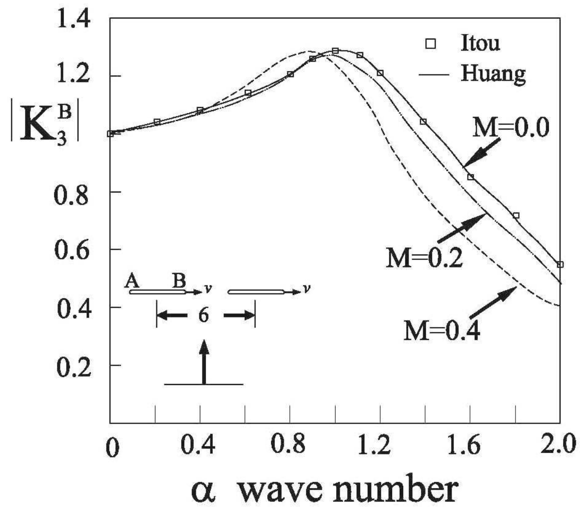

Fig.3. SIFs vs. wave numbers for colinear cracks subjected to normally incident wave.

The SIFs of the inner tips of two co-planar moving cracks for different horizontal distance and propagating velocities are presented in Fig.4. The values of SIFs increase rapidly as the distance between two cracks is very small. As the distance between two co-planar cracks become large enough, the calculated SIFs, which are in accordance with the results of Loeber [1] for a zero crack velocity condition, can be treated as those of a single crack subjected to normally incident wave.

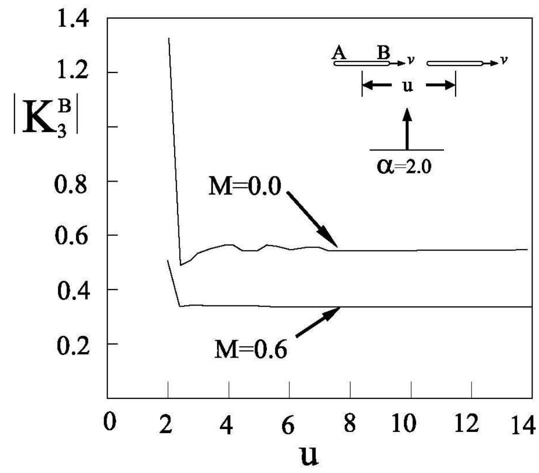

Fig.4. Effect of the distance between two colinear cracks on the SIFs for different crack velocities.

4 A finite length crack moving in a half-space with traction free boundary

Based on the image concept, the results for a crack in an infinite medium can provide an estimate of the stress field for a crack in a half-space solid. Let $D(x)$ be the density function of an array of continuous distribution of infinitesimal screw dislocations. By assuming an array of image dislocations with density function $-D(x)$ and at a distance ℓ above the free surface, the zero traction condition on the surface of the half-space is satisfied. Due to the existence of the boundary, not only the incident wave $\sigma_{yz}^{(i)}$ will impinge on the crack face, but also the stress wave reflected from the free surface $\sigma_{yz}^{(r)}$ will have effect on the crack. Again, when dealing with the moving crack problem, the total stress wave due to the crack movement can be obtained by replacing ω, α, θ with the apparent parameter ω^*, α^* and θ^*, respectively. To fulfill the requirements of the equation of motion of each dislocation on the crack within $|x| \leq 1$, we have

$$\int_{-1}^{1} D(s) A \alpha^{*2} \{[J_o(\alpha^*|x-s|) + J_2(\alpha^*|x-s|)]$$
$$\cos[(\omega^* t + p(s)] + [Y_o(\alpha^*|x-s|) + Y_2(\alpha^*|x-s|)]$$

$$\sin[(\omega^* t + p(s)]\}ds +$$

$$\int_{-1}^{1} -D(s')A\alpha^{*2}\{\{[\frac{2y'^2}{R^2}J_o(\alpha R) + (\frac{2}{\alpha R} - \frac{4y'^2}{\alpha R^3})$$

$$J_1(\alpha R)]\cos 2\phi + [\frac{2xy'}{R^2}J_o(\alpha R) - \frac{4xy'}{\alpha R^3}J_1(\alpha R)]$$

