The Gas-assisted Expelled Fluid Flow in the Front of a Long Bubble in a Channel

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Abstract: - Investigated in this study is the steady-state flow field of a long bubble penetrating into a region filled with a viscous fluid confined by two closely located parallel plates. Instead of using the complicated procedure for iteratively computing the free surface and flow patterns, we simply use a theoretical profile of the bubble so that the influence of bubble shape on the flow field can be examined directly. Due to the simplification, flow fields with higher Reynolds number are easier to be included and different flow phenomenon is found. The numerical techniques employed are finite difference method (FDM) with successive over-relaxation (SOR). The simulation results show coincidently with others the two typical flow patterns (complete bypass flow and recirculation flow). The gradually moving of the stagnation point in the front of the bubble tip between two typical flow patterns is clearly presented and explainable. Both of the position of the stagnation point parallel and perpendicular to the flow, x_{sp}^* and y_{sp}^* , depends on Reynolds number, Re, and λ , the ratio of asymptotic bubble width to the distance between two parallel plates. As λ or Re increases, y_{sp}^* increases too, but x_{sp}^* decreases. A quasi-linear relationship between λ and x_{sp}^* is found in a recirculation flow region. Be ware that the stagnation point is very sensitive with λ for Re>100. As Re increases, the maximum value of stream function ψ_{max}^* increases, and the recirculation zone near the bubble tip becomes bigger too.

Key-Words: - long bubble, expelled fluid flow, stagnation point, two-phase flow, inertia forces.

1 Introduction

Tsing long bubble to assist manufacture or medical processes is common in various industries, such as gas-assisted injection molding. bio-mechanic process and medical treatment process, etc. Particularly, the information about the expelled fluid flow in front of the bubble and the contour of the bubble front are of great interests in the simulating processes. Using the fundamental dynamic equations of the bubble by some theoretical or experimental deduced empirical equations can practically simplify simulation procedures. The flow patterns, and the migration of stagnation point and the influence of the inertia forces on the flow field are presented in this study.

The impacts of the bubble size on the flow field draw a great interest of many researchers [2,9-10]. Mavridis et al. [11] studied the movement of the polymer melts front in both two-dimensional channels and tubes. They computed the inner flow fields of both the Newtonian and the non-Newtonian fluids and compared the difference of these melt front shapes. Fairbrother and Stubbs [1] first studied the penetration of a long bubble in a tube. The empirical formula between the fractional coverage are obtained. Hsu et.al. [2] numerically 2 studied the Long Gas Bubble Front. Taylor [3] presented three fluid flow patterns, two kinds of re-circulating flows with a low capillary number, and a by-pass flow with high capillary number. Bretherton [4] developed a theoretical formula relating the fractional coverage and the pressure drop across the bubble. Goldsmith and Mason [5] visualized the flow pattern, the coating thickness and the shape of the bubble front.

Cox [6-7] extended Taylor's experiment and proposed a numerical solution of the momentum equation. Cox also developed an empirical equation of the bubble profile, verified the existence of the two flow patterns in the Poiseuille flow. In the Hele-Shaw cell model, Pitts [8] reported a theoretical bubble shape equation for λ smaller than 0.77.

However, the study of the effects of λ , using atheoretical bubble's profile is absent. In present work, the flow patterns previously considered by Hsu et al [9-10] in front of a semi-infinite gas bubble were computed. The streamlines demonstrate how the flow field changes when λ is varied and evidently exist recirculating flow and the migration of stagnation point.

