Swing-By Maneuvers for a Cloud of Particles with Planets of the Solar System

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Abstract: - The idea of the present paper is to study the swing-by maneuver between one of the planets of the Solar System and a cloud of particles. This is what happens when a fragmented comet crosses the orbit of a planet like Jupiter, Saturn, etc. We used the dynamical system that is formed by two main bodies (the Sun and one of the planets of the Solar System) and we assumed that they are in circular orbits around their center of mass and a cloud of particles is moving under the gravitational attraction of these two primaries. The motion is assumed to be planar for all the particles and the dynamics given by the “patched-conic” approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem. The main obejctive is to understand the change of the orbit of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud have semi-major axis a ± da and eccentricity e ± de before the close approach with the planet. It is desired to known those values after the close approach. In particular, we will study the effects of the periapsis distance in this maneuver.

Key-Words: - Astrodynamics, Orbital maneuvers, Swing-By, Gravity assisted maneuvers, Orbital motion.

1 Introduction

In the sutdy of space missions, it is very important to consider the problem of performing optimization in fuel expenditure when making orbital maneuvers. References [1] to [5] show some of the techniques that can be used to solve this problem.

One of the alternative techniques that can be used with this objective is the close approach between a spacecraft and a planet. This is a very popular technique used to decrease fuel expenditure in space missions. This maneuver modifies the velocity, energy and angular momentum of a spacecraft. There are many important applications very well known, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun, etc.

In the present paper we study the close approach between a planet and a cloud of particles. It is assumed that the dynamical system is formed by two main bodies (usually the Sun and one planet) that are in circular orbits around their center of mass and a cloud of particles that is moving under the gravitational attraction of the two primaries. The motion is assumed to be planar for all the particles and the dynamics given by the “patched-conic” approximation is used, which means that a series of two-body problems are used to generate analytical equations that describe the problem. The standard canonical system of units is used and it implies that the unit of distance is the distance between the two primaries and the unit of time is chosen such that the period of the orbit of the two primaries is 2π.

The goal is to study the change of the orbit of this cloud of particles after the close approach with the planet. It is assumed that all the particles that belong to the cloud have semi-major axis a ± da and eccentricity e ± de before the close approach with the planet. It is desired to known those values after the close approach.

Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, a modified version of the set used in the papers written by [6], [7] and [8] is used here. It is composed by the following three variables: 1) V_p, the velocity of the spacecraft at periapse of the orbit around the secondary body; 2) The angle ψ, that is defined as the angle between the line M1-M2 (the two primaries) and the direction of the periapse of the trajectory of the spacecraft around M2; 3) r_p, the distance from the spacecraft to the center of M2 in the moment of the closest approach to M2 (periapse distance). The values of V_p and ψ are obtained from the initial orbit of the spacecraft around the Sun using the “patched-conics” approximation and r_p is a free parameter that is varied to obtain the results.

2 Review of the Literature for the Swing-By

The literature shows several applications of the swing-by technique. Some of them can be found in Swenson[9], that studied a mission to Neptune using swing-by to gain energy to accomplish the mission; [10], that made a similar study for a mission to Pluto; [11], that formulated a mission to study the Earth's geomagnetic tail; [12], [13] and [14], that planned the mission ISEE-3/ICE; [15], that made the first studies for the Voyager mission; [16], that design a mission to flyby the comet Halley; [17], [18] that studied multiple flyby for interplanetary missions; [19] and [20] that design missions with multiple lunar swing-bys; [21], that studied the effects of the atmosphere in a swing-by trajectory; [22], that used a swing-by in Venus to reach Mars; [23], that studied numerically a swing-by in three dimensions, including the effects in the inclination; [24], that considered the possibility of applying an impulse during the passage by the periapsis; [25], that classified trajectories making a swing-by with the Moon. The most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics. Other models used to study this problem are the circular restricted three-body problem (like in [26]) and the elliptic restricted three-body problem ([27]).

3 Orbital Change of a Single Particle

This section will briefly describe the orbital change of a single particle subjected to a close approach with the planet under the "patched-conics" model. It is assumed that the particle is in orbit around the Sun with given semi-major axis (a) and eccentricity (e). The swing-by is assumed to occur in the planet Jupiter for the numerical calculations shown below, but the analytical equations are valid for any system of primaries. The periapse distance (r_p) is assumed to be known. As an example, for the numerical calculations, the following numerical values are used: a = 1.2 canonical units, e = 0.3, μ_J = 0.00094736, r_p = 0.0001285347 (100000 km = 1.4 Jupiter's radius), where μ_J is the gravitational parameter of Jupiter in canonical units (total mass of the system equals to one).

