Optimum determination of partial transmission ratios of three-step helical gearboxes

VU NGOC PI
Faculty of Mechanical, Maritime and Materials Engineering
Delft University of Technology
Mekelweg 2, 2628 CD Delft
THE NETHERLANDS
v.n.pi@tudelft.nl

Abstract: - In this paper, a new study on the applications of the optimization and regression techniques for optimum determination of partial ratios of three-step helical gearboxes in order to get different objectives including the minimum gearbox length, the minimum gearbox cross section dimension and the minimum mass of gears is presented. From the moment equilibrium condition of a mechanic system which includes three gear units and their regular resistance condition, three optimization problems for getting the minimum gearbox length, the minimum gearbox cross section dimension and the minimum mass of gears were conducted. In addition, explicit models for the prediction of the partial ratios of the gearbox were given by using regression analysis technique. With these models, the calculation of the partial ratios becomes accurate and simple.

Key-Words: - Gearbox design; Optimum design; Helical gearbox; Transmission ratio.

1 Introduction

It is known that, in gearbox design, the optimum prediction of the partial transmission ratios is one of the most important tasks. The reason of that is the partial ratios are main factors which affect the size, the mass, and the cost of the gearbox. Consequently, optimum determination of the partial ratios of gearboxes has been subjected to many studies.

Previously, for helical gearboxes, there have been many studies on the determination of the partial ratios. These studies can be summarized into three methods including graph method, "practical method", and modeling method.

In graph method, the partial ratios are predicted graphically. This method was used in several studies such as in the studies of V.N. Kudreavtev et al. [1], Trinh Chat [2] and A.N. Petrovski et al. [3] for the prediction of the partial ratios of two, three and four-step helical gearboxes. Figure 1 shows a graph which allows to determine the partial ratios u_1 and u_2 of the first and the second gear steps of a three-step helical gearbox. The partial ratio of the third step then can be found by:

$$u_3 = \frac{u_h}{u_1 \cdot u_2} \tag{1}$$

Where, u_h is the total transmission ratio of the gearbox.

G. Milou et al. [4] introduced a method called "practical method" for determining the transmission ratios of two step helical gearboxes. In this method, from the practical data, the authors gave a table for prediction of the partial transmission ratios.

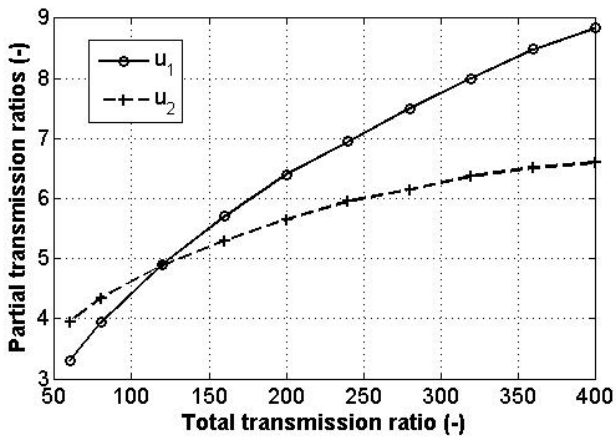

Fig. 1 Determination of partial ratios of three-step helical gearboxes [1]

In modeling method, from the results of optimization problems, models for the calculation of the partial ratios are found in order to get different objectives. Using this method, for three step helical gearboxes, Romhild I. and Linke H. [5] introduced the following models for the determination of the partial ratios of the first and the second step in order to get the minimum gearbox mass:

$$u_1 = 0.4643 \cdot u_h^{0.609} \tag{2}$$

$$u_2 = 1.205 \cdot u_h^{0.262} \tag{3}$$

The partial ratio of the third step u_3 is then determined by Equation 1.

With modeling method and also for three step helical gearboxes, Vu Ngoc Pi et al. [6] conducted a study on optimum determination the partial ratios for getting the minimum mass of gears. In the study, the partial ratios of gear steps 2 and 3 are predicted as follows:

$$u_1 = 0.314 \cdot \sqrt[3]{u_h^2} \tag{4}$$

$$u_2 = 1.33 \cdot \sqrt[4]{u_h} \tag{5}$$

For two step helical gearboxes, Vu Ngoc Pi [7] introduced the following model for determining the partial transmission ratio of the second step in order to get the minimum gearbox length:

$$u_2 \approx 1.1966 \cdot \left(K_c \cdot \frac{\psi_{ba2}}{\psi_{ba1}} \right)^{0.35} \cdot u_h^{0.2939} \tag{6}$$

Where, ψ_{ba1} and ψ_{ba2} are coefficients of helical gear face width of the first and the second gear units, respectively; k_c is a coefficient.

