

Comparison of the grey theory with neural network in the rigidity prediction of linear motion guide

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Abstract: The purpose of this paper is to compare the prediction models constructed through neural network and grey theory, and to apply the prediction model established to study of correlation between linear motion guide rigidity under the stress of tension and compression. Strain data of tension and compression are simultaneously obtained by the computer that is linked with the Universal testing machine and translated into rigidity values through the formula of $F = k\delta$. Through this study we can understand the differences in prediction of rigidity between neural network and grey theory. Experiment results will serve as reference for manufacturers and users, with the hope that based on fewer measurement data testing time can be reduced and the outcome can be more accurately predicted. Based on fewer measurement data, the outcome can be more accurately predicted, and that with a nondestructive test can accurately predict the rigidity of the linear motion guide. The outcome indicates that the prediction model established through neural network is superior to the prediction model established through the grey theory, and that the neural network model can accurately predict the result.

Key-words: grey prediction model ; rigidity ; linear motion guide ; neural network ; tension ; compression

1. Introduction

The linear motion guide can be seen as a special bearing. It is not a regular rotation bearing, but a automatic processing equipment, CNC machine tools, automated robot and nano-micro-processing device pertinent to linear motion. Any automatic equipment related to linear motion needs this important part. It employs balls as the transmission

platform between the rail and the block to engage in unlimited cycles of motions. The block is confined to the rail so the load platform can engage in high-speed, high-precision linear motions along the rail. Its major components include rail, block, end plate, ball and retainer (as shown in Fig.1). Normally the friction coefficient of rolling is only 2% of that of sliding. The ratio between the strain

and the load is the rigidity value of the linear motion guide ($F = k\delta$). The rigidity value is one of the important factors that determine the quality of a linear motion guide. Rigidity test is one of the key elements that determine the quality of a linear motion guide. Accordingly, this research attempts to test the rigidity change of a linear motion guide under stress and engage in prediction study.

Researches followed two methods for prediction modeling. The first is the grey modeling (GM). It has been expansively applied to demographic, industrial and economic predictions [1-9]. The second is neural network modeling, which is often employed in various predictions [10-16]. This paper employs both the grey prediction modeling and neural network modeling for prediction of the rigidity of linear motion guide. Outcomes of both models are compared. Results of rigidity prediction will serve as references for linear motion guide manufacturers and user. They can also serve as the basis for tests pertinent, with the hope that rigidity can be predicted based on fewer data and less time, so testing cost can be reduced and enhanced profitability.

2. Methodology of grey forecasting

The GM(1,1) is a time series forecasting model, encompassing a group of differential equations adapted for parameter variance, rather than a first order differential equation. Difference equation has structures that vary with time rather than being general difference equation. Although it is not necessary to employ all the data from the original series to construct the GM(1,1), the potency of the series must be more than four. In addition, the data must be taken at equal intervals and in consecutive order without bypassing any data [17].

The GM(1,1) model constructing process is

described below. The first-order differential equation of GM(1,1) model is

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \quad (1)$$

where t denotes the independent variables in the system, a the developed coefficient, b the Grey controlled variable, and a and b denote the parameters requiring determination in the model. The variables, including $x(1), x(2), \dots$ and $x(n)$, are used to construct the Grey forecasting model and accurately predict $x(n+1), x(n+2), \dots$, and $x(n+k)$.

Denote the original data sequence by $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)\}$ (2)

When a model is constructed, the Grey system must apply one-order accumulated generating operation (AGO) to the primitive sequence in order to provide the middle message of building a model and to suppress the variation tendency. Herein, $x(1)$ is defined as $x^{(0)}$'s one-order AGO sequence. That is,

$$x^{(1)}(i) = \sum_{j=1}^i x^{(0)}(j) \quad (3)$$

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(n)\}$$

From Eqs. (1) and (3), by the least-square method, coefficient \hat{a} becomes

$$\hat{a} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} \quad (4)$$

Furthermore, accumulated matrix B is

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad \text{where}$$

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$$

Meanwhile, the constant vector Y_n is

$$Yn = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Therefore, the solution of Eq.(1) can be obtained by using the least square method. That is,

