# Sub-Optimal Orbital Maneuvers for Artificial Satellites

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*Abstract:* - The determination of a specific orbit and the procedure to calculate orbital maneuvers for artificial satellites are problems of extreme importance in the study of orbital mechanics. Therefore, the transferring problem of a spaceship from one orbit to another, and the attention to this subject has increased during the last years. Many applications can be found in several space activities, for example, to put a satellite in a geostationary orbit, to change the position of a spaceship, to maintain a specific satellite's orbit, in the design of an interplanetary mission, and others. The orbit and the general data of the Brazilian Satellite SCD-1 (Data Collecting Satellite) will be used as example in this paper. It is the first satellite developed entirely in Brazil, and it remains in operation to this date. SCD-1 was designed, developed, built, and tested by Brazilian scientists, engineers, and technicians working at INPE (National Institute for Space Research) and in Brazilian Industries. During its lifetime we will study its orbital behavior and we will perform some orbital maneuvers that could be necessary, being this one either an orbital transferring, or just to make periodical corrections. The purpose of the transferring problem is to change the position, velocity and the satellite's mass to a new pre-determined state. This transfer can be totally linked (in the case of "Rendezvous") or partially free (free time, free final velocity, etc). In the global case, the direction, the orientation and the magnitude of the thrust to be applied must be chosen, respecting the equipment's limit. In order to make this transferring, either sub-optimal or optimal maneuvers may be used. In the present study, only the sub-optimal will be shown, since we want to study an algorithm for this type of maneuver. Hence, this method will simplify the direction of thrust application, to allow a fast calculation that may be used in real time, with a very fast processing. The thrust application direction to be applied will be assumed small and constant, and the purpose of this paper is to find the time interval that the thrust is applied. This paper is basically divided into three parts: during the first one the sub-optimal maneuver is explained and detailed, the second presents the Satellite SCD-1, and finally the last part shows the results using the sub-optimal maneuver applied to the Brazilian Satellite.

*Key-Words:* - Astrodynamics, Spatial Mechanics, Brazilian Satellites, Orbital Maneuvers, Space Trajectories.

## 1 Sub-optimal Maneuvers

### 1.1 Orbital Elements

The motion of a satellite around the Earth may be described mathematically by three scalar second-order differential equations. The Integration of these equations of motion yields six constants of integration. Those constants of integration are known as the orbital elements.

The Keplerian orbital elements are often referred as classical or conventional elements and are the simplest and the easiest to use. This set of orbital elements can be divided into two groups: the dimensional elements and the orientation elements.

The dimensional elements specify the size and the shape of the orbit and relate the position of the satellite in its orbit with a given time. They are as follows:

a = semi major axis, which specifies the size of the orbit;

e = eccentricity, which specifies the shape of the orbit.

τ = time for the perigee passage, which relates the position in orbit to time (τ is often replaced by M, the mean anomaly at some arbitrary time t, the mean anomaly τ is a uniformly varying angle).

The orientation elements specify the orientation of the orbit in space. They are as follows:

i = inclination of the orbital plane with respect to the reference plane, which is taken to be the Earth's equator plane for the orbits of the satellites (0 deg ≤ i ≤ 180 deg).

Ω = right ascension of the ascending node (often shortened to simply "node"); Ω is measured counterclockwise in the equator's plane, from the direction of the vernal equinox to the point at which the satellite makes its south-to-north crossing of the equator (0 deg ≤ Ω ≤ 360 deg).

ω = argument of perigee, ω is measured in the orbital plane in the direction of motion, from the ascending node to the perigee (0 deg ≤ ω ≤ 360 deg).

The angles *i* and Ω specify the orientation of the orbital plane in space. The angle ω then specifies the orientation of the orbit in its plane. The argument of latitude *u* defines the position of the satellite relative to the node line.