After determining the ratio u_2, the partial ratio of the first gear step u_1 is calculated as follows:

$$u_1 = \frac{u_h}{u_2} \tag{7}$$

It is understandable that the modeling method is more accurate and easier to determine the partial ratios than the other methods. Particularly, the modeling method can be usefully applied in computer programming and in other applications. Also, using this method, it is allowed to determine the partial transmission ratio for various objectives.

From previous studies, it is clear that there have been many studies on the determination of the partial transmission ratios of two, three and four step helical gearboxes. However, the optimum determination of the partial ratios of three step helical gearbox for different objectives (for example the minimum gear length, the minimum gearbox cross section dimension etc.) has not been investigated. In this paper, a study on optimum calculation of partial ratios for the gearboxes for getting the minimum gear length, the minimum gearbox cross section and the minimum mass of gears are presented.

2 Determination of the gearbox length

The length of a three-step helical gearbox is decided by the dimension of L which is determined by the following equation (see Figure 2):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + a_{w3} + \frac{d_{w23}}{2} \tag{8}$$

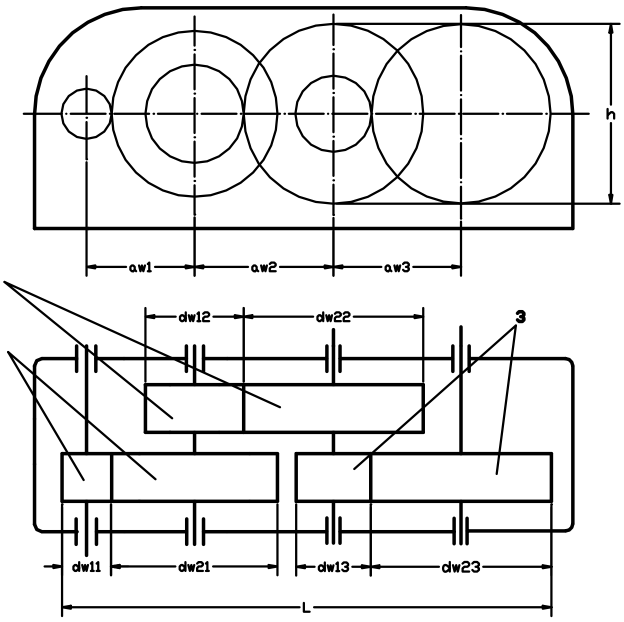

Fig. 2 Calculating schema for three-step helical gearboxes

The center distance of the first gear step can be determined as follows:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right)$$

From above equation we have

$$a_{w1} = \frac{d_{w21}}{2} \left(\frac{1}{u_1} + 1 \right) \tag{9}$$

Using the same way, the center distances of the second and the third steps are calculated by the following equations:

$$a_{w2} = \frac{d_{w22}}{2}\left(\frac{1}{u_2}+1\right) \quad (10)$$

$$a_{w3} = \frac{d_{w23}}{2}\left(\frac{1}{u_3}+1\right) \quad (11)$$

In which,

u_1, u_2 and u_3 –the transmission ratios of the first, the second and the third gear units, respectively;

d_{w11}, d_{w12} and d_{w13} are pitch diameters (mm) of driver gears of the first, the second and the third gear units, respectively;

d_{w21}, d_{w22} and d_{w23} are pitch diameters (mm) of driven gears of the first, the second and the third gear units, respectively;

ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are coefficients of helical gear face width of the first, the second and the third gear units, respectively;

a_{w1}, a_{w2} and a_{w3} are center distances (mm) of the first, the second and the third gear units, respectively.

From Equations 9, 10 and 11 and with the note that $d_{w11} = d_{w21}/u_1$, Equation 8 can be rewritten as follows:

$$L = \frac{d_{w21}}{2}\cdot\left(\frac{2}{u_1}+1\right)+\frac{d_{w22}}{2}\cdot\left(\frac{1}{u_2}+1\right)+\frac{d_{w23}}{2}\cdot\left(\frac{1}{u_3}+2\right) \quad (12)$$

Using the design equation for pitting resistance for the first helical gear unit, the following equation can be delivered [8]:

$$\sigma_{H1} = Z_{M1}Z_{H1}Z_{\varepsilon1}\sqrt{\frac{2T_{11}K_{H1}\sqrt{u_1+1}}{b_{w1}d_{w11}^2u_1}} \le [\sigma_{H1}] \quad (13)$$