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \quad (5)$$

When $\hat{x}^{(1)}(1) = \hat{x}^{(0)}(1)$, the acquired sequence one-order inverse-accumulated generating operation (IAGO) is acquired and the sequence that must be reduced as Eq. (6) can be obtained.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \quad (6)$$

Given $k = 1, 2, \dots, n$, the sequence of reduction is obtained as follows (Eq. (7)):

$$\hat{x}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(k+1)) \quad (7)$$

where $\hat{x}^{(0)}(k+1)$ is the Grey elementary predicting value of $x(k+1)$

The forecasted error value and actual value are necessary. To demonstrate the efficiency of the proposed forecasting model, this article adopts the residual method. Herein, Eqs. (8) is used to compute the residual error of Grey forecasting.

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \quad (8)$$

3. Neural networks

The neural networks have demonstrated great potential in the modeling of the input-output relationships of complicated systems [18], Consider that $X = \{x_1, x_2, \dots, x_m\}$ is the input vector of the system, where m is the number of input variables. And $Y = \{y_1, y_2, \dots, y_n\}$ is the corresponding output vector of the system where n is the number of output variables.

The back-propagation network shown in Fig. 2

is employed in this study. The neurons, of the input layer are used to receive the input vector X of the system and the neurons of the output layer are used to generate the corresponding output vector Y of the system. For each neuron Fig. 3, a summation function for all the weighted inputs is calculated as:

$$net_j^k = \sum_j w_{ji}^k o_i^{k-1} \quad (9)$$

where net_j^k is the summation function for all the inputs of the j -th neuron in the k -th layer, w_{ji}^k is the weight from the i -th neuron to the j -th neuron and o_i^{k-1} is the output of the i -th neuron in the $(k-1)$ -th layer.

As shown in Eq. (9), the neuron evaluates the inputs and determines the strength of each one through its weighting factor, the stronger is the influence of the connection. The result of the summation function can be treated as an input to an activation function from which the output of the neuron is determined. The output of the neuron is then transmitted along the weighted outgoing connections to serve as an input to subsequent neurons. In the present study, a hyperbolic tangent function with a bias b_j is used as an activation

function. The output of the j -th neuron o_j^k for the k -th layer can be expressed as:

$$o_j^k = f(net_j^k) = \frac{e^{(net_j^k + b_j)} - e^{-(net_j^k + b_j)}}{e^{(net_j^k + b_j)} + e^{-(net_j^k + b_j)}}$$

(10)

To modify the connection weights; properly, a supervised learning algorithm [19] involving two phases is employed. The first is the forward phase which occurs when an input vector X is presented

and propagated forward through the network to compute an output for each neuron. Hence, an error between the desired output y_j and actual output

o_j of the neural network is computed. The summation of the square of the error E can be expressed as:

$$E = \frac{1}{2} \sum_{j=1}^n (y_j - o_j)^2 \quad (11)$$

The second is the backward phase which is an iterative error reduction performed in a backward direction. The gradient descent method, adding a momentum term [19], is used. The new incremental change of weight $\Delta w_{ji}^k(n+1)$ can be expressed as:

$$\Delta w_{ji}^k(n+1) = -\eta \frac{\partial E}{\partial w_{ji}^k} + \alpha \Delta w_{ji}^k(n) \quad (12)$$

Where η is the learning rate, α the momentum coefficient and n the index of iteration.

Through this learning process, the network memorizes the relationships between input vector X and output vector Y of the system through the connection weights. Implementation steps are shown in Fig. 4. Through the test data obtained via the Universal testing machine in conjunction with the computer, we determine the data under tension and pressure as shown in Appendix.

