

Development of a Spiral Trajectory for High Speed Roughing of Light Alloy Aerospace Components

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Abstract: - A new spiral cutting method and its implementation within a CAD/CAM application software is proposed to save more than 16% in the roughing process of pockets for light alloy aerospace parts. The application prototype developed automatically generates the tool paths related to specific pocket profile geometry. It is shown, through experimentation using the prototype, that the parameters which define the spiral tool path geometry have a direct and important impact on cutting times. During the roughing operation, the proposed tool path takes into consideration the slant of the walls but also allows maximizing the quantity of material to be removed. The approach developed indicates great potential for roughing optimization of parts having multiple pockets.

Key-words: - CAD, CAM, roughing, spiral trajectories, tool path, machining.

1. Introduction

Recent progress in machining technologies have lead to numerous research works in different fields of application like surface finish simulation [1], [2], residual stress modeling and prediction [3],[4], as well as productivity improvement specifically related to High Speed Machining technologies (HSM) [5]. The Computer Aided Manufacturing software industry (CAD/CAM) however, did not progress as fast to fully benefit from these new developments related to the HSM technologies. As a result, the cutting strategies offered with numerous softwares are mainly poorly adapted to the HSM [6]. This is why numerous efforts in research are presently made in this specific field to develop new cutting methods which are better adapted to the performances now available with actual machines [7].

With HSM, the effects of acceleration and deceleration during a toolpath must be addressed to fully benefits from this technology [8]. These conditions lead to the development of smoother paths like the spiral type [9] in order to reduce the cycle time. A smoother tool path presents the advantage of making speed more constant and offers the possibility to machine the component at the highest available speed, when reachable.

As a result, these time savings allow to increase the production rate [10]. This approach has an

important advantage, particularly during the machining of pockets [11] largely found on aircraft wing components (Figure 1). Bieterman and Sandstrom [12] significantly contributed to this domain of application. Their results showed the great potential of spiral type toolpaths (figure 2).

This paper presents a new approach to develop a machining procedure which involves an adaptive spiral tool path. This technique is based upon commonly used programming methods in CAD/CAM software applications [13].

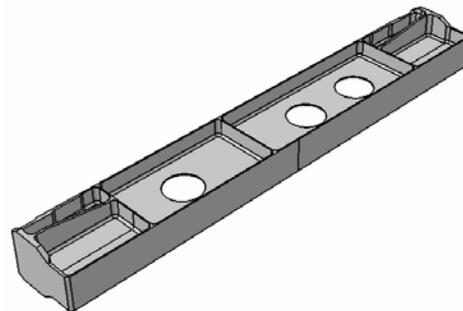


Fig.1: Aircraft wing component with pockets

The application prototype developed within commercial CAD/CAM software, using the Visual Basic programming language, allows to automatically generate a spiral adaptive cutting trajectory based on the following approach:

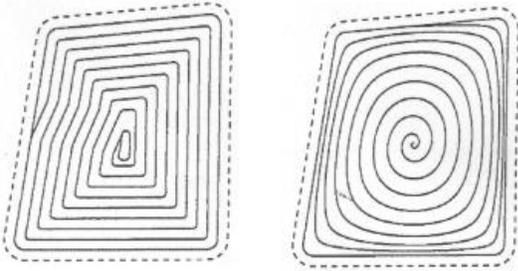


Fig.2: Conventional and spiral toolpaths [12]

- i. Calculation of the profile boundary offset of the pocket geometry;
- ii. Calculation of the central cutting trajectory at the centre of the pocket (central spiral);
- iii. Calculation of the cutting trajectory and adaptation of it to the pocket profile geometry;
- iv. Calculation of the appropriate radial depth of cut and feedrate.

The parameters defining the adaptive toolpaths are also studied in order to optimize this programming method. In order to appreciate the effect of the acceleration and deceleration of the machine axis, a comparison study of the cutting time for a square and a circle path of same lengths is also presented.

2. Adaptive spiral trajectory

2.1 Boundary offset calculation

The first step in the programming process consists in performing an offset of the pocket profile. This offset constitutes the tool path's final machining step. The calculations to offset the geometry are taking into account the tool's diameter and the stock allowance whenever there is a finishing operation.

The unit vector used to offset the pocket profile is calculated using the cross product between the tangent vector related to each element of the profile geometry and the tool axis. In the case of a line which is offset towards the inside of the pocket (line P1-P2), as illustrated in Figure 3, the cross product of the P1-P2 vector (vector a_1) and the tool axis (vector b) generates the normal vector and thereby offset the geometry so as to create the line P1'-P2'. The program proceeds in a similar way for

