Sommerfeld and Mass Parameter Identification of Lubricated Journal Bearings

Domenico Guida
University of Salerno
Department of Mechanical Engineering
via Ponte don Melillo, 84084 Fisciano
Italy
guida@unisa.it

Carmine M. Pappalardo
University of Salerno
Department of Mechanical Engineering
via Ponte don Melillo, 84084 Fisciano
Italy
cpappalardo@unisa.it

Abstract: In this paper we propose a method for identification of two parameters that define the structure of lubricated journal bearings. In rotordynamics literature these parameters are named *Sommerfeld number* and *Mass Number*, respectively. The system analyzed belongs to the class of systems that are described by n nonlinear second order differential equations. We have obtained the analytical solutions of these equations using Lie series expansion. This method can be applied to a vast class of dynamical systems, provided that they are described by smooth equations. Lie series expansion is a mathematical tool that allows us to obtain the system response in recursive way using symbolic PC-codes. Besides, this procedure involves only a few experimental data and makes the identification process very quick. It is worth noting that the application that we propose does not take into account neither the issue connected to experimental data noise nor the unavoidable disturbances that occurs in measurement process. Actually, in applied system identification one must always pay attention to the signal noise and to the robustness of the solution with respect to disturbances, but these issues are over the goal of this paper.

Key–Words: Nonlinear System Identification, Parameter Identification, Lubricated Journal Bearings

1 Introduction

Nonlinear system identification is the art of determining a model of a nonlinear dynamical process by combining information obtained from experimental data with that derived from an a priori knowledge of the physical behavior of the system. In general, the system or the physical process to be modeled can be of any kind, even though applied system identification usually considers only deterministic processes.

Nonlinear system identification is a discipline that can be studied at different levels. From the basic point of view, the purpose of identification is to determine just how many states or modes are needed to construct a model of the system. Once past this stage, one can begin an higher level system identification. On the other hand, the most refined level of identification is the parametric identification. In the case of nonlinear parametric identification one knows the mathematical form of the nonlinear differential equation describing the motion of the system but some parameters of the equations are unknown.

Between these two extremes there are the nonlinear system identification techniques whose purpose is to model the dynamic behavior of a physical system without determining its equations of motion. In general, there is a wide range of identification techniques for nonlinear systems and the choice of the technique to be used needs to be decided from time to time. In the field of nonlinear parametric identification there are basically three possible approaches: time-domain analysis, frequency-domain analysis and bifurcations analysis. In the case of time-domain analysis, one should find an approximate analytical expression, written in terms of unknown parameters, and compare it with experimentally measured data. In this case we need to measure only one output signal and then is possible to perform an efficient identification procedure.

We propose a method that allow us to construct an approximated analytical solution for nonlinear system equations of motion. This method is based on Lie series expansion. The method can be applied to a vast class of dynamical systems, provided that they are described by smooth equations. This tool allows us to obtain the system response in recursive way, using symbolic codes on PC. Besides, this procedure needs only a few experimental data and makes the identification process efficient.

In this paper we have used this method in order to identify of Sommerfeld and Mass parameters of a rotor on lubricated journal bearings. The model assumed is the *cavitated short bearing* whose details can be found in reference [1].

Dynamic characteristics of turbomachine are influenced strongly by bearings on which the rotor runs. This is because the stiffness of the rotor-bearing system is mainly determined by the bearing support stiffness acting in series with shaft stiffness and the damping of the system is usually determined almost entirely by the bearing damping properties. In some machines, there are also significant fluid forces developed on the rotor by the rotating seals and of the centrifugal or axial-flow process. The latter are traditionally outside the control of the machine designer. Analytical prediction and experimental measurement of these forces is the subject of current research, and it may soon be possible to modify rotor-dynamic characteristics by redesigning seals, impellers, and turbine stages.

One of the most important design parameter for rotordynamics is the ratio of bearing support stiffness to shaft stiffness. It is usually good design practice to keep this ratio as small as practical. Another important parameter is the ratio of support damping to internal damping in the rotor. This ratio should be kept as large as possible to insure rotor stability. In addiction to direct stiffness and damping, some types of fluid bearings, as well as seals and process wheels, also produce cross-coupled stiffness and damping. Cross-coupling can have profound effects on whirl instability. Finally, we can say that stiffness and damping of bearings influence stability of the rotor. Stiffness and damping depend on Sommerfeld and Mass parameter then the latter are very important parameters in design and maintenance in turbomachinery.

2 Overview on Mechanism of Load Support of Rotors on Lubricated Bearings

In hydrodynamic bearings, the fluid pressure is generated entirely by motion of the journal and depends on the viscosity of the lubricating fluid. The fluid supply pressure needs only to be high enough to maintain a copious supply of lubricant in the load-supporting clearance around the journal. This is normally accomplished by introducing the fluid through one or more supply holes or grooves located in the areas where the hydrodynamic pressure is low. If there is an insufficient supply of fluid, or if any other factor prevents the generation of a high enough pressure in the film fluid to support the load, then the hydrodynamic film breaks down and the journal contacts the bearing surface. Bearing in which this continuously occurs are called *boundary-lubricated bearings*. Hydrodynamic bearings often acts as boundary-lubricated bearings during the initial phase of the machine start-up, when the journal rotation speed is too slow to generate sufficient hydrodynamic pressure to support the load. Boundary lubrication is characterized by higher friction and a much greater potential for overheating than is hydrodynamic lubrication.