4. Setup of Experiment Equipment and Experiment Condition

Here we will introduce the rigidity of the linear motion guide because in most of the cases appropriate preloads are added to enhance the rigidity of the linear motion guide. Preloading is an approach to remove back cracks and reduce the elastic deformation between the balls and the contact surface. The preload of a linear motion

guide is adjusted through the size of the balls. Preloading via large balls creates something similar to the spring-like effect – knocking without vibrations. Generally speaking, preloading enhances rigidity by over 10 times. Too much preloading, however, will cause both the friction and the rising heat to increase the wear of the balls and it malfunction the preload effects. That tends to adversely affect positioning precision and durability. Table 1 shows comparison of different preloads. We choose linear motion guides under Z1 preload for the study.

For this paper's linear motion guide rigidity test, we design a jig (see Fig. 5) in conjunction with SHIMADZU UH100A universal testing machine. Due to the design of the jig, we can obtain the tensile and strain of the linear motion guide under tension. Under compression, the linear motion guide can be placed directly on the universal testing machine for testing. Through the stress and strain test and we can then obtain the rigidity. This experiment employs BRH25A linear motion guide manufactured by ABBA Linear Technology Company. Rigidity of 1,000kg-5,000kg is taken for modeling data. Rigidity is predicted to be between 5,500kg and 8,000kg. See Table 2.

Grey prediction models under compression and tension are shown below.

a. Steps for grey prediction modeling under tension are shown as:

1) Model's primitive sequence

$$\begin{aligned} x^{(0)} &= \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(9)\} \\ &= \{3620.1, 4054.1, 4318.6, \dots, 4752.9\} \end{aligned}$$

2) AGO

$$\begin{aligned} x^{(1)} &= \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(9)\} \\ &= \{3620.1, 7674.2, 11992.8, \dots, 39966.6\} \end{aligned}$$

3) Determine B and Yn through least square method

$$Y_n = \begin{bmatrix} 4054.1 \\ 4318.6 \\ \vdots \\ 4752.9 \end{bmatrix}, B = \begin{bmatrix} -5647.2 & 1 \\ -9833.5 & 1 \\ \vdots & \\ -37590.2 & 1 \end{bmatrix}$$

$$\hat{a} = (B^T B)^{-1} B^T Y_n$$

$$= \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.019925 \\ 4118.7 \end{bmatrix}$$

4) List the response equation

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}$$

5) Solve $\hat{x}^{(1)}(k)$ for 1-AGO

$$\begin{aligned} \hat{x}^{(0)}(k+1) &= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \\ &= (1 - e^a) \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \\ &= \{4054.1, 4318.6, 4480.3, \dots, 5484.5\} \end{aligned}$$

6) Error Examination

$$\begin{aligned} e(k) &= \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \\ &= \{-4.41, 0.01, 1.68, \dots, -23.2\} \end{aligned}$$

b. Steps for grey prediction modeling under compression are shown below:

1) Model's primitive sequence

$$\begin{aligned} x^{(0)} &= \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(9)\} \\ &= \{9221.3, 10766, 11981.5, \dots, 16304.4\} \end{aligned}$$

2) AGO

$$\begin{aligned} x^{(1)} &= \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \dots, x^{(1)}(9)\} \\ &= \{9221.3, 19987.3, 31968.9, \dots, 120395.9\} \end{aligned}$$

3) Determine B and Yn through least square method

$$Y_n = \begin{bmatrix} 10766 \\ 11981.5 \\ \vdots \\ 16304.4 \end{bmatrix}, B = \begin{bmatrix} -14604.3 & 1 \\ -25978.1 & 1 \\ \vdots & \\ -112243.8 & 1 \end{bmatrix}$$

$$\hat{a} = (B^T B)^{-1} B^T Y_n$$

$$= \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.0544 \\ 10591.2 \end{bmatrix}$$

4) List the response equation

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}$$

5) Solve $\hat{x}^{(1)}(k)$ for 1-AGO

$$\begin{aligned} \hat{x}^{(0)}(k+1) &= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \\ &= (1 - e^a) \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \\ &= \{11400.1, 12037.5, 12710.4, \dots, 23121.9\} \end{aligned}$$