The first step is to obtain the energy (EB) and angular momentum (CB) of the particle before the swing-by. They are given by:

$$EB = -\frac{(1-\mu_J)}{2a} = -0.4162$$

and

$$CB = \sqrt{(1-\mu_J)a(1-e^2)} = 1.0445$$

Then, it is possible to calculate the magnitude of the velocity of the particle with respect to the Sun in the moment of the crossing with Jupiter's orbit (V_i), as well as the true anomaly of that point (q). They come from:

$$V_i = \sqrt{(1-\mu_J)\left(\frac{2}{r_{SJ}} - \frac{1}{a}\right)} = 1.0796$$

and

$$\theta = \cos^{-1}\left[\frac{1}{e}\left(\frac{a(1-e^2)}{r_{SJ}} - 1\right)\right] = 1.2591,$$

using the fact that the distance between the Sun and Jupiter (r_{SJ}) is one and taking only the positive value of the true anomaly.

Next, it is calculated the angle between the inertial velocity of the particle and the velocity of Jupiter (the flight path angle δ), as well as the magnitude of the velocity of the particle with respect to Jupiter in the moment of the approach (V_∞). They are given by (assuming a counter-clock-wise orbit for the particle):

$$\gamma = \tan^{-1}\left[\frac{e\sin\theta}{1 + e\cos\theta}\right] = 0.2558$$

and

$$V_\infty = \sqrt{V_i^2 + V_2^2 - 2V_i V_2 \cos\gamma} = 0.2767,$$

using the fact that the velocity of Jupiter around the Sun (V_2) is one. Fig. 1 shows the vector addition used to derive the equations.

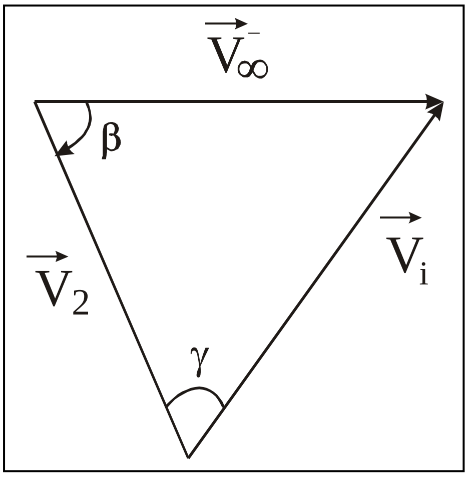


Fig. 1 – Vector addition during the close-approach.

The angle β shown is given by:

$$\beta = \cos^{-1}\left[-\frac{V_i^2 - V_2^2 - V_\infty^{-2}}{2V_2\,V_\infty^-}\right] = 1.7322$$

Those information allow us to obtain the turning angle (2δ) of the particle around Jupiter, from:

$$\delta = sin^{-1}\left(1 \Big/ 1 + \frac{r_p V_\infty^2}{\mu_J}\right) = 1.4272$$

The angle of approach (ψ) has two values, depending if the particle is passing in front (when we call Solution 1) or behind (when we call Solution 2) the planet. These two values will be called ψ1 and ψ2. They are obtained from:

$$\psi_1 = \pi + \beta + \delta = 6.3011$$

and

$$\psi_2 = 2\pi + \beta - \delta = 6.5882$$

The correspondent variations in energy and angular momentum are obtained from the equation $\Delta C = \Delta E = -2\,V_2\,V_\infty\,\text{sen}\,\delta\,\text{sen}\,\psi$, (since w = 1). The results are:

$$\Delta C_1 = \Delta\,E_1 = -0.009811$$

and

$$\Delta C_2 = \Delta\,E_2 = -0.1644$$

By adding those quantities to the initial values we get the values after the swing-by. They are: E1 = -0.4260, C1 = 1.0346, E2 = -0.5806, C2 = 0.8801.