Besides the friction coefficient varies with viscosity, speed and load for these two different types of lubrication. The temperature stability of the two cases can be considered as follows. With boundary lubrication, a rise in the temperature reduces the viscosity, which rises the friction factor, which further rises the temperature. This cycle, repeated, tends to induce overheating. With hydrodynamic lubrication, a rise of the temperature which reduces the viscosity will reduce the friction factor, thus reducing the heat generated. This cycle tends to produce a stable operating temperature. Boundary-lubricated bearings are used satisfactorily in small mechanisms and appliances with light loads and light duty cycles, but they obviously are not desirable in industrial turbomachinery applications. Thus the maintenance of a load-supporting fluid film is of prime importance for reliability of all the various types of hydrodynamic bearings used in turbomachinery. As has already been mentioned, the supply pressure to the bearing is of limited importance in this regard.

In the case of hydrodynamic lubricated bearings, the mechanism of load support can be explained as follows. The lubricating fluid is pulled by viscous shear into the converging wedge produced by the off-center displacement of the journal. Note that it is rotation of the journal that produces the relative velocity along the film wall and induces the viscous shear. As the fluid is pulled into the converging wedge, its pressure is raised. Conversely, the fluid pressure decreases as the viscous shear pulls it out into the diverging wedge downstream from the point of minimum film thickness. The net effect of the distribution of hydrodynamic pressure around the journal is to produce a force which reacts the applied load. The converging-diverging wedge effect becomes more pronounced as the off-center journal displacement, also called eccentricity in the bearing literature, is increased. Thus, as in the case of the hydrostatic bearing, there is an equilibrium position where the support force developed by the fluid-film pressure equals the applied load. Furthermore, since a change in applied load produces a new equilibrium position with an equal change in support force, it can be seen that the hydrodynamic bearing also acts like a spring. Translational velocity of the journal also induces hydrodynamic pressure in the film, with a resulting force, so the bearing acts like a damper as well. From the standpoint of rotordynamics, the most desirable feature of a hydrodynamic journal bearing is its damping, which is high compared to other types of bearings. The plain journal bearing is the simplest of all hydrodynamic bearings. Its geometry is that of a plain right circular cylinder, and it is the least expensive bearing to manufacture. Rotors running on this type of bearing are often speed limited by *oil whip*.Therefore more complex types of hydrodynamic bearings have been developed to remove or reduce this limitation. Most industrial turbomachines are designed to use some type of hydrodynamic bearing, and nearly all aircraft turbine engines employ some type of hydrodynamic bearing damper.

3 Fluid-Film Pressure Distribution

The basic problem of hydrodynamic bearing analysis is a determination of the fluid-film pressure distribution $p(x, z)$ for a given bearing geometry. Bearing designers normally accomplish this by solving various forms of the Reynolds' equation for special cases. The Reynolds' equation provides the basis of modern lubrication theory, and a number of its solutions for special cases of practical interest have been verified by experimental measurements. For any general thin film geometry, the equation is:

$$\begin{aligned} &\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)= \\ &=6\mu\left\{\frac{\partial}{\partial x}\left[\left(U_0+U_1\right)h\right]+2\frac{\partial h}{\partial t}\right\} \end{aligned} \tag{1}$$

where $p = p(x, z)$ is the pressure distribution; x and z are coordinates that locate a point in the film; h is the local film thickness; μ is the viscosity of the fluid; U_0 and U_1 are tangential velocities at the two walls bounding the film; and $\frac{\partial h}{\partial t}$ is the time rate of change of the local film thickness. Reynolds employed a number of simplifying assumptions in deriving the equation:

1. Viscous shear effects dominate, so the viscosity is the only important fluid parameter.

2. The fluid inertia forces are neglected.

3. The fluid is incompressible.

4. The fluid film is very thin, so that the pressure does not vary across the thickness of the film and any curvature of the film can be neglected.

5. The viscosity is constant throughout the film.

6. There is no slip at the wall.

Most bearings employed for rotor support in turbomachinery have cylindrical geometry, or some variation of it, to match a cylindrical journal. In cylindrical coordinates, Reynolds' equation becomes:

$$\begin{aligned} &\frac{\partial}{\partial\vartheta}\left\{\left[1+\varepsilon\cos(\vartheta)\right]^3\frac{\partial p}{\partial\vartheta}\right\}+ \\ &R^2\frac{\partial}{\partial z}\left\{\left[1+\varepsilon\cos(\vartheta)\right]^3\frac{\partial p}{\partial z}\right\}= \\ &=-6\mu\left(\frac{R}{C}\right)^2\left[\left(\omega-2\dot{\psi}\right)\varepsilon\sin(\vartheta)-2\dot{\varepsilon}\cos(\vartheta)\right] \end{aligned} \tag{2}$$