6) Error Examination

$$\begin{aligned} e(k) &= \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \\ &= \{-5.89, -0.47, 2.39, \dots, -22.92\} \end{aligned}$$

5. Experiment Outcomes

The first column Table 2 shows actual rigidity due to the applied load from 1,000kg to 8,000kg obtained from the test. This paper employs 1,000kg-5,000kg as data for grey forecasting model and uses it for prediction of rigidity of 5,500kg-8,000kg. The predicted results obtained by the grey forecasting model are shown in Table 2 and Fig. 6. The mean absolute percentage errors (MAPE) of the grey forecasting model and neural network from 1000kg to 5000kg are 2.15% and 0.47% respectively under compression, and 1.85% and 0.34% respectively under tension. And the MAPEs of the grey forecasting model and neural network from 5500kg to 8000kg are 13.29% and 0.22% respectively under compression, and 12.25% and 1.48% respectively under tension. Predictions above indicate that no matter it is under compression or tension, the errors of grey forecasting modeling are greater than that of the neural network in terms of

modeling sequence or forecasting sequence. Further, the errors of the neural network are very insignificant.

6. Conclusions

This paper employs both the grey forecasting model and neural network prediction model of the rigidity of linear motion guides. This study applies the neural network model to dual-input-dual-output situation. The GM(1,1) model is a model with a group of differential equations adapted for variance of parameters, and it is a powerful forecasting model, especially when the number of observations is not large. The neural network can be used to predict any loads between 5000kg and 8000kg, while the grey forecasting can only be employed to predict rigidity values between 5000kg and 8000kg at increments of 500kg. And the average error of neural network predictions is around 1%. So it says much about the fact that neural network prediction model is effective for prediction of rigidity of linear motion guides. The prediction of grey theory on rigidity of linear motion guides is not as accurate as that of neural network prediction modeling. In summary, neural network prediction model is suitable for prediction of rigidity of linear motion guides. Its accuracy is superior to that of grey theory modeling GM (1,1). The result is highly satisfactory

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Appendix:

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
1	1000	1000	9221.3	3520.1
2	1014	1008	9218.2	3625.9
3	1096	1040	9614.0	3662.0
4	1116	1076	9620.7	3710.3
5	1156	1112	9796.6	3731.5
6	1178	1148	9816.7	3776.3
7	1198	1184	9819.7	3819.4
8	1220	1220	10000.0	3836.5
9	1262	1256	10177.4	3876.5
10	1286	1288	10206.4	3903.0
11	1306	1324	10203.1	3917.2
12	1330	1360	10230.8	3953.5
13	1374	1396	10409.1	3965.9
14	1396	1432	10575.8	4000.0
15	1418	1468	10582.1	4033.0
16	1440	1500	10588.2	4054.1
17	1486	1536	10768.1	4063.5
18	1508	1568	10771.4	4083.3
19	1532	1608	10942.9	4102.0
20	1554	1640	10943.7	4120.6
21	1600	1676	11111.1	4148.5
22	1624	1712	11123.3	4175.6
23	1646	1748	11121.6	4201.9
24	1670	1784	11283.8	4207.5
25	1718	1816	11302.6	4223.3
26	1744	1856	11473.7	4256.9
27	1768	1892	11480.5	4261.3
28	1792	1928	11487.2	4284.4
29	1840	1968	11645.6	4315.8
30	1866	2004	11662.5	4319.0
31	1888	2040	11800.0	4340.4
32	1914	2076	11814.8	4343.1
33	1964	2112	11975.6	4363.6
34	1988	2148	11975.9	4383.7
35	2014	2184	11988.1	4385.5

