Finite element analysis of tunnel–soil–building interaction using displacement controlled model

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Abstract: Using some building affected by the river-crossing highway tunnels of Wuhan as engineering background, this paper investigates the interaction between the tunneling in soft soils and adjacent structures. A full three-dimensional finite element model, which takes into account the presence of the building during the excavation of the tunnel, is well analyzed. The soil behavior discussed in this paper is assumed to be governed by an elastic perfectly-plastic constitutive relation based on the widely adopted Mohr–Coulomb criterion with a non-associative flow rule. The paper consists of three parts. The first part presents the 3-D finite element numerical model, and the second part provides a full analysis of the construction of a shallow tunnel close to a five level building. Comparison between the full couple model analysis and the full 3D free-field analysis is given in the final part. The corresponding comparison results provide a fundamental guidance for the shield tunnel design and construction.

Keywords: Finite Element; Three-dimensional; Slurry shield tunneling; Structure; Excavation; Settlement

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1. Introduction

Due to the high interaction between tunneling and existing structures in urban areas, tunneling operations in urban areas draws much attention. This paper describes a thorough analysis of the tunneling influence in soft soils on adjacent building. A combination of in situ observations and numerical modeling was previously adopted to analyze such problem. Potts and Addenbrooke employed a coupled 2D finite element model to investigate the influence of a surface structure on the ground movement due to tunneling. Their numerical results proved that the ground movement was significantly influenced by the presence of the surface building[1].

It is difficult to rigorously analysis the tunneling–structure interaction problem, due to the (i) high interaction between tunneling in soft soils and adjacent structures, (ii) three-dimensional nature of this problem and, (iii) the non-linear behavior of geomaterials involved [2]. Such approach based on a full three-dimensional coupled modeling. Furthermore, the modeling needs to ascertain the presence of existing structures and the tunneling procedure employed [3, 4]. Burd et al. adopted this approach to analysis the interaction between the construction of an unlined tunnel and a masonry building. Their studies pointed out that the tunneling–building interaction causes a significant influence on the distribution of damage in the building[5]. Mroueh and Shahrour has explored the interactions of two level buildings with a single tunnel using 3D finite element analysis. They stated that self weight of buildings on the surface has a major role on determination of initial stresses in the ground and neglecting these results in underestimation of tunneling induced forces leading to less settlement predictions[6].

This paper investigates the interaction between the construction of the single and twin tunnel and adjacent structure, and develops the coupling effect models of lining and soil, and of soil, lining, foundations and upper structure. A full three-dimensional finite element analysis, which takes into consideration the elastoplastic behavior of the soil, the tunneling procedure and the presence of the structure, is employed to perform the study. The paper consists of three parts. The first part describes the numerical model used in this study. The paper is consisted of three parts. The first part presents the 3-D finite element numerical model, the second part provides a full analysis of the construction of a shallow tunnel close to a five level building. Comparison between the couple 3-D FEM analysis and the free 3-D FEM is given in the final part. The corresponding comparison results provide a fundamental guidance for the shield tunnel design and construction.

2 Installation procedures for FE-analysis

2.1. Deformations due to closed shield tunnelling

When a tunnel is planned,ground movements are an important topic of consideration. In closed shield tunneling.the aim is to minimize ground movements whereas in open face tunnelling ground movements tend to be allowed up to a certain extent. For closed shield tunnelling Mair and Taylor [7] consider several components of ground deformation.In case of adequate face support, ground movement towards the face will be relatively small, but radial movements towards the shield may be significant; in particular for a conical shield or in case of over-cutting. Ground movement towards the tail void can be minimized by grouting, but its effect is strongly influenced by the experience of the crew and the grout pressure control being implemented. In fact, the tail void is usually the major cause of settlements, whereas the deformation of the lining tends to be of minor importance[8] For closed shield tunnelling in homogeneous soil, ground loss ratios, i.e. the volume of ground that moves into the tunnel divided by the volume of the tunnel, of between 0.5% and 2% are realistic. In sands a loss of only0.5% can be achieved, whereas soft clays involve the range from 1% to 2% [9]. Considering data for mixed ground profiles with sands or fills overlaying tertiary clays, Mair and Taylor reported values between 2% and4%.[7] No doubt, tunnelling technology is continuously improving and smaller ground loss ratios might be achieved today.

