Sensitivity Analysis of Stability Problems of Steel Structures using Shell Finite Elements and Nonlinear Computation Methods

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Abstract: - The paper analyzes the influence of initial imperfections on the ultimate limit state of a slender strut, applying the ANSYS programme. The geometrical and material nonlinear finite element method was applied for the theoretical analysis. Modelling of the steel structure was performed using SHELL elements. The effect of input imperfections on the load-carrying capacity is evaluated by sensitivity analysis. This paper is devoted to a class of sensitivity analysis techniques that are known as the variance-based methods. Input imperfections are of random origin. The Sobol's sensitivity analysis was used to determine the sensitivity of load-carrying capacity of a strut with respect to the variance of initial imperfections. The sensitivity analysis results identify the imperfections the variability of which can influence the structure reliability. The Latin Hypercube Sampling method was applied for the evaluation of sensitivity indices. The computation model elaborated is unique with regard to its numerically demanding character.

Key words: reliability, steel, structures, imperfections, stability, sensitivity analysis, shell elements.

1 Introduction

Beams and columns find utilization as main members or secondary ones in structures such as rigid frames and bridges. During the stability design of steel structures, columns and beams are the basic elements that must be paid attention. A great deal of attention has been devoted to the research of flexural buckling of columns in simply supported and centrally loaded conditions. Differences in the buckling load of actual steel columns and idealized columns have been an investigation topic ever since the discussion of the critical load of a perfectly straight column by Euler took place more than 200 years ago [1]. Recently performed analyses on structural stability have been aimed at the elaboration and perfection of knowledge on the ultimate strength and actual behaviour of structural systems under load action. These serve as the basis of specified procedures for practical design [1]. Lately, analogous attention has been paid to better understanding of structural safety and reliability with respect to the perfection of the general probability based concept of the specified design criteria, see, e.g, [2], [3] and [4].

The properties of structures are influenced by a number of factors, which are of random character (material, geometry, effects of the surrounding environment, load action, etc.). If a structure is to reliably fulfil its function during its service life, it is necessary to make provision for this already during its design.

2 Reliability of Structures

In the limit state method acc. to the EUROCODE standards, the design load-carrying capacity value R_d is considered to be the lower quantile (for reliability index β=3.8, it is 0.1 percentile) evaluated from the random load-carrying capacity R, see Fig.1. Both R and A (load action) are random variables, characterised by their probability density functions, and it describes their statistical variance.

The variance of the load-carrying capacity of a structure is influenced by the variance of input imperfections. The variance of input imperfections is influenced by production quality, which should be monitored in all countries of the European Union. The availability of these data presents a problem. The basic methodology of reliability assessment utilizing random input characteristics is listed in EN1990 [5, 6].

In the general classification of initial structural imperfections, first published in [7] and [8], three fundamental categories of imperfections are considered. They include:

1. Geometrical imperfections: initial curvature of member axis, eccentricity of load action, deviation of the cross section (tolerance of dimensions and the cross section shape), from the theoretical layout, etc.
2. Material imperfections: dispersion of the material mechanical properties (inhomogeneousness of material characterised by the dispersion of the

yield strength, ultimate strength, Young's modulus, etc.), initial stress state (residual stress as a consequence of rolling, welding, straightening and other technological manufacturing processes).

3. Structural imperfections: imperfections in the realization of joints, connections, welds, anchorage and other structural details which are apparent in comparison with the theoretical assumptions introduced in the solution of idealized system, in deviations of the effects of the actual structural system.