$$\sin 2\phi\}\cos[(\omega^* t + p(s')] + \{[\frac{2y'^2}{R^2}Y_o(\alpha R)$$

$$+(\frac{2}{\alpha R} - \frac{4y'^2}{\alpha R^3})\ Y_1(\alpha R)]\cos 2\phi + [\frac{2xy'}{R^2}Y_o(\alpha R)$$

$$-\frac{4xy'}{\alpha R^3}\ Y_1(\alpha R)]\sin 2\phi\}\sin[(\omega^* t + p(s')]\}ds'$$

$$= 2\sigma_0 \frac{\beta}{\gamma^2}\{-\sin\theta^* \sin(\alpha^* \bar{y} \sin\theta^*)$$

$$\sin\left[\alpha^* x\cos\phi\cos\theta^* - (\alpha^* Mx - \omega^* t)\right]\cos\phi$$

$$+(\cos\theta^* - M)\cos(\alpha^* \bar{y}\sin\theta^*)$$

$$\cos\left[\alpha^* x\cos\phi\cos\theta^* - (\alpha^* Mx - \omega^* t)\right]\sin\phi\} \quad (16)$$

where

$$R = \sqrt{(2h + x\sin\phi - s'\sin\phi)^2 + (x\cos\phi - s'\cos\phi)^2}$$

$$y' = -(2h\cos\phi + x\sin 2\phi)$$

$$\bar{y} = -h - x\sin\phi$$

Note that, when the inclined angle of crack, ϕ, is not zero, the incident displacement potential still can be written in the same form as shown in Eq. (12), but the parameter β is now become $1+M\cos(\theta+\phi)$. The right hand side of Eq. (16) stands for the combination of the amplitudes of incident and reflection wave that impinge on the crack surface.

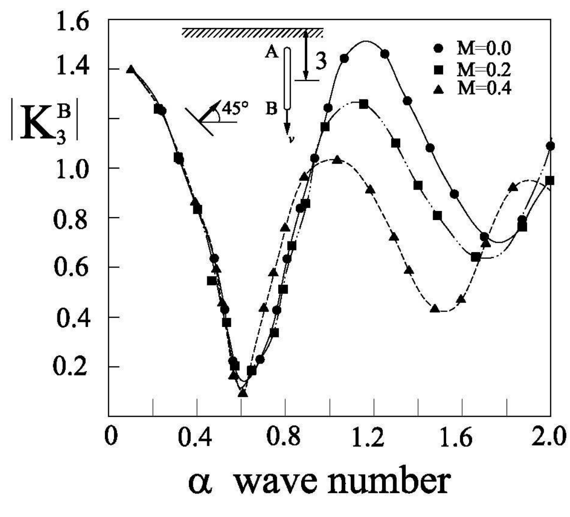

Fig.5. SIFs of propagating vertical cracks in a half-space subjected to inclined incident wave.

Applying the same approach as shown in reference [21], the dynamic SIFs of finite length crack moving in a half-space can be obtained. Since it is of absorbing interest for NDT purpose to investigate edge cracks or near surface defects, the related problem are also studied in this paper. In Fig. 5, the SIFs of a vertical crack near the free surface subjected to an inclined incident wave are presented. It appears that the SIFs decrease in magnitude with velocity.

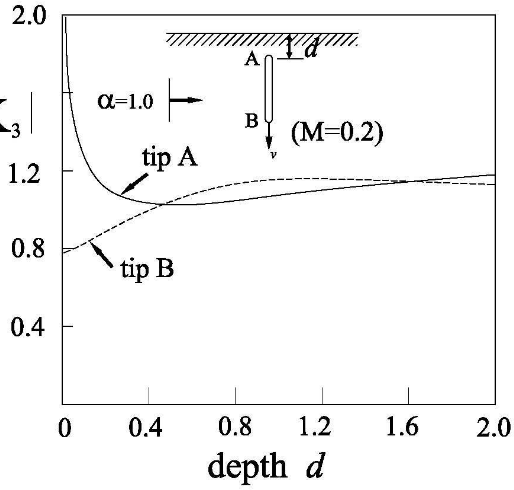

Fig.6. Variation of the SIFs with the depth of the crack beneath the free surface.