2.2. The Empirical settlement analysis

For the assessment of settlements a green field settlement trough is often assumed. Independent of the tunnelling method the green field settlements are well matched by a Gaussian function[10], as given by Eq. (1),

$$S(y) = S_{\max}.\exp(-\frac{y^2}{2i^2}), S_{\max} = \frac{V_s}{i.\sqrt{2\pi}}, i = K(z_0 - z) \quad (1)$$

where Smax is the settlement above the tunnel axis, y is the horizontal distance from the tunnel axis and i is the horizontal distance from the tunnel axis to the point of inflection of the settlement trough, and Vs is the settlement volume. For tunnelling in undrained ground, the settlement volume is more or less equal to the ground loss, but the settlement volume tends to be somewhat smaller for drained excavations. Indeed, dilation and swelling due to unloading may result in soil expansion, such that Vs < Vt, where Vt denotes the ground that moves into the tunnel. However, differences tend to remain small and it is often assumed that Vs ≈ Vt. Measured data on i from numerous tunnelling projects have amongst others been presented by Mair and Taylor[7] and they may als o be used to validate numerical calculations of the steepness of the settlement trough. O' Reilly and New amongst others have proposed a method for the assessment of tunnel induced horizontal ground displacements, which directly derives from the assumption of a Gaussian settlement distribution. Research on FEM analyses of both settlements and horizontal ground deformations will be considered in Sections2.3. [11]

2.3. Installation procedures for FE-analysis of for closed shield tunnelling

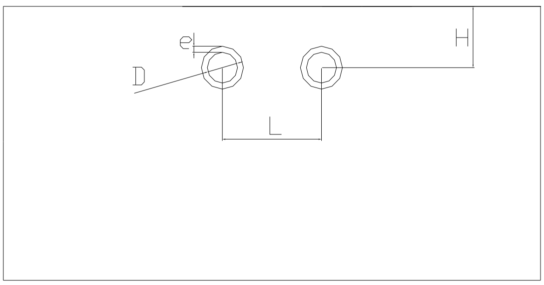
Addenbrooke et al. [12]combined the stress reduction method with a control of the volume of the surface settlement trough. Here the initial ground pressure is reduced stepwise until a prescribed ground loss ratio is reached and a lining is activated. Another approach to model installation processes of shield tunnels is the gap method, as first introduced by Rowe et al. [13]. Here the ground is initially unsupported and free to displace until contact to the lining is made. In a somewhat different gap method by Vermeer and Brinkgreve [14], the lining is contracted stepwise until its contraction matches a prescribed ground loss ratio. the novel grout pressure method will be used, which combines elements of the stress reduction method and the gap method. Here the lining is considered to be surrounded by a thin grout layer, which is taken into account as a gap with a known grout pressure. It will be shown that this grout pressure method has the advantage to predict both vertical and horizontal displacements as well as structural forces realistically.[15]. The key for a different approach lies in the displacement convergence pattern around a deforming tunnel boundary. Upon excavation, soil around the unsupported tunnel converges inwards in a radial fashion towards a point on the tunnel vertical line of symmetry. Previously, this pattern of convergence has been ideally assumed to be uniform in the analytical solutions proposed by Sagaseta as a means of simplifying mathematical derivations.[16] However, it is expected that the tunnel convergence is highly non-uniform with more crown settlement and less invert heave. displacement vector plots of soil deformation around the excavated tunnel for plane strain centrifuge experiments conducted by Mair [17]. The displacement vectors in the tests clearly show large crown settlement with very little invert heave. Centrifuge tests by Hagiwara et al. [18] and field measurements at the Heathrow trial tunnel by Deane and Bassett [19]also show that the area close to tunnel invert experienced very little movement compared to the crown. The above observations lead to the first assumption in the displacement controlled model (DCM) that convergence is non-uniform. Loganathan and Poulos [20] reported that such non-uniform convergence profiles lead to realistic predictions of ground displacements due to tunnelling. The second assumption for the DCM is that deformed tunnel shape is similar to the original excavated shape. Such an assumption is justified as deformations are usually small compared to tunnel size under working conditions. The third assumption for the DCM is that there exists a single point on the tunnel vertical line of symmetry to which all nodes on the excavated tunnel boundary converge to. There have been numerous studies, which propose that soil displacement vectors of the excavated tunnel boundary converge to the tunnel centre [12][21][22]. The latter is proposed based on field data while the former two are derived based on the following well established empirical relations: as given by Eq. (1)