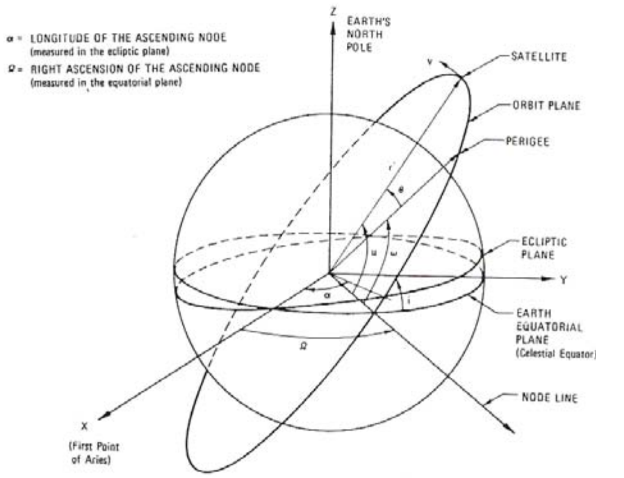


Fig. 1. Orientation of orbit in space.

### 1.2 REVISION OF THE LITERATURE

The Hohmann transfer[1], that study the transfer between two circular and coplanar orbits, is the first and one of the most important transfer orbits that exists in the literature. Several other important transfer can be found in books and papers published in the literature[2,3,4,5,6]. Going back in time, R. H. Goddard[7] was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption.

After him comes the very important work done by Hohmann[1]. He solved the problem of minimum ΔV transfers between two circular coplanar orbits. His results are largely used nowadays as a first approximation of more complex models and it was considered the final solution of this problem until 1959. A detailed study of this transfer can be found in Marec[8] and an analytical proof of its optimality can be found in Barrar[9].

The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal[10]. Smith[11] shows results for some other special cases, like coaxial and quasi-coaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits. A numerical scheme to solve the transfer between two generic coplanar elliptic orbits is presented by Bender[12]. Hohmann type transfers between non-coplanar orbits are discussed in several papers, like McCue[13], that study a transfer between two elliptic inclined orbits including the possibility of rendezvous; or Eckel and Vinh[14] that solve the same problem with time or fuel fixed. Another line of research studies the effects of the reality of finite thrust in the results obtained from the impulsive model. Zee[15] obtained analytical expressions for the extra fuel consumed to reach the same transfer and for the errors in the orbital elements and energy for a nominal maneuver (a real maneuver that uses the impulses calculated with the impulsive model). More recently, the literature studied the problem of a two-impulse transfer where the magnitude of the two impulses are fixed, like in Jin and Melton[16]; Jezewski and Mittleman[17]. Later, the three-impulse concept was introduced in the literature. Using this concept, it is possible to show that a bi-elliptical transfer between two circular orbits has a lower ΔV than the

Hohmann transfer, for some combinations of initial and final orbits. After that, Ting showed that the use of more than three impulses does not lower the ΔV, for impulsive maneuvers. Roth[18] obtained the minimum ΔV solution for a bi-elliptical transfer between two inclined orbits. Following the idea of more than two impulses, we have the work done by Prussing[19] that admits two or three impulses; Prussing[20] that admits four impulses; Eckel[21] that admits N impulses. Another line of research that comes from the Hohmann transfer is the study of multi-revolutions transfer with N impulses applied during N successive passages by the apses. Spencer, Glickman and Bercaw[22] shows equations and pictures to obtain the ΔV required for this transfer, as a function of the number of revolutions allowed for the transfer. After that, Redding[23] and Matogawa[24] would extend this concept of multi-revolution transfer to the non-impulsive case, by applying finite thrust around the apses. Some other researchers worked on methods where the number of impulses was a free parameter, and not a value fixed in advance. It is the case of the papers made by Lion and Handelsman[25], Jezewski and Rosendaal[26], Gross and Prussing[27], Eckel[28] and Prussing and Chiu[29]. Most of the research done in this particular case is based on the "Primer-Vector" theory developed by Lawden[30,31]. Several other publications are available treating this problem[32,33,34,35,36,37,38]. Similar topics in other fields can also be found in the literature[39,40].

## 1.3 The Maneuver Description

This topic has the objective to develop a sub-optimal method with computational high speed that calculates orbital maneuvers based in continuos thrust. The purpose is to have a method which generates fast results and low fuel consumption.