It follows Equation 13 that:

$$[T_{11}] = \frac{b_{w1}\cdot d_{w11}^2\cdot u_1}{2\cdot(u_1+1)}\frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon1})^2} \quad (14)$$

Where, b_{w1} and d_{w11} are determined by the following equations:

$$b_{w1} = \psi_{ba1}\cdot a_{w1} = \frac{\psi_{ba1}\cdot d_{w11}\cdot(u_1+1)}{2} \quad (15)$$

$$d_{w11} = \frac{d_{w21}}{u_1} \quad (16)$$

Substituting Equation 15 and 16 into Equation 14 we have

$$[T_{11}] = \frac{\psi_{ba1}\cdot d_{w21}^3\cdot[K_{01}]}{4\cdot u_1^2} \quad (17)$$

In which

$$[K_{01}] = \frac{[\sigma_{H1}]^2}{K_{H1}\cdot(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon1})^2} \quad (18)$$

From Equation 17 the pitch diameter d_{w21} can be calculated by

$$d_{w21} = \left(\frac{4[T_{11}]u_1^2}{\psi_{ba1}[K_{01}]}\right)^{1/3} \quad (19)$$

Calculating in the same way, the following equations for the second and the third gear units can be found

$$d_{w22} = \left(\frac{4[T_{12}]u_2^2}{\psi_{ba2}[K_{02}]}\right)^{1/3} \quad (20)$$

$$d_{w23} = \left(\frac{4[T_{13}]u_3^2}{\psi_{ba3}[K_{03}]}\right)^{1/3} \quad (21)$$

In the above equations:

$$[K_{02}] = \frac{[\sigma_{H2}]^2}{K_{H2}\cdot(Z_{M2}\cdot Z_{H2}\cdot Z_{\varepsilon2})^2} \quad (22)$$

$$\left[K_{03}\right]=\frac{\left[\sigma_{H3}\right]^2}{K_{H3}\cdot\left(Z_{M3}\cdot Z_{H3}\cdot Z_{\varepsilon 3}\right)^2} \quad (23)$$

Where,

Z_{M1}, Z_{M2}, Z_{M3} are coefficients which consider the effects of the gear material when calculate the pitting resistance of the first, the second and the third gear units, respectively;

Z_{H1}, Z_{H2}, Z_{H3} are coefficients which consider the effects contact surface shape when calculate the pitting resistance of the first, the second and the third gear units, respectively;

$Z_{\varepsilon 1}, Z_{\varepsilon 2}$, $Z_{\varepsilon 3}$ are coefficients which consider the effects of the contact ratio when calculate the pitting resistance of the first, the second and the third gear units, respectively;

σ_{H1}- contact stresses of the first gear unit (N/mm^2);

$\left[\sigma_{H1}\right], \left[\sigma_{H2}\right]$ and $\left[\sigma_{H3}\right]$ are allowable contact stresses (N/mm^2) of the first, the second and the third gear units, respectively.

From the moment equilibrium condition of the first gear step and the regular resistance condition of the system we have:

$$\frac{T_r}{T_{11}}=\frac{\left[T_r\right]}{\left[T_{11}\right]}=u_1\cdot u_2\cdot u_3\cdot\eta_{brt}^3\cdot\eta_o^3 \quad (24)$$

In which,

η_{brt}-helical gear transmission efficiency; η_{brt} is from 0.96 to 0.98 [8];

η_o -transmission efficiency of a pair of rolling bearing ; η_o is from 0.99 to 0.995 [8].

By choosing $\eta_{brt}=0.97$, $\eta_o=0.992$ and substituting them into Equation 24 we have

$$\left[T_{11}\right]=\frac{\left[T_r\right]}{0.8909\cdot u_1\cdot u_2\cdot u_3} \quad (25)$$

Substituting (25) into (19) we get

$$d_{w21}=\left(\frac{4.4898\cdot\left[T_r\right]\cdot u_1}{\psi_{ba1}\cdot\left[K_{01}\right]\cdot u_2\cdot u_3}\right)^{1/3} \quad (26)$$

For the second step, the following equation can be given

$$\frac{T_r}{T_{12}}=\frac{\left[T_r\right]}{\left[T_{12}\right]}=u_2\cdot u_3\cdot\eta_{brt}^2\cdot\eta_o^2 \quad (27)$$

As it was done with the first step, by choosing $\eta_{brt}=0.97$ and $\eta_o=0.992$ Equation 27 can be rewritten as follows:

$$\left[T_{12}\right]=\frac{\left[T_r\right]}{0.9259\cdot u_2\cdot u_3} \quad (28)$$

In which,

$\left[T_{11}\right]$, $\left[T_{12}\right]$ -torque (Nmm) on the driver shaft of the first and the second gear units, respectively;

$\left[T_r\right]$ –torque (Nmm) on the out put shaft.

