Manuscript received June 2, 2007; revised July 17, 2007

The unusual small wave velocities in structured bodies and instability of pore or cracked media by small vibration

SIBIRIAKOV B.P., PRILOUS B.I.
Trofimuk Institute of Oil and Gas
Geology and Geophysics
SB RAS
Novosibirsk, 630090
RUSSIA
SibiryakovBP@ipgg.nsc.ru prilbor4@mail.ru http://www.ipgg.nsc.ru

Abstract: - This paper studies properties of a continuum with structure. The characteristic size of the structure governs the fact that difference relations do not automatically transform into differential ones. It is impossible to distinguish an infinitesimal volume of a body, to which we could apply the major conservation laws, because the minimum representative volume of the body should contain at least a few elementary microstructures. This leads to motion equations of infinite order. Solutions of such equations include, along with sound waves, perturbations propagating with abnormally low velocities not bounded below. It is shown that in such media weak perturbations can increase or decrease without limit. Weak dispersion of structure sizes reduces the intensity of such increase and therefore stabilizes the medium, whereas unlimited growth of dispersion destabilizes it. The nonlinearity of the stress-strain curve is shown to cause a reduction of specific surface of a cracked medium, i.e. the medium undergoes fracture with the formation of a system of a small number of large cracks as though it ignores a large number of small cracks.

Key-words: - specific surface, operator of continuity, equation of motion, structured media, catastrophes.

1 Introduction

The characteristic range of structure leads to fact that the average distance between one of crack to another one or one pore to another is given by specific surface of sample. In the Fig.1 is shown an element of the volume of structured body where l_0 is the average distance between one pore to another. There is a theorem of integral geometry, which relates the specific surface σ_0 and l_0 with porosity f [1]:

$$\sigma_0 l_0 = 4(1-f) \quad (1)$$

Here f is the porosity. Hence, if there is a specific surface of sample, there is automatically the average range of microstructure l_0.

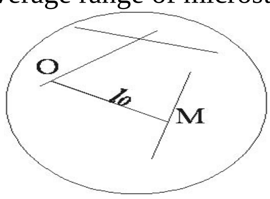

Fig. 1: The element of structured body. l_0 is an average distance between cracks.

The distinction of classic and structured continuums is clear on Fig.2. In the volume, which bounded by surface *C* there is equation of equilibrium, because all forces delete to each other, while in the volume bounded by surface *D*, there is no equation of equilibrium, because all forces concentrate on the one size of grain, and another size have no forces.

The idea of creation of new model of space is: we consider some finite volume of body. In this case the surface forces apply to sphere of radius l_0 while the inertial forces apply into center of structure. There is no possibility to tend an elementary volume into zero and coincides points of surface and the point of inertial forces like in classical continuum. We must consider a finite volume like representative volume of body and we have a problem of different positions of surface and inertial forces.

There is urgent necessary to translate surface forces into the center of structure by special translation operator. Obvious consequence of this action will be a possibility to apply the conservation law of ordinary mechanics into some image of structural continuum like in usual classical model of continuous media.

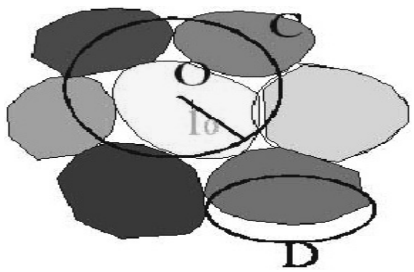

Fig. 2: The problem of creation of equilibrium equation into arbitrary element of discrete medium. On the surface *C* there is equation of equilibrium, but on the surface *D* there is not.