Kluever and Oleson[2], developed one method to calculate sub-optimal orbital maneuvers (with minimum time, therefore with minimum fuel consumption, given that the thrust magnitude is constant, and the time that the thrust is applied is proportional to the fuel consumption) for spaceships around the Earth, equipped with thrusters moved by solar electrical propeller. To solve the problem of optimal control, it is utilized a direct optimization technique, with approximations in the thrust's direction.

The equations for the spaceship, when the thrust is applied, are shown bellow. These equations are written in terms of non-singular equinoctial elements in order to cover circular orbits and planar orbits (i = 0º, 180º). The relationship between the equinoctial elements and the classical elements is given by:

$$h = esen(\omega + \Omega) \quad (1)$$

$$k = e\cos(\omega + \Omega) \quad (2)$$

$$p = \tan\left(\frac{i}{2}\right) sen\Omega \quad (3)$$

$$q = \tan\left(\frac{i}{2}\right)\cos\Omega \quad (4)$$

$$F = \Omega + \omega + E \quad (5)$$

For one spaceship moving in a gravitational field, and submitted to a propulsion force, the equations of motion are shown:

$$\dot{x} = a_t M \hat{\alpha} \quad (6)$$

The state vector for the equation (6) is:

$$x = [a, h, k, p, q]^T \quad (7)$$

The vector $\hat{\alpha}$ (3X1) is a canonical vector, in the thrust's application direction. The value of $a_t$ is the thrust acceleration magnitude, given by:

$$a_t = \frac{2\eta P_0}{mgI_{sp}} \quad (8)$$

Where $\eta$ is the thrust system efficiency, $P_0$ is the initial force given by the propulsion system, m is the vehicle's mass, g is the gravitational acceleration at the sea level, and $I_{sp}$ is the specific impulse. The state equation for F is not included because the average of classical elements is utilized, and therefore, only elements that vary slowly are considered.

The elements of the matrix M that appears in the equation (6) are:

$$M_{11} = \frac{2a}{nr}\left[hkb\cos F - (1 - h^2 b)\sin F\right] \quad (9)$$

$$M_{12} = \frac{2a}{nr}\left[(1-k^2b)\cos F - hkb\sin F\right] \quad (10)$$

$$M_{13} = 0 \quad (11)$$

$$M_{21} = \frac{G}{na^2}\left(\frac{\partial X}{\partial k} - hb\frac{\dot{X}}{n}\right) \quad (12)$$

$$M_{22} = \frac{k}{Gna^2}\left(\frac{\partial Y}{\partial k} - hb\frac{\dot{Y}}{n}\right) \quad (13)$$

$$M_{23} = \frac{k}{Gna^2}\left(qY - pX\right) \quad (14)$$

$$M_{31} = -\frac{G}{na^2}\left(\frac{\partial X}{\partial h} + hb\frac{\dot{X}}{n}\right) \quad (15)$$

$$M_{32} = -\frac{G}{na^2}\left(\frac{\partial Y}{\partial h} + hb\frac{\dot{Y}}{n}\right) \quad (16)$$

