Applicability analysis of 2D-models for masonry arch bridge assessment: Ring, Archie-M and the elasto-plastic model.

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Abstract: A lot of different tools are available for the assessment of masonry arch bridges:experimental formulae,2D-models and finite-element models and discrete-element models (2D as well as 3D) . Due to the complex behaviour of the arches and the disability of determine the material properties of the arch and the infill of these ancient constructions, many different suppositions have been made for the different models. Until now there is not yet a generally accepted method for assessment. This study covers a comparison of some 2D assessment tools. Three models are compared based on a case study: Ring, Archie-M and an elasto-plastic model. It is concluded that every model has advantages and disadvantages. Thus it cannot be stated which model should be preferred.

Key–Words: Masonry arch bridge, assessment methods, collapse load, comparison study, Ring, Archie-M, elasto-plastic model

1 Introduction

Masonry arch bridges have been built since antiquity. A lot of arch bridges are still in use today. For example, in Belgium 600 bridges are still in use as railway bridges and the oldest railway bridge dates from 1845 [1]. The UIC estimates that 60% of the railway bridges are masonry arch bridges [2]. With time, the structures have deteriorated and the requirements changed. The axle loads, number of axles and the vehicle speed the bridges are subjected to have changed significantly in the past century.
These facts mean there is still a need for arch bridge assessment tools.

The last decades meant a boost for the development of assessment tools. A variety of models have been developed ranging from experimental formulas, two-dimensional models to three-dimensional finite or discrete element models [1]. However, a generally accepted standard method is not yet available [3].

This paper represents the results of a comparison between three two-dimensional assessment models applied on single span masonry arch bridges: Archie-M [4], Ring [5] and an elasto-plastic model [6, 7].

2 State of the art

The classic approach to determining the stability of arch bridges dates back to the work of Pippard and Ashby [8] and Pippard [9]. It starts from a two-hinge arch for which is determined the minimum load applied to a fixed position, that causes the arch to turn into a mechanism, i.e. four hinges. The exact positions of the last two hinges are variables to be determined. More general approaches will show that this approach results in approximated solutions only. The elasto-plastic model [6, 7] and Ring [5] will confirm the findings of Boothby [10] that the hinges not necessarily occur in the supports.

This approach was further extended by Heyman introducing thrustlines [11]. This theory assumes that the thrustline must become tangential to intrados or extrados in four locations at which point the structure becomes a mechanism. The parameters required for accurate estimation of the collapse load, such as compressive strength and other mechanical properties of the material are only taken into account indirectly by the inclusion of correction factors. Hence, it only provides stability conditions or coarse estimates of the collapse load and contrary to the approach presented below, these methods do not allow the precise calculation of the behavior of arches under increasing loads.

More recent work based on the rigid-plastic approach is reported by Gilbert and Melbourn [12], Hughes and Blackler [13] and Boothby [10]. These rigid block theory is considered to be the basic model for the understanding of the fundamental behavior of brick arches. However, this study will show that a more sophisticated approach is needed to be able to reproduce the actual collapse mechanism. The rigid block theory uses too many simplifications and assumptions, which frequently deviate excessively from actual conditions. The importance of this method resides in its potential to obtain a first insight into the arch behavior. However, the method does not allow any movement of the abutments, which is a major limitation.

To arrive at realistic solutions it is important to take into account the elasto-plastic behavior of the material and determine yield surfaces as is done in plasticity theory. In the work of Clemente et al. [14] plasticity theory is applied based on a simplified model for the distribution of the tension. An experimental approach is taken by Taylor and Mallinder [15]-[17] to study in more detail the plastic nature of the hinge.

Currently, finite element methods are the ones used most often to study structural stability because of their high accuracy. The models range from 1-dimensional ,[18]-[20], over 2-dimensional ,[21]-[26], up to fully 3-dimensional models [27, 28]. The 1-dimensional models already provide sufficiently accurate results to allow both the study of the behavior of arch bridges as well as their design. The 2- and 3-dimensional models provide a more detailed view of local phenomenons, but they require the knowledge of many material properties. However, the computational complexity of such models results in large computation loads. Furthermore, to arrive at a good match between real and predicted behavior these models require a large amount of input data.