In order to reveal the effect of the free surface, the variations of SIFs versus the depth of the crack are depicted in Fig. 6. As the depth of the crack beneath the free surface become small, the SIFs of outer crack tip (tip A) will increase significantly. This means that when the subsurface crack is very close to a free surface, it will be easily extended to an edge crack. On the other hand, as the depth d is less than 0.01, the SIFs of the inner crack tip (tip B) will become a stable constant value. Therefore, we can draw the conclusion that the problem of the diffraction of elastic waves by edge cracks can be solved by assuming the distance between crack tip and free surface is less than 0.01.

The variations of SIFs of inclined edge crack versus the wave numbers are given in Fig. 7. It is noted that the locations of the peaks of SIFs move into the lower wave number as the crack velocity is increased.

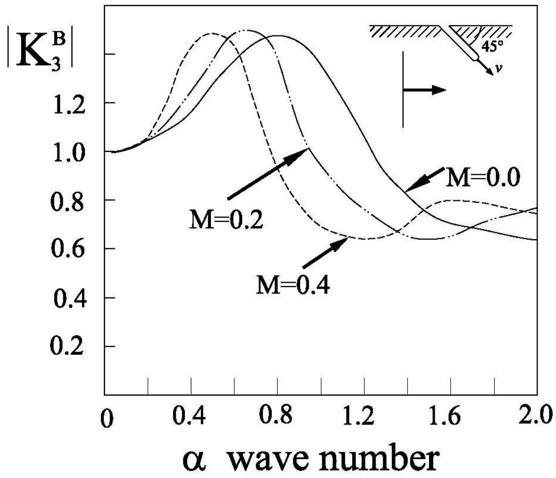

Fig.7. Variation of the SIFs of inclined edge cracks subjected to grazing wave.

5 Finite length cracks moving in a strip

When dealing with the scattering of waves in a strip-like structure, it is far more complicated because of multiple reflections. For assuring in-service safety of strip-like structures, the multiple subsurface and edge cracks developed in the structures were often detected by an ultrasonic non-destructive technique. Due to the guiding effect of the strip surface, waves can propagate over distance. These waves are usually generated by impinging the strip obliquely with a tone-burst from a transducer, and it is easy to test the entire structure in a single measurement. A guided wave consists of many different modes that propagate independently through the media, however, every curve in the figures presented in this paper is restricted to certain mode only.

The idea of imaging the primary source with respect to free surfaces can also be used to determine the Green's function for a region bounded by two planes. Consider a rectangular Cartesian coordinate system locates at the middle of a strip. Two non-planar cracks not in contact with each other are located at an arbitrary position in the strip. The origins of cracks are at a distance h_1 and h_2, respectively, below the upper free surface of the strip. Considering waves in the strip-like structure mentioned above, the solution of the displacement field can be expressed as [26]

$$u_Z = u_0\left[B_1 \sin(qy) + B_2 \cos(qy)\right] \exp\{i(\alpha x \cos\theta - \omega t)\} \quad (17)$$

where

$$q = n\pi / 2H$$

$$n = 2\alpha H \sin\theta / \pi$$

Here, H is half of the strip thickness, θ is the angle between x-axis and the normal of the incident wave; B_1=0 and, n=0,2,4...for symmetric modes, B_2=0 and n=1,3,5...for antisymmetric modes. The harmonic SH waves may propagate in a strip only under special conditions. Note that harmonic SH-waves in an elastic strip are dispersed except for n=0, which represents for $\theta = 0^{\circ}$, that is, there is no reflected waves for grazing incident. To degenerate the problem of diffraction of SH waves by a moving crack to that of a stationary crack, the solution of the displacement field should be of the form as Eq. (11). We have

$$u_Z = u_0\left[B_1 \sin(qy) + B_2 \cos(qy)\right] \exp\{i\alpha^* x \cos\theta^*\} \exp\{-i\beta[M\alpha^* x - \omega^* t]\} \quad (18)$$

Based on the dislocation model, the crack subjected to SH-waves can be simulated by superimposing an array of continuous distribution of vibrating infinitesimal screw dislocations. Let $D_i(x)$ be the distribution function of the dislocation density of crack No. i, i=1,2. Based on the concept of image method, the results of a vibrating screw dislocation in an infinite medium can give an estimate of the stress field for a vibrating screw dislocation in a strip.