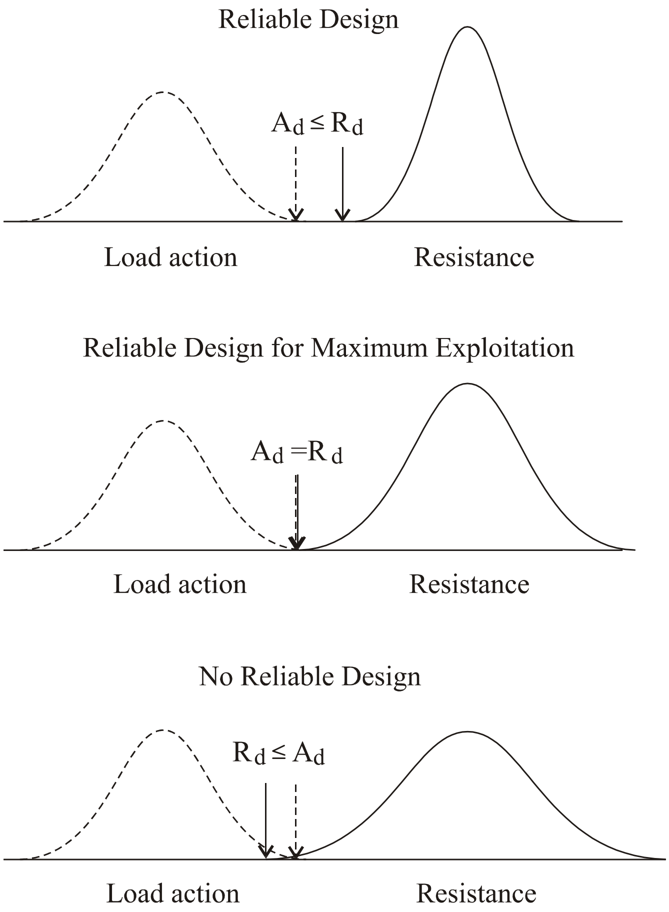

Fig.1 Statistical analysis of ultimate limit state

3 Initial Imperfections

In the field of structural steel imperfections, the research work has been traditionally focused on the variability analysis of material and cross-sectional characteristics. Current mechanical characteristics of steel are published in [8-12]. The most important results of experimental research are yield strength, tensile strength, ductility of steel plates and flanges of cross sections IPE, and geometrical characteristics of cross sections IPE found from 1989 till 2004 are published in [9]. The results of material tests on S235 steels manufactured in the Central Europe (Austria and the Czech Republic) are presented in [10, 11]. It is also necessary to mention the extensive experimental research realized in Hungary [4]. Mechanical characteristics of the U profile of steel S355 were published in [12].

3.1 Yield Strength

The yield strength is one among the most important tensile strength characteristics of structural steels; in beams, it often influences the load-carrying capacity of steel structures in a decisive manner. Yield strength is a dominant strength characteristic of members under tension, bending and torsion when elastic behaviour is presumed.

Real yield strength values have been obtained by experimental research [9, 12], and represent the basic input data of a number of reliability analyses [13-19]. Histograms of yield strength of steel S235 and S355 are depicted in Fig. 2 and Fig. 3.