The elasto-plastic model falls somewhere in between those two approaches on the scale of complexity and proposes a preliminary study to determine the critical points with a relatively small computational and modelling effort. Masonry is a very complex composed material. As shown by Cecchi [29], [30] and Milani [31], homogenization procedures exist to allow assessment of masonry as an homogenous material. For that reason a simplified homogeneous material model is used, namely the macro-model proposed by Lourenço [32] instead of a more detailed micro- or meso-model of the behavior of the material.

If required a full 3-dimensional FEM-model can then be used afterwards to refine the results provided by a preliminary 2D-analysis. A generally accepted standard model is not yet available,therefore comparison between the existing 2D-models is very valuable.

3 Arch bridge behaviour

An arch is statically indeterminate in the third degree. The arch collapses when a mechanism is formed.

Failure of an arch can occur due to three possible collapse mechanisms: a shear mechanism, a hinge-mechanism and a combined shear-hinge mechanism.

Boothby et al make the following conclusion for the occurrence of the different mechanisms [33, 10]:

- Shear mechanisms, Fig.1, are more likely to occur when the bridge has a thick arch ring or a low rise-span ratio

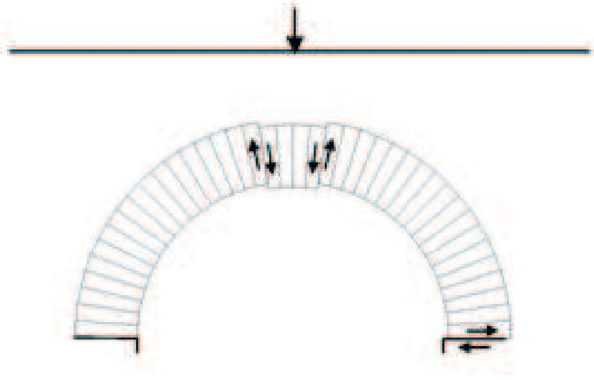


Figure 1: Shear mechanism

- Combined shear-hinge mechanisms, Fig.2, are more likely to occur when the bridge has a low rise-span ratio

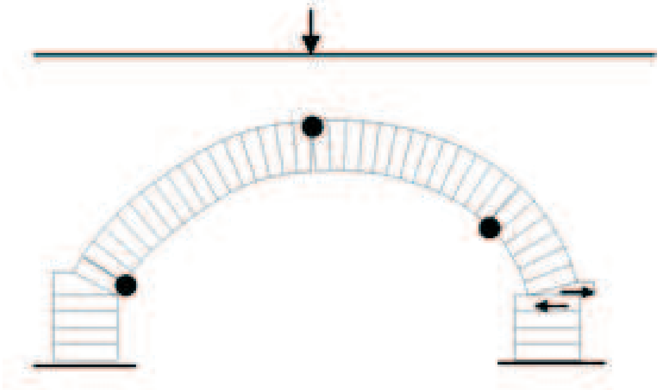


Figure 2: Combined shear-hinge mechanism

- Four hinge mechanisms, Fig.3, are more likely to occur when the bridge is well buttressed and has a high rise-span ratio.

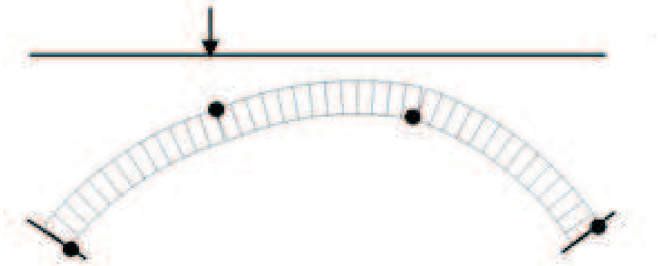


Figure 3: Hinge mechanism

Experiments by Hendry et al. [34] and by others show that, due to most arches being well-buttressed, the hinge-mechanism (Fig.4) can be considered as by far the most likely collapse mechanism for arches. Therefore this paper focusses on this mechanism.

The masonry arch bridges are almost in al cases well buttressed, thats the reason why the models focus on the four hinge collapse mode as failing mechanism for these constructions.

Ring, Archie-M and the elasto-plastic model are all able to determine the four hinge collapse mode. Sliding can only be detected by Ring and will not be used in this comparison

Figure 4: Collapse of the Bridgemill arch bridge. (Source:http: //archive.niees.ac.uk/talks/dem/nenad_bicanic.ppt)

4 Theoretical background

This section contains a brief theoretical background of the three models. For detailed information the provided papers in the reference can be consulted.