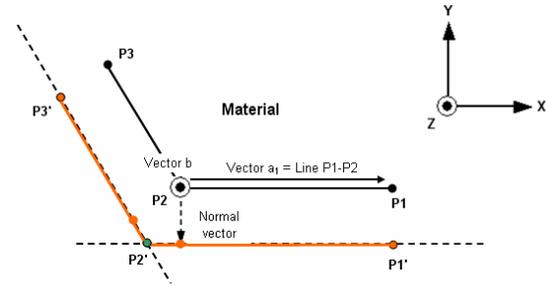


Fig.3: Offset of two lines and computation of the new intersection point

the following line and computes the new intersection point (P2') between both lines. In the case of a geometry comprising a circular arc, the main vector is depicted by the radius of the arc. As a result, the boundary points for this element are offset in the same direction as the radius. Some measurements need to be taken into consideration during the geometry offset. Depending on the radius of curvature in the pocket's corners, if the offset is smaller than the corner radii, there will be no overlap and therefore no element loss in the last stage of the tool path. In other words, if the tool's radius is smaller than the corner radius, there will always be a curve trajectory in the corners. Figure 4 illustrates the situation where the tool radius is smaller than the corner radius. No element loss is incurred and a circular trajectory (Arc P2'-P3') always exists. The opposite situation may also occur. When the tool's radius added to the stock allowance is greater than the corner radius, the circle's arc is eliminated. An intersection calculation between 2 lines needs to be done (see Figure 5).

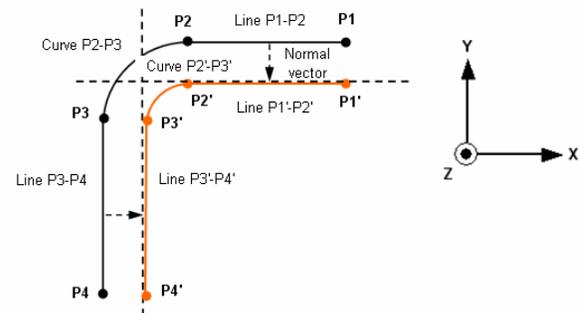


Fig.4: Offset with no element loss incurred

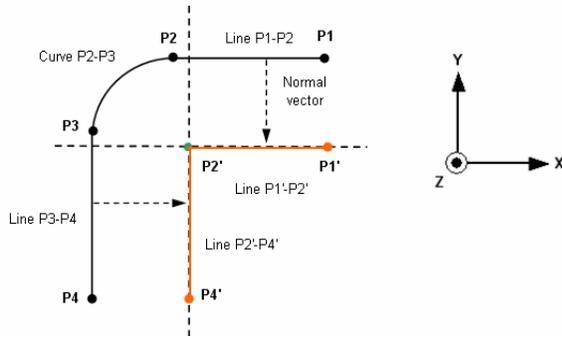


Fig.5: Elimination of a circle arc and intersection between 2 lines.

Depending on the pocket's geometry and the offset value, the program computes and redefines the geometric elements that are used to generate the final tool path which defines the boundary. In order to confirm that the programming allow an accurate geometry offset computation, several pocket profiles of various shapes were subjected to two (2) kinds of offsetting tests: an offset smaller than the corner radii and an offset greater than the corner radii. This simulation allows determining when to eliminate certain geometric elements so as to generate the final machining step required on the pocket boundary.

As previously mentioned, depending on the corner radii and the offset value, the final trajectory can require an additional calculation to redefine the last machining stage. This stage is defined by a large number of points. A polynomial curve allowing the creation of the final machining trajectory will go through these points which are constructed as control points.

2.2 Trajectory for the center of pocket

The second step of the procedure concerns the creation of the spiral tool path at the centre of the pocket. The cutting tool paths complexity is directly related to the part geometry to be machined. To be efficient, a spiral adaptative toolpath should gradually change to a path which becomes the profile shape of the pocket. In fact, the machining path starting at the center of the pocket has a circular shape rather than an offset of the current pocket geometry profile. Thus, the machining operation starts in the pocket center with a spiral evolution. The paths gradually adapt to the profile of the pocket. Data points are defined along the trajectory to generate the linear and curved

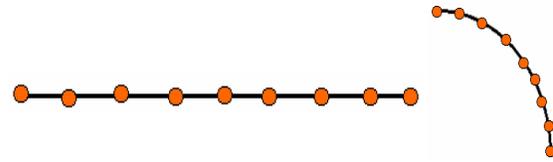


Fig.6: Linear and curvilinear elements

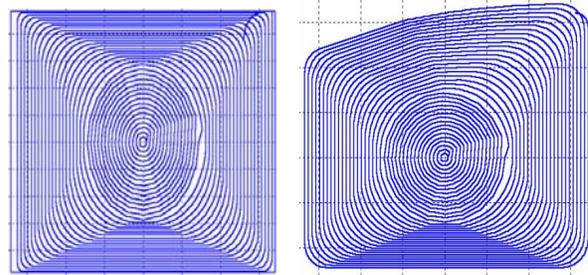


Fig.7: Generation of adaptive paths based on the pocket profile

elements which describe the cutting tool trajectory (figure 6). Any pocket shape can be processed with the technique as the figure 7 shows two examples. Among the required parameters to define the adapted spiral trajectory, let define "C" as the ratio of the pocket width for which the spiral starts to change into a combination of curvilinear elements which adapt to the profile of the pocket. This ratio "C" is related to the pocket width "l". A ratio C=70% would generate 70% of true spiral trajectory with the remaining 30% as a trajectory which gradually adapts to the profile offset of the pocket geometry (figure 8):