$$M_{33} = -\frac{h}{Gna^2}(qY - pX) \quad (17)$$

$$M_{41} = 0 \quad (18)$$

$$M_{42} = 0 \quad (19)$$

$$M_{43} = \frac{KY}{2Gna^2} \quad (20)$$

$$M_{51} = 0 \quad (21)$$

$$M_{52} = 0 \quad (22)$$

$$M_{53} = \frac{KX}{2Gna^2} \quad (23)$$

$$X = a\left[(1-h^2b)\cos F + hkb\sin F - k\right] \quad (24)$$

$$\dot{X} = \frac{a^2n}{r}\left[hkb\cos F - (1-h^2b)\sin F\right] \quad (25)$$

$$\frac{\partial X}{\partial h} = -2ahb\cos F + kb\sin F \quad (26)$$

$$\frac{\partial X}{\partial k} = hb\sin F - 1 \quad (27)$$

$$Y = a\left[(1-k^2b)\sin F + hkb\cos F - h\right] \quad (28)$$

$$\dot{Y} = \frac{a^2n}{r}\left[(1-k^2b)\cos F - hkb\sin F\right] \quad (29)$$

$$\frac{\partial Y}{\partial h} = kb\cos F - 1 \quad (30)$$

$$\frac{\partial Y}{\partial k} = -2akb\sin F + hb\cos F \quad (31)$$

$$G = \sqrt{1-k^2-h^2} \quad (32)$$

$$b = \frac{1}{1+G} \quad (33)$$

$$n = \sqrt{\frac{\mu}{a^3}} \quad (34)$$

$$r = a(1 - k\cos F - h\sin F) \quad (35)$$

$$K = 1 + p^2 + q^2 \quad (36)$$

$$E - e\sin E = n(t - T) \quad (37)$$

$$\dot{E} = \frac{n}{1 - e\cos(F - \omega - \Omega)} \quad (38)$$

The processing time is reduced when orbital averages are used. Since all the orbital elements utilized are variables that vary slowly, due to the fact that the thrust force have a small magnitude, large integration steps can be used. The equation of motion for the spaceship can be approximated, calculating the increment of each orbital element within one period and dividing by this time. Hence, the variations of the equinoctial elements with respect to time, are:

$$\bar{x}' = \frac{1}{T}\int_{-\pi}^{\pi} \bar{a}_t \overline{M}\,\hat{\alpha}\,\frac{dt}{dF}\,dF \quad (39)$$

Where x is the approximation of the state, and T is the orbital period. The strip above the variables means that they were availed using the average state vector. The integral represents the change of the orbital elements in one turn, keeping the orbital elements as constants.

## 1.4 Continuous optimal maneuvers

Another possibility to study this problem is to consider optimal control. This approach is based on Optimal Control Theory. First order necessary conditions for a local minimum are used. These equations can give us the following information:

a) One set of differential equations for the Lagrange multipliers. They are called "adjoint equations". Together with the equations of motion they complete the set of differential equations to be integrated numerically at each step;

b) The "Transversality Conditions", that are the conditions to be satisfied by the Lagrange multipliers at the final time. Together with the constraints of the transfer (start in a point that belongs to the initial orbit and finishes in a point that belongs to the final orbit) these end conditions complete the set of boundary conditions to be satisfied. This problem is known as the "Two Point Boundary Value Problem" (TPBVP), because there are boundary conditions to be satisfied at the beginning and at the end of the interval of integration;

c) Maximum Principle of Pontryagin. This principle says that the magnitude of the scalar product of the Lagrange multiplier by the right-hand side of the equations of motion has to be a maximum. Working out the algebra involved we will end up with a condition for the angles of "pitch" and "yaw" that can be solved to give us their numerical values at each time.

Then, the problem becomes a problem of non-linear programming with finite dimension. This problem is then solved using the following algorithm:

i) Choose an estimate for the initial and final "range angle" (the variable that replaces the time as the independent variable) and for the initial values of the Lagrange multipliers;

ii) Integrate the adjoint equations and the equations of motion simultaneously, obtaining the instantaneous values of the "pitch" and "yaw" angles from the Maximum Principle of Pontryagin;

iii) At the end of the maneuver, verify if the boundary conditions are satisfied. If they are not satisfied update the initial values following the procedure described in the next session and go back to step i. If the constraints are satisfied the procedure is finished.

This treatment is called hybrid approach because it uses direct searching methods for minimization together with first order necessary conditions for a local minimum.

With this approach, the problem is again reduced to parametric optimization, as in the suboptimal method, with the difference that the angle's parameters are replaced by the initial values of the Lagrange multipliers, as the variables to be optimized.