2 Governing Equations

This paper is closely followed Saffman and Geoffrey Taylor [2] and Hsu et al [9-10] assumptions, studies a bubble steadily with moving speed U expels the viscous fluid confined in two closely parallel plates as illustrated in Fig. 1, and the channel is open. The continuity equation is expressed as

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \tag{1}$$

The momentum equations are

$$\begin{cases} u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \upsilon \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right) \\ u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \upsilon \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} \right) \end{cases}$$
(2)

The stream function $\psi(x, y)$ of the flow is defined as

$$u_x = \frac{\partial \psi}{\partial y}, \ u_y = -\frac{\partial \psi}{\partial x}$$
 (3)

and vorticity ω is defined as

$$\omega = \operatorname{curl} \vec{\mathbf{V}} = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}$$
(4)

Substituting equation (3) and (4) into (2), the

vorticity equation can be obtained as

$$\frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \upsilon \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)$$
(5)

By introducing the following dimensionless variables

$$x^{*} = \frac{x}{H}, \quad y^{*} = \frac{y}{H},$$

$$u_{x}^{*} = \frac{u_{x}}{U(1-\lambda)}, \quad u_{y}^{*} = \frac{u_{y}}{U(1-\lambda)}$$

$$\omega^{*} = \frac{\omega}{U(1-\lambda)/H} = \left(\frac{\partial u_{y}^{*}}{\partial x^{*}} - \frac{\partial u_{x}^{*}}{\partial y^{*}}\right), \quad \psi^{*} = \frac{\psi}{UH(1-\lambda)}$$

$$\operatorname{Re} = \frac{UH(1-\lambda)}{U}$$

where v is the kinematic viscosity and Re is the Reynolds number respectively, equations (4) and (5) can be written as

$$\frac{\partial^2 \psi^*}{\partial x^{*2}} + \frac{\partial^2 \psi^*}{\partial y^{*2}} + \omega^* = 0$$
(6)

$$\frac{\partial^2 \omega^*}{\partial x^{*2}} + \frac{\partial^2 \omega^*}{\partial y^{*2}} - \operatorname{Re}\left(\frac{\partial \psi^*}{\partial y^*}\frac{\partial \omega^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial \omega^*}{\partial y^*}\right) = 0 \quad (7)$$

An inverse bubble velocity U is added in the flow to fix the bubble in the computation domain so that the boundary conditions are expressed as follows.

No slip condition on the channel wall(\overline{AB})

$$\psi^* = -1; \quad \omega_{i,j_s}^* = \frac{2\left(-1 - \psi^* \Big|_{i,j_{s-1}} + \frac{\Delta y}{(1-\lambda)}\right)}{(\Delta y)^2}$$
(8)

The Hagen-Poiseuille flow at the far upstream (\overline{BC})

$$\psi^{*} = \frac{3}{2} \frac{\lambda \left(y^{*} - \frac{y^{*}}{3} \right)}{(1 - \lambda)} - \frac{y^{*}}{(1 - \lambda)}; \quad \omega^{*} = \frac{3\lambda y^{*}}{(1 - \lambda)}$$
(9)

The symmetry conditions at the centerline (\overline{CD}) $\psi^* = 0; \quad \omega^* = 0$ (10)

At the interface between the two fluids (\widehat{DE}).

In general, for a given bubble profile, the net force acting on the interface should be zero due to the balance between the surface tension on the interface and stresses on both sides of the interface. By using the theoretical bubble profile equation proposed by Pitts [8], the present study imposes two conditions suggested by Cox[6]: no diffusion across the interface (Eq. 11), and zero tangential stress on

the interface between gas and the fluid (Eq. 12). Therefore, two conditions at the interface can be deduced as

$$u_{y}^{*}\sin\theta + u_{x}^{*}\cos\theta = 0 \tag{11}$$

$$\left(\frac{\partial u_y^*}{\partial y^*} - \frac{\partial u_x^*}{\partial x^*}\right) 2\sin\theta\cos\theta + \left(\frac{\partial u_x^*}{\partial y^*} + \frac{\partial u_y^*}{\partial x^*}\right) \left(\cos^2\theta - \sin^2\theta\right) = 0 \ (12)$$

By combining equations (11) and (12), the equation of vorticity on the interface can be obtained as

$$\omega^* = 2 \tan \theta \frac{\partial \psi^*}{\partial x^*} \frac{d\theta^*}{dx^*}$$
(13)