4.1 Ring

Ring[1] uses a rigid-plastic block formulation where the collapse load is found by maximizing the load factor with linear programming [35].Although limit analysis, or plastic (mechanism) analysis techniques were originally developed for steel components and structures, it has since been shown that these can be applied to masonry gravity structures, such as piers and arches. The constraints are: equilibrium, no-tension, no-sliding and crushing constraints. The Ring02 software allows to analyse the ultimate limit state, determining the amount of live load that can be applied before structural collapse; and permit investigation of the mode of response when supports undergo small movements.

The live load is distributed through the fill with the Boussinesq distribution with a maximum spreading angle.

The passive earth pressure is the only lateral pressure used and is modeled with uniaxial horizontal elements working in compression [36].

[1]LimitState Ltd., www.limitstate.com, version 2.0.k.10834

4.2 Archie-M

Archie-M[2] applies the traditional thrust line analysis combined with a zone of thrust for modelling finite crushing strength. The thrust line is found with a three hinge system. The hinge positions are found by assuming that there must be a minimum total energy in the system [4]. Although the purpose of Archie-M is to show that a required load can be supported, the collapse load can be estimated. This is achieved by varying the load factor until the thrust line touches an extra border of the arch and making a fourth hinge [37].

The live load is distributed through the fill with a sine form as simplified model [38].

The lateral earth pressure is always the at rest pressure for the fill. The live load lateral earth pressure can be set to the active or the at rest pressure. A proportion of passive restraint pressure can be added to ensure the thrust remains in the arch.

4.3 Elasto-plastic model

The elasto-plastic model assumes a hinge to behave in a perfect plastic manner. The load factor is increased until a hinge is formed and the boundary conditions are changed so that the moment in the hinge stays constant. This process is repeated until a fourth hinge has been formed [6]. A major advantage of the elasto-plastic method is his ability to carry out details sensitivity analysis [7]. The influence of geometry, material properties and different loadings can be evaluated easily by this analytical model.

The external load is considered to be a normal point load directly on the arch. The load of the infill is modeled as a distributed load.

Lateral earth pressure is not considered in this model.

4.4 Summary

Apart from the model method the lateral earth pressure and the live load distribution through the fill are the main points of difference. These issues are considered difficult and have been leading to suppositions for most of the models. One could say that the lateral earth pressure is subjective [39] because the models used are predefined.

This study tries to avoid most of these differences. In doing so it tries to link the differences found to the basic theory of the models.

[2]Obvis Ltd., www.obvis.com, version 2.4.1

5 Case study

The three models are compared by using a case study. The case study is a fictitious bridge. The details are specified in table 1 and figure 5. A parameter study was carried out for the comparison.

Variable	Value
Span	8 m
Rise	2 m
Ring thickness	400 mm
Bridge width	1 m
Fill height	3 m
Fill weight	20 kN/m^3
Fill cohesion	0 kPa
Fill shear resistance	30°
Masonry Weight	22 kN/m^3
Masonry crushing strength	8 N/mm^2
Masonry E-modulus	8 GPa
Number of blocks	40