The main difficulty involved in this method is to find good first initial guesses for the Lagrange multipliers, because they are quantities with no physical meaning. This problem can be solved by using the method proposed by Biggs. He proposes a transformation called "adjoint-control", where one guess control angles and its rates at the beginning of thrusting instead of the initial values of the Lagrange multipliers. A set of equations is developed that allow to obtain the Lagrange multipliers from the values of the initial angles of "pitch" and "yaw" and its rates. By performing this transformation it is easier to find a good initial guess, and the convergence is faster. This hybrid approach has the advantage that, since the Lagrange multipliers remain constant during the "ballistic arcs" (arcs that have the thrusts inactive), it is necessary to guess values of the control angles and its rates only for the first "burning arc". This transformation reduces the number of variables to be optimized and, in consequence, the time of convergence.

To solve the nonlinear programming problem, the gradient projection method was used.

It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla \mathbf{f}^{\mathbf{T}} . \left[ \nabla \mathbf{f} . \nabla \mathbf{f}^{\mathbf{T}} \right]^{-1} \mathbf{f} \tag{40}$$

where f is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \alpha \frac{\mathbf{d}}{|\mathbf{d}|} \tag{41}$$

where:

$$\alpha = \gamma \frac{\mathrm{J}(\boldsymbol{u})}{\nabla \mathrm{J}(\boldsymbol{u}) . \mathbf{d}} \tag{42}$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla\mathbf{f}^{T}\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{T}\right]^{-1}\mathbf{f}\right)\nabla J(\boldsymbol{u}) \qquad (43)$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and $\gamma$ is a parameter determined by a trial and error technique. The possible singularities in equations (40) to (43) are avoided by choosing the error margins for tolerande in convergence large enough.

This procedure continues until $\left|\boldsymbol{u}_{i+1} - \boldsymbol{u}_{i}\right| < \varepsilon$ in both equations (40) and (41), where $\varepsilon$ is a specified tolerance.

## 2 The Brazilian Satellite SCD-1

The first Data-Collecting Satellite (Satélite de Coleta de Dados) was launched on February 9, 1993. It is the first satellite developed entirely in Brazil and it remains in operation in orbit to this date. SCD-1 was designed, developed, built, and tested by Brazilian scientists, engineers, and technicians working at INPE (National Institute of Space Research) and in Brazilian industries.

Fig. 2. The Satellite SCD-1

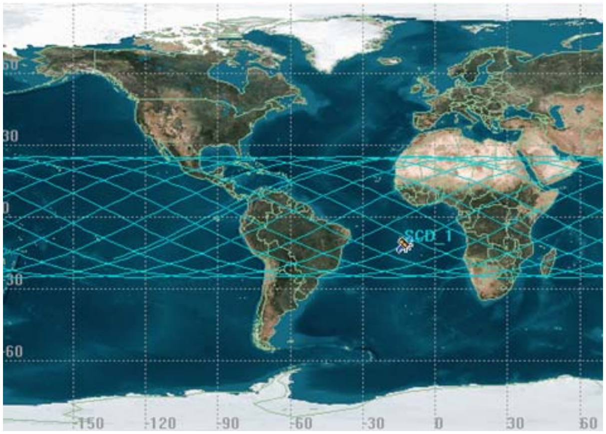


Fig. 3. The Area Coverage by SCD-1 (STK®)

SCD-1 is an experimental satellite with an environmental mission. It receives data collected on the ground or at sea by hundreds of automatic data-collecting platforms (DCPs) and retransmits all the information in a combined real-time signal back to tracking stations on Earth. Applications include hydrology, meteorology, and monitoring of the environment in general. The data are used by agencies such as the Weather Forecasting and Climate Studies Center (Centro de Previsão do Tempo e Estudos Climáticos - CPTEC), hydroelectric power managers, and both private and governmental institutions with many different interests. An example is the meteorological and environmental data collected in the Amazon region, including the levels of carbon monoxide and carbon dioxide in the atmosphere. These data are transmitted to INPE and are used for monitoring forest fires.