where e is the journal eccentricity, c is the radial clearance, R is the journal radius; Also $h = c\,(1+\varepsilon\cos\vartheta)$ is the local film thickness; ω, $\dot{\psi}$ are the angular velocities of the journal and line of centers, respectively. and $\dot{\varepsilon} = \dot{e}/c$ is the dimensionless radial velocity. For a plain journal bearing, open to the atmosphere at the ends $z = 0$ and $z = L$, and with an uncavitated fluid film, the boundary conditions are:

$$\begin{cases} p(\vartheta, z=0) = p(\vartheta, z=L) = p_a \\ p(\vartheta=0, z) = p(\vartheta=\pi, z) = p_0 \end{cases} \tag{3}$$

where p_a is atmospheric pressure and p_0 is determined by the supply pressure to the bearing. Closed-form solutions to equation (2) in functional form, with realistic boundary conditions such as (3), have not been obtained to date, except for the special case of small eccentricities. There are also additional factors, such as film cavitation, which must be included for a realistic model in some cases and which make a functional solution even more difficult. To obtain useful solutions, two successful alternative approaches have been employed:

1. Simplification of the Reynold's equation for special cases of practical interests, so that functional solutions can be obtained.

2. Reformulation of the equation into finite-difference ore finite-element form, for numerical solution.

For journal bearings, the first approach has yielded to solutions for two notable special cases: a) the *long bearing* and b) the *short bearing*. For the long bearing, the major simplifying assumption is that the second term in equation (2) is of negligible magnitude compared with the first; that is, the pressure distribution around the bearing is invariant along the length of the bearing. Thus the second term in the equation is omitted, which makes the equation integrable. Physically, this means that there is no flow in the axial direction. For the short bearing, the first term of the equation is omitted on the basis that it has a negligible effect on the flow compared to the other terms in the equation. Practically, this will occur when $L/D \leq 0.25$, where L is the axial length of the bearing and $D = 2R$ is the journal diameter. In this case the film pressure turns out to be a parabolic function of the axial coordinate.

4 Bearing Stability Characteristics

It is common practice in the rotordynamics literature to describe whirl stability characteristics of a hydrodynamic bearing with a stability map, which gives the dimensionless threshold speed of instability versus the equilibrium eccentricity. Typically, these curves are generated by computing the dimensionless speed at which the real pert of the eigenvalue becomes positive, for various values of static eccentricity.

The fully cavitated journal bearing is stable for all speeds if it is very highly loaded. Consequently, modified versions of the simple journal bearing are designed to supplement the actual supported load with hydrodynamic pressure in the oil film and thereby increase the operating eccentricity at high speed. It should be realized that stability maps are usually generated from a rigid rotor model and therefore do not accurately represent the stability characteristics of the bearing on a flexible rotor. Also, the whirl stability characteristics of a turbomachine are determined by a number of factors in addition to the bearing coefficients. Yet there have been a number of cases in the field where turbomachines with tilt-pad bearings were observed to be unstable with violent sub-synchronous rotor whirling. This is due to other sources of cross-coupled stiffness or negative damping not in the bearing, such as internal friction in the rotor assembly, seals, or aerodynamic forces on process wheels.

5 Experimental Verification of Bearing Characteristics

Measurement of hydrodynamic bearing characteristics have been made on special test apparatus by engineering researchers. Agreement with the Reynolds theory just presented has been good, in most cases, or the discrepancies can be explained in terms of unknowns errors. Direct measurements of the bearing damping coefficients is especially difficult since both the force and journal motion are time-varying quantities. Parkins measured journal bearing coefficients by shaking the test journal in horizontal and vertical straight line harmonic motions. Nordmann and Shollhorn used an impact impedance method, converting the vibratory response to frequency-dependent transfer functions to measure journal bearing coefficients. Tripp and Murphy measured the stiffness of tilt-pad bearings by applying a known static load at various shaft speed and measuring the change in the static eccentricity. In a number of cases, the measured critical speeds and response to imbalance of turbomachines running on very large journal bearings, when compared to computer predictions based on theoretical bearings coefficients, have suggested that the Reynolds theory overpredicts the direct bearing coefficients. Nicholas and Barret have shown that the discrepancies may actually be due to the additional flexibility of the journal bearing housing or supporting structure, which is often neglected in a computer model. The effect of the housing flexibility is to reduce the effective stiffness and damping of the journal bearing and lower the critical speed.

6 Journal Bearing Model

The system analyzed is showed in figures 1 and 2.

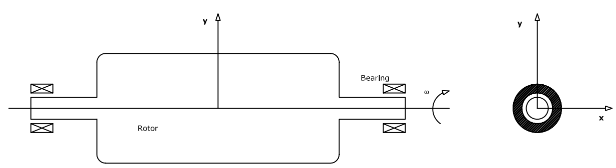

Figure 1: Physical System

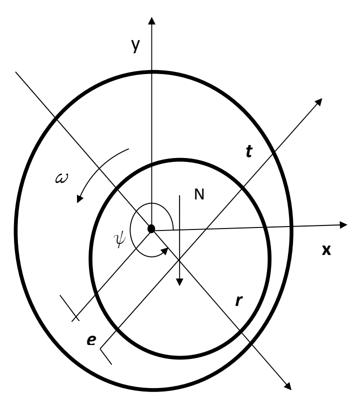

Figure 2: Journal Bearing Model

It is composed of a rotor on two lubricated journal bearings. This configuration is used very often for studying the dynamic behavior of many machine basic components like: turbines, internal combustion engines, electric motors, mechatronic devices, medical devices and so on. The goal of lubricated journal bearings is to minimize the friction loss between rotor and stator substituting the oil friction to the dry friction that occurs between the surfaces in relative motion.