3.Three-dimensional couple analysis

3.1.Numerical modeling

Fig.1(b)depicts the problem under consideration which is used to quantify the interaction between tunneling in Wuhan soft ground and the classroom in Wuhan University of Technology in the freefield analysis . The shield tunnel is characterized by its depth H, diameter D, lining thickness e, while the building is neglected

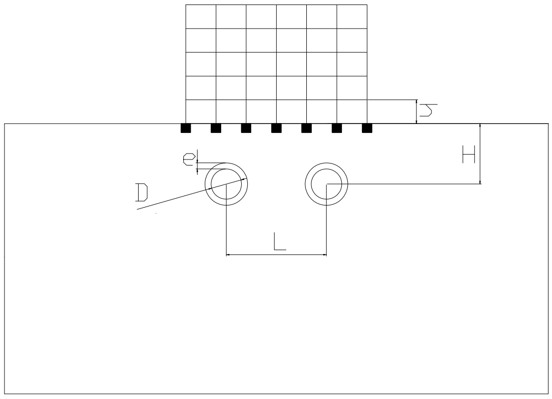
Fig.1(b)depicts the problem under consideration which is used to quantify the interaction between tunneling in Wuhan soft ground and the classroom in Wuhan University of Technology in the coupled analysis . The shield tunnel is characterized by its depth H, diameter D, lining thickness e, while the building is modeled by a spatial reinforced concrete framed structure characterized by the level height h and column's spacing a and b. [23]



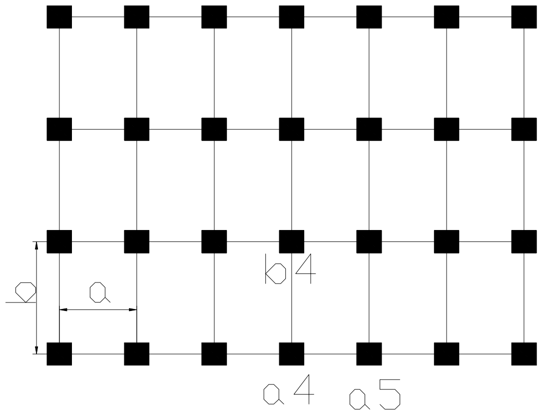
(a)

The behavior of the building is assumed to be linear-elastic [6]. The soil behavior is assumed to be governed by an elastic perfectly-plastic constitutive relation based on the Mohr–Coulomb criterion with a non-associative flow rule. The yield function and the plastic potential are given by:

$$f = p\sin\varphi + \sqrt{J_2}\cos\theta - \sqrt{\frac{J_2}{3}}\sin\varphi\sin\theta - C\cos\varphi \qquad (2)$$



(b)



(c)

Fig.1Geometry （a）tunneling-soil interaction geometry in the freefield analysis(b)tunneling-building-soil interaction geometry in the coupled analysis (c)building geometry in the coupled analysis.

$$g = p\sin\psi + \sqrt{J_2}\cos\theta - \sqrt{\frac{J_2}{3}}\sin\varphi\sin\theta \qquad (3)$$

C, Φand ψ designate the soil cohesion, friction angle and dilatancy angle, respectively; p, J2 and θ stand for the mean stress, second invariant of the deviatoric stress tensor and Lode angle, respectively. Their expressions are given by:

p=σii/3 (4)

$$J_2 = \frac{1}{2} s_{ij}.s_{ij} \quad \text{and} \quad s_{ij} = \sigma_{ij} - p\delta_{ij} \qquad (5)$$

$$\theta=\frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}}{2}.\frac{J_3}{J_2^{\frac{3}{2}}}\right) \text{ and yet } J_3=\frac{S_{ij}.S_{jk}.S_{ki}}{3} \quad (6)$$