$$C = 70\% : (\phi) = 0.7 \times l \quad (1)$$

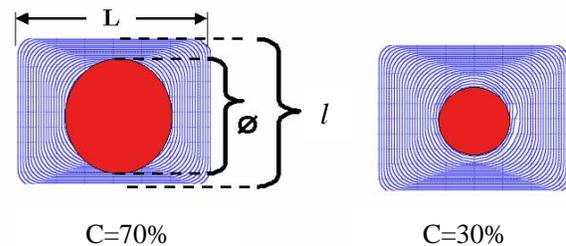


Fig.8: Parameters for the generation of the spiral trajectory [14]

With :

- ϕ : end diameter related to the spiral trajectory
- l : pocket width
- L: pocket length

2.2.1 Estimation of an optimal transition ratio

To better understand the tool paths automatic generation algorithm, square and rectangular pockets were first tested during the simulations. The test part with such simple pockets selected for the tests is showed at figure 9.

It is a simple base example for the roughing operations of aerospace structural parts. The machining experiments, in a "dry run" mode, were performed using a CNC vertical machine Huron model with a Siemens 840D controller. The experimental tests performed with the square geometry are showed in table 1. The machining time results are presented as a function of the "C" ratio and the cutting speed specified.

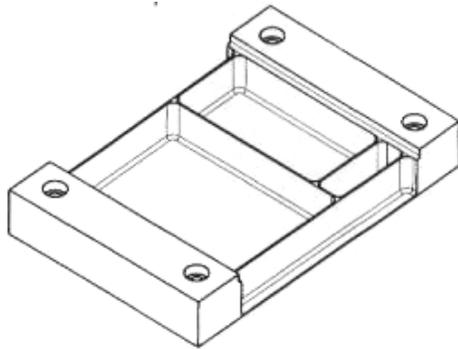


Fig.9: Test part

Spiral type trajectories varying from C=20% to C=70% have been calculated and translated in G codes to be processed with the Siemens control at feedrates varying from 200 to 600 inches per minute. The table 1 presents the results for all tests in terms of "dry run" machining times. It also presents the time savings in terms of improvement rate of the spiral type paths compared to the conventional type paths.

$$\%Gain = \frac{t_{conv.} - t_{spiral}}{t_{conv.}} \times 100 \quad (2)$$

With :

- Gain : % savings in machining time
- $t_{conv.}$: conventional path machining time
- t_{spiral} : spiral path machining time

To verify the potential of the approach, it is required to compare these times to reference times related to conventional parallel trajectories for the same roughing geometry and cutting parameters.

| Feed Type | Time (sec.) Gain (%) | | | | | | |
|-----------|-------------------------|---------------|--------------|--------------|--------------|--------------|--------------|
| | 200 | 300 | 350 | 400 | 450 | 500 | 600 |
| C =20% | 141,6 9,10 | 99,2 10,4 | 88,0 10,8 | 80,1 10,9 | 74,3 10,6 | 70,0 10,0 | 64,0 9,1 |
| C =30% | 138,5 11,40 | 96,28 12,9 | 84,8 13,4 | 76,6 13,9 | 70,5 14,1 | 65,9 14,0 | 60,0 13,2 |
| C =40% | 136,4 13,40 | 94,27 15,2 | 83,0 15,6 | 74,8 15,8 | 68,7 16,0 | 64,0 16,0 | 57,7 15,7 |
| C =50% | 136,5 19,50 | 94,04 21,5 | 82,1 21,8 | 74,7 21,7 | 68,6 21,4 | 64,0 21,1 | 57,6 20,4 |
| C =60% | 134,9 18,20 | 92,71 20,1 | 81,4 20,5 | 73,4 20,2 | 67,6 19,6 | 63,1 18,7 | 57,0 17,3 |
| C =70% | 133,6 19,80 | 91,93 21,5 | 81,1 21,5 | 73,4 21,0 | 68,0 20,1 | 64,0 18,7 | 58,6 15,9 |

Table 1: Cutting time (sec.) and Gain (%) Vs Feedrate (in./min) and "C" transition

The results are showed graphically in figure 10 where the machining time is presented with respect to parameter "C". Each curve relates to a specific feedrate. As expected, an increase in the feedrate results in a lower machining time. However, the time savings become significant when the speed increases from 200 IPM to 300 IPM.

The study concerning the variation of the spiral transition ratio "C" reveals that the increase in the size of the spiral generally decreases the machining time. The smoothness of the paths is the source of this improvement. Less small corners are found as the spiral size gets bigger. Short linear paths with corners are the source of acceleration and deceleration and the specified feedrate may not be reached in some cases. For high feedrates, an optimum cutting time is reached at transition C=55%. This becomes clear for 600 IPM feedrate.