Finally, to obtain the semi-major axis and the eccentricity after the swing-by it is possible to use the equations:

$$\text{a} = -\frac{\mu}{2\text{E}}$$

and

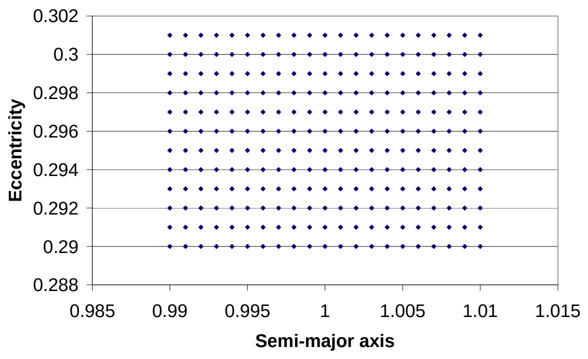
$$\text{e} = \sqrt{1 - \frac{\text{C}^2}{\mu\text{a}}}$$

The results are: a_1 = 1.1723, e_1 = 0.2937, a_2 = 0.8603, e_2 = 0.3144.

4 Orbital Change of a Cloud of Particles

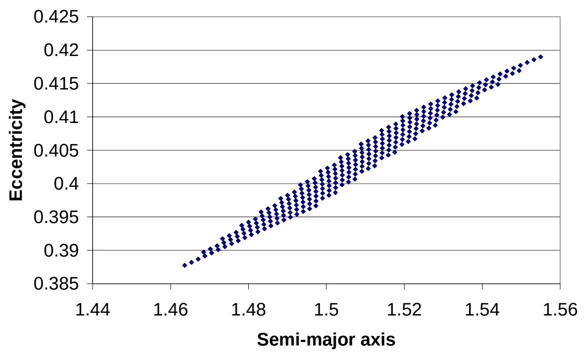
The algorithm just described can now be applied to a cloud of particles passing close to Jupiter and Saturn. The idea is to simulate a cloud of particles that have orbital elements given by: a ± da and e ± de. The goal is to map this cloud of particles to obtain the new distribution of semi-major axis and eccentricities after the swing-by. Fig. 2(a-f) show some results for Jupiter, for the case da = de = 0.001, r_p = 1.1 R_J and 5.0 R_J.

Those figures allow us to get some conclusions. The solution called “Solution 1” has a larger amplitude than the Solution 2 in both orbital elements, but it concentrates the orbital elements in a line, while the so called “Solution 2” generates a distribution close to a square. The area occupied by the points is smaller for “Solution 1”. Both vertical and horizontal lines are rotated and become diagonal lines with different inclinations. The effect of increasing the periapsis distance is to generate plots with larger amplitudes, but with the points more concentrated, close to a straight line.



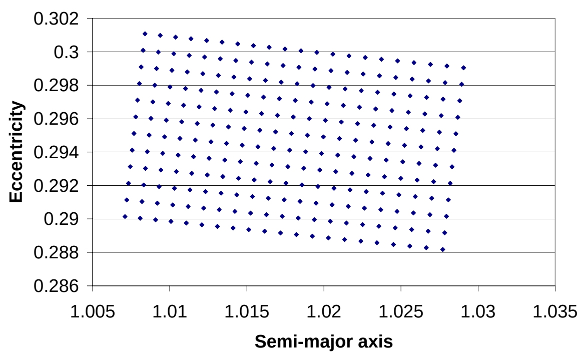
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 2(a) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

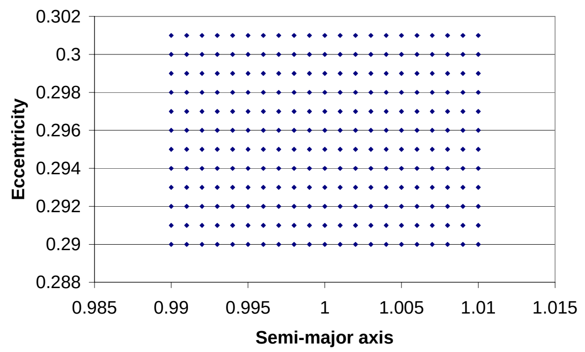
Fig. 2(b) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

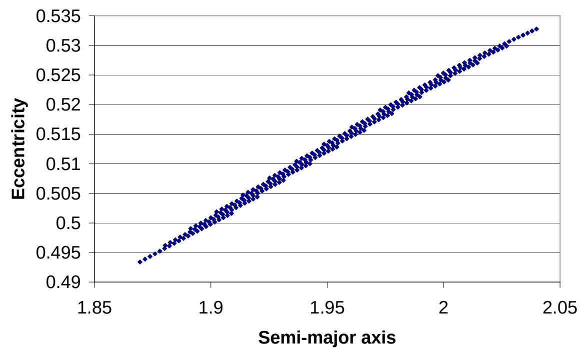
Fig. 2(c) – Swing-by for a cloud of particles with Jupiter for rp = 1.1 R_J.