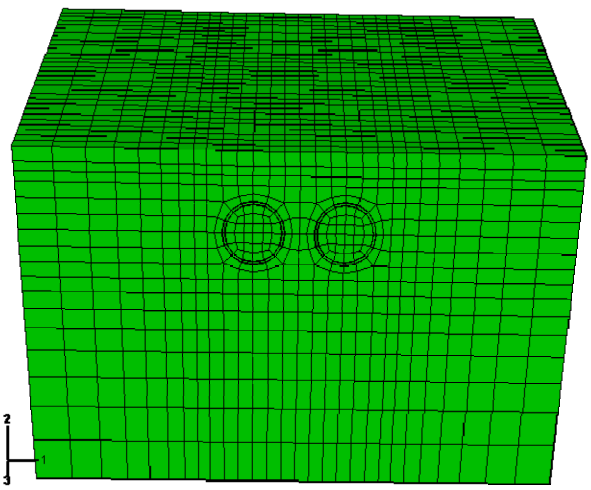
It is worth noting that such analysis can be improved by employing a more realistic soil material constitutive relation, which takes into account soil hardening and stress-dependant elastic properties.

In this paper, numerical simulations were performed by means of the finite element program Abaqus[24].

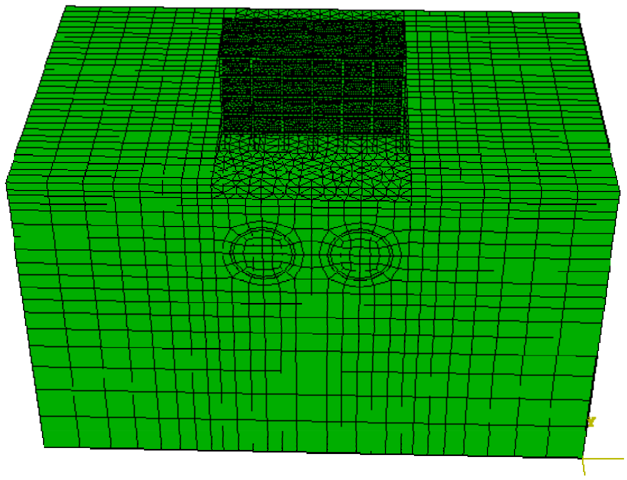
Analysis of the tunneling–structure interaction problem is performed with two steps [6]. The first step is concerned with the determination of initial stresses in the soil mass prior to the tunnel construction. It is performed using a finite element calculation considering the self-weight of both the soil and the structure. Displacements are reset to zero at the end of this stage; consequently, results referred to hereafter are due to the tunnel construction. The second step deals with the numerical simulation for the construction of the tunnel in presence of the structure. The tunnel construction process is modeled by kill of soil elements located in the excavated zone and activation of lining elements.

3.2.Full 3-D free analysis and 3-d couple analysis

The full three-dimensional coupled approach is adopted in this paper to study the influence of the twin tunneling



(a)



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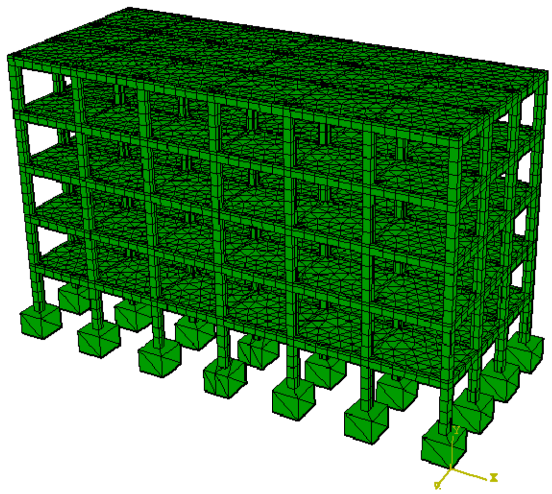


(c)

Fig.2. Full 3-D coupled analysis: presentation of the example;(a) Finite Element Mesh adopted in the Free-field analysis;(b) Finite Element Mesh adopted in the coupled analysis(c) finite element mesh for building in the coupled analysis

on the building. The longitudinal section of the twin tunnels is assumed to coincide with that of the building. The tunnels and structure characteristics are given by: tunnel diameter D=11 m, lining thickness e=0.5 m, tunnel depth H=12.7m, the twin center longL=16.34m,Lcolumn's spacing a=5m,b=4m, and height of each level h=3.6m. Material properties for the soil, lining and structure are listed in table1.