Thus, the conditions on the interface are

$$\psi^* = 0; \quad \omega^* = 2 \tan \theta \frac{\partial \psi^*}{\partial x^*} \frac{d\theta^*}{dx^*}$$
 (14)

The flow in the region is rectilinear at the far downstream (\overline{EA})

$$\psi^* = \frac{-y^* + \lambda}{1 - \lambda}, \ \lambda \le y^* \le 1; \ \omega^* = 0$$
(15)

2.1 Numerical Procedure

The finite-difference method (FDM) with successive over-relaxation (SOR) is employed in the present study. The finite difference formulas are obtained from equation (6) as

$$\frac{\omega_{i,j,j} - 2\omega_{i,j} + \omega_{i-1,j}}{(\Delta x)^2} + \frac{2}{\Delta y (dy + \Delta y)} \omega_{i,j+1}^* - \frac{2}{\Delta y dy} \omega_{i,j}^* + \frac{2}{dy (dy + \Delta y)} \omega_{i,j-1}^* \quad (16)$$
$$- \operatorname{Re} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\omega_{i+1,j}^* - \omega_{i-1,j}^*}{2\Delta x} - \frac{\partial \psi^*}{\partial x^*} \frac{dy^2 \omega_{i+1,j}^* - (dy^2 - \Delta y^2) \omega_{i,j}^* - \Delta y^2 \omega_{i,j-1}^*}{dy \Delta y (dy + \Delta y)} \right) = 0$$

where *i*, *j* denote indices at *x*-direction and *y*-direction, Δx and Δy are even grid sizes at *x*-direction and *y*-direction, and *dy* is the uneven grid size along the interface ($0 < dy < \Delta y$), respectively. The uneven grid dy is introduced in order to fit the curvilinear interface as shown in Figure 2. Two different grids, one is curvilinear conformal to the interface and the other is rectilinear parallel to the solid boundaries, are adopted in the numerical computational scheme. The finite difference equation with SOR is solved by using Gauss-Seidal iteration method. Rearrange equation (16), the Gauss-Seidal iteration form with SOR is obtained as

$$\omega_{i,j}^{*^{\kappa+1}} = \frac{\alpha}{e} \Big(a \omega_{i+1,j}^{*^{\kappa}} + b \omega_{i,j+1}^{*^{\kappa}} + c \omega_{i-1,j}^{*^{\kappa+1}} + d \omega_{i,j-1}^{*^{\kappa+1}} \Big) + (1-\alpha) \omega_{i,j}^{*^{\kappa}} (17)$$

where

$$\begin{aligned} \mathbf{a} &= \frac{1}{\left(\Delta x\right)^2} - \operatorname{Re}\frac{\partial \psi^{*^{\kappa}}}{\partial y^*} \frac{1}{2\Delta x}, \quad \mathbf{b} = \frac{2}{\Delta y \left(dy + \Delta y\right)} + \operatorname{Re}\frac{\partial \psi^{*^{\kappa}}}{\partial x^*} \frac{dy}{\Delta y \left(dy + \Delta y\right)} \\ \mathbf{c} &= \frac{1}{\left(\Delta x\right)^2} + \operatorname{Re}\frac{\partial \psi^{*^{\kappa}}}{\partial y^*} \frac{1}{2\Delta x}, \quad \mathbf{d} = \frac{2}{dy \left(dy + \Delta y\right)} - \operatorname{Re}\frac{\partial \psi^{*^{\kappa}}}{\partial x^*} \frac{\Delta y^2}{dy \Delta y \left(dy + \Delta y\right)} \\ \mathbf{e} &= \frac{2}{\left(\Delta x\right)^2} + \frac{2}{dy\Delta y} + \operatorname{Re}\frac{\partial \psi^{*^{\kappa}}}{\partial x^*} \frac{dy^2 \Delta y^2}{dy \Delta y \left(dy + \Delta y\right)} \\ \\ \begin{cases} \frac{\partial \psi^{*^{\kappa}}}{\partial x^*} = \frac{\psi^{*^{\kappa}}_{i,j} - \psi^{*^{\kappa}}_{i,j-1}}{\Delta x} \\ \frac{\partial \psi^{*^{\kappa}}}{\partial y^*} = \frac{\psi^{*^{\kappa}}_{i,j} - \psi^{*^{\kappa}}_{i,j-1}}{dy} \end{cases} \end{aligned}$$