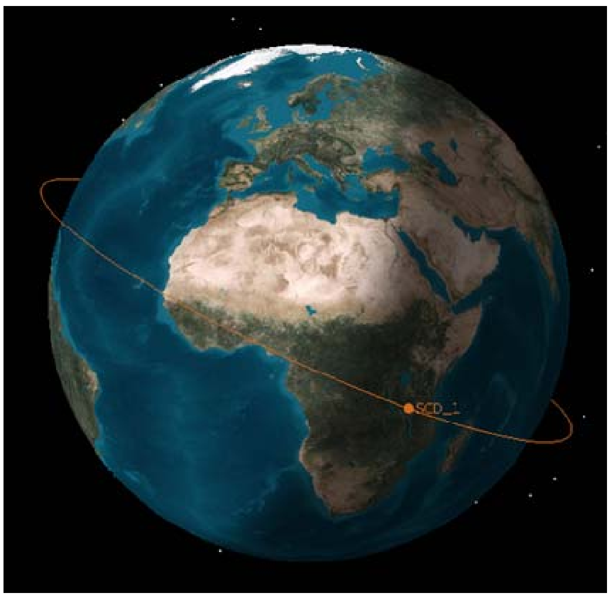


Fig. 4. The SCD-1 Orbit (STK®)

SCD-1 weighs approximately 110 kg and goes around the Earth every 100 minutes on a nearly circular orbit at about 760 km altitude. The inclination of the orbit with respect to the plane of the equator is 25 degrees, providing excellent coverage of equatorial, tropical, and subtropical regions (up to about 35 degrees of latitude) around the world. The spin-stabilized spacecraft has the shape of an octogonal prism, with a diameter of 1 meter and a height near 70 cm without the antennas that are mounted on both base surfaces. It was originally designed for a life of one year with 80% probability, but it has survived 15 years in operation without any crippling functional failure. However, since its chemical (nickel-cadmium) batteries are now completely run down, the satellite can no longer be used while it is in the Earth's shadow.

## 3 Results

The results consist of plots that show the orbital element's variation, according to the time.
Due to the very fast processing, this method could be utilized in real time to make any correction maneuver.