Table 1: Case study variables

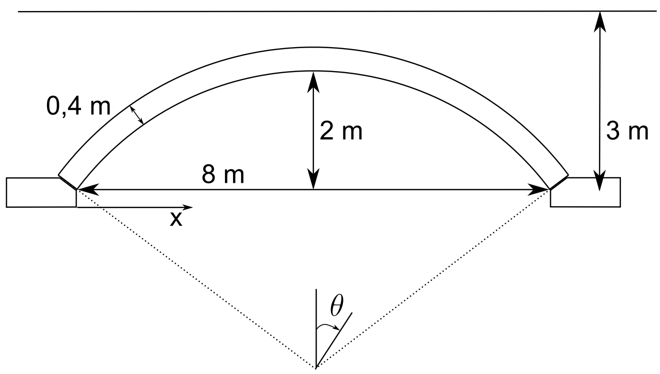


Figure 5: Geometry

6 Comparison

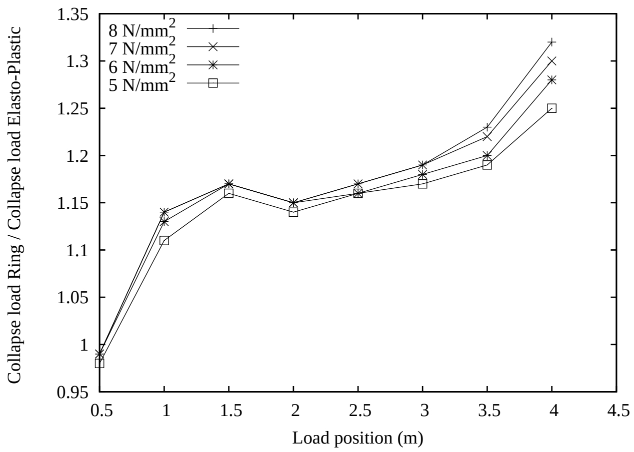
Every model makes different suppositions for the lateral fill forces. Active pressure and passive restraint pressure have their influence on the arch behaviour. For easy comparing the lateral pressure is disabled. Archie-M does not have this option, but the lateral earth pressure can be neglected by using the at rest pressure with a shear resistance of 89°. The lateral earth pressure coefficient would be $K_0 = 1,5 \cdot 10^{-4}$ and has therefore not a big influence on the results.

It is an evidence that for real assessments the lateral earth pressure may not be neglected but in this case it is useful.

Another main difference is the distribution of the live load through the fill. The elasto-plastic model does not distribute the live load through the fill. In a first comparison the case study is assessed with the three models. Afterwards a same comparison has been set up for Ring and Archie-M with a distributed live load.

6.1 Three models, without live load distribution

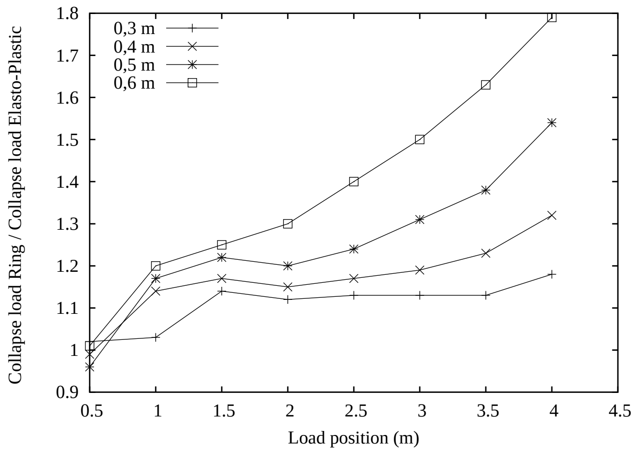
The collapse load of the case study is calculated for different load positions. The results are summarized in graphs 1 and 2 for Ring and the elasto-plastic model and graphs 3 and 4 for Ring and Archie-M. The collapse load ration is plotted for various loading positions. The thickness is varied by changing the extrados.



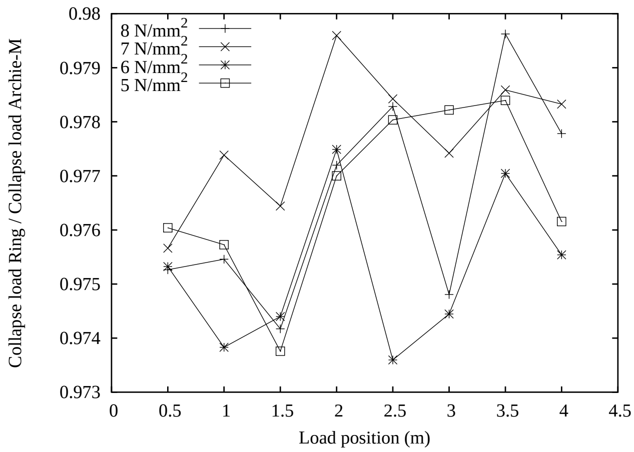
Graph 1: Collapse load ratio for different positions and crushing strength

As shown in the graphs the collapse load differs. At the abutments the collapse load provided by the elasto-plastic is higher than the collapse load from Ring and vice versa for the middle of the arch. The collapse loads at the abutments will not be considered due to unrealistic values from the elasto-plastic model. Thus, it can be stated that the elasto-plastic collapse load is lower than the Ring collapse load.

