

The Effect of Frequency and Amplitude of Vibration on the Coefficient of Friction for Metals

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Abstract:- Experiments were conducted to determine the effects of frequency and amplitude of vibration on friction. The experimental analysis also seeks to take into account a variety of factors influencing the coefficient of friction such as normal load and surface roughness. An in-house pin-on-disc apparatus was constructed with a spindle speed control and applied forced-feedback to perform the tests. The response surface methodology is utilized to investigate the effects of the factors and their cross influence on the coefficient of friction for the Steel C1020 and Aluminum 7079. The analysis of variance is done on the experimental data to evaluate the statistical significance of the model. The response equation for the coefficient of friction of Steel C1020 and Aluminum 7079 were determined. The ranges of frequency and amplitude of vibration were 120 – 600 Hz and 15 – 225 μm , respectively. Studies have shown that the coefficient of friction decreases with the increase of frequency and amplitude of vibration within the observed ranges for both metals whereas the in case of Aluminum 7079 the coefficient of friction is about 13 % when it compares with the coefficient of friction of Steel C1020.

Key-Words: - Frequency and amplitude of vibration, Coefficient of friction, Pin on disc.

1 Introduction

The quantity known as the coefficient of friction has long been used in science and engineering. It is easy to define but not easy to understand on a fundamental level. The ratio of the friction force to normal interface force is defined as the coefficient of friction. The friction force is not constant if normal vibrations are present. Hence, if the coefficient of friction is constant, then the normal force must have the same velocity dependence as the friction force. This statement, in general, is not strictly correct. Many researchers, for example Bristow [1], Kato et al. [2] Antoniou et al. [3], Tolstoi [4], Madakson [5], and Dimnet et al. [6], have shown that, at least, at very low relative interface velocities the ratio of friction force to normal contact force is not constant. However, the results of some researchers also show that this ratio becomes approximately constant when the relative interface velocity is above a certain value.

It has been recognized that friction and vibration have a mutual influence [7-12]. Friction generates vibration in various forms, while vibration effects friction in turns. A number of researchers have used

the term “feedback” in studies involving friction-vibration relations. If one views the effect of frictional contact on the structural behavior of a mechanical system as the first effect then it has been shown that the vibration behavior of the mechanical system will in turn effect the frictional contact; the system feedback on friction.

An apparent reduction of friction by vibration is common. Several authors observed the reduction of friction force with the vibration, amplitude of vibration, relative sliding speed, roughness of rubbing surfaces, type of material, humidity, temperature, lubrication [7-19].

In view of the findings described in the literature, it is apparent that, as far as short dynamic events are concerned, the coefficient of friction may be regarded as independent of a slow wear process. However, its dependence on relative interface velocity, nominal normal interface pressure, surface roughness, molecular attraction between surfaces (surface energy), temperature, surface deformation and vibration is obvious. However, the combined effect of these factors mainly the effect of the frequency and amplitude of vibration on the coefficient of friction has not been yet investigated.

In this study the combined effect of frequency, amplitude of vibration, surface roughness, and sliding speed on coefficient of friction of different Aluminum 7079 and Steel C1020 is investigated by utilizing experimental design technique.

2 Materials and Methods

2.1 Apparatus and measuring instruments

The problem of establishing exactly which attributes of the contact conditions and the material contribute most to the coefficient of friction is a major one for developing friction tests and analytical friction models. Since the number of potential friction effecting factors is large, it is necessary to identify the set of key variables to each particular case in order to construct appropriate apparatus and to select the appropriate test method or simulation. An in-house pin-on-disc apparatus shown in Fig. 1 was constructed and utilized to determine the effect of vibration, amplitude of vibration, surface roughness, and sliding speed on coefficient of friction for different metals