In this analysis we have assumed that the rotor is symmetric in reference to bearing planes so that we can study only one bearing. The dynamics of the rotor can be described starting from the pressure field in the gap between rotor and bearing. This pressure field is obtained integrating the Reynolds lubrication equation. This is a very complicated task. Reynolds lubrication equation is a bi-dimensional partial differential equation and needs boundary conditions to be solved. These conditions are not easy to impose because they require a deep knowledge about the fluidynamics of the system. It is worth noting that boundary conditions can make discontinue the force field on the journal and they can also make unsuitable the method proposed because it requires smooth equations, so as we have mentioned in the introduction.

In this paper we have assumed the theory of the *cavitated short bearing* for describing in explicit form the force field $f_r(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t))$ and $f_t(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t))$. Referring the notations indicated in figure 2 and the hypothesis assumed in [1] we can express the journal motion equations by following relations (4-7). We have indicated with ω rotor angular speed, *R* journal radius, *C* the difference between bearing and journal radius, *m* rotor mass, μ oil dynamic viscosity, p_0 inlet pressure and N external load. While with $\epsilon(t)$ we have indicated the dimensionless eccentricity of the journal and with $\psi(t)$ the attitude angle, so as showed in figure 2. With P we have indicated the inlet dimensionless pressure while whit Q and W the dimensionless parameters which have to be identified. In rotordynamics these parameters are named *Sommerfeld number* and *Mass number*.

$$\begin{cases} \ddot{\varepsilon}(t) - \varepsilon(t)\dot{\psi}(t)^2 + Q\, f_r(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t)) + \\ + W \sin(\psi(t)) = 0 \\ \\ \varepsilon(t)\ddot{\psi}(t) + 2\dot{\varepsilon}(t)\dot{\psi}(t) - Q\, f_t(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t)) + \\ + W \cos(\psi(t)) - P = 0 \end{cases} \tag{4}$$

where

$$\begin{cases} f_r(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t)) = \left[\left| 1 - 2\dot{\psi}(t) \right| \frac{\varepsilon(t)^2}{(1-\varepsilon(t)^2)^2} + \right. \\ \left. + \frac{\pi}{2} \frac{\left(1+2\varepsilon(t)^2\right)\dot{\varepsilon}(t)}{(1-\varepsilon(t)^2)^{5/2}} \right] \\ \\ f_t(\varepsilon(t), \psi(t), \dot{\varepsilon}(t), \dot{\psi}(t)) = \left[\left(1 - 2\dot{\psi}(t)\right) \frac{\pi\, \varepsilon(t)}{4(1-\varepsilon(t)^2)^{3/2}} + \right. \\ \left. + \frac{2\varepsilon(t)\dot{\varepsilon}(t)}{(1-\varepsilon(t)^2)} \right] \end{cases} \tag{5}$$

and

$$Q = \frac{\mu R L^3}{m\, C^3 \omega}\, ,\; W = \frac{N}{m\, C\, \omega^2} \tag{6}$$

$$P = \frac{2RLp_0}{m\, C\, \omega^2}\, ,\; \epsilon = \frac{e}{C} \tag{7}$$

7 Identification of Journal Bearing Parameters

Nonlinear system identification is the art of determining a model of a nonlinear dynamical process by combining information obtained from data with that of physical insight or a priori knowledge. The system to be modeled may be an experiment, a natural process, or even a large-scale computer simulation. Dynamic responses of such deterministic systems may be periodic or non-periodic. Nonlinear system identification is a broad subject. At the most basic level, the goal might be to merely identify how many states or modes are needed to construct a model of the system.With such information at hand, a more detailed system identification can begin. At the more refined extreme is parametric system identification, for which the form of the differential equations of motion that model the system is known, but unknown parameters need to be identified. In between these two extremes lie techniques of nonparametric identification and nonlinear prediction, where the goal might range from revealing a nonlinear restoring force characteristic, to modeling the dynamic behavior without determining the differential equations of motion. This article summarizes some ideas spanning this range of problems. We start with the most basic case of estimating the number of active states or modes, since in the most raw situation, this is where the analyst may start. We then discuss nonparametric identification methods, and finally, parametric system identification. There are many approaches to system identification in the literature, and we only touch on some of them. The goal here is to introduce basic ideas. Our focus is on deterministic systems, although methods are available for systems with random components. The tools discussed should enable system identification for a variety of nonlinear response regimes, be they periodic, quasiperiodic, or chaotic. Periodic responses may be more tractable for standard analyses, while nonperiodic responses tend to explore the response space and produce a large amount of information.