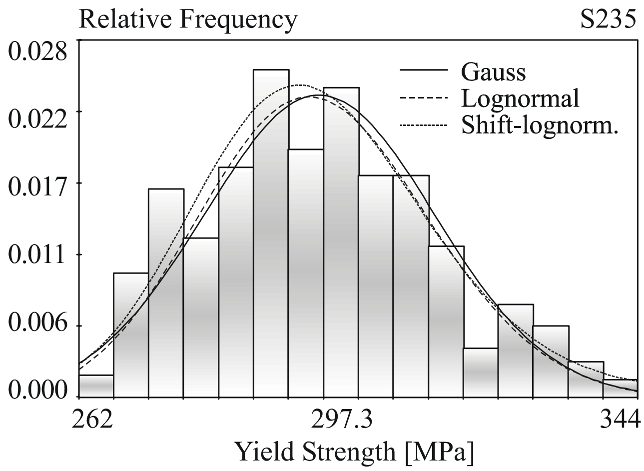

Fig.2 Histogram of yield strength of steel S235

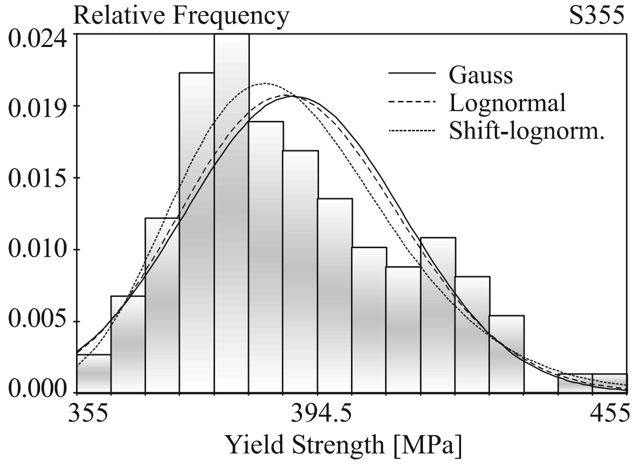

Fig.3 Histogram of yield strength of steel S355

3.2 Geometrical Imperfections

The observance of geometric dimensions and the weight of bars are eminently checked in production. Tolerances on geometrical shape and dimensions are listed in the Tolerance Standard EN 10034:1993. The permitted limit deviation of the actual weight from the theoretical one is given as + 4 % for individual bars. The tolerance limits of weight and geometry are in discordance. The variables *h*, *b*, t_1, t_2, see Fig.4, were statistically evaluated from experimentally obtained data.

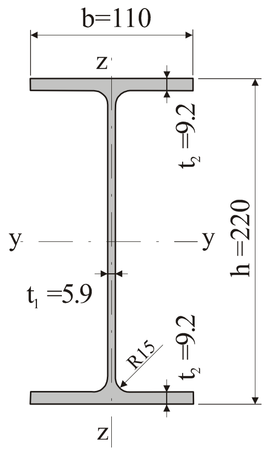

Fig.4 IPE220 Cross section dimensions

One of the main geometrical characteristics is the relative cross-sectional area which is given as the ratio of the real cross-sectional area obtained from the measurement of the cross-sectional geometry to the nominal value of the cross-sectional area. The results of the relative area evaluated for hot-rolled steel profiles IPE 160 to IPE 240 are depicted in Fig.5.

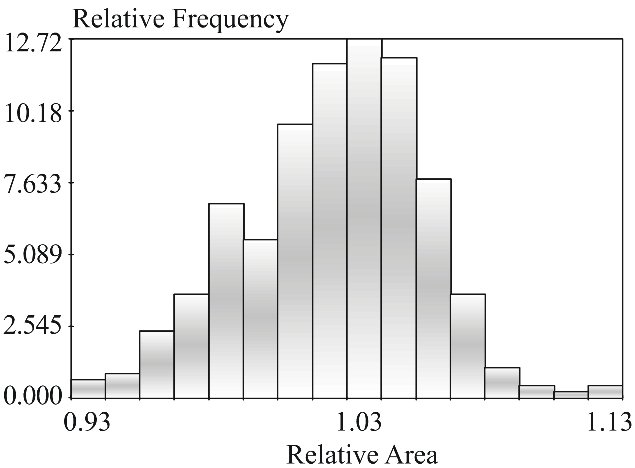

Fig.5 Histogram of relative cross section area

The member was modelled applying the programme ANSYS by the shell (thin-walled) elements called SHELL 181, see Fig.6 and Chapter 4.

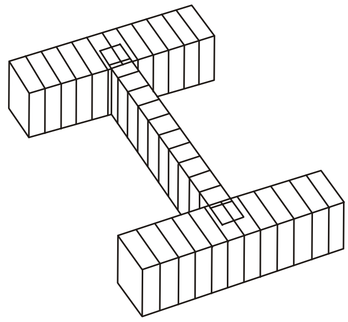

Fig.6 Shell elements mesh of IPE220 cross section

3.3 Initial Curvature of Member Axis

The initial curvature of member axis was modelled in the form of one half sine curve. According to the results of experimental research [20], the dominant shape of initial curvature is given as one half-wave of the sine function. It was considered with initial curvature in the plane of primary bending in the direction of axis *y*, see Fig.7.