To understand better the importance of the paeriapsis distance, we also made simulations using the value of rp = 5.0 R_J. The results are shown below.



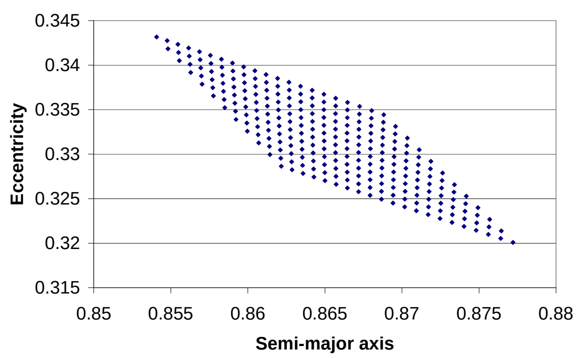
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 2(d) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

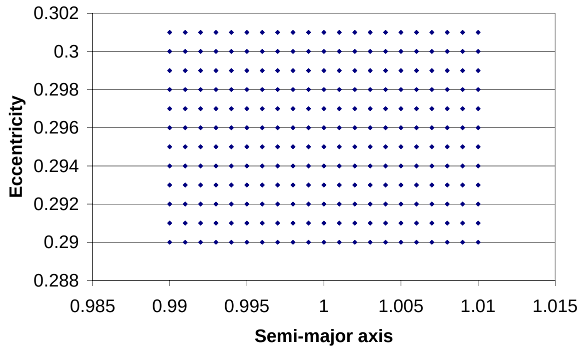
Fig. 2(e) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

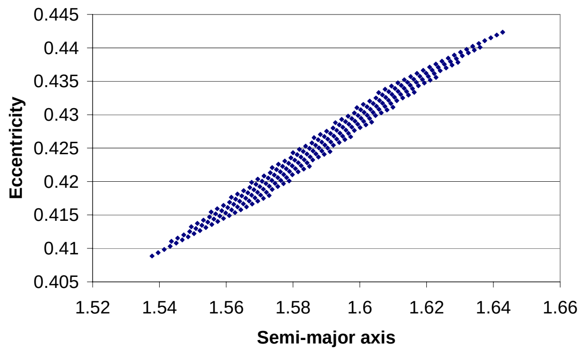
Fig. 2(f) – Swing-by for a cloud of particles with Jupiter for rp = 5.0 R_J.

The following figures 3(a-f) show the equivalent results for Saturn. The idea is to verify the differences that are obtained when the main planet is changed, which means that we see the effects of a diferent mass for the planet.



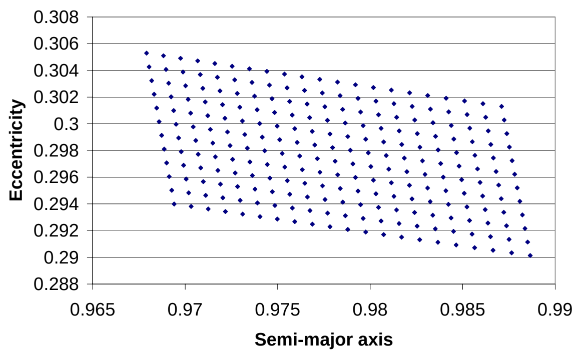
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 3(a) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

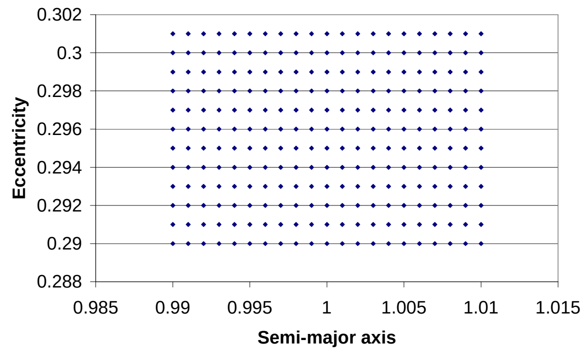
Fig. 3(b) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