7.1 Parametric Nonlinear System Identification

In this scenario, the ordinary differential equations of motion are known, and the forms of nonlinear terms are known. However, parameters in the equations of motion remain unknown and need to be identified. Among the approaches to consider in this situation are time-domain analysis, frequency-domain analysis, and bifurcation analysis. The basic idea in time domain analysis is to take measured time histories of displacements, velocities and accelerations, and find parameters such that the equations of motion best accommodate the measured response for all time samples. This ultimately amounts to a least-squares solution with a minimized cost function based on a residual. One perspective is the direct evaluation of terms in the differential equation based on measured quantities. Here, the optimal parameters can be chosen to best balance the equation of motion at each time sample, for example by singular value decomposition. Another method is to use an analytical expression of the time response, written in terms of unknown parameters, and compare it with the measured response of the system to be identified. Then just one measurement signal is needed. Such methods have been reviewed by Stry and Mook, who in turn proposed the use of a correction term to accommodate modeling errors, which is then recast into a two-point boundary value problem for the solution of the correction term. The correction term is then used to fit the nonlinear functions to be identified. The idea in frequency-domain analysis is to take the Fourier transforms of the measured time histories, insert them into the differential equations of motion, and find parameter values that balance harmonics in the least-squares sense. If the system is linear in its parameters, i.e., the parameters are external coefficients on the nonlinearities in the equations of motion, then the balance equations are linear in the parameters, and a straightforward least squares solution suffices. Differentiation can be done by multiplying the transformed signal by io. As such, for example, only measurements of displacements are needed, and velocities and accelerations can be obtained in the frequency domain. If the system response is periodic, then the Fourier coefficients can be computed by Fourier series or Fourier transforms. If the system response is chaotic, then numerous saddle-type unstable periodic orbits are 'visited' during the response. These periodic orbits can be approximately extracted and treated similarly to the way the stable periodic responses were treated, thus extending the idea to handle chaotic responses as well as stable periodic responses. The numerous periodic orbits from the chaotic response provide ample redundancy for the least-squares approximation of the parameters. Bifurcation behavior can be exploited by finding parameters such that bifurcation events occur at the right parameter values. Bifurcations are considered to be rather sensitive to parameters, which is good for parameter identification. The bifurcation behavior is usually determined by using perturbation methods, such as multiple scales, averaging, and normal forms; to obtain analytical expressions of the bifurcation events as functions of parameters, which can then be used for the purposes of identification. In order to use bifurcations in system identification, bifurcations need to be observed experimentally. This means that the parameter space must be explored in the experiments, and parameter values for which sudden qualitative changes in the dynamical behavior must be recorded. The type of bifurcation must be recognized. As a cautionary note, bifurcation behavior is usually analyzed for steady-state, constant-parameter behavior. Experimental sweeps of parameter space really mean that the system has non-constant, slowly varying parameters. Systems with slow varying parameters may have significantly different bifurcation occurrences than systems with truly constant parameters. Thus, the experimenter must be patient in the exploration of parameter space.

7.2 Nonparametric Nonlinear System

In nonparametric identification, vibration behavior is modeled or predicted by means other than differential equations. Some approaches include Volterra series (or Weiner series) modeling, neural networks, and nonlinear time-series prediction, or merely the identification nonlinear stiffness characteristics. First we mention the Volterra series approach. The response of a linear system to an input can be represented by a convolution integral, in which the input is convolved with the impulse response function. In the Volterra series approach, this convolution integral is a first-order Volterra functional. The impulse response is the kernel. For modeling nonlinear inputoutput dynamics, a series of Volterra functionals is constructed, the k_{th} term consisting of a k-fold nested convolution integral involving k delays of the input and a k_{th} order kernel, which acts as a weighting function in the integral. In identifying the system, these kernels are to be identified, typically with a least-squares fit in the time-series response. (The time-series values of the kernals or weighting values are identified). The Weiner G-functionals, which are orthogonal functionals constructed from the Volterra functionals, can be used in place of the Volterra functionals in the system identification. Backpropagation artificial neural networks can be applied to model discrete-time dynamics of sampled dynamic systems. In the learning process, the inputs and outputs are applied to the neural network, and a steepest descent (for example) with respect to network parameters, is used to minimize the error. The neural networks are highly nonlinear, and may have many local minima in the error. The aim would be to settle on a suitable local minimum. The problem of identifying h in the inputoutput description is akin to nonlinear time-series prediction, for which the sampled output represents the time series to be predicted. In time-series prediction, the goal is to take recent samples of an output, and predict the input for the near future. Time-series prediction can be done in the phase space reconstruction of the time series. When the reconstruction is in the appropriate dimension, as described earlier, and trajectories do not cross, then there is a unique shorttime dynamical evolution for any point in the reconstructed phase space. That is, there is a well defined function that maps a given point in the phase space to its iterate, a few samples later. This function is to be identified. One way to do it is to find localized dynamics near reference trajectories in the dynamics. These localized dynamics are good within a specified distance of the reference trajectory, and are often modeled with linear functions. This can also be done by dividing the phase space into cells. Thus, when identified, the dynamics are approximated with piecewise linear maps. The nonlinear dynamics of single-input/single-output dynamical systems and control systems can be modeled by setting up equivalent reverse multipleinput/ single-output linear systems to represent the system at hand. When the system input and output are reversed, the nonlinearities can then be treated as additional correlated inputs [13,14]