Finite element analysis for the free-field model is carried out using the mesh presented in Fig. 2(a) .

The finite element mesh was 60m long, 60m high, and 120m wide. It consisted of 19080 elements and 22134nodes.

Table1 Mechanical properties of encountered materials

No.	category	γ(KN/m3)	depth(m)	Φ(º)	C(kp)	E(Mpa)	ν
1	miscellaneous fill	19.80	2.00	26.80	19.50	7.50	0.30
2	silty clay	18.90	1.50	21.45	30.25	6.53	0.30
3	mucky soil	18.32	7.30	10.10	14.25	4.99	0.43
4	silty clay	18.69	3.20	21.90	24.80	5.76	0.37
5	silt	18.98	5.50	31.18	12.50	10.55	0.30
6	silty clay	18.71	3.00	26.23	19.06	8.00	0.30
7	Silt Fine sand	19.90	37.5	32.38	8.73	14.29	0.30
8	lining	25	0.5			34500	0.15
9	beam board pillar	25				25500	0.2
10	foundation	25	1.5			30000	0.15

Finite element analysis for the coupled model is carried out using the mesh presented in Fig. 2(b) . The finite element mesh is adopted in the analysis. The finite element mesh was 60 m long, 60 m high, and 120 m wide. It consisted of 47563 elements and 43714 nodes. Eight-noded brick elements and four-noded shell elements were used to model the soil and the concrete lining. The structure is modeled using 4-nodes tetrahedron elements. The structure is modeled using 4-nodes tetrahedron elements.

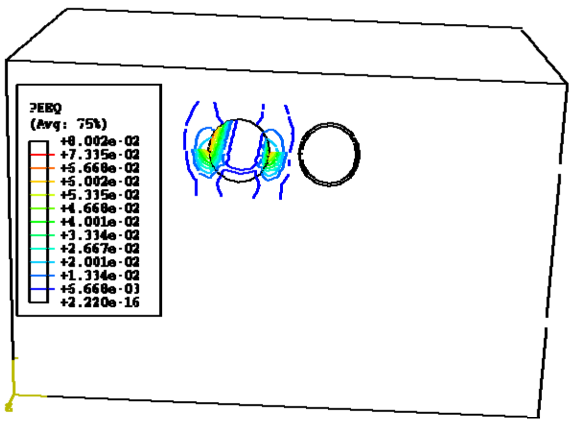
The boundary conditions adopted for the finite element mesh are composed of vertical sides and bottom side conditions. Roller supports were applied on all vertical sides of the mesh, and pin supports were assigned to the base of the mesh. Therefore, the movement in the direction normal to all vertical sides of the mesh and the movements in all directions at the base of the mesh were restrained. The transverse boundaries of the mesh are located a distance 60m from the central frame in order to minimize their impact on the tunneling-building interaction.

The entire analysis is performed in undrained condition. Computation is carried out in 30 successive steps using the following parameters for the excavation modeling, of the excavated section at each step Llin=2m. Based on a preliminary study, these parameters are fixed in order to reproduce realistic tunneling-induced soil movement in free-field condition (in the free-field analysis).