When increasing compression strength or increasing arch thickness the difference ratio increases as well. Comparing the linear programming constraints for the stresses from Ring [5] with the hinge condition for the elasto-plastic model [3] leads to the same



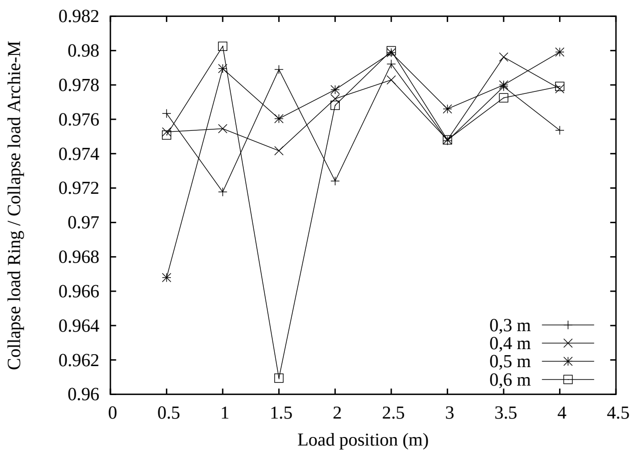
Graph 2: Collapse load ratio for different positions and thickness



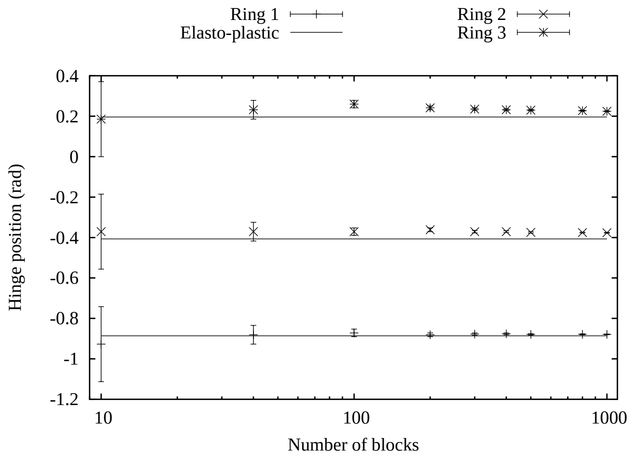
Graph 3: Collapse load ratio for different positions and crushing strength

boundary for the internal forces. The internal force distribution calculated with the elasto-plastic model based on the elastic material properties must satisfy the equilibrium constraints used by Ring. The elasto-plastic internal force distribution is, therefore, a possible distribution which can be the one maximizing the objective function used by Ring. Thus, the elasto-plastic collapse load must be smaller than or equal to the Ring collapse load, which is the case for this case study. For thick arches or high compression strengths the difference gets significantly higher because the Ring boundary conditions allow more internal force distributions which are not considered by the elasto-plastic model.

The Ring and Archie-M collapse loads are approximately equal. The Ring collapse load is always



Graph 4: Collapse load ratio for different positions and thickness



Graph 5: Hinge positions, increasing number of blocks

smaller than the Archie-M one. When comparing the internal force distribution it is found that the internal forces calculated by Archie-M are a greater, although not much, than the Ring internal force distribution. Thus, it can be stated that both models have found almost the same collapse load but the internal force distribution for the limit state is slightly different.

x	Ring (rad)				Elasto-plastic (rad)			
0,5	-0,93	-0,70	0,09	0,93	-0,93	-0,77	0,05	0,93
1	-0,93	-0,60	0,14	0,93	-0,93	-0,64	0,09	0,93
1,5	-0,93	-0,46	0,19	0,93	-0,93	-0,52	0,14	0,93
2	-0,88	-0,37	0,23	0,93	-0,89	-0,41	0,20	0,93
2,5	-0,79	-0,28	0,28	0,93	-0,79	-0,30	0,25	0,93
3	-0,70	-0,19	0,32	0,93	-0,68	-0,20	0,31	0,93
3,5	-0,56	-0,09	0,37	0,93	-0,56	-0,10	0,37	0,93
4	-0,46	0,00	0,46	0,93	-0,93	-0,42	0,00	0,93

Table 2: Hinge positions (radians) for different load positions

Table 2 shows for Ring and the elasto-plastic model the hinges for different load positions and the angles where the hinge occurs. The calculated hinge positions are almost equal for both models except for a mid span loading. At mid span loading the collapse mode is a five hinge mechanism. Both models only consider four hinges. But further investigation shows that the five hinges are detected.