7.3 Dynamic Response Using Lie Series

The system (1) can be rewritten in the following vector form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), Q) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases} \tag{8}$$

with:

$$\begin{cases} x_1(t) = \varepsilon(t) \\ x_2(t) = \psi(t) \\ x_3(t) = \dot{\varepsilon}(t) \\ x_4(t) = \dot{\psi}(t) \end{cases} \tag{9}$$

Where with **x** we have denoted the state vector, with $\mathbf{x}_0$ the initial conditions while with **f(x)** the vector force field due to the oil pressure. Lie Series expansion is a powerful method that allow us to express the solution of nonlinear ordinary differential equation by means of an operator used in recursive way [1,2,3,4,5 and 6]. From a mathematical viewpoint the solution is given by:

$$\begin{aligned} \mathbf{x}(t) &= [e^{t\,[D(Q)]}](\mathbf{x}_0) = \\ &= [\sum_{k=0}^{\infty} \tfrac{t^k}{k!}\,[D(Q)]^k](\mathbf{x}_0) = \\ &= \mathbf{x}_0 + t\,[D(Q)](\mathbf{x}_0) + \tfrac{t^2}{2}[D(Q)]^2(\mathbf{x}_0) + ... \end{aligned} \tag{10}$$

with

$$[D(Q)] = [\mathbf{f}^{\mathrm{T}}(\mathbf{x}(t), Q)\frac{\partial}{\partial \mathbf{x}(t)}]\Big|_{\mathbf{x}(t)=\mathbf{x}_0} \tag{11}$$

In practice, the use of this operator is crucial in order to find solution by means of Lie series expansion. If Lie operator is built in an appropriate way, it allows us to obtain a very quick approximate analytical solution of the system equations of motion. From the operative viewpoint, we must truncate the solution (7) at $k = n$:

$$\begin{aligned} \hat{\mathbf{x}}(t) &= [\sum_{k=0}^{n} \tfrac{t^k}{k!}\,[D(Q)]^k](\mathbf{x}_0) = \\ &= \mathbf{x}_0 + t\,[D(Q)](\mathbf{x}_0) + \tfrac{t^2}{2}[D(Q)]^2(\mathbf{x}_0) + ... + \\ &+ \tfrac{t^n}{n!}[D(Q)]^n(\mathbf{x}_0) \end{aligned} \tag{12}$$

Where with $\hat{\mathbf{x}}(t)$ we have indicated the approximated analytical solution of (7). It is worth noting that (9) can be built in recursive way using PC code for symbolic calculus. Substituting the (1) and (2) in (5) we obtain the following relations:

$$\begin{cases} f_1(\mathbf{x}(t), Q) = x_3(t) \\ f_2(\mathbf{x}(t), Q) = x_4(t) \\ f_3(\mathbf{x}(t), Q) = +x_1(t)x_4(t)^2 + \\ -Q\,f_r(x_1(t), x_2(t), x_3(t), x_4(t)) - W\sin(x_2(t)) \\ f_4(\mathbf{x}(t), Q) = -2\frac{x_3(t)}{x_1(t)}x_4(t) + \\ +\frac{Q\,f_t(x_1(t),x_2(t),x_3(t),x_4(t))}{x_1(t)} - \frac{W\cos(x_2(t))}{x_1(t)} + \frac{P}{x_1(t)} \end{cases} \tag{13}$$

Substituting the (10) in (8) and the (8) in (9) obtain the solution of (5) written in explicit form:

$$\begin{aligned} &[D(Q)] = \left[\mathbf{f^T}(\mathbf{x}(t), Q)\frac{\partial}{\partial \mathbf{x}(t)}\right]\Big|_{\mathbf{x}(t)=\mathbf{x}_0} = \\ &= \Big[f_1(\mathbf{x}(t), Q)\cdot\frac{\partial}{\partial x_1(t)} + f_2(\mathbf{x}(t), Q)\cdot\frac{\partial}{\partial x_2(t)} + \\ &+f_3(\mathbf{x}(t), Q)\cdot\frac{\partial}{\partial x_3(t)} + f_4(\mathbf{x}(t), Q)\cdot\frac{\partial}{\partial x_4(t)}\Big]\Big|_{\mathbf{x}(t)=\mathbf{x}_0} = \\ &= \Big\{x_3(t)\cdot\frac{\partial}{\partial x_1(t)} + x_4(t)\cdot\frac{\partial}{\partial x_2(t)} + \\ &+ \big[x_1(t)x_4(t)^2 - Q\,f_r(x_1(t), x_2(t), x_3(t), x_4(t)) + \\ &-W\sin(x_2(t))\big]\cdot\frac{\partial}{\partial x_3(t)} + \\ &+ \Big[-2\frac{x_3(t)}{x_1(t)}x_4(t) + \frac{Q\,f_t(x_1(t),x_2(t),x_3(t),x_4(t))}{x_1(t)} + \\ &-\frac{W\cos(x_2(t))}{x_1(t)} + \frac{P}{x_1(t)}\Big]\cdot\frac{\partial}{\partial x_4(t)}\Big\}\Big|_{\mathbf{x}(t)=\mathbf{x}_0} \end{aligned} \tag{14}$$