The one-dimensional operator of field translation from point *x* into point $x \pm l_0$ is given by symbolic formula [2]:

$$u(x \pm l_0) = u(x)e^{\pm l_0 D_x} \quad (2)$$

In this formula: $D_x = \frac{\partial}{\partial x}$

The formal expansion in Taylor series is given a finite increment of field as a series of infinite number of all order derivatives with different powers of l_0. The factor l_0 relates with specific surface of the sample. The three-dimensional operator of field translation for some cube with range of l_0 may be constructed as follows:

$$P[u(x)] = \frac{1}{6}u(x)[\exp(\frac{l_0}{2}D_x) + \exp(-\frac{l_0}{2}D_x) + \quad (3)$$
$$+\exp(\frac{l_0}{2}D_y) + \exp(-\frac{l_0}{2}D_y) + \exp(\frac{l_0}{2}D_z) + \exp(-\frac{l_0}{2}D_z)]$$

The analogous operator of translation for some sphere is given by expression:

$$P(D_x, D_y, D_z; l_0) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\exp[l_0(D_x \sin\theta\cos\varphi + \quad (4)$$
$$+D_y \sin\theta\sin\varphi + D_z\cos\theta)]\sin\theta d\theta d\varphi = \frac{sh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} =$$
$$= E + \frac{l_0^2}{3!}\Delta + \frac{l_0^4}{5!}\Delta\Delta + ...$$

According to Poisson formula we have [3]:

$$\int_0^{2\pi}\int_0^{\pi} f(\alpha Cos\theta + \beta Sin\theta Cos\varphi + \gamma Sin\theta Sin\varphi)Sin\theta d\theta d\varphi = \quad (5)$$
$$2\pi\int_0^{\pi} f(RCosp)Sinpdp = 2\pi\int_{-1}^{1} f(Rt)dt, where\ R = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$$

P operator may be rewritten as follows**:**

$$P(D_x, D_y, D_z) = \frac{1}{2}\int_{-1}^{1}\exp(l_0\sqrt{\Delta}\cdot t)dt = \quad (6)$$
$$= \int_0^1 Ch(l_0\sqrt{\Delta}\cdot t)dt = \frac{Sh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} = E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + ...$$

2 Problem Formulation

By using operator ***P*** we can write equation of motion of microinhomogeneous body because for average stresses in structure the law of impulse conservation takes usual form, namely

$$\frac{\partial}{\partial x_k}[P(\sigma_{ik})] = \rho\ddot{u}_i \quad (7)$$

In more detailed form (7) can be rewritten as follows:

$$\frac{\partial}{\partial x_k}(E + \frac{l_0^2}{3!}\Delta + \frac{l_0^4}{5!}\Delta\Delta + ...)\sigma_{ik} = \rho\ddot{u}_i \quad (8)$$

In particularly, if we take into account 1st and 2nd terms of (8) only, there is situation of the 4th order equilibrium equation, as in [4].
In one dimensional case equation (8) takes more simple expression:

$$u''(E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + ...) + k_S^2 u = 0 \quad (9)$$

$$\frac{\sin(kl_0)}{kl_0} - \frac{k_S^2}{k^2} = 0 \quad (10)$$

The equation (9) by means of substitution
u=Aexp(ikx) gives us the dispersion equation (10) for unknown wave number *k*, or for unknown wave velocity, which depends on range of structure l_0 or specific surface of sample σ_0. It is evident that by $l_0 \to 0$ the wave number $k \to k_S$, i.e. the wave velocity is equal to V_P or V_S elastic wave velocity. However if l_0 is not very small value, the wave velocity decreases up to zero by $kl_0 \to m\pi$, if *m* is integer number.
Hence this model describes along with usual seismic waves a lot of waves of very small velocities, which not bound below. This effect is more substantial for the *P* waves than for the *S* ones.
Hence, if the Poisson ratio measured on samples by velocities V_P and V_S, their ratio V_S/V_P is growing by growing l_0, and this effect can produces abnormally small Poisson ratio, up to negative volume of it. A

lot of velocities which are describe by (10) closely relates to infinite large degrees of freedom in structured media. The analogous approach for case of finite degrees of freedom was recently published in the work [5].