 α and K denote the over-relaxation factor and iterative time, respectively. Discrete and rearrange the stream-vorticity equation (7), the Gauss-Seidal iteration form with SOR is obtained as

$$\psi_{i,j}^{*^{k+1}} = \frac{\alpha}{e_1} \left(a_1 \psi_{i+1,j}^{*^k} + b_1 \psi_{i,j+1}^{*^k} + c_1 \psi_{i-1,j}^{*^{k+1}} + d_1 \psi_{i,j-1}^{*^{k+1}} + \omega_{i,j}^{*^{k+1}} \right) + (1 - \alpha) \psi_{i,j}^{*^k} (18)$$

where
$$a_1 = c_1 = \frac{1}{\Delta x^2}$$
$$b_1 = \frac{2}{\Delta y (dy + \Delta y)}$$
$$d_1 = \frac{2}{\Delta y (dy + \Delta y)}$$

$$e_{1} = \frac{2}{\Delta x^{2}} + \frac{2}{dy\Delta y}$$

Starting with an initial guess of values of vorticity and stream function (all set at zero values in the computation domain) and introducing boundary conditions, the value of ω^* is computed from equation (17) at each grid point, and then substituting the ω^* value into equation (18) to obtain new ψ^* value. The iteration process is repeated to calculate the values of ω^* and ψ^* until the relative residual reaches the convergent criterion. The criterion is defined as

$$\frac{\left|\frac{\psi_{i,j}^{*_{k+1}} - \psi_{i,j}^{*_{k}}}{\psi_{i,j}^{*_{k}}}\right| \le 10^{-7}$$
(19)

The vorticity on the channel wall is derived from the wall values of stream function, ψ_{i,j_s}^* in backward form by using the Taylor expansion. The expansion becomes

$$\psi_{i,j_s-1}^* = \psi_{i,j_s}^* - \Delta y \frac{\partial \psi^*}{\partial y^*} \bigg|_{i,j_s} + \frac{1}{2} \frac{\partial^2 \psi^*}{\partial y^{*^2}} \bigg|_{i,j_s} \Delta y^2 + O\left(\Delta y^3\right) (20)$$

where the index j_s denotes the y-position of the channel wall.

Substituting the known value $\psi_{i,j_i}^* = -1$ on the boundary and rearranging equation (20), the boundary value of the vorticity can be obtained as

$$\omega_{i,j_{s}}^{*} = \frac{2\left(-1-\psi^{*}\big|_{i,j_{s-1}}+\frac{\Delta y}{(1-\lambda)}\right)}{(\Delta y)^{2}}$$
(21)

3 The numerical results

Many previous studies verified two typically flow patterns [5,7]. Especially, Hsu [9,10] displayed the third pattern which suggested by Taylor [3]. Present study discusses the impacts of the values of λ and Reynolds number Re on the position of stagnation point and the maximum ψ^* value on the flow field.

The influences of the upstream boundary condition on the fluid flows are discussed below. It is known from Cox [7] that the upstream distribution of can be expressed as

$$\psi^* = \frac{3}{2} \frac{\lambda \left(y^* - \frac{1}{3} y^{*^3} \right)}{(1 - \lambda)} - \frac{y^*}{(1 - \lambda)}$$
(22)

By setting in equation (22), the relative equation of and can be obtained as

$$\lambda = \frac{2}{3 - y^{*^2}}$$
(23)

From equations (22) and (23), one can find that the location of varies as a function of and. If one set that the position of locates on , it results in . The increases from to 1 as increases from zero to unit, i.e., the recirculating flow pattern will be observed if the value of is higher than . Thus the critical situation will signify the two typical flow patterns proposed by Taylor [3], Cox [7].