The Green's function for a region bounded by two planes can be obtained by taking the image of the primary source with respect to both free surfaces, say $y = -H$ and $y=H$. The image with respect to $y=H$ is an array of vibrating image screw dislocations located at $3H > y > H$ and which has the same, but opposite sign, density functions. However, the existence of this array of image dislocations will destroy the symmetry of the primary source and its image with respect to $y = -H$. Thus, another source in the region $-3H < y < -H$ must be added to restore symmetry, in another word, to keep the free surface at $y = -H$ traction-free. The system of sources must also again be symmetric with respect to $y=H$, therefore, another source at $5H > y > 3H$ is needed, and so forth. Consequently, to satisfy the conditions of traction-free at $y = -H$ and $y = H$, infinite number of arrays of dislocations are needed. Therefore, the problem of diffraction of SH waves by non-planar cracks embedded in a strip degenerates into diffraction of SH waves by two set of infinite non-planar array of vibrating screw dislocations with their images interlacing in an infinite space. To fulfill the requirements of the equation of motion of each dislocation on crack No.1, the following equation for all points within crack No.1 must be satisfied.

$$\sum_{\ell=-\infty}^{+\infty}\Big\{\int_{-1}^{1}D_1(s)\Big[\big(a_{1,\ell}-a_{2,\ell}\cos 2\phi_1-e_{2,\ell}\sin 2\phi_1\big)\cos(\omega t+P_1)+\big(b_{1,\ell}-b_{2,\ell}\cos 2\phi_1-f_{2,\ell}\sin 2\phi_1\big)\sin(\omega t+P_1)\Big]ds+\sum_{i=3}^{4}\int_{-1}^{1}D_2(s')\Big[(-1)^{i+1}(a_{i,\ell}\cos\phi_i+e_{i,\ell}\sin\phi_i)\cos(\omega t+P_2)+(b_{i,\ell}\cos\phi_i+f_{i,\ell}\sin\phi_i)\sin(\omega t+P_2)\Big]\Big\}ds'$$

$$=\sigma_0 G_k(x_1,y_1,\phi_1,t)/A\alpha^{*2} \qquad (19)$$

While for crack No.2, the following equation must be satisfied.

$$\sum_{\ell=-\infty}^{+\infty}\Big\{\int_{-1}^{1}D_1(s)\sum_{i=7}^{8}(-1)^{i+1}\Big[(a_{i,\ell}\cos\phi_{i-4}+e_{i,\ell}\sin\phi_{i-4})\cos(\omega t+P_1)+(b_{i,\ell}\cos\phi_{i-4}+f_{i,\ell}\sin\phi_{i-4})\sin(\omega t+P_1)\Big]ds+\int_{-1}^{1}D_2(s')\Big[(a_{5,\ell}-a_{6,\ell}\cos 2\phi_2-e_{6,\ell}\sin 2\phi_2)\cos(\omega t+P_2)+(b_{5,\ell}-b_{6,\ell}\cos 2\phi_2-f_{6,\ell}\sin 2\phi_2)\sin(\omega t+P_2)\Big]\Big\}ds'$$

$$=\sigma_0 G_k(x_2,y_2,\phi_2,t)/A\alpha^{*2} \qquad (20)$$

Where

$$G_k(x_i,y_i,\phi_i,t)=(-1)^{k+1}\Big\{\frac{n\pi}{2H\alpha}\sin(q\bar{y}_i+\frac{k\pi}{2})\cos\big[\alpha^*\cos\theta\bar{x}_i+(M\alpha^*\bar{x}_i-\omega^*)t\big]\cos\phi_i+\frac{\beta}{\gamma^2}\cos(q\bar{y}_i+\frac{k\pi}{2})\sin\big[\alpha^*\cos\theta\bar{x}_i+(M\alpha^*\bar{x}_i-\omega^*)t\big]\sin\phi_i\Big\}$$

k = 0 for symmetric modes, and k = 1 for antisymmetric modes.