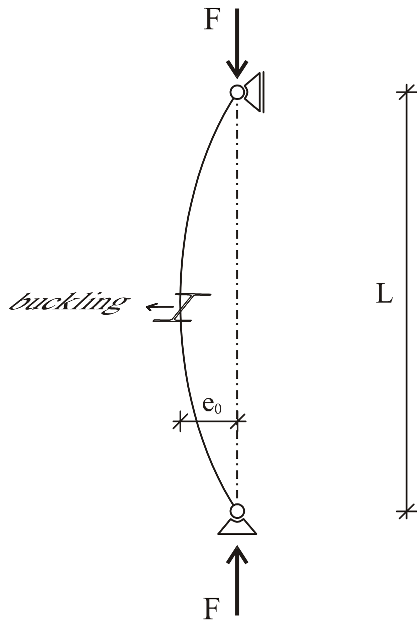

Fig.7 Member with imperfection e_0 under compression

The amplitude e_0 of maximal initial imperfection of the axis was introduced as the random quantity with Gauss density function. In case that the amplitude e_0 is measured for a higher number of struts, both the positive and negative realizations should occur with the same frequency. The Gauss probability density function was selected for the random variable e_0, so that 95 % of the realizations are found within the tolerance limits of the standard EN 10034.

3.4 Young's Modulus
According to experimental measurements, the Young's modulus *E* can be considered with the Gaussian distribution, mean value being 210 GPa, and standard deviation, 12.6 GPa [20, 21].

3.5 Residual Stress
In case of hot-rolled steel girders, the unsteady cross section cooling down takes place after hot rolling. The regions at the flange edge and at the web middle cool down more rapidly, the primary tendency being the stabilization of volume changes during shrinkage. The following, slower cooling down and shrinkage of thermally more exposed internal cross section parts in the contact region of flanges and web causes the compressive stress in the regions previously cooled down and volume stabilized. Simultaneously, shrinkage stress takes place at the flange edge and at the web middle. These primary technology states of stress components can continue redistributing due to beam loading and unloading.

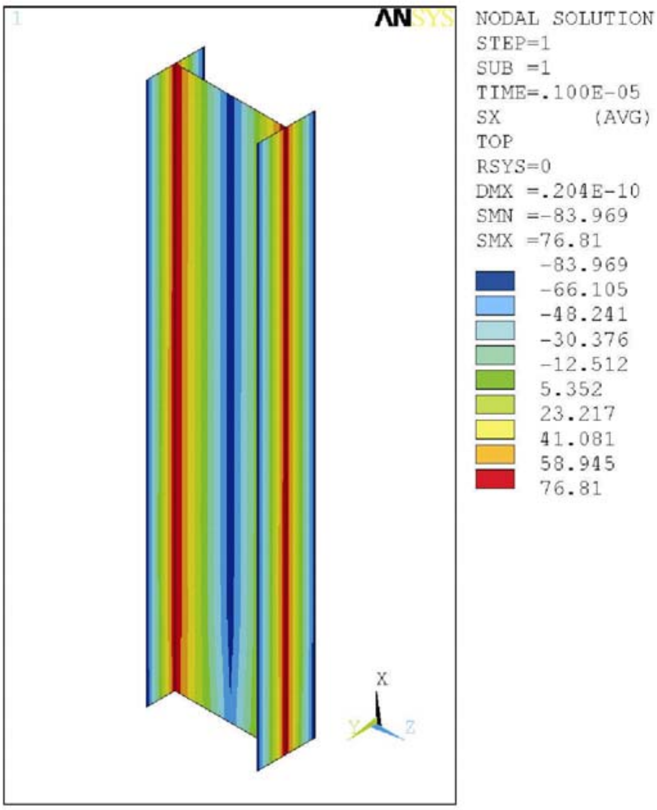

Fig.8 Model of residual stress

The results of experimental research are published in [1] and [22]. The residual stress was considered as a random quantity having Gaussian density function with value 80 MPa and standard deviation 40 MPa. Triangular distribution was considered for both flanges and web, see [13]. The residual stress of 80 MPa of a straight strut is depicted in Fig.8. The symmetry of geometry and symmetrical distribution of residual stress were considered. For lucidity's sake, only half of the strut length is depicted.

4 Computation Model
4.1 Shell Finite Elements
The beam was meshed applying the programme ANSYS. In general, the beam was modelled using thin-walled elements, type SHELL 181, i.e., the thin-walled effect was taken into account. The symmetry was used with regard to the very demanding character of the problem solved. In the bar half on the symmetry plane, we supposed the shift prevention in all cross section nodes in direction of axis *X*, and rotation around axes *Y* and *Z*. On the second edge of the bar half solved, we prevented the shifts of nodes in direction of the axis *Y* on the flange of profile IPE240. On the lower flange of that edge, we prevented the shifts in the direction of the axis *Z*. The upper flange was left free.