The pin-on-disc machine mainly consists of the carriage, spindle, and arrangement to generate vertical vibration. It is connected to a custom PC, that has the capability for both spindle motion control and carriage vertical applied load control, and data acquisition and display unit (monitor). The machine has a linear vertical and horizontal motion system for positioning the pin and a rotational motion control system that controls the spindle speed through the control of the motor speed. The pin is attached directly to the friction and load sensors; the latter provides feedback to the force versus displacement servo-loop. The disc specimen is attached to the spindle and has rotational movements through a compound V-pulley above the top supporting square plate and fixed with the shaft to transmit rotation to the shaft from the motor. The vertical shaft passes through two close-fit bush bearings which are rigidity fixed with two-square plates and clamped with screw from the bottom surface of the rotational plate. The pin-on-disc apparatus is designed so it has the capability of vibrating the disc at different frequencies and amplitudes. A compression spring is fitted with the shaft between the first and the second supporting plates in order to restrain any vertical movement of the shaft. There are two circular plates near the bottom of the shaft. One is fixed with the shaft end and another is fixed with the base plate with the help of a height adjusted screw. The upper circular plate

has a spherical ball extended from the lower surface of this plate. On the top surface of the lower circular plate there are a number of slots. Rotating the screw will bring up the lower circular plate to touch the ball. When the shaft rotate, the ball will slide on the slotted surface and due to spring action, the shaft along with the plate will vibrate. The mode of vibration is sinusoidal and the direction is vertical. Different values of frequency are generated by varying the number of slots. The amplitude of vibration values are varied by adjusting the height of the slotted plate. The rotational speed of the shaft is controlled and programmed to have different values. A machinable type Aluminum 7079 and Steel C1020 disc are used to perform the tests. The surface topography of the discs is controlled. The Mahr profilometer is used to obtain the profile of aluminum and steel discs. The profile measurement consists of 512 traces with trace taken over a 10-mm-long distance. The 512 traces are separated so to occupy a width of 1.25 mm, providing a reasonable aspect ratio for the sampled area. The constructed machine has capability of applied loads servo-controlled with a closed-loop feedback from 1 mN to 50 N and controlled speed from 0.001 to 2500 rpm with precisely controlled accelerations and positions. The measured friction force, normal force, CoF, amplitude of vibration and vibration, speed, and applied load are displayed on the monitor.

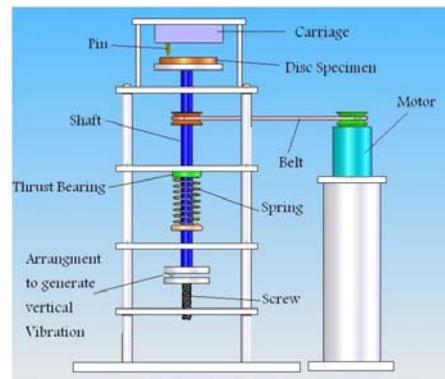


Fig. 1 Schematic diagram of the pin-on-disc machine

2.2 Response surface method (RSM) and central composite design (CCD)

RSM is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objectives is to optimize this response. By careful design and analysis of experiments, it seeks to relate a response,

or output, variable to the levels of a number of predictor, or input, variables. RSM defines the effect of the independent variables, alone or in combination, on the process. In addition to analyzing the effects of the independent variables, this experimental methodology also generates a mathematical model.

In most of RSM problems, the form of relationship between the response and the independent variables is unknown. Thus, the first step in RSM is to find a suitable approximation for the true functional relationship between y and the set of the independent variables. Usually, a low-order polynomial in some region of the independent variables space is appropriate. In many cases, the second-order model that includes the interaction term is required. It is widely used because of its flexibility. The model used in RSM is generally a full quadratic equation or diminished form of this equation. The behavior of the system can be describe by the following second-order polynomial equation

$$response(CoF) = \beta_0 + \sum \beta_i x_i + \sum \beta_{ii} x_i^2 + \sum \sum \beta_{ij} x_i x_j \quad (1)$$

Where the response is the predicted response, β_0 is the interception coefficient, β_i are the linear terms, β_{ii} are the quadratic terms, β_{ij} are the interaction terms, and x_i and x_j represent the coded levels of the independent variables.