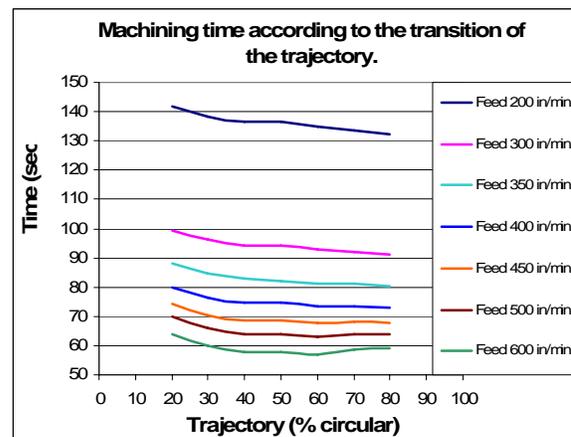


Fig.10: Machining time for a 200 to 600 IPM feedrate range

Each curve is presented on a separate graph to emphasize the feedrate and the "C" transition parameter effect on the cutting time. A Polynomial equation fitting is proposed to help predict machining cutting times as a function of the transition "C" for a given feedrate (equation 3). The figure 11 illustrates these results for respective feedrates of 300, 400, 500 and 600 IPM. The figure 12 presents the machining time with respect to the feedrate for C=40% and C=70%. The decreasing rate of the time for both cases is very similar.

$$t_{300} = 6 \cdot E - 8c^5 - 1 \cdot E - 5c^4 + 0,001c^3 - 0,03c^2 - 0,07c + 106$$

$$t_{400} = -8 \cdot E - 8c^5 + 2 \cdot E - 5c^4 - 0,002c^3 + 0,11c^2 - 3,3c + 113$$

$$t_{500} = -1 \cdot E - 7c^5 + 3 \cdot E - 5c^4 - 0,003c^3 + 0,17c^2 - 4,3c + 112 \quad (3)$$

$$t_{600} = -1 \cdot E - 7c^5 + 3 \cdot E - 5c^4 - 0,003c^3 + 0,16c^2 - 4,1c + 103$$

With:

- t_i : machining time for a specific feedrate "i"
- c: transition ratio (%)

2.3 Peripheral trajectory for the pocket

Based upon the experimental results presented, the "C" transition ratio for the spiral is set at the optimal value of 50% for the calculation of the remaining "peripheral" trajectory. This one is generated by means of a "measuring" method which consists in performing a 360° scan of the pocket geometry boundary. Considering that the pocket centre is initially translated at the coordinate system's (0,0) position, a fictitious line going from the point (0,0) and linking the last stage of the central spiral to the final stage, is used to distribute the control points. As a result, it is possible to complete the tool path for all the points which delineate the boundary. Figure 13 illustrates this process.

Once all the points are created, the program removes some of those located in the corners (see Figure 14). In this figure, which shows a rectangular-shaped pocket, one can notice the removal of some of the control points located in the corners. The interpolation curves replacing those points are splines. This technique allows the smoothing of the trajectory when the final polynomial curve used to delineate the tool path is generated. This is particularly beneficial to reduce sharp direction changes in the corners. It is widely used in trajectory path planning problems [15].