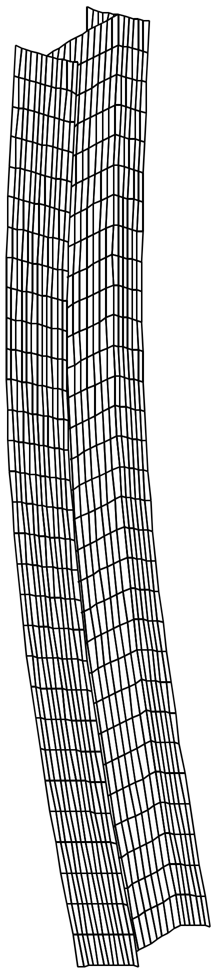

Fig.9 Strut under compression (elements SHELL 181)

Within the framework of each run of the Latin Hypercube Sampling (LHS) method [23], the load-carrying capacity was solved by nonlinear computation by means of the programme system ANSYS. A very detailed FEM model was used, see Fig.9. Also the influence of local imperfections which could contribute to the load-carrying capacity loss was thereby taken into consideration.

4.2 Nonlinear Computation Methods

In geometrically and material nonlinear FEM solution, the Euler method was applied based on proportional loading in combination with the Newton-Raphson method. We determined the load-carrying capacity as the loading constant at which the matrix determinant of tangential stiffness K_t of the structure would approach zero with certain accuracy. As we required the determination of load-carrying capacity with accuracy 0.1 %, it was necessary to use, with the Euler method, automatic control of the loading step. We supposed the bilinear kinematic material strengthening. Further on, we also have supposed that the initial steel plastification occurs when Mises stress exceeds yield strength.

4.3 Input Random Quantities

All the input random quantities were considered with the Gauss density function, their statistical characteristics being described in the Chapter 2. For geometrical characteristics of cross section dimensions, it was supposed that the nominal (characteristic) value was equal to the mean value.

Table 1 Statistic characteristics of the input variables

Random variables		Mean value	Standard Deviation
Yield strength of S235	f_{y1}	297.3 MPa	16.8 MPa
Yield strength of S355	f_{y2}	394.5 MPa	19.809 MPa
Young's modulus	E	210 GPa	12.6 GPa
Cross-sectional depth	h	220 mm	0.975 mm
Cross-sectional width	b	110 mm	1.093 mm
Web thickness	t_1	5.9 mm	0.247 mm
Flange thickness	t_2	9.2 mm	0.421 mm
Residual stress	rs	80 MPa	40 Mpa
Amplitude of initial axis curvature	e_0	0 mm	0.767 L/ 1000

It has been assumed that 95 % of the realizations of random imperfection e_0 of the strut of profile IPE 220 are found within the tolerance limits 0.15 % L mm of the standard EN 10034.

The aim of the study is a comparison of the influence of imperfections from Table 1 on the load-carrying capacity of struts made of steels S235 and S355 with identical non-dimensional slenderness. The non-dimensional slenderness is given in EUROCODE 3 by:

$$\overline{\lambda} = \sqrt{\frac{A \cdot f_y}{F_{cr}}} \tag{1}$$

where A is the cross-sectional area, f_y is the characteristic value of yield strength, and F_{cr} is the Euler critical force of bilaterally hinged strut [1].

5 Sensitivity Analysis

Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, on different sources of variation, and of how the given model depends upon the information fed into it [24, 25, 26]. On this basis, we contend that the sensitivity analysis is a prerequisite for model building in any setting, be it diagnostic or prognostic, and in any field where models are used [26].