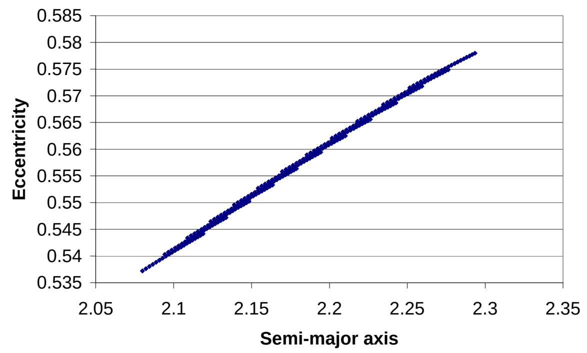
Fig. 3(c) – Swing-by for a cloud of particles with Saturn for rp = 1.1 R_S.

To understand better the importance of the paeriapsis distance, we also made simulations using the value of rp = 5.0 R_S for Saturn. The results are shown below.



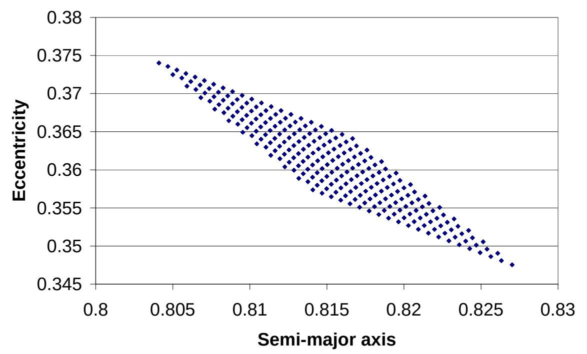
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 3(d) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

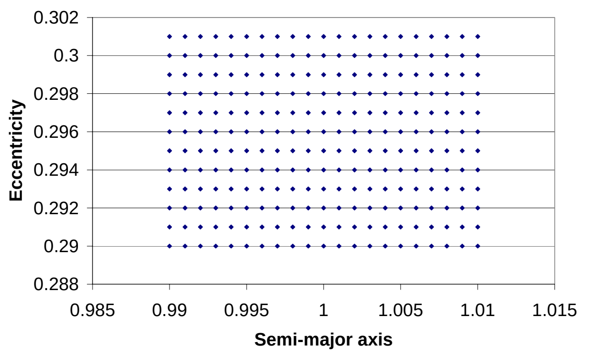
Fig. 3(e) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

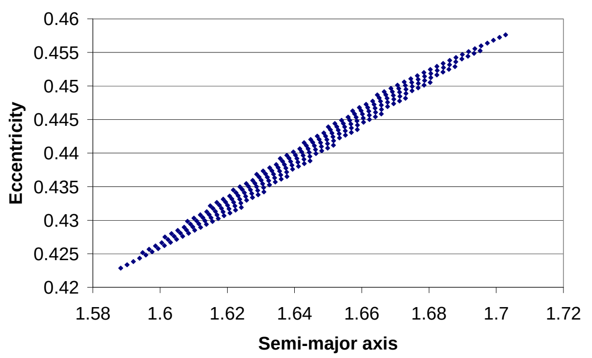
Fig. 3(f) – Swing-by for a cloud of particles with Saturn for rp = 5.0 R_S.

In order to make this study more general, we also made some simulations for the planet Uranus. The following figures 4(a-f) show the equivalent results for this planet. The idea is also to verify the differences that are obtained when the main planet is changed, which means that we see the effects of a diferrent mass for the planet.