2.3 Experimental design and data collection

Because the number of possible variables for use in predictive friction coefficient quite large, it is necessary to identify the set of key variables applicable to each particular case and to conduct screening experiments to reduce the number of the independent variables. These independent variables are identified and their names, units and levels are shown in Table 1. The process with a standard RSM design, central composite design (CCD), is utilized to ascertain the effect of the four factors and their cross influence on the CoF between dry surfaces of different metals. CCD is well suited for fitting a quadratic surface, which usually well work for process optimization. Four factors are investigated in this study. Their names, units and levels are shown in Table 1. The effect of speed and roughness on coefficient of friction under no vibration was investigated first and then the results

were compared with the results obtained under different conditions according to the levels shown in Table 1.

The design matrix is established using CCD based on the number of variables. For accuracy reasons, two replicates are made and averaged for each combination of the factors. This resulted in a total of 124 tests. The units of the independent variables differ from one another. Even if some of the parameters have the same units, not all of these parameters will be tested over the same range. Since parameters have different units and/or ranges in the experimental domain, the regression analysis should not be performed. Instead, one must first normalize the parameters before performing a regression analysis. Each of the coded variables is forced to range from -1 to 1, so that they all effect the response more evenly, and so the units of the parameters are irrelevant. The normalized parameter is designed to exit in the range -1 to +1. For a physical variable x , a normalized parameter I may be defined as follows:

$$I = \frac{2(x - x_{\min})}{x_{\max} - x_{\min}} - 1 \quad (2)$$

where x_{\min} is the low value of x and x_{\max} is its high value. Therefore $I = -1$ corresponds to $x = x_{\min}$ and $I = +1$ to $x = x_{\max}$. In RSM design all factors are represented in terms of their normalized representation and all the equations relating the dependent variable to the normalized factors are obtained. Such an equation may then be written in terms of the physical factors by de-normalizing the equations using the equation above.

2.4 Test procedure

The tests are performed on the in-house pin-on-disc machine. The machine is easily interchangeable compatible rotary and linear drives allow for the combination of rotary and linear motions of test specimens. The applied loads are servo-controlled with a closed-loop feedback. The applied load is kept at 15 N for all tests. The speed is programmed to have different values according to table 1.

In this study the machine is used to measure the friction force, normal force and the CoF. A Steel C1020 and Aluminum 7079 with radii 49 mm used to perform the tests. The surface topography of the discs is controlled. Testing is done in a pin-on-surface mode where the pin is located at any radius up to 48 mm from the centerline of the spindle. During testing, parameters are monitored, displayed

and stored in the computer for further analysis. Data is displayed in real time on the monitor. The normal and tangential loads are also recorded from which the COF ratios are computed. The COF is then collected and analyzed using analysis of variance. During the analysis of variance, the main and interaction effects are computed. Those effects that are statistically insignificant are screened out. The significant effects are retained. A model is generated in terms of the significant main and interaction effects.

The screening experiments were carried out by varying one of the parameters while keeping the rest of them at constant values. Some other parameters were excluded even though they might be significant parameters such as temperature because they are not a key variable for this particular study. The levels of the parameters were decided based on the machine capability and particular case of automotive brake system.

3 Results and Discussions

The experiments were designs to determine the effect of the four factors, their quadratic terms and their cross influence on CoF between dry surfaces. The analysis is done to extract all the information present in the data, taking account of variability and measurement error. The effects of the factors as linear, quadratic, cross-products coefficients, or cubic on responses were tested for adequacy. The results show that the quadratic model is the most significant.

3.1 Response residual and regression analysis

In order to confirm the adequacy of the model obtained, the confirmation run experiments are performed for the friction function. The percentage error ranges between the experimental and the predicted value of the CoF lie within -2.6 to 2.4%. All the experimental values for the confirmation run are within 95% prediction interval. This indicates that the quadratic model of friction function is accurate. The violation of the basic assumptions and model adequacy is also investigated by inspection of the normal probability plot of the standardized residuals and the Box-Cox plot for power transforms. There is no indication of non-normality, nor there is no evidence pointing to possible outliers and the equality of variance assumption does not seem to be violated.