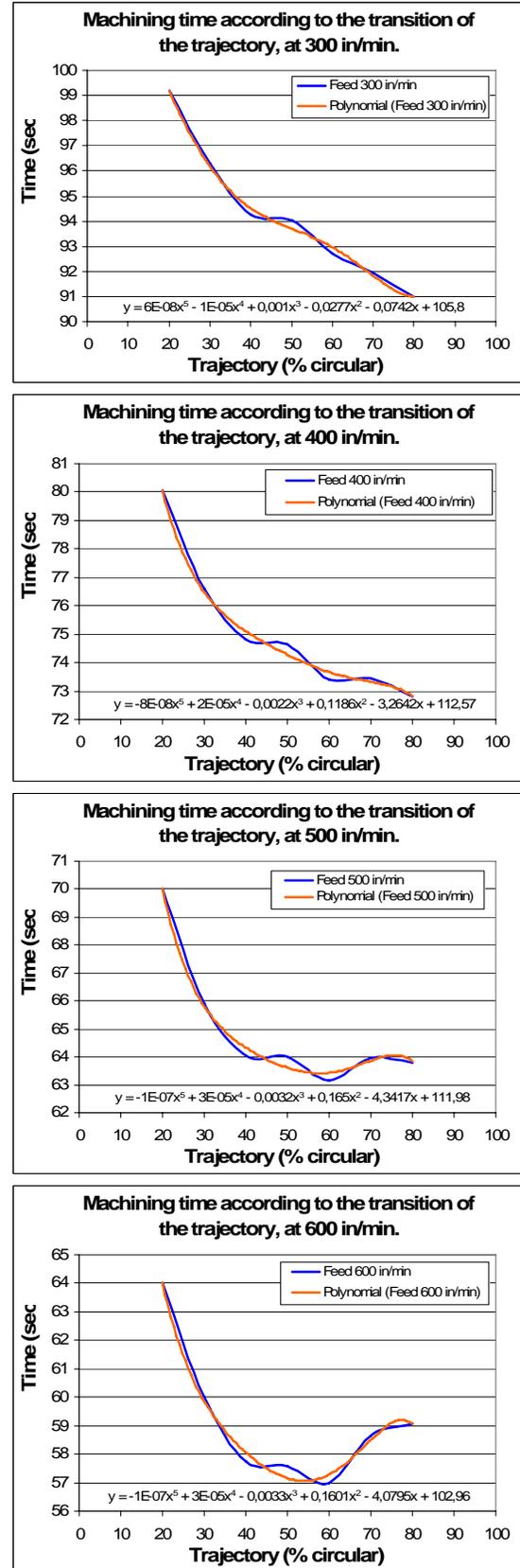


Fig.11: Machining time Vs C (%)

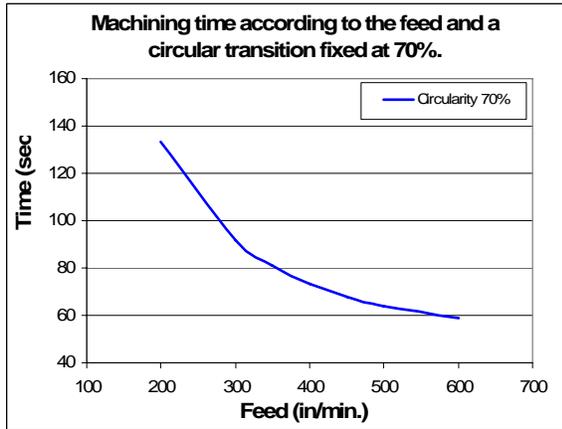
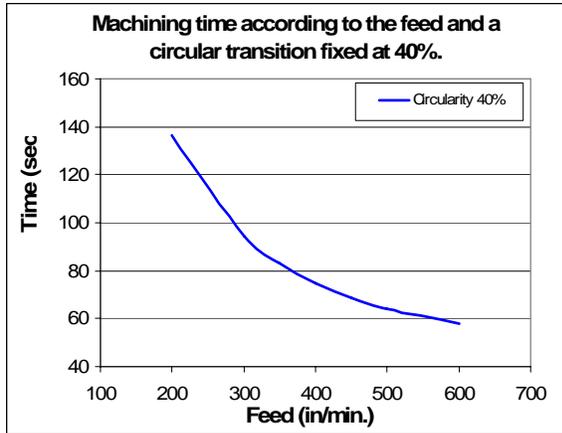


Fig.12: Machining time for C=40% and C=70%

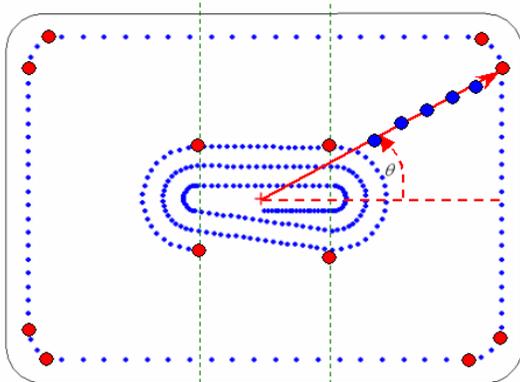


Fig.13: Boundary inspection and distribution of the control points

2.4 Calculation of the radial depth of cut and feedrate

Once the trajectory has been generated, the program computes the radial depth of cut. Because the tool path is kind of an adaptive spiral, the cutting thickness cannot always be constant. The

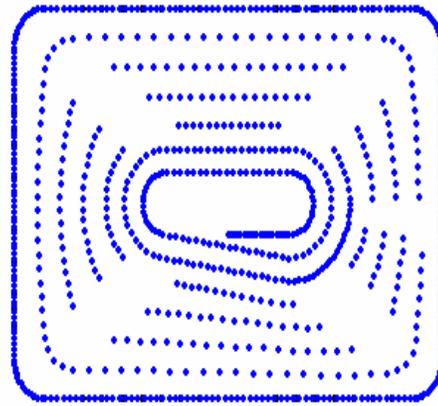


Fig.14: Removal of the control points in the corners

method used to determine the thickness consists in measuring the distance between two control points located in the same place in the trajectory's following stage. The cutting thickness was calculated by using the least squares approach.

Most machining programs suggest slowing down the feedrate in the pocket's corners. Since the tool is almost entirely immersed in the material, a reduced speed contributes to an easier chip removal and more stable machining. As a result, the VB procedure can also take into account changes in terms of speed during machining, since the cutting thickness may vary.