The numerical results presented here verify the flow fields mentioned in the previous studies. Phenomena in the creeping flow (Re = 0) and the effects of the inertia force (Re > 0) to the flows are discussed. The effects of λ and Re to the position of the stagnation point and the value of the maximum ψ^* are also presented.

3.1 The inertial effects

We assume the profile of the bubble still obey the result proposed by Pitts [8].

Figure 3 illustrates three kind of flow patterns with five different λ values. The complete bypass flow is shown in Fig. 3(a) and 3(b). The flow pattern suggested by Taylor [3] is shown in Fig. 3(c), and the recirculation flow is presented in Fig. 3(d) and 3(e). All streamlines in the figures are drawn from $\psi^* = -1$ to ψ^*_{max} with an increment of 0.1.

In Figure 3(a) and 3(b), the streamlines are quite uniform and parallel in the rear region within $-3 \le x^* \le -1.8$. It shows that the flow velocity in the region is nearly equal. Also, the contour of the bubble is nearly parallel to the channel wall in far downstream, as proposed by Cox [6] and Pitts [8]. Comparing Fig. 3(a) and 3(b), the results show that the flow region in the front of the bubble tip, which is affected by the bubble, decreases as λ value increases.

The third flow pattern, which is the flow with the stagnation point located in the front of the bubble on the centerline, is shown in Fig. 3(c). It is very interesting to notice that a recirculation region is formed when λ value is just a little higher than 2/3. The recirculation region lays between the centerline and the streamline, $\psi^* = 0$, including the stagnation point. In addition, the third flow pattern seems to be a transition state between the complete bypass flow pattern and recirculation flow pattern, because this flow phenomenon only can be observed when λ value is just a little higher than 2/3. More details will be discussed in Figure 12.

Figures 3(d) and 3(e) are the typical recirculation flow pattern verified by many previous studies such as Cox [7]. The ψ_{max}^* value in the recirculation region in both Fig. 3(d) and 3(e) is greater than zero. There is a stagnation streamline $(\psi^* = 0)$ starts from the front of the bubble and outreaches into the fluid with a distance away from Between the centerline and the the centerline. stagnation streamline, the fluid flows reversely away from the bubble. The downstream bypass path of the main flow becomes narrower and the size of the recirculation zone grows larger while the λ value increases. It is worth noting that the stagnation point moves downstream along the bubble profile and the streamlines show notable changes in both main flow region and recirculation region while the λ value increases.

Figure 3 present the distribution of streamline with different λ under zero Reynolds number value in complete bypass flow ($\lambda < 2/3$) region, the transient region and recirculation flow ($\lambda \ge 2/3$) region, respectively. the stagnation point locates on the centerline between the upstream and the bubble tip when λ is just above the critical value 2/3.

Figure 4 shows the corresponding velocity distributions of Figure 3. It is found that the fluid in the region with $-1 \le \psi^* \le 0$ flows downstream in the

same direction toward the bypass path at the rear part of the bubble. The reverse flow is in the region with $\psi^* > 0$. The value of λ dominates the present of the complete bypass flow or recirculation flows. As shown in Fig. 4(a) and 4(b), the velocity in the front of bubble tip decreases while λ increases up to 2/3. The recirculation flow is present with $\lambda > 2/3$. The maximum recirculation velocity and the recirculation zone increase while λ increases from 2/3 to 1, and reach their maximum values when $\lambda = 1$. The stagnation point moves along the centerline, as shown in Fig. 4(c), and it moves downstream along the bubble profile, as observed in Fig. 4(d) and 4(e), while λ increases. In other words, the stagnation point moves downstream along the centerline and the bubble profile while the cross-sectional area of the bubble increases. Also, while the crosssectional area of the bubble increases, it is difficult for the viscous fluid at the center region to move downstream. Finally, the fluid reaches a stagnant state and then starts to move upstream. As mentioned, the maximum recirculation velocity increases while λ increases. However, the effect of λ to the velocity distribution in the range with $-1 \le \psi^* \le 0$ is minor. The velocity distributions near the stagnation point in Fig. 4(c), 4(d) and 4(e) are enlarged to show in Fig. 5(a), 5(b) and 5(c) for further insight.