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
36	2040	2220	12142.9	4404.8
37	2092	2256	12162.8	4406.3
38	2118	2292	12314.0	4424.7
39	2144	2324	12321.8	4435.1
40	2170	2360	12329.6	4452.8
41	2222	2392	12483.2	4446.1
42	2248	2428	12629.2	4463.2
43	2274	2464	12633.3	4463.8
44	2302	2500	12648.4	4480.3
45	2354	2532	12793.5	4489.4
46	2382	2568	12806.5	4505.3
47	2408	2604	12946.2	4505.2
48	2434	2640	12946.8	4520.5
49	2490	2676	12968.8	4535.6
50	2516	2708	13104.2	4528.4
51	2544	2748	13113.4	4549.7
52	2570	2780	13247.4	4557.4
53	2626	2816	13262.6	4571.4
54	2652	2852	13260.0	4570.5
55	2680	2888	13400.0	4584.1
56	2708	2924	13405.9	4583.1
57	2762	2960	13539.2	4596.3
58	2790	2996	13543.7	4609.2
59	2818	3032	13548.1	4607.9
60	2846	3068	13552.4	4620.5
61	2892	3104	13514.0	4632.8
62	2914	3136	13616.8	4625.4
63	2936	3172	13592.6	4637.4
64	2960	3208	13703.7	4649.3
65	3008	3240	13798.2	4641.8
66	3032	3276	13781.8	4653.4
67	3056	3308	13765.8	4646.1
68	3084	3344	13891.9	4657.4
69	3136	3380	13876.1	4668.5
70	3162	3412	13991.2	4661.2

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
71	3188	3444	13982.5	4666.7
72	3216	3480	14105.3	4677.4
73	3272	3512	14103.5	4670.2
74	3300	3548	14224.1	4680.7
75	3328	3580	14222.2	4685.9
76	3356	3612	14220.3	4678.8
77	3412	3644	14336.1	4683.8
78	3442	3680	14341.7	4693.9
79	3470	3712	14438.3	4686.9
80	3498	3744	14454.6	4691.7
81	3556	3780	14573.8	4701.5
82	3586	3812	14577.2	4706.2
83	3614	3844	14572.6	4699.3
84	3644	3880	14693.6	4708.7
85	3700	3912	14800.0	4701.9
86	3730	3948	14801.6	4711.2
87	3760	3980	14803.2	4715.6
88	3792	4012	14929.1	4720.0
89	3848	4048	14914.7	4717.9
90	3878	4080	15031.0	4722.2
91	3908	4116	15030.8	4731.0
92	3936	4148	15022.9	4724.4
93	3998	4180	15143.9	4728.5
94	4026	4216	15135.3	4737.1
95	4058	4248	15255.6	4730.5
96	4088	4280	15253.7	4734.5
97	4148	4312	15363.0	4728.1
98	4176	4348	15352.9	4736.4
99	4208	4380	15357.7	4719.8
100	4240	4416	15474.5	4728.1
101	4300	4448	15467.6	4742.0
102	4330	4476	15575.5	4731.5
103	4362	4504	15578.6	4731.1
104	4392	4532	15574.5	4730.7
105	4452	4564	15676.1	4734.4

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
141	5708	5844	16988.1	4743.5
142	5740	5876	17083.3	4746.4
143	5774	5904	17082.8	4746.0
144	5808	5964	17082.4	4740.9
145	5842	5996	17182.4	4743.7
146	5874	6024	17175.4	4735.8
147	5908	6056	17174.4	4738.7
148	5940	6116	17267.4	4733.7
149	5974	6144	17265.9	4733.4
150	6008	6172	17264.4	4733.1
151	6040	6200	17356.3	4725.6
152	6072	6260	17348.6	4728.1
153	6108	6288	17352.3	4720.7
154	6142	6320	17448.9	4723.5
155	6174	6348	17440.7	4723.2
156	6208	6404	17438.2	4715.8
157	6240	6432	17528.1	4715.5
158	6276	6460	17530.7	4708.5
159	6310	6488	17527.8	4708.3
160	6344	6544	17622.2	4701.1
161	6376	6572	17613.3	4701.0
162	6408	6604	17604.4	4703.7
163	6444	6628	17703.3	4700.7
164	6478	6688	17699.5	4696.6
165	6512	6716	17695.7	4689.9
166	6546	6744	17788.0	4689.8
167	6578	6776	17778.4	4692.5
168	6612	6828	17774.2	4683.1
169	6646	6860	17865.6	4685.8
170	6680	6888	17861.0	4679.3
171	6712	6912	17851.1	4676.6
172	6746	6968	17941.5	4676.5
173	6780	6996	17936.5	4670.2
174	6812	7024	17926.3	4670.2
175	6846	7052	18015.8	4670.2