Substituting (28) into (20) we can have d_{w22} as follows:

$$d_{w22}=\left(\frac{4.3201\cdot\left[T_r\right]\cdot u_2}{\psi_{ba2}\cdot\left[K_{02}\right]\cdot u_3}\right)^{1/3} \quad (29)$$

Using the same way, the pitch diameter for the third step d_{w23} is calculated by the following equation:

$$d_{w23}=\left(\frac{4.1571\cdot\left[T_r\right]\cdot u_3}{\psi_{ba3}\cdot\left[K_{03}\right]}\right)^{1/3} \quad (30)$$

Substituting (26), (29) and (30) into Equation 12 with the note that $u_1=u_h/\left(u_2\cdot u_3\right)$, the length of the gearbox can be rewritten as follows:

$$L=\frac{1}{2}\left(\frac{\left[T_r\right]}{\left[K_{01}\right]}\right)^{1/3}\left[\left(\frac{4.4898\cdot u_h}{\psi_{ba1}\cdot u_2^2\cdot u_3^2}\right)^{1/3}\cdot\left(\frac{2\cdot u_2\cdot u_3}{u_h}+1\right)+\right.$$
$$\left.+\left(\frac{4.3201\cdot u_2}{\psi_{ba2}\cdot K_{c2}\cdot u_3}\right)^{1/3}\cdot\left(\frac{1}{u_2}+1\right)+\left(\frac{4.1571\cdot u_3}{\psi_{ba3}\cdot K_{c3}}\right)^{1/3}\cdot\left(\frac{1}{u_3}+2\right)\right] \quad (31)$$

Where, $K_{c2}=\frac{\left[K_{02}\right]}{\left[K_{01}\right]}$ and $K_{c3}=\frac{\left[K_{03}\right]}{\left[K_{01}\right]}$ are coefficients.

3 Determination of the dimension of the gearbox cross section

In practice, the cross section of a three-step helical gearbox is decided by the dimension of A which is determined as follows (see Figure 2):

$$A = L \cdot h \tag{32}$$

Where

L-the length of the gearbox (mm) which is determined by Equation 31.

h-the height of the gearbox (mm); From Figure 2, it is clear that *h* is determined by the maximum value among pitch diameters d_{w21}, d_{w22} and d_{w23} or it can be predicted by the following equation:

$$h = \max\left(d_{w21}, d_{w22}, d_{w23}\right) \tag{33}$$

From Equations 26, 29 and 30, Equation 32 can be rewritten as follows:

$$h = \max\left(\frac{4.4898 \cdot u_h}{\psi_{ba1} \cdot u_2^2 \cdot u_3^2}; \frac{4.3201 \cdot u_2}{\psi_{ba2} \cdot k_{c2} \cdot u_3}; \frac{4.1571 \cdot u_3}{\psi_{ba3} \cdot k_{c3}}\right) \tag{34}$$

4 Determination of the mass of gears

For a three-step helical gearbox, the mass of gears is calculated by the following equation:

$$G = G_1 + G_2 + G_3 \tag{35}$$

In which, G_1, G_2 and G_3 are the mass of gears of unit 1, unit 2 and unit 3, respectively (see Figure 2). In practice, G_1 can be determined as follows:

$$G_1 = \frac{\pi \cdot \rho \cdot d_{w11}^2 \cdot b_{w1} \cdot (e_1 + e_2 \cdot u_1^2)}{4} \tag{36}$$

Where,

ρ -the density of the gear material (kg/m^3);

e_1, e_2 -volume coefficients of gear 1 and 2, respectively; the volume coefficient of a gear is ratio of the actual volume to the theoretical volume of the gear. For a helical gear unit, we can have e_1=1; e_2=0.6 [2].