Fig. 5(a), 5(b) and 5(c) show that the positions of stagnation point with $\lambda = 0.67, 0.70$ and 0.77 are located at (0.75811, 0), (-0.02668, 0.15268) and (-0.16555, 0.38062), respectively. The velocity near the stagnation point, as shown in Fig. 5(a), decreases to zero when y^* coordinate varies from larger y^* to $y^* = 0$. The figure also shows a tiny recirculation flow region, which is not clearly presented in Fig. 3(c) and 4(c). The reason for not showing the tiny recirculation zone is that, for Fig 3(c), the value of ψ^*_{max} is less than 0.1, which makes a difficulty to plot streamlines and, for Fig. 4(c), only a zero velocity is shown. The velocity between the bubble tip and stagnation point near the centerline also presents in Fig. 5(a). The flow velocities move nearly parallel to the centerline only in a thin y^* range not over 0.05, while the fluid move abruptly upward along the bubble. Because the position of the stagnation point is located on the bubble profile with $\lambda > 2/3$, there is no fluid velocity parallel to the centerline in Fig. 5(b) and 5(c). In Fig. 5(b), the fluid in the front of the bubble tip shows a stagnant state, which is not clearly observed in Fig. 3(d) and 4(d). Fig. 5(c) shows that the fluid in the range, $y^* \le 0.4$, moves abruptly toward the centerline in the region near the bubble. In conclusion, the fluid in the front of the stagnation point shows the transition phenomena when λ increases. The stagnation point moves downstream along the centerline to the bubble tip, and then moves further downstream along the bubble profile. In the recirculation region, the fluid moves gradually toward the centerline, turns around and then flows upstream.

In order to reduce the loading on calculation, many previous studies such as Cox [6], added an inverse bubble velocity, -U, in the flow to set the bubble fixed in the calculation domain. Cox[7] used same concept to measure the velocity profile in order to verify the coincidence of experiments with the theoretical and numerical simulations. The present study also adopts the same concept to simplify the computations. In addition, the bubble is assumed to extend downstream to infinity $(x^* \rightarrow -\infty)$. One can set a zero velocity at the far downstream section, i.e., the viscous fluid either adheres to the channel wall or moves upstream away from the bubble. As a result, if a constant bubble velocity U is added back, which means to subtract the velocity presents in Fig. 4(a-e) with the bubble velocity, the fluid only moves upstream. The concept was also used in Mavridis et al. [11] to investigate the fountain flow phenomenon in the airassisted polymer injection process. Figures 6(a-e) show the results of adding U to the velocity distributions presented in figures 4(a-e), respectively. These figures show that the velocity distribution in the front of the bubble tip, $x^* > 0$, is less affected by the bubble, the velocity distribution are the same in various cross section along x -coordinate under same λ value. The velocity variations are evident in the region of $x^* < 0$. This also indicates that the expelling effect of the bubble to the viscous fluid flow fields is only a local effect. Furthermore, the velocity distributions in the region from the bubble tip ($x^* = 0$) to downstream section ($x^* = -3$) can be divided into two patterns. One is in the region surrounded by the downstream section, bubble profile and channel wall, in which shows a nearly stagnant flow, and the other is near the bubble tip where shows a bubble driven flow. The stagnant range decreases while λ increases.