The analysis of variance (ANOVA) is done on the experimental data (table 2) to evaluate the statistical significance of the model. The ANOVA confirms

the adequacy of the quadratic model. The ANOVA confirms the adequacy of the quadratic model. The Model F-value of 146.05 implies that the model is significant. There is only a 0.01% chance that a 'Model F-Value' this large could occur due to noise. Values of 'Probability, P > F' less than 0.0500 indicate model terms are significant. For Steel C1020 the vibration (A), amplitude of vibration (B), surface roughness (C), speed (D), quadratic term of surface roughness (D)² and quadratic term of speed (C)² are significant model terms. The final response equation for the CoF for Steel C1020 is obtained in terms of coded factors as follows,

$$\text{CoF} = 0.313 - 0.142 * A - 0.0356 * B - 0.0458 * C - 0.1210 * D - 0.0822 * C^2 + 0.0561 * D^2 \quad (3)$$

For Aluminum 7079 the vibration (A), amplitude of vibration (B), surface roughness (C), speed (D), quadratic term of surface roughness (D)², quadratic term of speed (C)² and the interaction between vibration and speed are significant model terms. The final response equation for the CoF for Aluminum 7079 is obtained in terms of coded factors as follows,

$$\text{CoF} = 0.269 - 0.118 * A - 0.0476 * B - 0.0395 * C - 0.0953 * D - 0.0669 * C^2 + 0.0471 * D^2 + 0.0289 * (A * D) \quad (4)$$

Equations 3 & 4 are fitted regression model representations of the RSM experiments for CoF.

3.2 Model graphs and analysis

The mathematical models furnished in previous section can be employed to predict the coefficient of friction of Steel C1020 and Aluminum 7079 for the range of factors used in the investigation by substituting their respective values in coded form.

Interaction plots illustrate the significance of interactions of various factors on the response. Such a plot involves dependent variable represented on the ordinate and one of the factors on the abscissa. Fig.2 shows the variation of the CoF of Steel C1020 as a function of the frequency (A) and the amplitude (B) of vibration with the roughness (C) and the speed (D) are held constant at their middle values that is coded (0, 0) and corresponded to the actual factor levels of 1.5 μm and 1.0 m/s, respectively. It is observed that the frequency and the amplitude of vibration have no interaction effect on the CoF since their lines don't cross as shown in Fig.2. However, they are both having significant effects on the CoF. The results in Fig.2 agree with predicted Steel C1020 predicted equation (eq. 3). It is observed that

the CoF is non-linearly decreased with the increase of the frequency and amplitude of vibration. Fig.2 also shows that the amplitude of vibration is less significant (less effect on the CoF) than the frequency of vibration as the interaction lines indicated. When the roughness and the speed are changed the CoF also changed as shown in Fig.3 and 4. The decreases of the roughness and the speed increase the CoF as indicated in Fig.3. The roughness and the speed in Fig.3 are kept at their lowest levels that is coded (-1, -1) and corresponded to the actual factor levels of 0.5 μm and 0.8 m/s, respectively. On the other hand, the increases of the roughness and the speed decrease the CoF as shown in Fig. 4. Fig.4 shows that CoF is at its lowest value

when the roughness and speed are at highest values (coded: +1, +1).

Figs. (5, 6, 7) show the variation of the coefficient of friction of Aluminum 7079 as factors moves from low to high levels. It is shown that frequency of vibration, amplitude of vibration, roughness and speed all effect the CoF of Aluminum 7079 with different degree of significant. The behavior of the CoF of the Aluminum 7079 is very similar to the CoF of Steel C1020 but the CoF values are reduced by about 13%. The amplitude of vibration has higher effects on the CoF of Aluminum 7079 than that of Steel C1020. Note that the results with reference to no vibration and no amplitude conditions are not included in Figs 2- 7.