Besides of it, the equation (10) has also complex roots. Really, if we put $k_S l_0=\varepsilon$, $kl_0=p$, the equation (10) takes a form:

$$pSin(\varepsilon p) = \varepsilon \qquad (11)$$

Assuming $\varepsilon p=x+iy$ and dividing real and imaginary parts, we can write two independent equations for determination real and imaginary parts of complex wave numbers, namely:

$$\begin{aligned} xSinxChy - yCosxShy &= \varepsilon^2 \\ ySinxChy + xCosxShy &= 0 \end{aligned} \qquad (12)$$

Equations (12) may be rewritten as:

$$\frac{tgx}{x} = -\frac{thy}{y} \qquad (13)$$

$$Sin^2 x + Sh^2 y = \frac{\varepsilon^4}{x^2 + y^2}$$

First of the equation (13) means, that complex roots do not arise at small values x because expression of the right hand has negative value. For occurrence of complex roots the obvious condition for negative value $tgx>\pi/2$ is necessary.

The physical sense of this necessary condition implies that the complex roots occur under the condition that the wavelength must be magnitude of four or less times greater than a range of structure. These complex roots mean that amplitude of oscillations may be increasing or decreasing up to infinity or even to zero. So these roots are responsible for catastrophic and disastrous behavior of structured bodies.

Hence, if there is a source of sufficient energy, even some small oscillations can produce catastrophes. It is interesting, that nonlinear deforming of samples increases this effect, because a wave velocity for rocks is decreasing by growing amplitude of wave. It means that the wave number is growing by the same frequency compare to pure elastic process. In figures 3-5 are shown complex roots of dispersion equation (10). The vertical axis represents a dimensionless frequency (ε), while horizontal axis represents us real and imaginary parts of wave numbers. Every point in these figures is a position of some root, namely its real part, imaginary one and dimensionless frequency. The more is a spreading of ε values, the greater the number of complex roots appears.

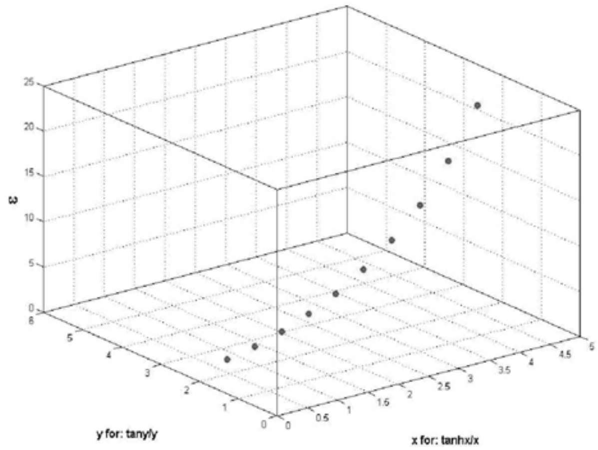

Fig. 3: Dispersion Equation Roots (1st range of ε)

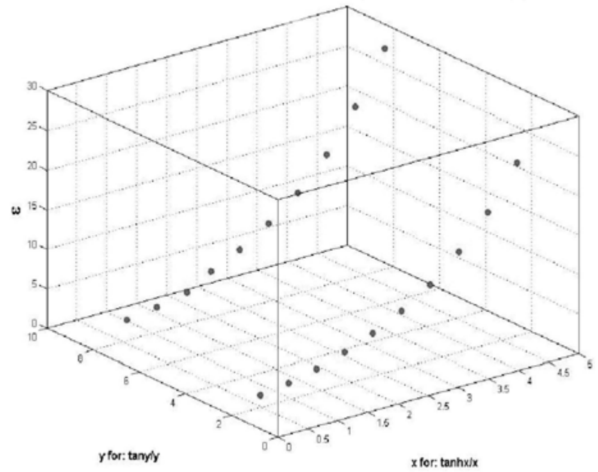

Fig. 4: Dispersion Equation Roots (2nd range of ε)