Figures 7 and 8 present the distribution of

streamline with different Reynolds numbers under constant λ value in complete bypass flow ($\lambda < 2/3$) region and recirculation flow ($\lambda \ge 2/3$) region, respectively. In Figure 7, the increasing of Reynolds number doesn't have notable effect on the flow field in complete bypass flow region. In Figures 8, as Reynolds number increases, the maximum ψ^* value increases, and the recirculation zone near the bubble tip become bigger, too. The range of the recirculation flow region increase as the Reynolds number further increases, this shows Reynolds number has notable effect on the recirculation flow and the maximum ψ^* value.

The velocity distributions of Figure 8 in both moving frame of reference and fixed frame of reference are shown in Figure 9 and Figure 10, respectively.

The velocity distributions in Figure 9 are similar to those of creeping flow shown in Fig. 4(e). In Figure 10, fast fluid particles in the central region of the fluid flow will induce the fluid near the channel wall to move toward the central core and upstream region. This tendency increases while the Reynolds number increases. In conclusion, the variation of the Reynolds number does not have notable effect in the complete bypass flow. But in the recirculation flow, the fluid near the bubble tip is expelled to move toward the core region while the Reynolds number increases.

3.2 The relations between λ and the location of stagnation point, maximum ψ value

Based upon Pitts [8] which reported that the experimental results and theoretical profiles agree well under the condition $\lambda \leq 0.77$, thus the present results all calculated until the λ reaches the maximum value 0.8 to avoid deviation from the experimental results.

Figure 11 depicts maximum streamline value as a function of λ for various Re values. The value of ψ^*_{max} in Figure 11 is larger than zero and it means the recirculation flow occurred. If ψ^*_{max} is less than zero, the bypass flow shown. It is found that ψ^*_{max} converges to the same λ values ($\lambda \le 0.685$) for all Re values, then it deviates obviously as increasing λ value. The higher λ value and the higher Re values results in the higher ψ^*_{max} value. In the case of λ increasing, the flow rate at far downstream gets decreasing, so the losing mass flow rate in the far downstream should be supplied from the far

upstream. Furthermore, ψ^*_{\max} is monotonous increasing while the Reynolds number increasing at the same λ value and the ψ^*_{\max} gets closer to each other at higher Reynolds number ($200 \le \text{Re} \le 300$) and deviates for lower Re.

Figure 12(a) shows the location of stagnation point on x -direction versus the λ , and Figure 12(a) presents the stagnation point corresponding to bubble tip locates on the centerline ($x^* > 0$). In Figure 12(a) the stagnation point moves from $x^* = 3$ toward the bubble tip area along the centerline for all Re as the increment deviates from $\lambda = 2/3$. In other words, the increment of λ stimulate the movement of stagnation point from $x^* = 3$ to $x^* = 0$ is not over 0.02 ($2/3 < \lambda < 0.6839$) in creeping flow region, and in relative high Reynolds number region (Re > 100) the stagnation point is very sensitive to any increment of λ , a small increment of λ will cause stagnation point to move toward the bubble tip area. The situation of stagnation point moves in the range $0 \le x^* \le 3$ is a transient state between complete bypass flow and recirculation flow. It is found that the $x_{sp}^* \approx 0.00502$ at $\lambda = 0.6838$.

The relation between x_{sp}^* and λ with various Re in the typical recirculation flow region ($\lambda \ge 2/3$) is shown in Figure 10(b). The value of x_{sp}^* decreases when or Re increases. In addition, the value of x_{sp}^* decreases almost linearly while λ increases with fixed. The stagnation point moves further downstream away from the bubble tip (i.e., x_{sp}^* is more negative) while the Reynolds number increases with fixed λ . A quasi-linear relation between x_{sp}^* and λ is found in the recirculation flow region.

The relation between the *y*-coordinate (i.e., y_{sp}^*) of the stagnation point and λ is shown in Figure 13. The solid line represents the *y*-position of the streamline $\psi^* = 0$ on the far upstream section with various λ . All the other lines represent the relation between y_{sp}^* and λ with various Reynolds number. The value of y_{sp}^* increases while Re or λ increases, but the gradient of curves decreases while λ increases. Comparing y_{sp}^* to the *y*-coordinate (y^*) of $\psi^* = 0$ on the far upstream section, the difference between them decreases while λ decreases. Also, the difference between y_{sp}^* and y^* of $\psi^* = 0$ decreases while Re increases with fixed λ .