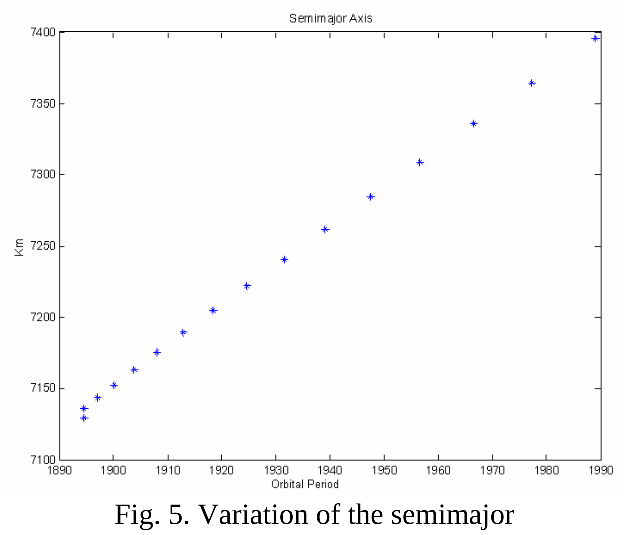


Fig. 5. Variation of the semimajor

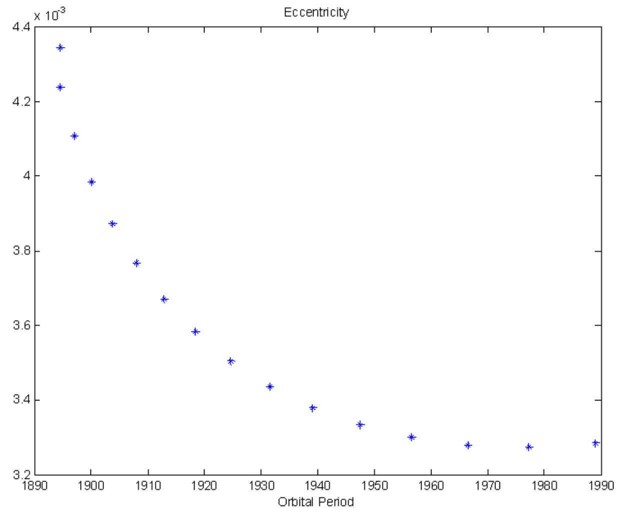


Fig. 6. Variation of the Eccentricity

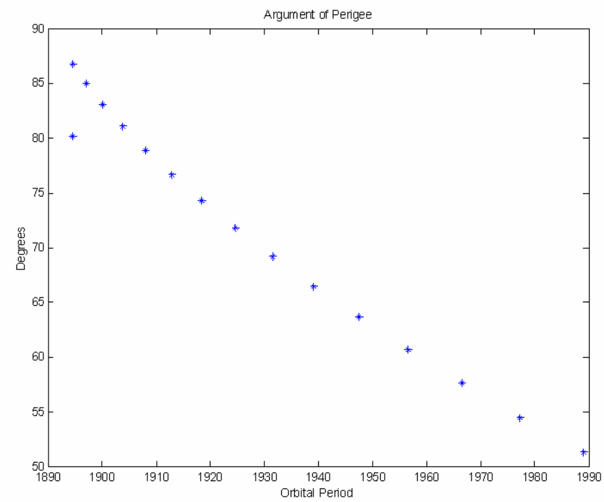


Fig. 7. Variation of the Argument of perigee

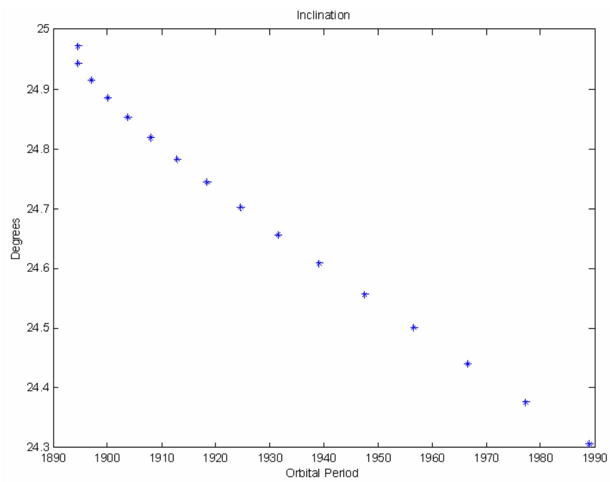


Fig. 8. Variation of the Inclination

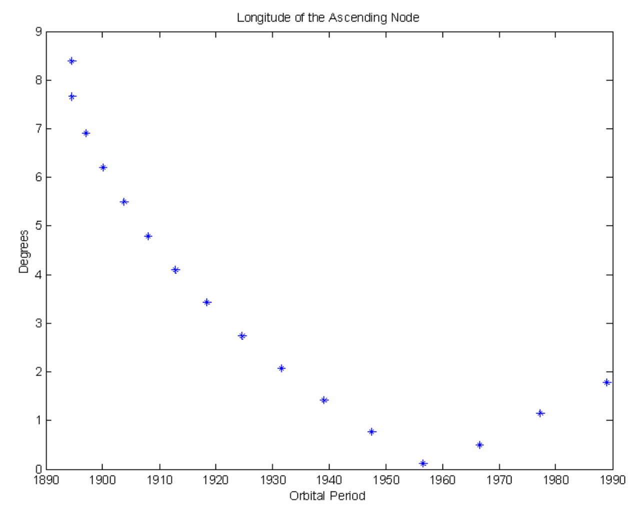


Fig. 9. Variation of the Longitude of the Ascending Node

# 4 Conclusion

A numerical algorithm was developed to calculate a transferring problem, and the purpose is to change the position, velocity and the satellite's mass to a new pre-determined state. Hence, this method simplifies the directions of the application of the thrust, to allow a fast calculation that may be used in real time. The level of the thrust to be applied is assumed to be small and constant.

The Brazilian Satellite SCD-1 was used as an example in this paper. It is the first satellite developed entirely in Brazil and it remains in operation in orbit to this date.

In real applications, as in the case of SCD-1, it might be necessary to do a complementary maneuver, being this one either to an orbital transferring, or just to make periodical corrections.

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