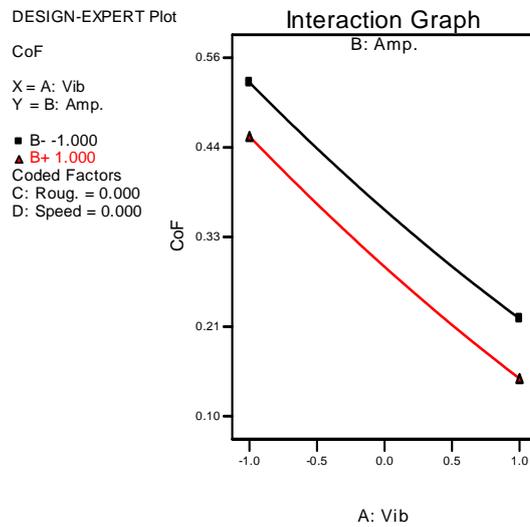


Fig. 2 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.=1.5 μm (coded: 0.0), speed=1.0 m/s (coded: 0.0)); normal load 12 N; test sample: Steel C1020.

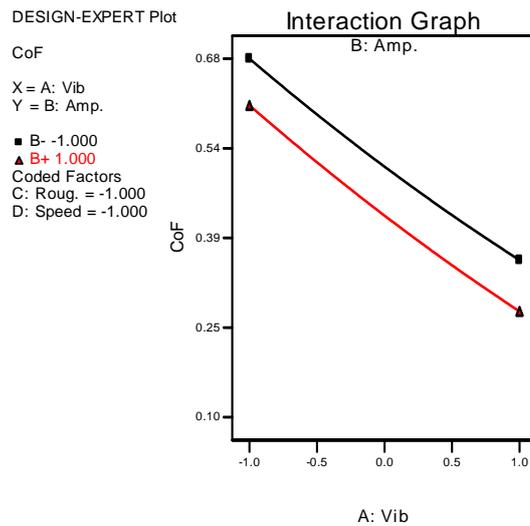


Fig. 3 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.= 0.5 μm (coded: -1.0), speed = 0.8 m/s (coded: -1)); normal load 12 N; test sample: Steel C1020.

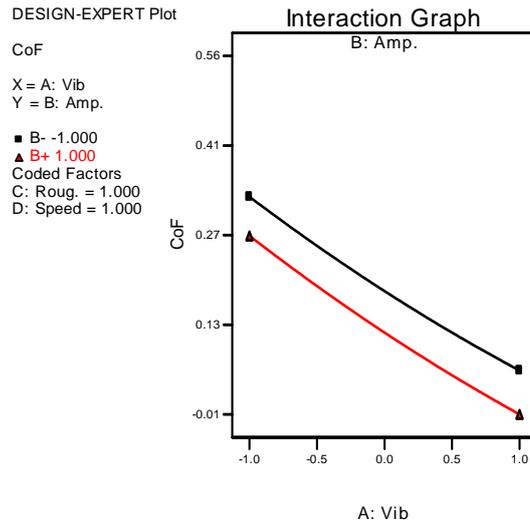


Fig. 4 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.= $2.5 \mu m$ (coded: +1.0), speed = 1.2 m/s (coded: +1)); normal load 12 N; test sample: Steel C1020.

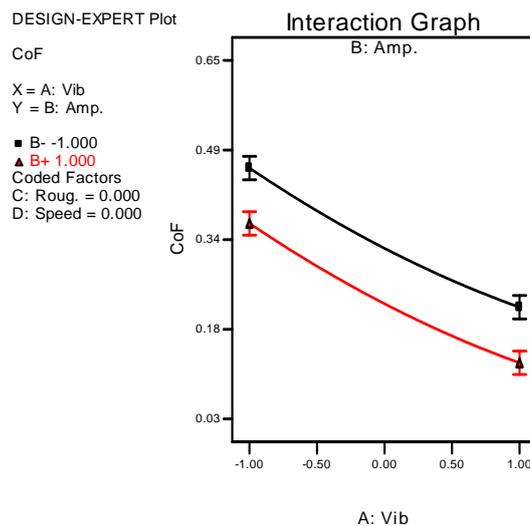


Fig. 5 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.= $1.5 \mu m$ (coded: 0.0), speed=1.0 m/s (coded: 0.0)); normal load 12 N; test sample: Aluminium 7079.