From Equation 36, with $\psi_{bd1} = b_{w1} / d_{w11}$ is diameter coefficient of the first helical gear unit, the mass of gears of unit 1 can be determined by the following equation:

$$G_1 = \frac{\pi \cdot \rho \cdot \psi_{bd1} \cdot d_{w11}^3 \cdot (1 + 0.6 \cdot u_1^2)}{4} \tag{37}$$

For the first helical gear unit, the allowable torque on the driving shaft 1 can be predicted as follows [8]:

$$\left[T_{11}\right] = \frac{\psi_{bd1} \cdot d_{w11}^3 \cdot u_1 \cdot [K_{01}]}{2 \cdot (u_1 + 1)} \tag{38}$$

From Equation 38 we can have:

$$\psi_{bd1} \cdot d_{w11}^3 = \frac{2 \cdot (u_1 + 1) \cdot [T_{11}]}{u_1 \cdot [K_{01}]} \tag{39}$$

Substituting (39) into (37) the mass of gears of the first gear unit is calculated as follows

$$G_1 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{11}] \cdot (u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{4 \cdot [K_{01}] \cdot u_1} \tag{40}$$

Calculating in the same way, the following equations were found for the determination of the mass of gears of the second gear unit G_2 and the third gear unit G_3:

$$G_2 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{12}] \cdot (u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{4 \cdot [K_{02}] \cdot u_2} \tag{41}$$

$$G_3 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{13}] \cdot (u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{4 \cdot [K_{03}] \cdot u_3} \tag{42}$$

In the above equations, $\left[K_{01}\right]$, $\left[K_{02}\right]$ and $\left[K_{03}\right]$ are predicted by Equation 18, 22 and 23, respectively.

Based on the moment equilibrium condition of the mechanic system which includes the gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{T_{11}} = \frac{\left[T_r\right]}{\left[T_{11}\right]} = u_1 \cdot u_2 \cdot u_3 \cdot \eta_{brt}^3 \cdot \eta_o^3 \tag{43}$$

As it was done in Section 3, by

choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into Equation 43 we have

$$[T_{11}] = \frac{[T_r]}{0.8909 \cdot u_1 \cdot u_2 \cdot u_3} \quad (44)$$

Substituting (44) into (40) the following equation can be found:

$$G_1 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \cdot \frac{(u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{0.8909 \cdot u_1^2 \cdot u_2 \cdot u_3} \quad (45)$$

For the second helical gear unit it can be written as:

$$\frac{T_r}{T_{12}} = \frac{[T_r]}{[T_{12}]} = u_2 \cdot u_3 \cdot \eta_{brt}^2 \cdot \eta_o^2 \quad (46)$$

By choosing $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ Equation 47 becomes

$$[T_{12}] = \frac{[T_r]}{0.9259 \cdot u_2 \cdot u_3} \quad (47)$$

From Equations 47 and 41, the mass of gear of the second unit is determined by the following equation:

$$G_2 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{02}]} \cdot \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{0.9259 \cdot u_2^2 \cdot u_3} \quad (48)$$

In exactly similar manner, we can determine the mass of gears of unit 3 as follows:

$$G_3 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{03}]} \cdot \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{0.9622 \cdot u_3^2} \quad (49)$$

Substituting (45), (48) and (49) into (35) with the note that $u_1 = u_h / (u_2 \cdot u_3)$, the mass of gears of the gearbox is determined as follows:

$$G = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \left\{ \frac{(u_h + u_2 \cdot u_3) \cdot \left[1 + 0.6 \cdot u_h^2 / \left(u_2^2 \cdot u_3^2\right)\right]}{0.8909 \cdot u_h^2} + \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{0.9259 \cdot k_{C2} \cdot u_2^2 \cdot u_3} + \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{0.9622 \cdot k_{C3} \cdot u_3^2} \right\} \quad (50)$$

where, $k_{C2} = [K_{02}]/[K_{01}]$ and $k_{C3} = [K_{03}]/[K_{01}]$.

5 Optimization problems and results

5.1 For finding the optimum partial ratios for getting the minimum gearbox length

5.1.1 Optimization problem

From Equations 31, the optimum problem for determining the partial transmission ratio in order to get the minimum gearbox length can be expressed as follows:

The objective function is:

$$\min L = f(u_h; u_2; u_3) \quad (51)$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

$$K_{c2\min} \le K_{c2} \le K_{c2\max}$$

(52)

$$K_{c3\min} \le K_{c3} \le K_{c3\max}$$

$$\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}$$

$$\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}$$

$$\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}$$

A computer program was built in order to solve the above optimization problem. The used data in the program were: K_{c2} and K_{c3} are from 1 to 1.3, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are from 0.25 to 0.4 [8], u_2 and u_3 are from 1 to 9 [1]; u_h is from 30 to 120.