4 Conclusion

The study not only verifies the flow pattern proposed by previous studies but also detailed investigates the third flow pattern between the complete bypass flow and recirculation flow. It shows that the third flow pattern only observed in a small range ($2/3 < \lambda < 0.6839$) and low Reynolds number.

In a moving frame of reference, it is found that the velocity near the front of bubble tip decreases while the λ increases until the λ reaches the 2/3, and as the λ is larger than 2/3 the velocity increases (toward the upstream). The stagnation point divides the fluid flow in two directions: flow toward the upstream or downstream. When the Reynolds number increases, the velocity near the bubble tip increases and the affection in the region is similar to the case of high λ in creeping flow.

In a fixed frame of reference, it is found that the velocity is only affected in a small region in front of bubble tip. The velocity in the region from bubble tip ($x^* = 0$) to the downstream ($x^* = -3$) can be divided into two regions, one is the velocity shows nearly stagnant in the downstream and the other is the velocity shows the obviously driven by the bubble near the bubble tip. The stagnant range decreases as the λ increases, and it also verifies the real physical phenomenon of zero velocity at the far downstream. As the Reynolds number increases, the fluids near the bubble tip expelled to move toward the centerline and the velocities increase in recirculation flow region. But the increasing of Reynolds number does not have notable effect on the flow field in complete bypass flow region.

Increasing of Reynolds number, the affection of inertial forces, the range of the recirculation flow region increases for the same λ value and the range of λ in the transient flow becomes smaller, but no notable affections were found in the complete bypass flow. When the λ or Re increases, the ψ^*_{max} and y^*_{sp} get increasing and the x^*_{sp} gets decreasing.

Nomenclature:

- *H* The half distance between the two parallel plates
- *U* The constant velocity of the bubble

- *n* The normal unit vector on the bubble interface
- *t* The tangential unit vector on the bubble interface
- *x* The axial direction in coordinate system
- y The radial direction in coordinate system

Greek letters

- α The over-relaxation factor
- θ The angle between the normal of the interface and the axial direction
- λ The ratio of asymptotic bubble width to half distance of the two parallel plates
- ψ Stream function
- *ω* Vorticity

Dimensionless parameters

- Re Reynolds number
- * Dimensionless form

Subscript

- *i* The number of grid in axial direction
- *j* The number of grid in radial direction
- s Solid wall
- sp Stagnation point

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Figure 1. Diagram of the gas bubble steadily expelled viscous fluid and the coordinate system



Figure 2. Diagram of the uneven grid near the interface



Figure 3. Distributions of streamline with different λ and Re = 0



Fig. 4 Distributions of velocity in the moving frame of reference with different λ and Re = 0

Fig. 6 Distributions of velocity in the fixed frame of reference with different λ and Re = 0

λ = 0.65 Re = 0



Fig. 5 Distributions of velocity near the stagnation point in the moving frame of reference with different λ and Re = 0



Figure 7. Distribution of dimensionless streamlines for various Reynolds numbers in complete bypass flow region



Figure 8. Distribution of dimensionless streamlines for various Reynolds numbers in recirculation flow region



Figure 9. Distribution of velocity of the recirculation flow in the moving frame of reference with different Re

Figure 11. λ versus the maximum ψ^*_{max} value with

upstrea Re=0

Re=10 Re=25

Re=50 Re=100

Re=300

0.80

0.70

0.60

0.50

0.40

0.30

0.20

0.10

0.00

0.66

0.68

point on y -direction with various Re

0.70

0.72

Figure 13. λ versus the location of stagnation

0.74

λ

0.76

0.78

0.80

y-coordinate

Figure 12. λ versus the location of stagnation point on *x* -direction with various Re