5.1 Why Carry Out a Sensitivity Analysis?

Within the scope of modelling, the notion "sensitivity analysis" has different meaning to different people, see, e.g., [27-40]. Other examples as to why we carry out the sensitivity analysis are listed in [26]. For a reliability engineer, it could involve the process of changing components in the design of a plan in order to investigate changes in the fault tree analysis of that plan. For a chemist, it could mean the analysis of the strength of the relation between kinetic or thermodynamic inputs and measurable outputs of a reaction system. For a software engineer, it could be related to the robustness and reliability of the software with respect to different assumptions. For an economist, it could mean the appraisal of the stability of estimated parameters of a model derived via regression with respect to all factors excluded from the regression, thereby determining whether parameter estimation is robust or fragile. For a developer of expert systems, it could be the measure of sensitivity with respect to the quantities of prior distributions. For a statistician dealing with statistical modelling, it is pertinent mainly to the robustness analysis. Various disciplines have different utilization and applications and a universal procedure for the analysis of sensitivity does not exist [26].

Sensitivity analysis methods in application concerning the analysis of the influence of imperfections of steel structures on their reliability are presented in the paper. The aim of sensitivity analysis is the assessment of the influence of input random variables on the variability of output random variables. With regard to the limit states of structures, the load-carrying capacity (or deflection at selected section) is frequently considered to be the output variable [27, 28, 29]. In mass produced products (e.g. steel hot–rolled profiles, etc.), it is possible to determine, according to the relative sensitivity, those variables that the monitored output (e.g. load-carrying capacity) is especially sensitive to. It is then possible to concentrate the control activity on these variables with aim at securing sufficient stability in statistical parameters or at lowering their random variability. The sensitivity analysis can be generally divided into two groups: (i) deterministic sensitivity analysis and (ii) stochastic sensitivity analysis.

5.2 Deterministic Sensitivity Analysis

The deterministic sensitivity analysis (or also design sensitivity) is a relatively known, in structure design currently used method. This analysis accompanies the design procedures for which a calculation model is applied. In this context, we usually speak about a parametric study (sometimes called "what-if-study"). Such a study may be organized in a simple way as a sequence of calculations with gradually varying values of a certain input parameter X_i in each calculation step j(j=1, 2,…,M) in a certain real range. By comparing the results of calculations R_j (the set of the structure response), it is possible to discover the influence of parameter X_i to response R_j. Some of the more advanced computational programmes include such a possibility. Everything is carried out automatically. However, quantified data on the sensitivity are not obtained.

5.3 Stochastic Sensitivity Analysis

The stochastic sensitivity analysis offers a more complex (and quantified) information on the influence of parameters. However, it is necessary to use more sophisticated stochastic numerical methods. The input parameters are considered to be random variables. The process of determining the sensitivity is, to a certain degree, similar to that in the deterministic sensitivity analysis. The change of parameter X_i is also carried out (here, however, within the framework of the applied simulation technique), and we observe how this change will manifest itself in the output variable. This variable may be represented by the data on the structure reliability; too, and this is also utilized within the framework of the paper presented.

In recent years, a number of different methods of the stochastic sensitivity analysis have been evolved [26]. Stochastic methods are based upon various assumptions, and it is difficult to compare the results. However, each of the methods has its informative capability of a different kind. These are briefly as follows: (i) the method based upon the observation of the correlation coefficient, (ii) variance-based methods and (iii) Fourier amplitude sensitivity texts [26]. This paper is devoted to (ii).

5.4 Sobol's Sensitivity Analysis

The coherent sensitivity analysis concept enabling an analysis of the influence of arbitrary subgroups of input factors (doubles, triples, etc.) on the monitored output was worked out by the Russian mathematician Ilja M. Sobol [41, 42].

The sensitivity analysis of load-carrying capacity (random output Y) to input imperfections (random inputs X_i from Table 1) was evaluated in our study. Sobol's first order sensitivity indices may be written in the form:

$$S_i = \frac{V\left(E\left(Y|X_i\right)\right)}{V(Y)} \tag{2}$$

Sobol proposed an alternate definition $S_i = corr\left(Y, E\left(Y|X_i\right)\right)$ based on the evaluation of correlation between the output random variable Y and the conditional random arithmetical mean $E\left(Y|X_i\right)$. Analogously, we can write the second order sensitivity indices:

$$S_{ij} = \frac{V\left(E\left(Y|X_i, X_j\right)\right)}{V(Y)} - S_i - S_j \tag{3}$$

Sensitivity index S_{ij} expresses the influence of doubles on the monitored output. Other Sobol's sensitivity indices enabling the quantification of higher order interactions may be expressed similarly.