5.1.2 Results and discussions

Figure 3 shows the relation between the partial transmission ratios and the total transmission ratio when calculating with the following values of coefficients: K_{c2}=1.2, K_{c3}=1.3, ψ_{ba1}=0.25, ψ_{ba2}=0.3 and ψ_{ba3}=0.35.

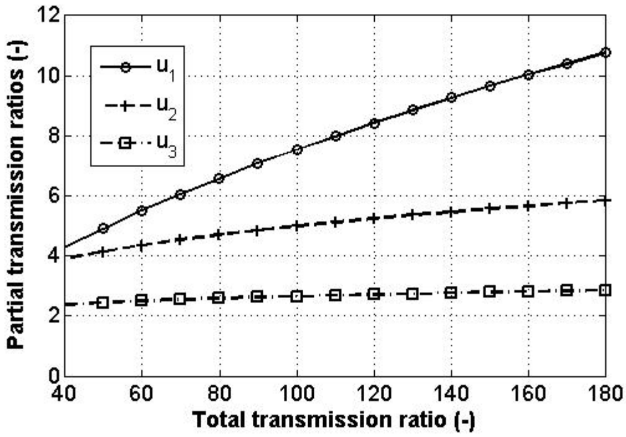

Fig. 3: Partial transmission ratios versus the total transmission ratio for getting minimum gearbox length

It can be seen from the result that with the increase of the total ratio u_h the partial ratios increase. In addition, it is found that with the increase of the total ratio, the increase of partial ratio of the first step u_1 is much larger than that of the third step u_3. The reason of that is with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driver shaft of the first gear unit T_{11}. Therefore, the increase of the partial ratio u_3 should be much smaller than that of u_1 in order to reduce the gearbox length.

It can be seen from Figure 3 that the partial transmission ratio of the first gear unit is larger than 9 - the limit of the transmission ratio of a helical gear set [8] when the total transmission ratio is beyond about 130. Accordingly, for getting the minimum gearbox length, the maximum total transmission ratio of the gearbox is 130.

Based on the results of the optimization program, regression analysis was carried out in order to find the models for the determination of the partial transmission ratios. The following regression models then were found for finding the optimum values of partial ratios u_2 and u_3 of the second and the third step, respectively:

$$u_2 \approx 1.3503 \cdot \frac{K_{C2}^{0.4397}}{K_{C3}^{0.1341}} \cdot \frac{\psi_{ba2}^{0.4398}}{\psi_{ba3}^{0.1342}} \cdot \frac{u_h^{0.2677}}{\psi_{ba1}^{0.3056}} \qquad (53)$$

$$u_3 \approx 1.3287 \cdot \frac{K_{C3}^{0.4395}}{K_{C2}^{0.3027}} \cdot \frac{\psi_{ba3}^{0.4396}}{\psi_{ba2}^{0.3027}} \cdot \frac{u_h^{0.1201}}{\psi_{ba1}^{0.1373}} \qquad (54)$$

The above regression models fit quite well with the calculated data (with the coefficients of determination for the models for the second and the third units were $R^2 = 0.9998$ and $R^2 = 0.9999$, respectively).

Equations 53 and 54 can be used for the prediction of the partial transmission ratios u_2 and u_3 of gear units 2 and 3, respectively. After determining u_2 and u_3, the partial transmission ratio of the first gear unit u_1 then can be calculated by the following equation:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \qquad (55)$$

5.2 For finding the optimum partial ratios for getting the minimum cross section dimension of the gearbox

5.2.1 Optimization problem

Based on Equations 32, 31 and 34, the optimum problem for determining the partial transmission ratios for getting the minimum cross section dimension of the gearbox can be expressed as follows:

The objective function is:

$$\min A = f(u_h; u_2; u_3) \qquad (56)$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

$K_{c2\min} \le K_{c2} \le K_{c2\max}$
(57)

$K_{c3\min} \le K_{c3} \le K_{c3\max}$

$\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}$

$\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}$

$\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}$

To solve the above optimization problem a computer program was built. The following data were used in the program: K_{c2} and K_{c3} are from 1 to 1.3, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are from 0.25 to 0.4 [8], u_2 and u_3 are from 1 to 9 [1]; u_h is from 30 to 120.

5.2.2 Results and discussions

Figure 4 describes the relation between the partial transmission ratios and the total transmission ratio (the following data were used in the calculation: K_{c2}=1.2, K_{c3}=1.3, ψ_{ba1}=0.25, ψ_{ba2}=0.3 and ψ_{ba3}=0.35).