$$\begin{aligned} &\hat{x}_1(t) = \hat{\varepsilon}(t) = [\sum_{k=0}^{6}\frac{t^k}{k!}[D(Q)]^k](x_{1,0}) = \\ &= [\sum_{k=0}^{6}\frac{t^k}{k!}[D(Q)]^k](\varepsilon_0) = \\ &= x_{1,0} + t\,[D(Q)](x_{1,0}) + \frac{t^2}{2}[D(Q)]^2(x_{1,0}) + \\ &+\frac{t^3}{3!}[D(Q)]^3(x_{1,0}) + \frac{t^4}{4!}[D(Q)]^4(x_{1,0}) + \frac{t^5}{5!}[D(Q)]^5(x_{1,0}) + \\ &+\frac{t^6}{6!}[D(Q)]^6(x_{1,0}) = \\ &= \varepsilon_0 + t\,[D(Q)](\varepsilon_0) + \frac{t^2}{2}[D(Q)]^2(\varepsilon_0) + \\ &\frac{t^3}{3!}[D(Q)]^3(\varepsilon_0) + \frac{t^4}{4!}[D(Q)]^4(\varepsilon_0) + \frac{t^5}{5!}[D(Q)]^5(\varepsilon_0) + \\ &+\frac{t^6}{6!}[D(Q)]^6(\varepsilon_0) \end{aligned} \tag{15}$$

The (12) and (13) are the approximated analytical solutions $\hat{\epsilon}(t)$ and $\hat{\psi}(t)$. It is worth noting that these relations depend on the unknown parameters that have to be identified.

$$\begin{aligned} &\hat{x}_2(t) = \hat{\psi}(t) = [\sum_{k=0}^{6}\frac{t^k}{k!}[D(Q)]^k](x_{2,0}) = \\ &= [\sum_{k=0}^{6}\frac{t^k}{k!}[D(Q)]^k](\psi_0) = \\ &= x_{2,0} + t\,[D(Q)](x_{2,0}) + \frac{t^2}{2}[D(Q)]^2(x_{2,0}) + \\ &\frac{t^3}{3!}[D(Q)]^3(x_{2,0}) + \frac{t^4}{4!}[D(Q)]^4(x_{2,0}) + \frac{t^5}{5!}[D(Q)]^5(x_{2,0}) + \\ &+\frac{t^6}{6!}[D(Q)]^6(x_{2,0}) = \\ &= \psi_0 + t\,[D(Q)](\psi_0) + \frac{t^2}{2}[D(Q)]^2(\psi_0) + \\ &\frac{t^3}{3!}[D(Q)]^3(\psi_0) + \frac{t^4}{4!}[D(Q)]^4(\psi_0) + \frac{t^5}{5!}[D(Q)]^5(\psi_0) + \\ &+\frac{t^6}{6!}[D(Q)]^6(\psi_0) \end{aligned} \tag{16}$$

7.4 Parameter identification

If we indicate with $\mathbf{x}_j$ the measured values of the rotor eccentricity and the attitude angle, we can define the error functions $e_1(Q)$ and $e_2(Q)$. The minimum of these functions give us the estimated values of the unknown parameter Q despite of the presence of the unavoidable noise that occurs in all measurement process [7,8 and 9].

$$\begin{aligned} e_1(Q) &= \sum_{j=1}^{N}\left[x_{1,j} - \hat{x}_1(t = j\,\Delta t)\right]^2 = \\ &= \sum_{j=1}^{N}\left[\varepsilon_j - \hat{\varepsilon}(t = j\,\Delta t)\right]^2 \end{aligned} \tag{17}$$

$$\begin{aligned} e_2(Q) &= \sum_{j=1}^{N}\left[x_{2,j} - \hat{x}_2(t = j\,\Delta t)\right]^2 = \\ &= \sum_{j=1}^{N}\left[\psi_j - \hat{\psi}(t = j\,\Delta t)\right]^2 \end{aligned} \tag{18}$$

8 Results and discussion

We have performed the proposed identification procedure assuming as data the results obtained from the numerical integration of the system equations of motion (1). In figure 3 is showed the journal orbit obtained for Q=100 and W=500 and P=1.

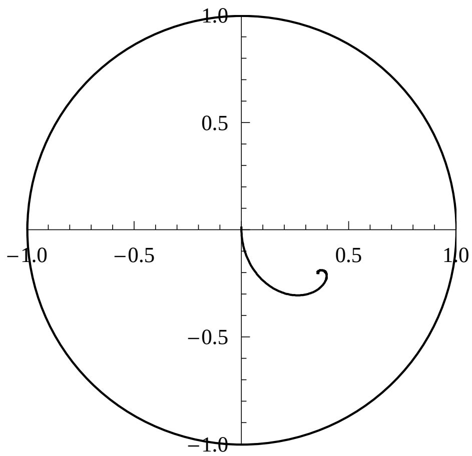

Figure 3: Journal orbit

In figure 4 and 5 are showed the analytical and numerical solutions on which we have performed the identification procedure.