Ring only considers a hinge on a contact. If the number of blocks is increased the hinge positions from Ring, as shown in graph 5, are moving closer to the elasto-plastic hinge positions. Thus the collapse mode can be considered approximately equal. The elasto-plastic has an advantage by providing the evolution of the hinge formation process.

x	Ring				Archie-M			
0,5	0	5	22	40	0	5	22	40
1	0	7	23	40	0	7	23	40
1,5	0	10	24	40	0	10	24	40
2	1	12	25	40	1	12	25	40
2,5	3	14	26	40	3	14	26	40
3	5	16	27	40	5	16	27	40
3,5	8	18	28	40	8	18	28	40
4	0 (10)	20	30	40	0 (10)	20	30	40

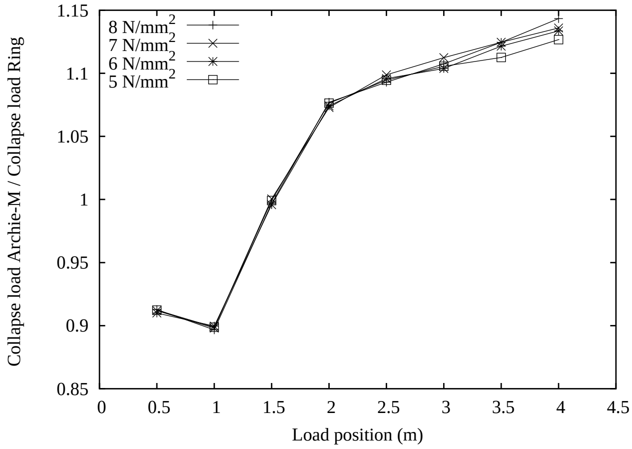
Table 3: Hinge positions (contact surface) for different load positions

Table 3 shows for Ring and Archie-M the contact surface where the hinges occur for different load positions. The collapse mode is equal for all load cases.

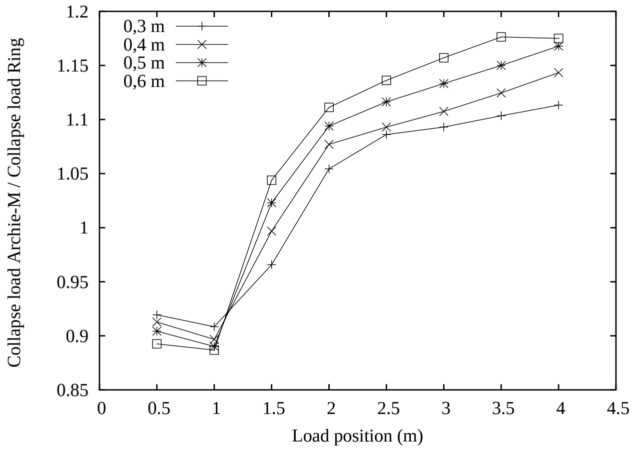
6.2 Ring and Archie-M, with live load distribution

A similar comparison is done for Archie-M and Ring with a distributed live load. As shown in graphs 6 and 7 the collapse load differs about 20% maximum for this case study. The Archie-M collapse load is greater than Ring at mid span positions and visa versa at the abutments for both models using the standard distribution.

The live load distribution has a significant difference. The case study is assessed with the default cutoff angle of 30 degrees for the Ring Boussinesq distribution. Graph 8 compares the Archie-M results with the Ring results for different cutoff angles. If the cutoff angle increases the Boussinesq distribution looks more like the sine distribution. In the middle of the bridge the collapse loads have a significantly smaller difference, which is not the case at the abutments. At



Graph 6: Collapse load ratio for different positions and crushing strength (Distributed live load)



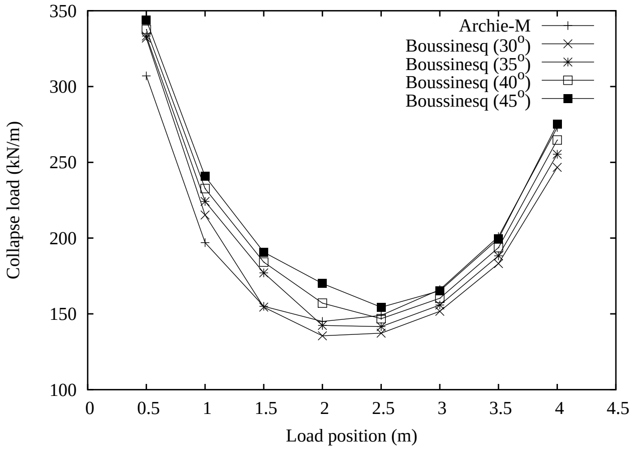
Graph 7: Collapse load ratio for different positions and thickness (Distributed live load)

the abutments the Archie-M distribution keeps more load on the bridge because the distribution length is shorter.