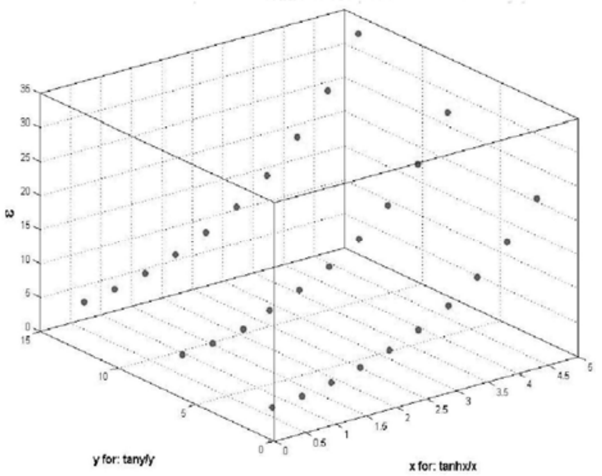

Fig. 5: Dispersion Equation Roots (3rd range of ε)

3 *P* operator for random structures

In the case of random distance values between cracks i.e. $l=l_0+\xi l_0$, where ξ is the random value with zero mean value and normal distribution, the *P* operator takes a form:

$$P(D_x,D_y,D_z;l_0)=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\exp[l_0(D_in_i)+ \quad (14)$$
$$+\xi(D_in_i)]Sin\theta d\theta d\varphi$$

$$<\exp(\omega\xi)>=\exp(\frac{1}{2}\sigma^2\omega^2) \quad (15)$$

$$P(D_x,D_y,D_z;l_0,\sigma)=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\exp[l_0(D_in_i)+ \quad (16)$$
$$+\frac{l_0^2\sigma^2}{2}(D_in_i)^2]Sin\theta d\theta d\varphi$$

$$\frac{1}{kl_0}\int_0^{kl_0}Cos(x)\exp[-x^2\sigma^2/2]dx=\frac{k_s^2}{k^2} \quad (17)$$

There is a formula concerning of exponent of random variable with normal distribution [6], which is presented in (15). There σ is the variance of random variable ξ. In this case *P* operator can be rewritten in the form (16), and a dispersion equation takes a form (17). It is evident that by variance $\sigma=0$ *P* operator in (16) and an equation (17) are coincide with corresponding expressions, which were demonstrated earlier. The numerical investigation was made in [7]. There was shown, that the small value of σ causes the decreasing of imaginary part of roots. It means that the small chaos stabilizes the cracked or porous media. However, for large variances of σ there is no the decreasing of imaginary part of roots. There is opposite effect. It means that the large chaos destabilizes the medium again.

4 The nonlinear elasticity

We can represent the stress-strain diagram (Fig.6) as a cubic polynomial with respect to strain u_x, namely:

$$\sigma_{xx}=(\lambda+2\mu)(1+au_x+bu_x^2)u_x \quad (18)$$

The classical equation of motion by $P=E$ is follows:

$$u_{xx}(1+2au_x+3bu_x^2)=\frac{1}{c^2}u_{tt} \quad (19)$$

We can try to solve it by expansion of solution in the form:

$$u=u_1\exp(i\omega t)+u_2\exp(2i\omega t)+ \quad (20)$$
$$+u_3\exp(3i\omega t)+...$$