Once the thickness is calculated for the whole trajectory, the program determines the feed correction. By specifying the feed and the "slowdown" value, a linear equation allows to compute the feedrate correction that is required. For the specified cutting thickness, the feed is entirely applied, whereas for the maximum cutting thickness, the reduced value of the feedrate is used at this point on the trajectory. The lead is therefore computed in a linear manner for the radial depths of cut lying between the specified thickness and the maximum thickness. Figure 15 shows how the feed is corrected according to the cutting thickness, for a rectangular-shaped pocket with a feed specified at 600 in/min and a 75% slowdown. The feed is specified with respect to the location (control points) on the tool path (figure 16).

The first machining step is carried out at a speed reduced to 450 in/min. The tool begins in slotting mode and as a result, the slowdown is entirely applied (75% * 600 in/min = 450 in/min). The central spiral is then determined by a constant cutting thickness, hence the 600 in/min feedrate.

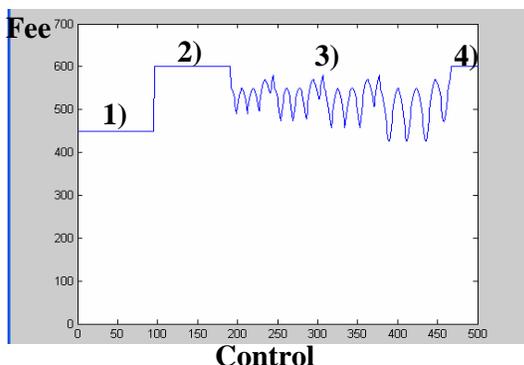


Fig.15: feed variation

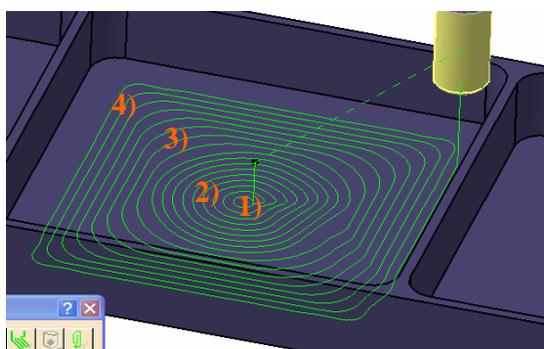


Fig.16: Spiral trajectory of a pocket

The rest of the trajectory which needs to adapt to the geometry boundary has a variable cutting thickness and a feed suitable for this thickness. Geometrically, a rectangular-shaped pocket has 4 corners where the cutting thickness will be at its maximum. This results in undulations of the feed as can be seen in Figure 15.

The following section presents an application example of the prototype implemented within the commercial software. It shows the generation of adaptive spiral trajectories for the machining of pockets of various shapes.

3. Application example of adaptive spirals within a CAM software

The machining procedure which is programmed using the Visual Basic language, allows generating an adaptive spiral tool path for pockets of various shapes typically found for aircraft parts. The following sections describe the tests performed on a real part and the corresponding results.

3.1 Machining test involving a typical aerospace structural part

An implementation of the spiral cutting strategies was conducted for the machining of an entire aerospace structural part including 9 pockets (Figure 17). In this example, the "adaptive spiral" cutting paths are compared against the "conventional" type of tool paths commonly found in different CAD/CAM systems (Figure 18).

The machining test was conducted on a CNC machine tool to estimate the time reduction resulting from the new strategies as compared to the conventional paths. The time saved by resorting to this machining operation was reduced by more than 16%.

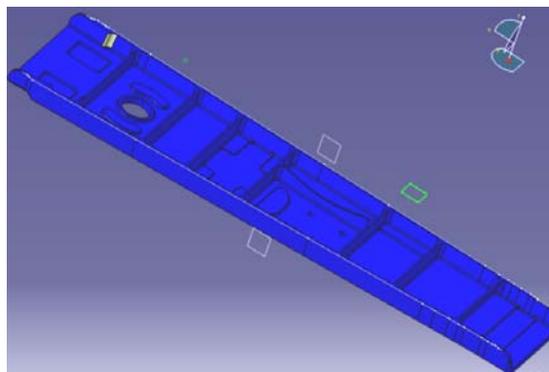


Fig.17: structural part

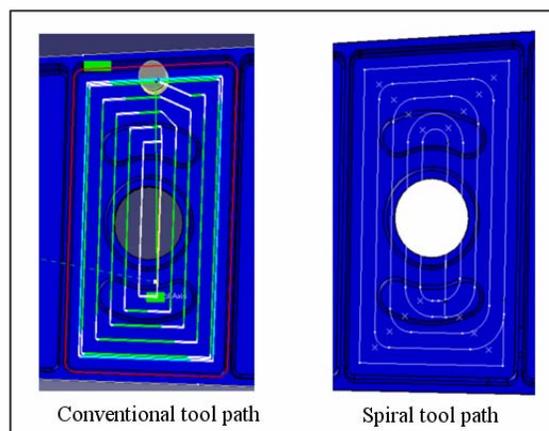


Fig.18: Conventional path and spiral tool paths

3.2 Pockets with inclined walls

When pockets with inclined walls are machined, the tool path is sometimes generated by using the geometry of the pocket's bottom surface (see Figure 19). The roughing sequences do not allow machining the excess material located above the

inclined wall or outside the extruded bottom. For those cases, alternative or additional machining sequences are required.