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
106	4484	4596	15678.3	4738.1
107	4516	4632	15790.2	4736.2
108	4548	4664	15791.7	4739.8
109	4578	4696	15786.2	4743.4
110	4640	4732	15890.4	4741.5
111	4672	4764	15891.2	4745.0
112	4706	4796	16006.8	4748.5
113	4734	4828	15993.2	4742.6
114	4798	4864	16100.7	4750.0
115	4828	4896	16093.3	4744.2
116	4862	4928	16099.3	4747.6
117	4926	4960	16204.0	4751.0
118	4958	4992	16202.6	4754.3
119	5000	5000	16304.4	4752.9
120	5020	5024	16298.7	4748.6
121	5054	5056	16303.2	4751.9
122	5086	5088	16406.5	4746.3
123	5118	5120	16403.9	4749.5
124	5150	5184	16401.3	4756.0
125	5182	5212	16503.2	4746.8
126	5214	5248	16500.0	4753.6
127	5248	5276	16607.6	4753.2
128	5280	5344	16603.8	4754.4
129	5312	5376	16600.0	4757.5
130	5346	5408	16706.3	4743.9
131	5378	5440	16701.9	4746.9
132	5410	5504	16697.5	4753.0
133	5444	5528	16802.5	4749.1
134	5478	5560	16803.7	4752.1
135	5510	5592	16798.8	4747.0
136	5542	5656	16896.3	4752.9
137	5576	5688	16897.0	4747.9
138	5608	5720	16891.6	4750.8
139	5642	5752	16994.0	4753.7
140	5676	5812	16994.0	4748.4

No	compression (kg)	Tension (kg)	compression K (kg/mm)	tension K (kg/mm)
176	6882	7108	18015.7	4664.0
177	6914	7136	18005.2	4664.1
178	6950	7164	18099.0	4658.0
179	6982	7192	18088.1	4658.0
180	7018	7244	18087.6	4649.6
181	7050	7272	18170.1	4649.6
182	7086	7296	18169.2	4641.2
183	7120	7320	18163.3	4638.8
184	7154	7372	18250.0	4630.7
185	7186	7396	18238.6	4628.3
186	7220	7420	18324.9	4625.9
187	7256	7448	18323.2	4626.1
188	7290	7496	18316.6	4615.8
189	7324	7520	18310.0	4607.8
190	7358	7544	18395.0	4605.6
191	7392	7568	18388.1	4603.4
192	7426	7612	18472.6	4591.1
193	7460	7636	18465.4	4588.9
194	7496	7660	18463.1	4581.3
195	7530	7680	18455.9	4576.9
196	7566	7724	18544.1	4570.4
197	7598	7744	18531.7	4560.7
198	7632	7764	18614.6	4556.3
199	7666	7784	18606.8	4552.0
200	7702	7824	18603.9	4538.3
201	7736	7840	18596.2	4531.8
202	7772	7860	18682.7	4522.4
203	7804	7876	18669.9	4516.1
204	7838	7912	18751.2	4500.6
205	7874	7932	18747.6	4496.6
206	7908	7948	18739.3	4485.3
207	7944	7984	18735.9	4470.3
208	7978	7996	18816.0	4457.1
209	8000	8000	18810.5	4451.9

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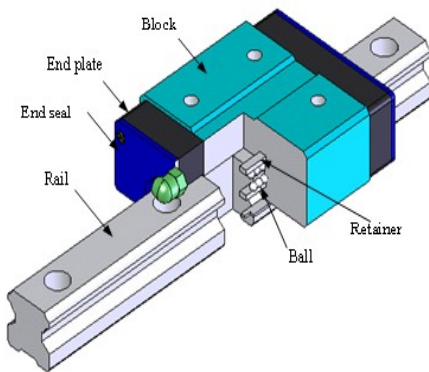


Fig.1 Structure of Linear Motion Guide.