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + ... + S_{123...M} = 1 \tag{4}$$

The number of members in (4) is 2^{M-1}, i.e., for M=3, we obtain 7 sensitivity indices S_1, S_2, S_3, S_{12}, S_{23}, S_{13}, S_{123}; for M=10, we obtain 1023 sensitivity indices; it is excessively large quantity for practical usage. The main limitation in the determination of all members of (4) is the computationally demanding character.

The possible motivations for sensitivity analysis are [26]:

- Model corroboration. Determination of the robustness of inference. Dependence of model on fragile assumptions.

- Research prioritization. Estimation of the factors which deserve further analysis or measurement ⇒ Factor prioritization setting.
- Model simplification. Which factors or model components can be fixed or simplified? ⇒ Factor fixing setting.
- Identification of critical or otherwise interesting regions in the space of input factors. Identification of factors which interact and may thereby generate extreme values; it is important, e.g., in structural reliability.
- Prior to parameter estimation, to help in experiment setup (actual or numerical) in conditions where output sensitivity to the estimated factor is the greatest.

A simplification of models underpinned by the "factor fixing" setting can be significant during model auditing, e.g., during policy assessment or in the face of scientific controversy [26].

5.5 Latin Hypercube Sampling

The Latin Hypercube Sampling (LHS) method was applied for calculation of sensitivity indices. LHS is the Monte Carlo type method, see [23]. The model output Y is the load-carrying capacity calculated in each run of the LHS method.

The calculation process by the LHS method can be practically clarified on the calculation of the first order sensitivity indices (2). In the first step, N realizations of the quantity X_i, i.e. $X_i(1),\ldots, X_i(N)$ were generated. After that, K realizations of vector $X_{\sim i}$ (all except for the i^{th} one), i.e.. $X_{\sim i}(j, 1),\ldots, X_{\sim i}(j, K)$ were generated for each realization $X_i(j)$, $j = 1,\ldots, N$. Let us notice that K can but need not be equal to N. Further on, $E(Y|X_i)$ must be determined for each j:

$$E(Y|X_i) \approx m(j) = \frac{1}{K}\sum_{k=1}^{K} f\left(X_i(j), X_{\sim i}(j,k)\right) \qquad (5)$$

Approximately, the numerical value $V(E(Y|X_i))$ can be obtained according to the relation:

$$V(E(Y|X_i)) \approx \frac{1}{N-1}\sum_{j=1}^{N}\left(m(j)-\overline{m}\right)^2 \qquad (6)$$

where $\overline{m}$ is an assessment of arithmetical mean. In this study, N=K=30000 simulation runs of the LHS method were applied. The variance $V(Y)$ of load-carrying capacity is calculated under the assumption that all the input imperfections are considered to be random ones; 30000 simulation runs were applied, as well. It was proceeded similarly when calculating the second order indices (3).

6 Sensitivity Analysis Results

The results of the load-carrying capacity sensitivity analysis are presented in Fig.10 to Fig.13. The sensitivity analysis results of struts with slenderness $\overline{\lambda} = 0.6$ and $\overline{\lambda} = 1.0$ are compared. The partial variances are applied to compute sensitivity indices which, in turn, are used to measure the importance of each factor.

In connection with the nonlinear FEM, extremely demanding calculations are concerned requiring very much computer time even on the most advanced multiprocessor computers.