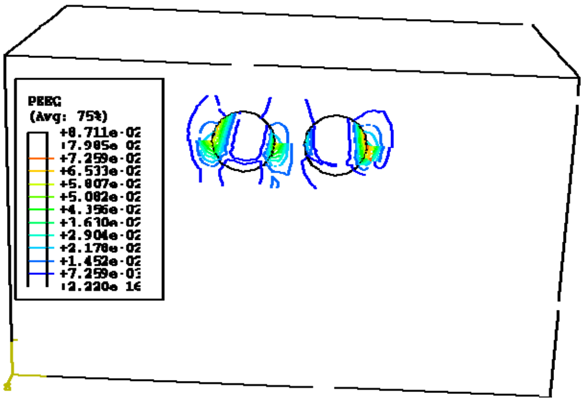
4. Tunnelling-building interaction analysis

4.1Plasticity analysis

Fig. 3 illustrates the distribution of the tunneling-induced soil plasticity zone. It shows that slurry shield tunneling induces plasticity around the tunnel is located in a region that extends up to 1.5D from the tunnel centre. The peak PEEQ values of Fig.3 (a), (b), (c), (d) are 0.080,0.087,0.081,0.091 respectively. It can be seen that the PEEQ difference between single tunnel or twin tunnel of free-field model and couple model is very small, while the PEEQ difference between the free-field models or couple models for single tunnel and twin tunnel is comparably large.



(a)



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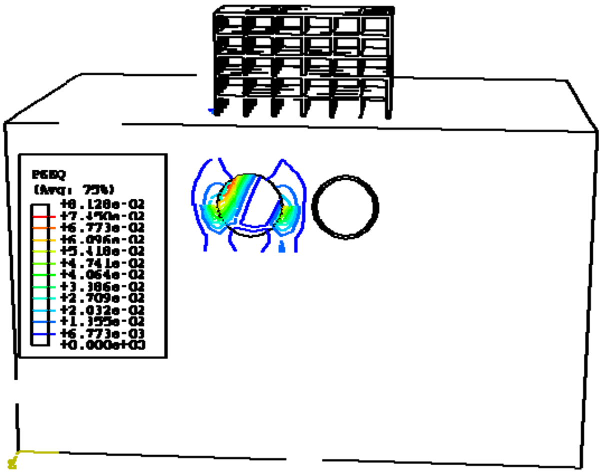

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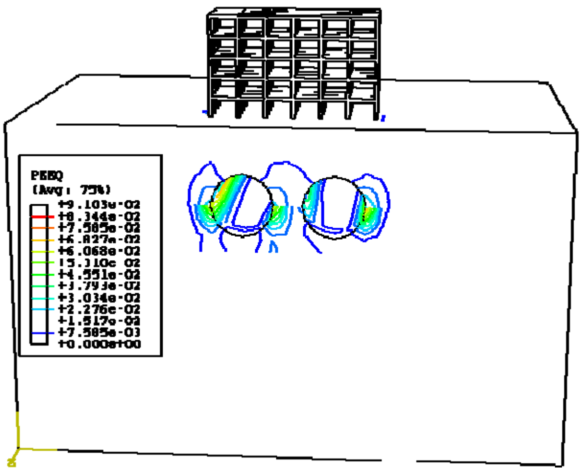
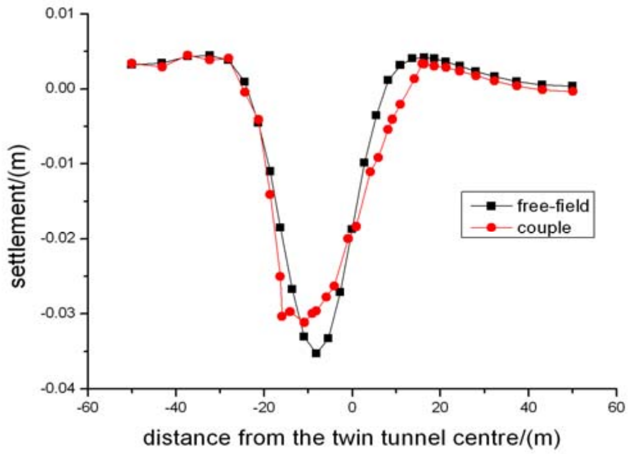