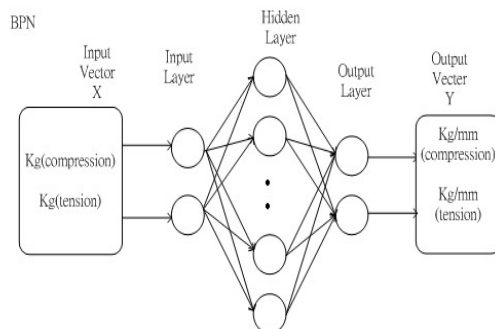


Fig.2 Configuration of the Back-propagation Network for the Modeling of the Linear Motion Guide Rigidity Test.

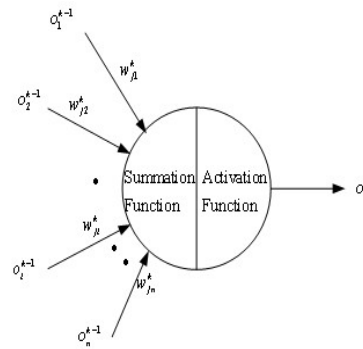


Fig.3 Artificial Neuron with an Activation Function.

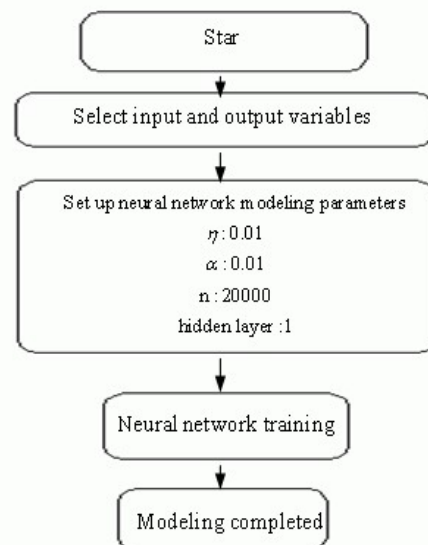


Fig.4 Neural Network Implementation Steps

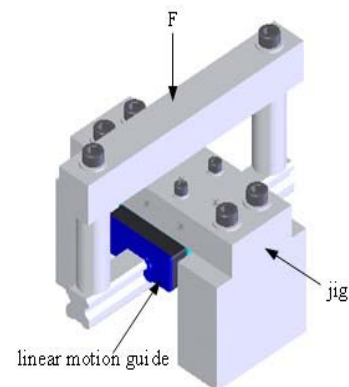


Fig.5 Jig for Testing

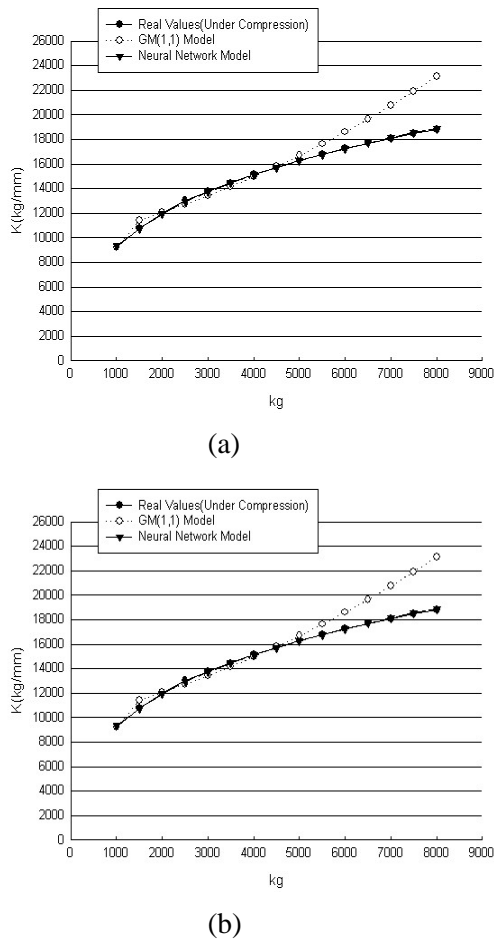


Fig.6 Real Values and Model Values for Linear Motion Guide Rigidity from 1000kg to 8000kg.a) Under Tension; b) Under Compression

Table 1 Preload

Grade	Symbol	Preload force
Clearance	ZF	0
No Preload	Z0	0
Light Preload	Z1	0.02C
Middle Preload	Z2	0.05C
Heavy Preload	Z3	0.07C

C: Basic static load rating

Table 2 Model Values and Forecast Errors(unit: K=kg/mm). a) Under Tension; b) Under Compression.

a. Model values and forecast errors (under tension)

Load kg (F)	Real value Rigidity (K=F/δ) kg/mm	GM(1,1) (kg/mm)		Neural network (kg/mm)	
		Model value	error (%)	Model value	error (%)
1000	3620.1	—	—	3634.5	-0.4
1500	4054.1	4054.1	-4.41	4047.1	0.17
2000	4318.6	4318.6	0.01	4336.9	-0.42
2500	4480.3	4480.3	1.68	4521.2	-0.91
3000	4609.1	4609.1	2.51	4630.3	-0.46
3500	4681.9	4681.9	2.09	4691.1	-0.2
4000	4718.4	4718.4	0.89	4722.4	-0.09
4500	4731.2	4731.2	-0.83	4736.4	-0.11
5000	4752.9	4752.9	-2.39	4740	0.27