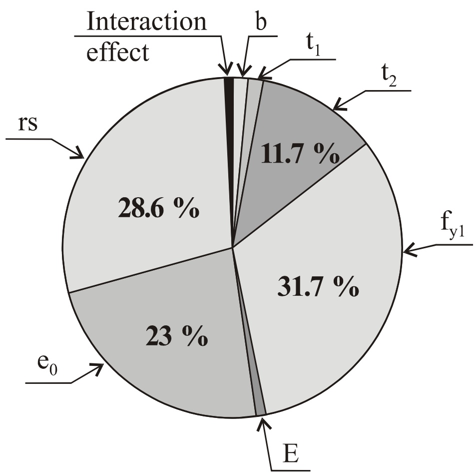

Fig.10 Sensitivity analysis for $\overline{\lambda} = 0.6$, S235

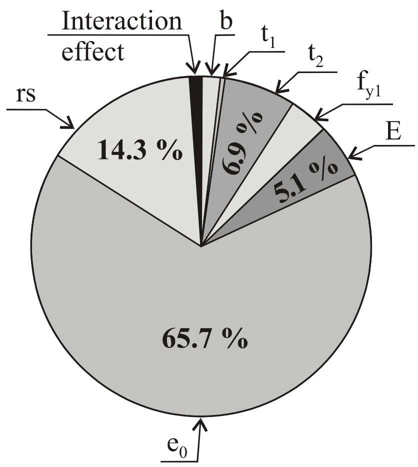

Fig.11 Sensitivity analysis for $\overline{\lambda} = 1.0$, S235

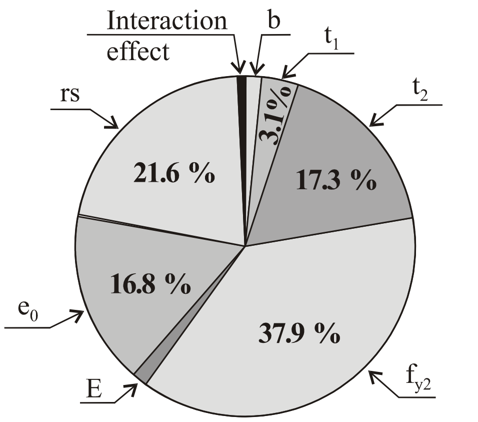

Fig.12 Sensitivity analysis for $\bar{\lambda} = 0.6$, S355

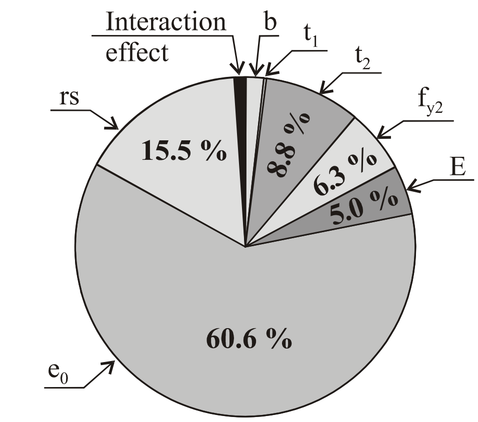

Fig.13 Sensitivity analysis for $\bar{\lambda} = 1.0$, S355

7 Conclusion

For the struts with slenderness $\bar{\lambda} = 0.6$, yield strength is the dominant quantity; for the beams with slenderness $\bar{\lambda} = 1.0$, the amplitude of initial curvature of the strut axis represents the dominant imperfection. The flange thickness represents another dominant quantity. The sensitivity indices of beams with different slenderness are clearly differing whereas the results concerning a steel of one steel grade are approximately similar.

The major advantage of the study elaborated is the fact that it enables the residual stress influence evaluation. Although the average yield strength is, for the steel grade S355, by one third higher than for the steel S235, the values of the residual stress sensitivity indices in Fig.11 and Fig.13 are very close. The same conclusion holds if we compare Fig.10 and Fig.12. The results are to be completed by further studies for other values of slenderness. The objective of analyses will be finding the dependence between the Sobol's sensitivity indices and the non-dimensional slenderness, dealt with analogously and in the paper [8] (see Fig.2 in [8]) but including the residual stress. The computation model elaborated is unique with regard to its numerically demanding character.

For each result presented in Fig.10 to Fig.13, the sum of the first order sensitivity indices is lower than 0.98; it bears witness to the fact that higher order interaction effects are negligible.

The sensitivity analysis provides the steel manufacturers with valuable information on the necessity of the imperfections to be controlled, during the manufacturing process, with increased attentiveness and accurateness. The studies published here link up with the long-term theoretical and experimental research the results of which were published in [44-53].

It has been found by comparison of the studied characteristics both of a dominant Czech manufacturer and an Austrian producer that the Czech steel is fully competitive within the EU framework. The yield strength is one of important quantities. The results of experimental investigations into material and geometrical characteristics have been published for a long-time period. In addition to the yield point analysis, also tensile strength and ductility have been studied in course of a long-term research work.

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