$$\bar{y}_j=H-h_j-x\sin\phi_j,$$

$$\bar{x}_j=x\cos\phi_j+(-1)^j u/2,\ j=1,2$$

$$h_+=h_2+h_1,\ h_-=h_2-h_1,$$

$$\phi_3=|\phi_2-\phi_1|,\ \phi_4=\phi_2+\phi_1$$

$$a_{i,\ell}=(2y_{i,\ell}^2/R_{i,\ell}^2)J_0(\alpha R_{i,\ell})+\big[2/\alpha R_{i,\ell}-4y_{i,\ell}^2/\alpha R_{i,\ell}^3\big]J_1(\alpha R_{i,\ell})$$

$$b_{i,\ell}=(2y_{i,\ell}^2/R_{i,\ell}^2)Y_0(\alpha R_{i,\ell})+\big[2/\alpha R_{i,\ell}-4y_{i,\ell}^2/\alpha R_{i,\ell}^3\big]Y_1(\alpha R_{i,\ell})$$

$$e_{i,\ell}=(2x_{i,\ell}y_{i,\ell}/R_{i,\ell}^2)J_0(\alpha R_{i,\ell})-(4x_{i,\ell}y_{i,\ell}/R_{i,\ell}^3)J_1(\alpha R_{i,\ell})$$

$$f_{i,\ell}=(2x_{i,\ell}y_{i,\ell}/R_{i,\ell}^2)Y_0(\alpha R_{i,\ell})-(4x_{i,\ell}y_{i,\ell}/R_{i,\ell}^3)Y_1(\alpha R_{i,\ell})$$

$$R_{i,\ell}^2=x_{i,\ell}^2+y_{i,\ell}^2$$

$$y_{1,\ell}=-4\ell H\cos\phi_1$$

$$y_{2,\ell}=-2h_1\cos\phi_1-x\sin 2\phi_1-4(\ell-1)H\cos\phi_1$$

$$y_{3,\ell}=-\sqrt{h_-^2+u^2}\sin\big[\phi_2-\tan^{-1}(h_-/u)\big]+x\sin\phi_3-4\ell H\cos\phi_2$$

$$y_{4,\ell}=-\sqrt{h_+^2+u^2}\sin\big[\phi_2-\tan^{-1}(h_+/u)\big]+x\sin\phi_4-4(\ell-1)H\cos\phi_2$$

$$y_{5,\ell}=-4\ell H\cos\phi_2$$

$$y_{6,\ell}=-2h_1\cos\phi_2-x\sin 2\phi_2+4(\ell-1)H\cos\phi_2$$

$$y_{7,\ell}=\sqrt{h_-^2+u^2}\sin\big[\phi_1-\tan^{-1}(h_-/u)\big]-x\sin\phi_3-4\ell H\cos\phi_1$$

$$y_{8,\ell}=-\sqrt{h_+^2+u^2}\sin\big[\phi_1+\tan^{-1}(h_+/u)\big]-x\sin\phi_4+4(\ell-1)H\cos\phi_1$$

$$R_{1,\ell}^2=\big[(x-s)\cos\phi_1\big]^2+\big[4\ell H+(x-s)\sin\phi_1\big]^2$$

$$R_{2,\ell}^2=[(x-s)\cos\phi_1]^2+[4(\ell-1)H+2h_1+(x+s)\sin\phi_1]^2$$

$$R_{3,\ell}^2=(u+s\cos\phi_2-x\cos\phi_1)^2+(4\ell H-h_--s\sin\phi_2+x\sin\phi_1)^2$$

$$R_{4,\ell}^2=(u+s\cos\phi_2-x\cos\phi_1)^2+\big[4(\ell-1)H+h_++s\sin\phi_2+x\sin\phi_1\big]^2$$