The spiral tool path takes into account for the presence of inclined walls and adapts to the trajectory, in order to remove the excess of material. Figure 20 shows how the trajectory needs to be generated should there be inclined walls defining the pocket.

By specifying the pocket's top and bottom dimensions, the algorithm can determine a number of axial steps to be performed and compute the boundary geometry by generating several intersection plans with the part. Once the boundary geometry is well defined at each step, the program works as described in the previous sections and draws a spiral trajectory which is adapted to all cutting thicknesses.

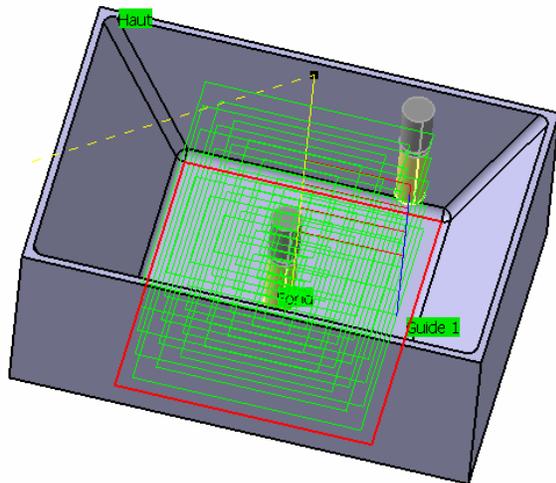


Fig.19: Inclined cutting trajectory

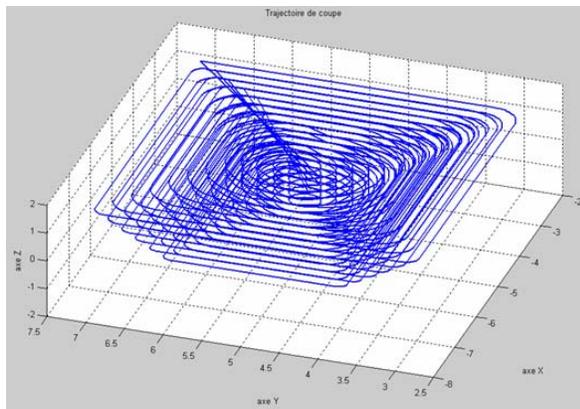


Fig.20: Enhancement of an inclined pocket cutting trajectory

4. Adaptive spiral paths behaviour

In order to appreciate the effect of the acceleration and deceleration of the machine axis during the "dry run" machining, the comparison of the cutting time for a square and a circle path of same lengths have been performed. The figure 21 shows the two paths used to study the dynamic effect on the cutting time.

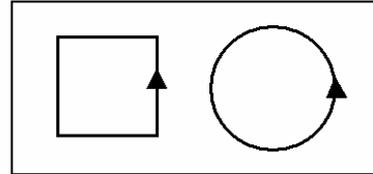


Fig.21: Square and circular path with same lengths

The table 2 presents the experimental time measured with a Siemens control to perform the squared and circular paths of same lengths with varying feedrates of 200 to 400 IPM. For a square path with 0.5 inch of side length, 0.82 seconds are required to travel the entire path at 200 IPM while 0.79 seconds are required for the circular path of same length (0,318 inch). With a 3.7% improvement rate, the effect of acceleration and deceleration is low at this feedrate. However, for a 400 IPM, this improvement rate goes up to about 30%. This experiment is performed for path lengths of 0.5 inch through 6 inches.

| Feed (in/min) | Length / Radius (in) | Time for the toolpath (sec) | | Gain (%) |
|---------------|----------------------|-----------------------------|--------|----------|
| | | Square | Circle | |
| 200 | 0,5 / 0,318 | 0,82 | 0,79 | 3,7 |
| | 1,0 / 0,636 | 1,44 | 1,41 | 2,1 |
| | 2,0 / 1,273 | 2,62 | 2,59 | 1,1 |
| | 4,0 / 2,546 | 5,04 | 4,99 | 1,0 |
| | 6,0 / 3,819 | 7,44 | 7,4 | 0,5 |
| 300 | 0,5 / 0,318 | 0,75 | 0,60 | 20,0 |
| | 1,0 / 0,636 | 1,16 | 1 | 13,8 |
| | 2,0 / 1,273 | 1,96 | 1,81 | 7,7 |
| | 4,0 / 2,546 | 3,55 | 3,41 | 3,9 |
| | 6,0 / 3,819 | 5,15 | 5 | 2,9 |
| 400 | 0,5 / 0,318 | 0,74 | 0,52 | 29,7 |
| | 1,0 / 0,636 | 1,08 | 0,81 | 25,0 |
| | 2,0 / 1,273 | 1,63 | 1,41 | 13,5 |
| | 4,0 / 2,546 | 2,88 | 2,66 | 7,6 |
| | 6,0 / 3,819 | 4,04 | 3,82 | 5,4 |

Table 2: Squared path compared to circular path

Graphically, the relationship between the path geometry and the time improvement rate can be appreciated (figure 22). We see that the time improvement rate is significant when the paths are of smaller size. Even though the path lengths are the same for the square and the circle, we note that the time improvement rate decreases as the paths lengthen. Considering these results in relation to the machining of squared pockets with spiral paths, one can see that the shorter the paths are at the beginning and center of the pocket, the higher are the time savings. When the tool approaches the transition area of the toolpath, its length becomes larger, and consequently the time improvement rate decreases.