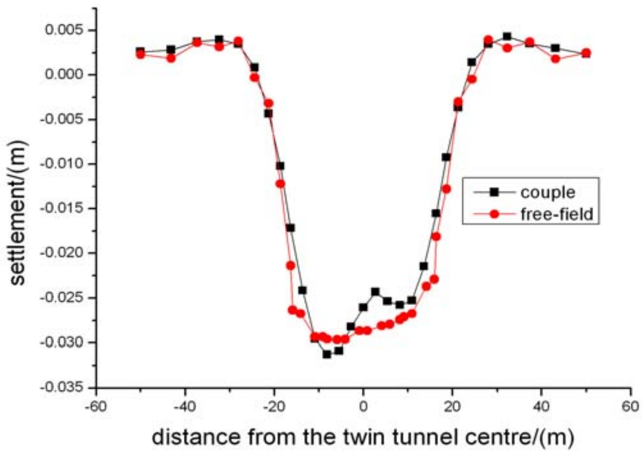
(d)

Fig.3 extension of plasticity (a)Free-field single excavation tunnel(b)Free-field twin excavation tunnel(c)couple single excavation tunnel(d)couple twin excavation tunnel

4.2 Soil-movement



(a)

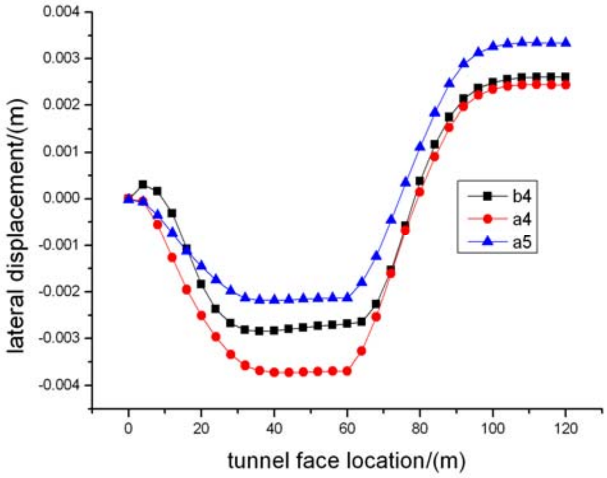


(b)

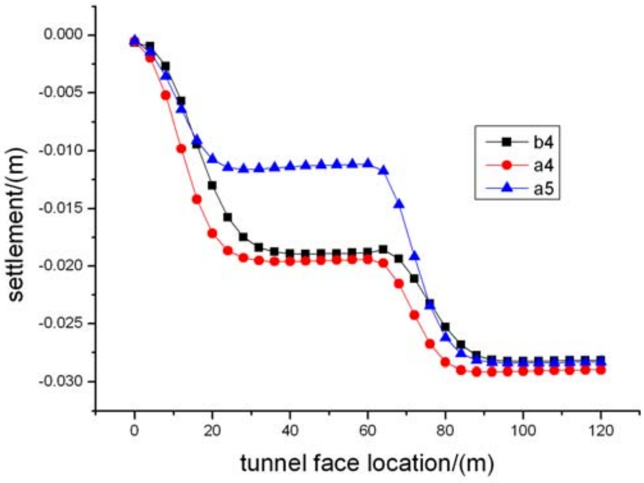
Fig.4（a）settlement during single tunnel excavation（b）settlement during twin tunnel excavation

Fig.4 (a) and (b) present the settlement induced by the couple excavation tunnel and its comparison to the free-field excavation tunnel. It can be observed from Fig. 3(a) that the presence of the structure affects the soil surface settlement profile. Obviously, the structure stiffness causes a reduction in the soil settlement profile. It is similar for the twin tunnel. However, we observe sharp increases in soil settlement for the twin tunnel in the vicinity of the foundations. These increases are due to the plasticity induced in this zone by both the structure's self-weight and tunneling as illustrated in Fig.3. However, it is expected that a mesh refinement may lead to a smoother surface settlement profile.