Table 4 gives the contact surface where a hinge occurs for different load positions. Comparing the hinges positions gives equal positions for loads at the middle of the bridge. When the load gets closer to the abutments there is at least one hinge placed differently. This difference is caused by the distribution because the hinge positions are equal for a non distributed live load.

6.3 Worst positions

For the analysis of a masonry arch bridge the worst position for the live load is the most important one. The three models have the possibility to determine the



Graph 8: Distribution effect

x	Ring (cont.)				Archie-M (cont.)			
0,5	0	8	23	40	0	8	24	40
1	0	9	24	40	0	9	25	40
1,5	0	10	25	40	0	11	25	40
2	0	12	25	40	0	12	26	40
2,5	2	14	27	40	2	14	27	40
3	5	16	28	40	5	16	28	40
3,5	7	18	29	40	7	18	29	40
4	10	20	30	(0) 40	0 (40)	10	20	30

Table 4: Hinge positions (contact surface) for different load positions (Distributed live load)

worst load position.

Model	Distribution	Position (m)	Collapse load (kN)
Archie-M	ArchieM, BD21/97	2,32	147
Archie-M	Archie/Multi	2,32	152
Ring	Boussinesq (30° cutoff)	2,21	134,4
Ring	Boussinesq (35° cutoff)	2,23	139,7
Ring	Boussinesq (40° cutoff)	2,46	146,5
Archie-M	None	1,82	113
Ring	None	1,82	109,8
Elasto-plastic	Dirac	1,76	97,2

Table 5: Worst position for one point load

The results show that the three models have a comparable result for the collapse load and the worst position. Archie-M produces the fastest result through a wizard while Ring needs to define multiple load cases. It may be stated that Archie-M has therefore an advantage.

The results are quite similar if the live load is not distributed through the fill.

6.4 Summary

The collapse load differences can be summarized as follows: collapse load elasto-plastic $\leq$ collapse load Ring $\leq$ collapse load Archie-M. The three models produce almost the same collapse mode.

The three models have their advantages and disadvantages. The elasto-plastic model gives a real insight in the formation of the mechanism because each hinge is related to the load factor required for the formation (including the preceding hinges).

For a new load position the load factor must be recalculated when using the elasto-plastic model or Ring which makes Archie-M a faster tool because the thrust line is drawn immediately on the screen to prove a safe arch. Even if not considered in this comparison the lateral pressure is a main point of difference and remains subjective because no other models than the predefined ones are available for Ring and Archie-M. Ring has an advantage because it is able to detect sliding failure which can occur for certain structures.

The three models produce a similar result for the worst load position.

7 Conclusions

The elasto-plastic model gives a more realistic insight in the arch behaviour by providing the evolution of the hinges, including the forces needed for each hinge. At the same time, the predicted collapse load is smaller and therefore much safer than Ring and Archie-M. This difference is caused by the theoretical background of the two models.

The collapse load calculated by Ring and Archie-M has a difference depending on the distribution of the live load. At mid span positions the Archie-M collapse load is higher than Ring because the distribution from Ring is more concentrated. When the cutoff angle increases the collapse loads are almost equal. Thus, it is concluded that at mid span positions the collapse load from Ring and Archie-M are approximately equal if the same distribution would be used which is exactly the case for a non distributed live load. The Ring collapse load at the abutments is higher because the distribution allows more load of the bridge. For a live load without distribution collapse loads are approximately equal but the Ring collapse load is smaller than the Archie-M collapse load.

The three models produce the same collapse mode. The hinge positions are equal or approximately equal for almost every load positions. Small differences were found for loads close to the abutments with Ring and Archie-M if the live load is distributed with the standard models.

A comparison between 2D arch assessment tools has been established. Choosing one model which should be the preferred model for performing the calculations is not possible because every model has advantages and disadvantages.

8 Future work

The elasto-plastic model has significantly smaller results. This means it is the safest model to apply. In case of a high infill distribution of the load through the infill should be incorporated, which will lead to higher collapse loads. It might be interesting to validated the model with some laboratory collapse tests.

Sliding can be the cause of failure so it might be interesting if the theory used by Archie-M and the elasto-plastic model would be updated to include sliding failure.

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