The figure 23 shows the time saving rate varying with the side length of a square. Obviously, the time improvement rate increases with the cutting feedrate. With high axis speeds, the smooth paths are less sensible from deceleration and acceleration than cornered paths. These results show the efficiency of the adaptive spiral toolpaths while these are of maximum benefits at the beginning of the roughing operation.

5. Conclusion

The development of the adaptive spiral machining procedure optimizes the cutting strategies in CAD/CAM software applications.

This new approach significantly reduces machining time. As opposed to the traditional parallel circle method, the spiral trajectory technique reduces the time waste caused by the acceleration and deceleration of the machine axes when sudden direction changes occur.

Our study shows that significant time benefits are seen for feedrates higher than 200 inches per minute. We also noted that the beginning of the transition from a spiral to the profile of the pocket was optimal at 55% of a rectangular type pocket width. Depending on the pocket geometry, the new spiral technique leads to a reduction in machining time by approximately 16% for the roughing operations that are involved in the manufacturing of aircraft wing components. Furthermore, the generation of spiral trajectories on a prototype part as well as a virtual machining simulation, have shown its usefulness in an industrial context.

The program, which generates the spiral trajectories, was used for various pocket geometries that are commonly found in the industry.

Although the machining procedure is better adapted to simple pockets, benefits can however be realized for most geometries. The method is recommended for any pocket geometries having corner angles for which deceleration and acceleration can be reduced using the proposed spiral trajectories.

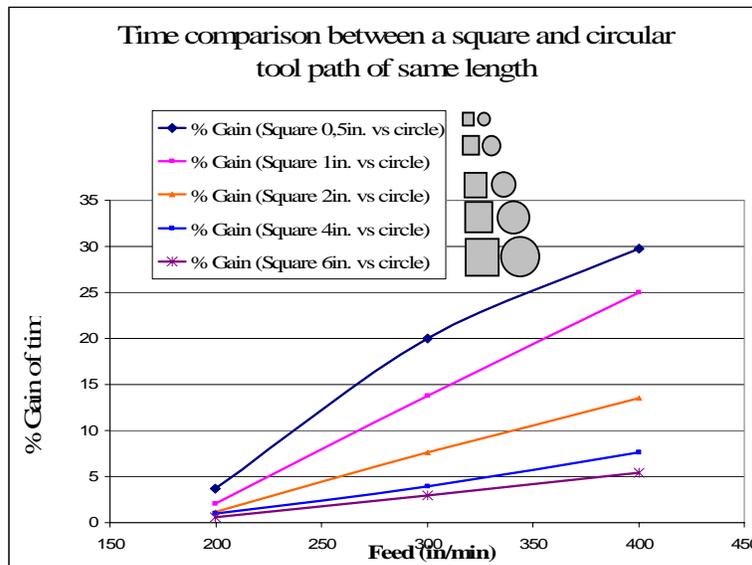


Fig.22: Time comparison for the square and circle geometry

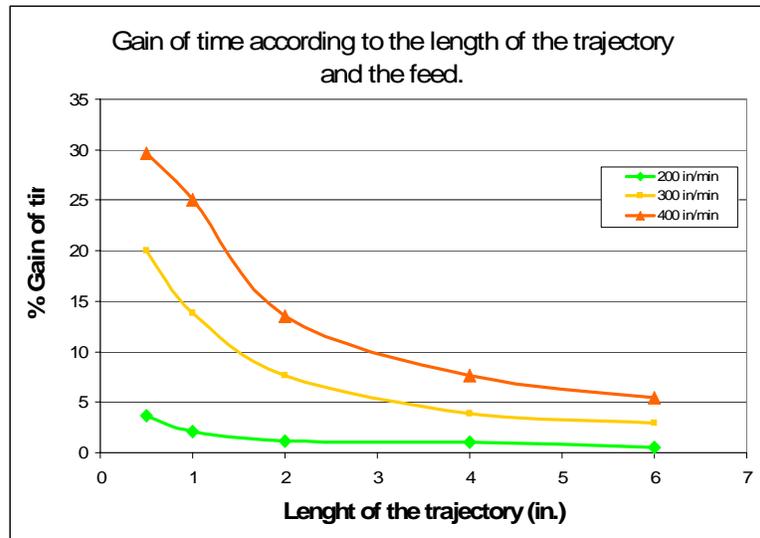


Fig.23: Effect of feedrate on time improvement

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