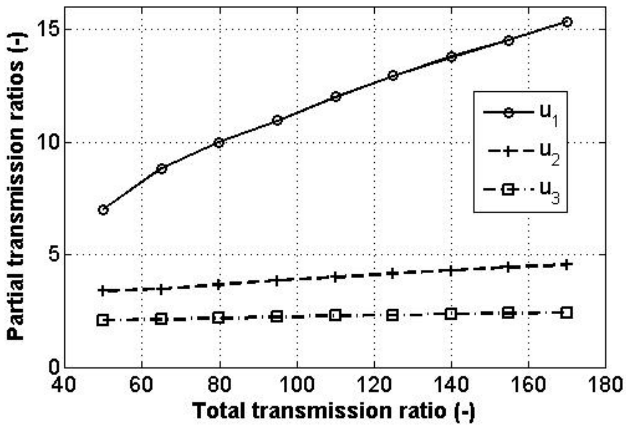

Fig. 4: Partial transmission ratios versus the total transmission ratio for getting gearbox cross section dimension

As in the optimization problem for getting the minimum gearbox length (see Subsection 5.1.2), with the increase of the total transmission ratio u_h all of the partial transmission ratios increase. In addition, while the partial transmission ratio of the first step u_1 increases very much, the partial transmission ratio of the third step u_3 nearly does not change when the total transmission ratio increases. The reason of that is with the increase of the total ratio u_h, the output torque T_r is much larger than the torque on the driver shaft of the first gear unit T_{11}. Consequently, the partial transmission ratio u_3 should be much smaller than the partial transmission ratio u_1 in order to reduce the gearbox size as well as the gearbox cross section dimension.

It is found that when the total transmission ratio is beyond about 70, the partial transmission ratio of the first gear unit is larger than 9 (see Figure 4) - the maximum transmission ratio of a helical gear unit [8]. As a result, the objective for getting the minimum gearbox cross section dimension should be used in cases that the total transmission ratio is less than 70.

Based on the calculated results of the optimization program, the following regression models were found for the prediction of the optimum values of partial ratios u_2 and u_3 of the second and the third steps, respectively:

$$u_2 \approx 1.0134 \cdot \frac{K_{c2}^{0.4291}}{K_{c3}^{0.144}} \cdot \frac{\psi_{ba2}^{0.4294}}{\psi_{ba3}^{0.1442}} \cdot \frac{u_h^{0.2848}}{\psi_{ba1}^{0.2853}} \quad (58)$$

$$u_3 \approx 1.0269 \cdot \frac{K_{c3}^{0.4279}}{K_{c2}^{0.2854}} \cdot \frac{\psi_{ba3}^{0.428}}{\psi_{ba2}^{0.2854}} \cdot \frac{u_h^{0.1423}}{\psi_{ba1}^{0.1426}} \quad (59)$$

The above regression models fit very well with the calculated data. The coefficients of determination for the models for the second step and the third step were $R^2 = 0.9996$ and $R^2 = 0.9998$, respectively.

Equations 58 and 59 are used to calculate the transmission ratios u_2 and u_3 of steps 2 and 3, respectively. After determining u_2 and u_3, the transmission ratio of the first step u_1 then can be calculated by the following equation:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \quad (60)$$

5.3 For finding the optimum partial ratios for getting the minimum mass of gears

5.3.2 Optimization problem

From Equation 50, the optimum problem for determining the partial transmission ratios for getting the minimum mass of the gears is expressed as follows:

The objective function is:

$$\min G = f(u_h; u_2; u_3) \qquad (61)$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max} \qquad (62)$$

$$k_{C2\min} \le k_{C2} \le k_{C2\max}$$

$$k_{C3\min} \le k_{C3} \le k_{C3\max}$$

To perform the above optimization problem, a computer program was built. The following data were used in the program: u_2 and u_3 were from 1 to 9 [1], k_{C2} and k_{C3} were from 1 to 1.3 [8] and u_h was from 30 to 180.

5.3.2 Results and discussions

The relation between the partial transmission ratios and the total transmission ratio when the coefficients K_{c2} and K_{c3} equal 1.1 was shown in Figure 5.