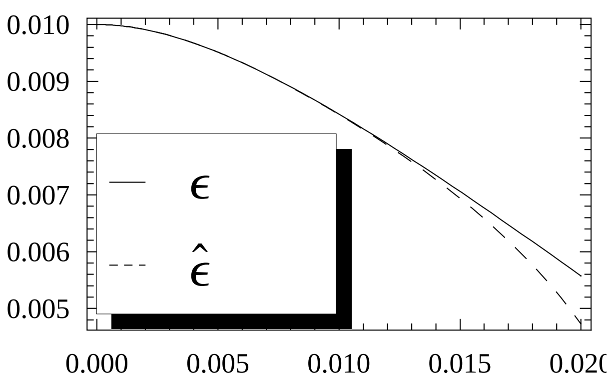

Figure 4: ϵ and $\hat{\epsilon}$, Analytical and numeric result vs. time

Then we have used a solution obtained using Lie series expansion truncated at the sixth order. We have displayed the two solution, analytical and experimental response, obtained assuming the following initial conditions:

$$\begin{cases} \varepsilon(0) = 0.01 \\ \psi(t) = \frac{\pi}{2} \\ \dot{\varepsilon}(t) = 0 \\ \dot{\psi}(t) = 0 \end{cases} \tag{19}$$

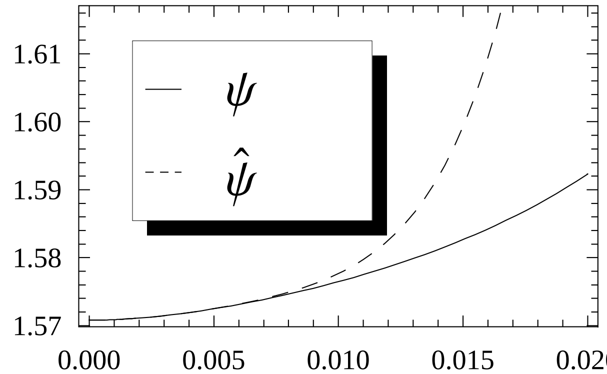

Figure 5: ψ and $\hat{\psi}$, Analytical and numeric result vs. time

In figure 3 is indicated the journal orbit obtained numerically. In the figures 4 and 5 are showed the polar coordinates of the journal orbit obtained by Lie series expansion $\hat{\epsilon}(t)$ and $\hat{\psi}(t)$.

We have performed the identification procedure for dimensionless values of the time between 0 and 0.005 and we have found that the relative error is less than 1 per cent.

$$Q^{true} = 100 \quad , \quad \begin{cases} Q_{\hat{\varepsilon}}^{identified} = 100.036 \\ Q_{\hat{\psi}}^{identified} = 98.5198 \end{cases} \tag{20}$$

The authors hope to improve this method a make it more and more efficient.

References:

[1] John M. Vance, Rotordynamics of Turbomachinery, *Wiley*, (1988).

[2] V. D'Agostino, D.Guida, A Ruggiero, Non linear Parameter Identification of a Rotor on Journal Bearings, *International Conference on Engineering Optimization*, 1 (2008) 35-42.

[3] D. de Falco, D.Guida, E. Pennestri, Non linear Parameter Identification of Rotors, *Recent Advances in Analytical Dynamics and Nonlinear Control*, 1 (2008) 67-73.

[4] S. D'Ambrosio, D. Guida, C. M. Pappalardo, J. Quartieri,Nonlinear Identification Method of Multibody System Parameters, *IPMM-2007 The 6th International Conference*, 1 (2007) 27-31.

[5] S. D'Ambrosio, C. Guarnaccia, D. Guida, T. L. L. Lenza, J. Quartieri, System Parameters Identification in a General Class of Non-linear Mechanical Systems, *International Journal of Mechanics*, 1 (2007) 76-79.

[6] D. Guida, J. Quartieri, S. Steri, A Procedure for Calculating an Approximate Analytical Response in a Large Class of Mechanical Systems, *WSEAS MInternational Conference on Non-Linear Analysis*, Tenerife 2006.

[7] D. Guida, (2005). On the simulation of crankshaft dynamics. In: WSEAS SYSTEM SCIENCE and SIMULATION in ENGINEERING (ICOSSSE 2005). Tenerife (Spain), p. 12-16

[8] Guida D., Durso L. (2005). Dry Friction Influence on the Stability of a Mechanical System with Two Degree of Freedom. In: 7th WSEAS International Conference on Non-Linear Analysis, Non-Linear Systems and Chaos. Sofia, p. 56-60

[9] V.N. Pilipchuk, C.A. Nan, Non linear system identification based on Lie series solutions, *Mechanical System and Signal Processing*, 19 (2005) 71-86.

[10] Jer-Nan Juang, Minh Q. Phan, Identification and Control of Mechanical Systems, *Cambridge University Press*, (2001).

[11] L. Ljung, System Identification - Theory for the User, *Prentice Hall*, (1999).

[12] Jer-Nan Juang, Applied System Identification, *Prentice Hall*, (1994).

[13] S. D. Fassois, Identification Model-Based Methods, Encyclopedia of Vibration, *ACADEMIC PRESS*, (2002).

[14] B F Feeny, Non Linear System Identification, Encyclopedia of Vibration, *ACADEMIC PRESS*, (2002).