$$R_{5,\ell}^2=\big[(x-s)\cos\phi_2\big]^2+\big[4\ell H+(x-s)\sin\phi_2\big]^2$$

$$R_{6,\ell}^2 = \left[(x-s)\cos\phi_2\right]^2 + \left[4(\ell-1)H + 2h_2 + (x+s)\sin\phi_2\right]^2$$

$$R_{7,\ell}^2 = (u + s\cos\phi_2 - s\cos\phi_1)^2 + \left[4\ell H + h_- + x\sin\phi_2 - s\sin\phi_1\right]^2$$

$$R_{8,\ell}^2 = (u + s\cos\phi_2 - s\cos\phi_1)^2 + \left[4(\ell-1)H + h_+ + x\sin\phi_2 + s\sin\phi_1\right]^2$$

According to the relation *nπ=2αHsinθ*, the parameters including the wave number, wave mode, strip thickness and angle of incidence are related. Once the values of strip thickness and angle of incidence are fixed, the number of mode of guided wave will be a function of the wave number. The SIFs of cracks subjected to incident wave of angle 45° are depicted in Fig.8.

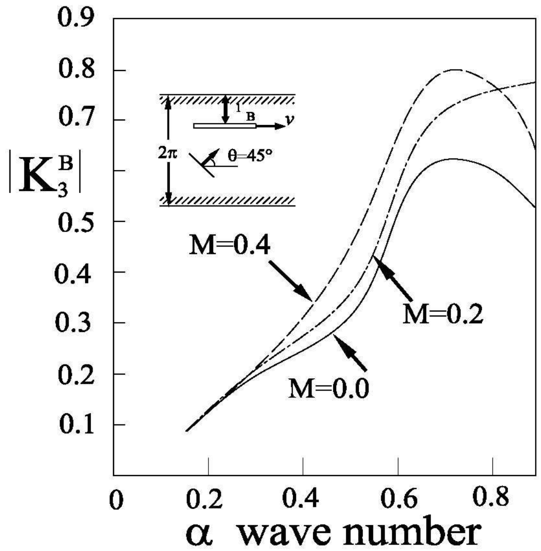

Fig.8. Variation of SIFs with wave numbers for crack subjected to inclined incident wave.

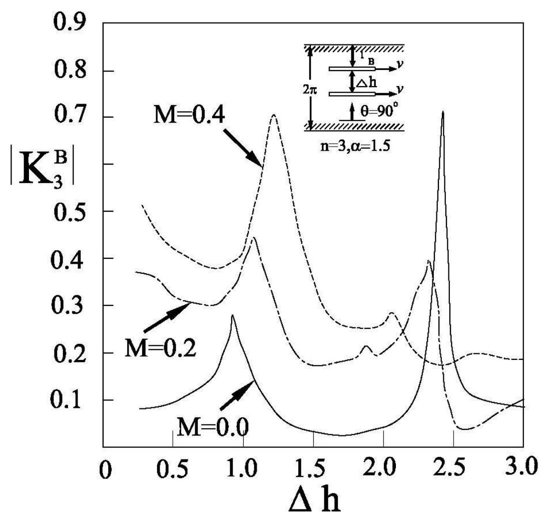

Fig.9. SIFs vs. the distance between two parallel horizontal subsurface cracks subjected to normally incident wave.

Since the stress waves are diffracted and reflected from boundary to boundary within the surfaces of strip and cracks, resonance vibrations of the layers between the cracks and the free surfaces will occur as shown in Fig. 9. It is noted that the values of the SIFs of higher crack velocity are greater than that of lower crack velocity as the distance between two parallel cracks is small.

6 Conclusions

The dislocation model is proposed to analyze the scattering of SH wave by moving finite cracks in an elastic medium of different boundary conditions. By replacing *ω*, *α, θ* with the apparent parameters *ω**, *α** and *θ**, respectively, the formulation of the problem will lead to a system of singular integral equations which is similar to that of the stationary one. Numerical results of the SIFs are displayed versus wave numbers for different crack velocities and angle of incidence. With the approach, the SIFs of propagating edge cracks can also be derived by assuming one of the crack tips to be nearly in contact with the free surface. Resonance vibrations of the layer between the cracks and the free surface are observed, which substantially give rise to high elevation of local stresses.

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