MAPE (1000-5000)			1.85		0.34
5500	4753.1	4964.331	-4.45	4737.5	0.33
6000	4742.6	5064.239	-6.78	4731.2	0.24
6500	4708.2	5166.157	-9.73	4722.3	-0.3
7000	4670.2	5270.127	-12.85	4711.7	-0.89
7500	4614.5	5376.189	-16.51	4699.9	-1.85
8000	4451.9	5484.386	-23.20	4687	-5.28
MAPE (5500-8000)			12.25		1.48

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left[\frac{|f^{(0)}(k) - x^{(0)}(k)|}{x^{(0)}(k)} \right]$$

b. Model values and forecast errors
(under compression)

Load kg (F)	Real value Rigidity (K=F/δ) kg/mm	GM(1,1) (kg/mm)		Neural network (kg/mm)	
		Model value	error (%)	Model value	error (%)
1000	9221.3	—	—	9322.3	-1.1
1500	10766.0	11400.1	-5.89	10736.3	0.28
2000	11981.5	12037.5	-0.47	11917.9	0.53
2500	13020.8	12710.4	2.39	12895.9	0.96
3000	13761.5	13421	2.48	13722.5	0.28
3500	14462.8	14171.4	2.02	14444.4	0.13
4000	15143.3	14963.6	1.19	15093.8	0.33
4500	15734.3	15800.2	-0.42	15690.2	0.28
5000	16304.4	16683.5	-2.32	16244.6	0.37
MAPE (1000-5000)			2.15		0.47
5500	16800.3	17616.3	-4.85	16763	0.22
6000	17264.7	18601.1	-7.74	17248.4	0.1
6500	17697.0	19641.1	-10.98	17702.6	-0.03
7000	18087.9	20739.1	-14.65	18126.7	-0.22
7500	18462.2	21898.6	-18.61	18521.6	-0.32
8000	18810.5	23122.9	-22.92	18888.4	-0.41
MAPE (5500-8000)			13.29		0.22

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left[\frac{|f^{(0)}(k) - x^{(0)}(k)|}{x^{(0)}(k)} \right]$$