4.3Foundations-movement



(a)



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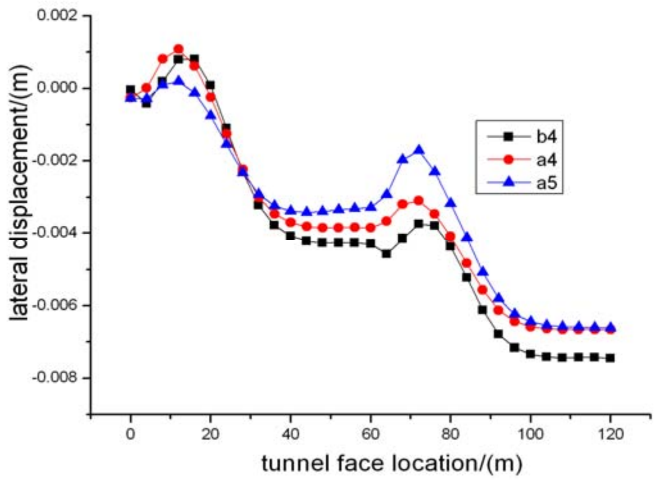

(c)

Fig. 5 Full 3-D coupled analysis: displacement of foundation during the single and twin tunnel excavation

（a）Lateral displacement（b）settlement（c）Longitudinal displacement

Fig.5(a) depicts the evolution of the lateral displacement of the structure foundations during single tunneling excavation and twin tunneling excavation. It shows that the lateral displacement of each foundation increases away from the twin tunnel centre during the single tunneling excavation. However, the transverse displacement of each foundation increases close to the twin tunnel centre during the twin tunneling excavation. It starts when the tunnel face is about 2D behind the foundation, attains about 40–45% of the total displacement when the tunnel face crosses the foundation section and then decreases when the tunnel face moves away from the foundation and .stabilizes when the tunnel face is about 2D from the foundation section. It is similar for the twin tunnel. The maximum lateral displacement is observed at the rear foundation A4 for the single tunnel; it is equal to 3.7 mm which is about twice the lateral displacement of front foundations A5 and B4. However, the maximum lateral displacement is observed at the rear foundation A4 for the twin tunnel; it is equal to 3.1 mm which is about 1.5 the lateral displacement of front foundations A5 and B4.

Fig. 5（b）shows that the settlement of each foundation increases during the single tunneling; the foundation settlement reaches about 55–60% of its final value when the tunnel face crosses the foundation section. It is similar for the twin tunnel. The maximum settlement is observed at the central front foundation a4; it is equal to 19 mm which is about 10% higher than the settlement observed at the rear foundation b4. It can be observed that the single tunneling causes a differential settlement of about 8.2mm between the centre frame foundations a4 and a5 which are spaced at 4 m. However, the foundation settlement reaches about 30mm of its final value.

Referred to Fig.5(c), it can also be observed that the longitudinal displacement of each foundation increases when the tunnel face becomes close to the foundation section and then decreases when the tunnel faces moves away from the foundation. The longitudinal displacements of the front and rear foundations are very close (about 4 mm). It is similar for the twin tunnel. However, the longitudinal displacements of the front and rear foundations are very close (about 7 mm)

5. Conclusion

Using some building affected by the river-crossing highway tunnels of Wuhan as engineering background, this paper investigates the interaction between tunneling in soft soils and adjacent structures. A full three-dimensional finite element model, which takes into account the presence of the building during the excavation of the tunnel, is well analyzed as the following,

5.1 The PEEQ difference between single tunnel or twin tunnel of free-field model and couple model is very small, while the PEEQ difference between the free-field models or couple models for single tunnel and twin tunnel is comparably large.

5.2 The presence of the structure affects the soil surface settlement profile. Obviously, the structure stiffness causes a reduction in the soil settlement profile. It is similar for the twin tunnel. However, we observe sharp increases in soil settlement for the twin tunnel in the vicinity of the foundations.

5.3 The transverse displacement of each foundation increases away from the twin tunnel centre during single excavation tunneling. However, the lateral displacement of each foundation increases close to the twin tunnel centre during the twin excavation tunneling.

5.4 The settlement of each foundation increases during the single and twin tunneling.

5.5 The longitudinal displacement of each foundation increases when the tunnel face becomes close to the foundation section and then decreases when the tunnel faces moves away from the foundation during the single and twin tunneling.

The corresponding comparison results provide a fundamental guidance for the shield tunnel design and construction.

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