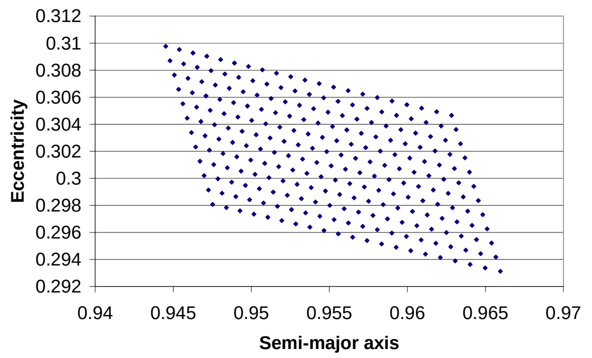
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 4(a) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

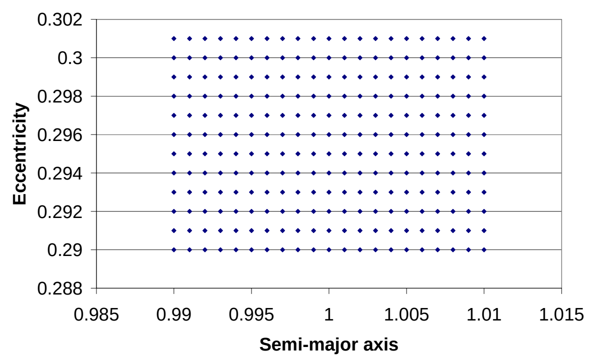
Fig. 4(b) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

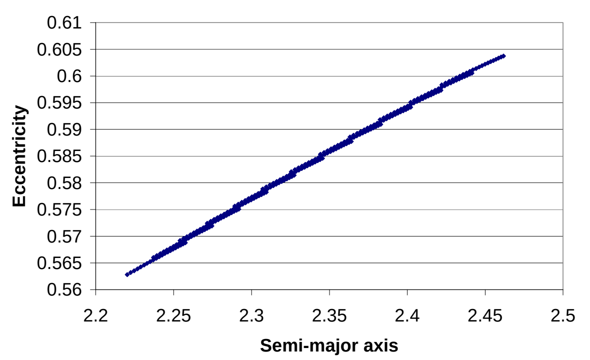
Fig. 4(c) – Swing-by for a cloud of particles with Uranus for rp = 1.1 R_U.

Once more, to understand better the importance of the paeriapsis distance, we also made simulations using the value of rp = 5.0 R_U for Uranus. The results are shown below.



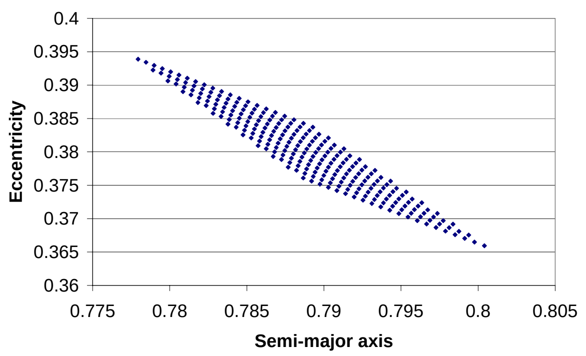
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 4(d) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

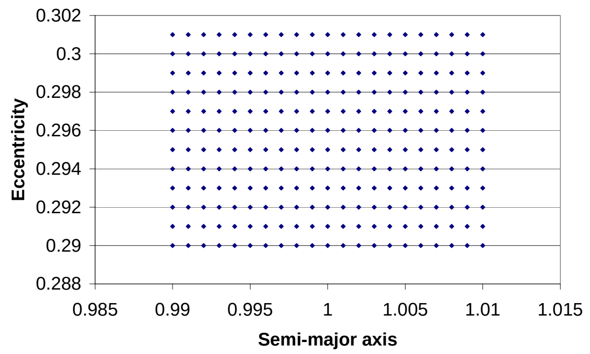
Fig. 4(e) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

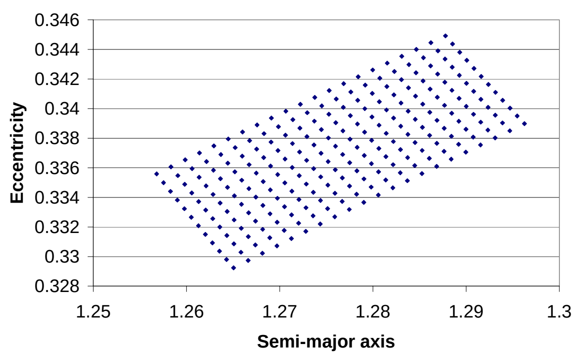
Fig. 4(f) – Swing-by for a cloud of particles with Uranus for rp = 5.0 R_U.

Now, in order to investigate the effects in a planet with a smaller mass, we perform some simulations using the planet Mars as the main body for the Swing-By. The results are shown in Figs 5(a-f).