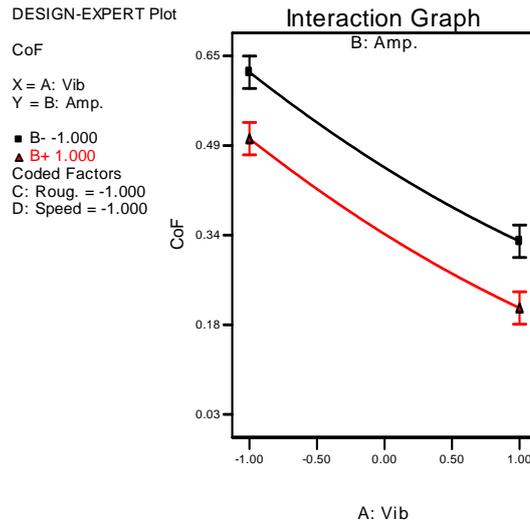


Fig. 6 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.= $0.5 \mu\text{m}$ (coded: -1.0), speed = 0.8 m/s (coded: -1)); normal load 12 N; test sample: Aluminium 7079.

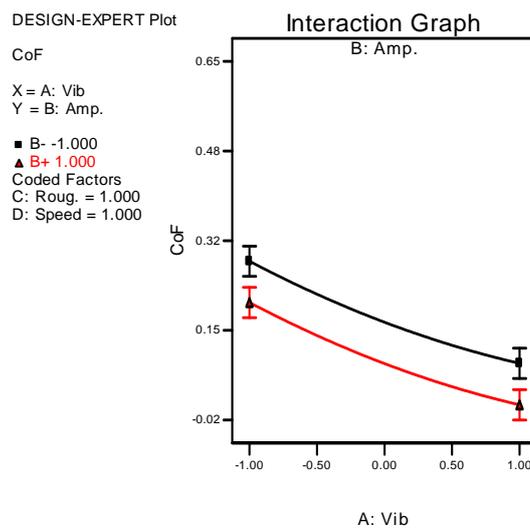


Fig. 7 Variation of coefficient of friction as a function of frequency and amplitude of vibration (roug.= $2.5 \mu\text{m}$ (coded: +1.0), speed = 1.2 m/s (coded: +1)); normal load 12 N; test sample: Aluminium 7079.

Table 1 Levels of the independent factors and coding identification

<i>Independent Factors</i>	<i>Units</i>	<i>level 1 actual & (coded)</i>	<i>level 2 actual & (coded)</i>	<i>level 3 actual & (coded)</i>	<i>level 4 actual & (coded)</i>	<i>level 5 actual & (coded)</i>	<i>No vibration level actual</i>
Frequency of vibration (A)	Hz	120 (-1)	240 (-0.5)	360 (0)	480 (+0.5)	600 (+1)	0
Amplitude of vibration (B)	μm	15 (-1)	75 (-0.5)	125 (0)	175 (+0.5)	225 (+1)	0
		<i>Low level actual & (coded)</i>		<i>Middle level actual & (coded)</i>		<i>High level actual & (coded)</i>	<i>No vibration level actual</i>
Surface roughness (C)	μm (RMS)	0.5 (-1)		1.5 (0)		2.5 (+1)	0
Relative surface speed (D)	m/s	0.8 (-1)		1.0 (0)		1.2 (+1)	0