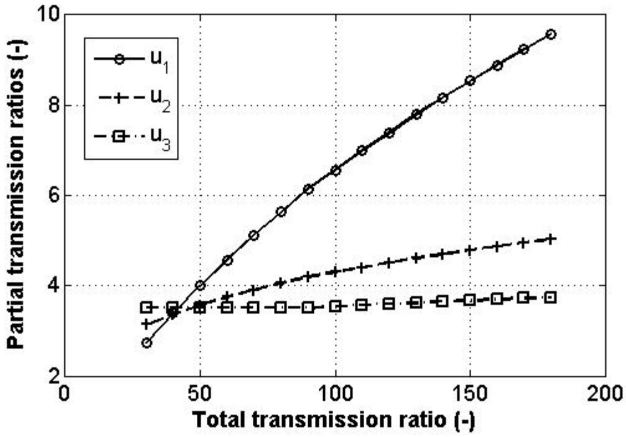

Fig. 5: Partial transmission ratios versus the total transmission ratio for getting minimum mass of gears

As in Sections 5.1 and 5.2, the partial ratio of the first step u_1 increases significantly when the total ratio u_h increases. In contrast, the partial transmission ratio of the third step u_3 increases very slowly. The reason of that can be explained as the same in the above sections.

It is detected that when the total transmission ratio is beyond about 160, the partial transmission ratio of the first gear unit is larger than 9 (see Figure 5) - the limit of the transmission ratio of a helical gear set [8]. Therefore, the target of getting the minimum gearbox cross section dimension should be used when the total transmission ratio is less than 160.

Based on the results of the optimization program, regression analysis was carried out and the following models were found for calculating the optimum values of the partial ratios of the second and the third step:

$$u_2 \approx 1.3104 \cdot \frac{k_{C2}^{0.3714}}{k_{C3}^{0.0977}} \cdot u_h^{0.2533} \qquad (63)$$

$$u_3 \approx 2.3417 \cdot \frac{k_{C3}^{0.3455}}{k_{C2}^{0.2492}} \cdot u_h^{0.088} \qquad (64)$$

The above regression models (63) and (64) fit quite well with the calculated data (the coefficients of determination were $R^2 = 0.9993$ and $R^2 = 0.9986$ for model (63) and (64), respectively).

Models 63 and 64 are used to determine the transmission partial ratios u_2 and u_3 of the second and the third helical gear units. After determining u_2 and u_3, the transmission ratio of the first gear unit u_1 is calculated as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \qquad (65)$$

6 Conclusion

It can be concluded that the minimum gearbox length, the minimum gearbox cross section dimension and the minimum mass of gears of a three-step helical gearbox with second-step double gear-sets can be obtained by optimum splitting the total transmission ratio of the gearbox.

For three step helical gearboxes, the maximum total transmission ratio is 160 for getting the minimum mass of gears and is 70 for getting minimum gearbox cross section dimension.

Models for prediction of the optimum partial ratios of the gearboxes in order to get the minimum gearbox length, the minimum gearbox cross section dimension and the minimum mass of gears have been found.

Using explicit models, the partial ratios of the gearboxes can be calculated accurately and simply by.

References:

[1] V.N. Kudreavtev; I.A. Gierzaves; E.G. Glukharev, *Design and calculus of gearboxes* (in Russian), Mashinostroenie Publishing, Sankt Petersburg, 1971.

[2] Trinh Chat, Optimal calculation the total transmission ratio of helical gear units (in Vietnamese), *Scientific Conference of Hanoi University of Technology*, 1996, pp. 74-79.

[3] A.N. Petrovski, B.A. Sapiro, N.K. Saphonova, About optimal problem for multi-step gearboxes (in Russian), *Vestnik Mashinostroenie*, No. 10, 1987, pp. 13-14.

[4] G. Milou; G. Dobre; F. Visa; H. Vitila, Optimal Design of Two Step Gear Units, regarding the Main Parameters, *VDI Berichte* No 1230, 1996, pp. 227-244.

[5] Romhild I. , Linke H., Gezielte Auslegung Von Zahnradgetrieben mit minimaler Masse auf der Basis neuer Berechnungsverfahren, *Konstruktion* 44, 1992, pp. 229- 236.

[6] Vu Ngoc Pi, Nguyen Dang Binh, Vu Quy Dac, Phan Quang The, Optimal calculation of total transmission ratio of three-step helical gearboxes for minimum mass of gears *(In Vietnamese)*, *Journal of Science and Technology of Technical Engineering Universities*, Vol. 55, 2006, pp. 91-93.

[7] Vu Ngoc Pi, A new study on optimal calculation of partial transmission ratios of two-step helical gearboxes, *2nd WSEAS International Conference on Computer Engineering and Applications (CEA'08)*, Acapulco, Mexico, January 25-27, 2008, pp. 162-165.

[8] Trinh Chat, Le Van Uyen, *Design and calculus of Mechanical Transmissions* (in Vietnamese), Educational Republishing House, Hanoi, 1998.