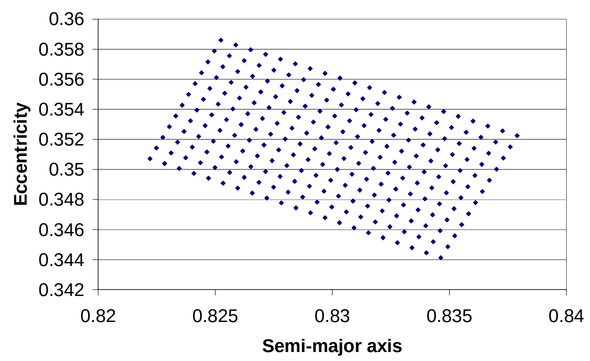
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 5(a) – Swing-by for a cloud of particles with Mars for rp = 1.1 R_M.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

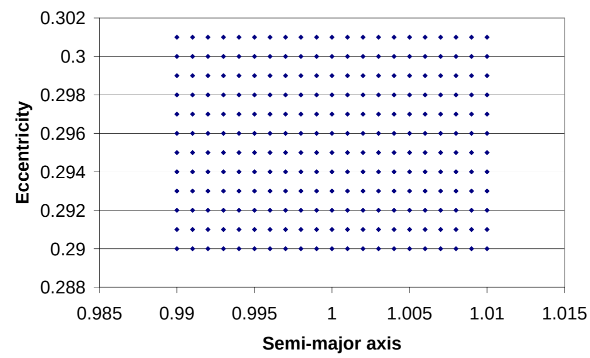
Fig. 5(b) – Swing-by for a cloud of particles with Mars for rp = 1.1 R_M.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

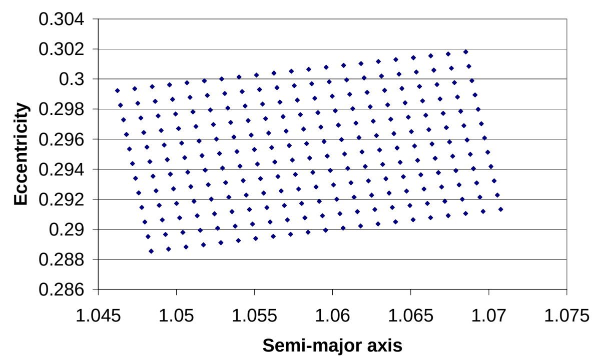
Fig. 5(c) – Swing-by for a cloud of particles with Mars for rp = 1.1 R_M.

And, to follow the simulations made with the other planets, we change the periapsis distance to 5.0 R_M.



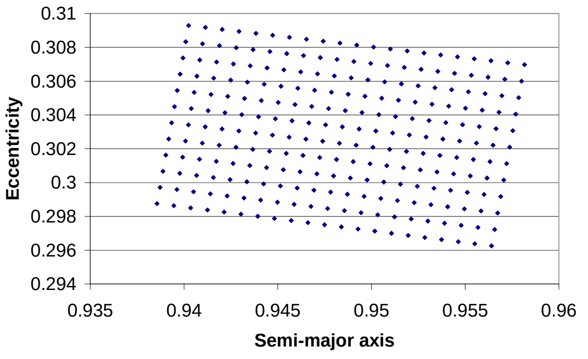
Eccentricity vs. Semi-major axis before the Swing-By

Fig. 5(d) – Swing-by for a cloud of particles with Mars for rp = 5.0 R_M.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 1"

Fig. 5(e) – Swing-by for a cloud of particles with Mars for rp = 5.0 R_M.



Eccentricity vs. Semi-major axis after Swing-By for "Solution 2"

Fig. 5(f) – Swing-by for a cloud of particles with Mars for rp = 5.0 R_M.

After performing a large number of simulations using different values for the periapsis distance, it is possible to study the effects of this parameter in the whole maneuver. In Fig. 6(a-d), Man1 refers to the solution 1 and Man2 refers to the solution 2.

From those results we can see that different planets show different behaviors regarding this maneuver.