Table 2 Analysis of variance table for steel C1020

Source	Sum of Squares	DF	Mean Squire	F Value	Prob > F
Model	5.41	14	0.39	146.05	< 0.0001
A	2.59	1	2.59	978.54	< 0.0001
B	0.15	1	0.15	55.55	< 0.0001
C	0.33	1	0.33	123.44	< 0.0001
D	1.82	1	1.82	688.91	< 0.0001
A2	5.388E-003	1	5.388E-003	2.03	0.1552
B2	2.177E-004	1	2.177E-004	0.082	0.7746
C2	0.33	1	0.33	124.25	< 0.0001
D2	0.16	1	0.16	59.01	< 0.0001
AB	2.014E-004	1	2.014E-004	0.076	0.7830
AC	1.480E-003	1	1.480E-003	0.56	0.4555
AD	0.024	1	0.024	8.89	0.0032
BC	5.115E-006	1	5.115E-006	1.932E-003	0.9650
BD	1.082E-003	1	1.082E-003	0.41	0.5234
CD	7.646E-003	1	7.646E-003	2.89	0.0907
Residual	0.56	210	2.648E-003		
Cor Total	5.97	224			

In investigating of the effect of all factors at all levels on the friction function for both Steel C1020 and Aluminum 7079 the vibration effect is shown to be most significant factor. The lower the friction function the higher the vibration might be due to the reduction of actual rubbing surface, because there is always more separation between the rubbing surfaces due to reduction in the mean contact area of

the two sliding objects for vibration. In addition vibration reduces load momentarily, which causes of effective normal force resulting in reduction of metal-to-metal contact and hence friction coefficient. The decreases of the friction function with the increases of amplitude of vibration is due to the fact that the greater the amplitude of vibration, the higher the actual rubbing time. Therefore, the

separation of contact surfaces as the higher the amplitude the higher the separation of rubbing surfaces. As the amplitude increases, keeping the frequency of vibration constant, the acceleration of vibration also increase and that might cause momentary vertical load reduction, which causes the reduction of effective normal force resulting in reduction of friction function with the increase of amplitude of vibration. The reduction of coefficient of friction with the increase of sliding speed is due to the reduction of actual rubbing surface and to the fact that the disc material interfaces softened and decreased in shear strength as temperature increased, leading to lower friction force. As temperature increases, the possibility of forming oxides increases as well. Certain oxides have lubricating characteristics and could assist for further reduce friction. Friction function is high at low to moderate roughness because of the growth of real of area of contact; it tends to be high at very high roughness because of mechanical interlocking. The non-linear effect of sliding speed and surface roughness on CoF are due to the significance of the quadratic terms of these factors in eq. 3. The nonlinearity effect of frequency and speed are due to the significant of the interaction term between frequency and sliding speed as indicated in eq. 4.

4 Conclusion

This work examines how the coefficients of frictions are affected by amplitude of normal vibration at different frequencies. Variation of coefficient of friction with the amplitude of normal vibration is investigated experimentally when mild steel pin slides on Steel C1020 and Aluminum 7079. The approach presented in this paper provides an impetus to develop analytical model, based on experimental results for obtaining friction function model using response surface methodology. The validity of the model has been enhanced by screening out the non-significant factors. The coefficient of friction with the variation of the factor levels are investigated experimentally on a homemade pin-on-disc machine. Coefficient of friction (CoF) is analyzed as a nonlinear function of the factors and predicted by a second-order polynomial equation. The investigations of this study indicate that the factors vibration, amplitude of vibration, surface roughness and its quadratic form, sliding speed and its quadratic form are the primary factors influencing the coefficient of friction of Steel C1020 with different degree of significant. The factors vibration, amplitude of vibration, surface roughness and its quadratic form,

sliding speed and its quadratic form, and the interaction between vibration and sliding speed are the primary factors influencing the friction function of Aluminum 7079 with different degree of significant. The friction function linearly decreases with the increases of vibration and amplitude of vibration, non-linearly decreases with the increases of sliding speed and linearly increases with the increases of the surface roughness until the middle range is reached and then there is non-linearly decrease thereafter. Similar trends of friction functions are observed for Aluminum 7079 with a reduction of almost 13% except for the case with amplitude of vibration where the variation showed more significant effect on the coefficient of friction for Aluminum 7079.

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