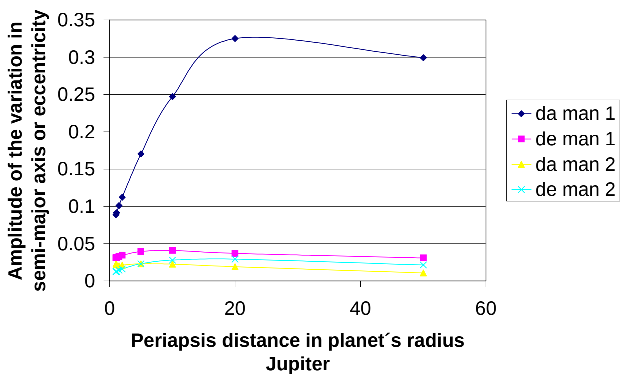


Fig. 6(a) – Effects of the periapsis distances for Jupiter.

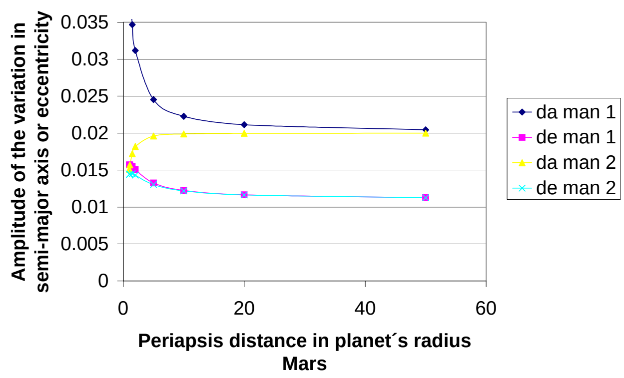


Fig. 6(b) – Effects of the periapsis distances for Mars.

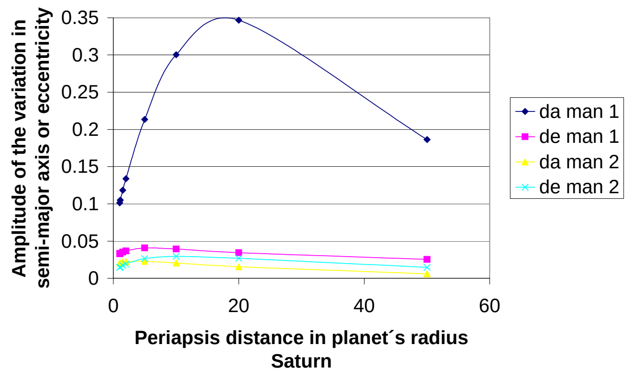


Fig. 6(c) – Effects of the periapsis distances for Saturn.

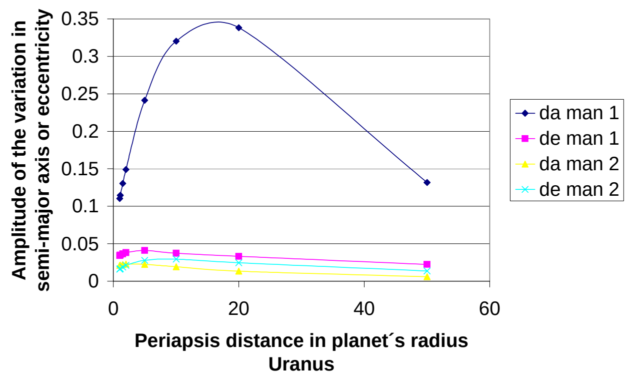


Fig. 6(d) – Effects of the periapsis distances for Uranus.

For the large planets, the amplitude has a sharp maximum for the variation in amplitude. It means that there is a specific value for the periapsis distance that maximizes the dispersion of the particles. This information can be used for practical applications, like in the case that you can control the periapsis distance (like a satellite that will explode, but that you can control its trajectory before the explosion), and then you can choose the position that causes larger amplitudes. For Mars the behavior shows a decreasing amplitude, so if you want to maximize the amplitude you need to send the spacecraft as close as possible to the planet.

5 Conclusion

The figures above allow us to get some conclusions. The solution called "Solution 1" has a larger amplitude than the Solution 2 in both orbital elements, but it concentrates the orbital elements in a line, while the so called "Solution 2" generates a distribution close to a square. The area occupied by the points is smaller for "Solution 1". Both vertical and horizontal lines are rotated and become diagonal lines with different inclinations. The effect of increasing the periapse distance is to generate plots with larger amplitudes, but with the points more concentrated, close to a straight line. In general, results like those ones shown here can be used to understand better the effects of the periapsis distance.

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