Substituting this expansion into (19) we can write supposed solution in more detailed form [7]:

$$[u_{1xx}\exp(i\omega t)+u_{2xx}\exp(2i\omega t)+ \quad (21)$$
$$+u_{3xx}\exp(3i\omega t)+...]\cdot[1+2a\{u_{1x}\exp(i\omega t)+$$
$$+u_{2x}\exp(2i\omega t)+u_{3x}\exp(3i\omega t)+...\}+$$
$$+3b\{u_{1x}^2\exp(i\omega t)+u_{2x}^2\exp(2i\omega t)+$$
$$+u_{3x}^2(3i\omega t)+2u_{1x}u_{2x}\exp(3i\omega t)+$$
$$+2u_{1x}u_{3x}\exp(4i\omega t)+2u_{2x}u_{3x}\exp(5i\omega t)+...\}]=$$
$$=-k_s^2u_1\exp(i\omega t)-(2k_s)^2u_2\exp(2i\omega t)-$$
$$-(3k_s)^2u_3\exp(3i\omega t)-.....$$

Collecting terms with the same exponents it is possible to write some consequence of linear equations kind of:

$$u_{1xx}+k_s^2u_1=0 \quad (22)$$

$$u_{2xx}+(2k_s)^2u_2=-2au_{1x}u_{1xx}= \quad (23)$$
$$=2ak_s^2u_{1x}u_1$$

$$u_{3xx}+(3k_s)^2u_3=2ak_s^2(4u_{1x}u_2+u_1u_{2x})+ \quad (24)$$
$$+3bk_s^2u_1u_{1x}^2$$

In this consequence of heterogeneous equations all terms, except second ones in the left hand are produced by volume forces which are created by stresses. It means that in microinhomogeneous media these terms must undergo to *P*-operator. The second terms in the left hand is created by inertial forces and these terms don't undergo by *P* operator. The result of action of *P*-operator into equation (22) was demonstrated earlier (10). Namely, if the solution represents in stationary oscillations kind of:

$$u_1=\frac{1}{k_s}\exp(ikx) \quad (25)$$

i.e. dispersion equation with respect to unknown wave number *k*, which was examined earlier (10):

$$u_1\left(\frac{Sinkl_0}{kl_0}-\frac{k_s^2}{k^2}\right)=0 \qquad (26)$$

As to second generation of field, pay your attention that the operator *P* from right hand of equation represents in the form:

$$P(2ak_s^2u_{1x}u_1)=-\frac{2ai}{k}\frac{Sin2kl_0}{2kl_0}\exp(2ikx) \qquad (27)$$

Hence, the intensity of the second field generation for microinhomogeneous medium is:

$$\bar{u}_2=-2ai\frac{k_s}{k}\frac{Sin2kl_0/2kl_0}{\frac{Sin2kl_0}{2kl_0}-\frac{k_s^2}{k^2}}\bar{u}_1 \qquad (28)$$

In this formula $\bar{u}_1, \bar{u}_2$ represent amplitudes of oscillations i.e. for example $u_2=\bar{u}_2\exp(2ikx)$.

In this case $\frac{Sinkl_0}{kl_0}=\frac{k_s^2}{k^2}$, and the denominator is not equal to zero in (28).

In case $l_0 \to 0$, it is evident that $k \to k_s$ and formula (28) for long waves takes a form:

$$\bar{u}_2=\frac{4ia}{(k_Sl_0)^2}\bar{u}_1 \qquad (29)$$

In spite of very small value a in (29) the field u_2 is not very small because a denominator in (29) very small too. This is very interesting phenomenon of generation of nonlinear effects by even sufficiently small oscillations.

The analogous formula there is for third generation, namely:

$$\bar{u}_3=-\bar{u}_1\frac{4a^2Sin(3kl_0)/3kl_0}{[\frac{Sin(2kl_0)}{2kl_0}-\frac{k_S^2}{k^2}][\frac{Sin(3kl_0)}{3kl_0}-\frac{k_S^2}{k^2}]}- \qquad (30)$$

$$-\bar{u}_1\frac{3bk^2[Sin(2kl_0)/2kl_0-k_S^2/k^2]/k_S^2}{[\frac{Sin(2kl_0)}{2kl_0}-\frac{k_S^2}{k^2}][\frac{Sin(3kl_0)}{3kl_0}-\frac{k_S^2}{k^2}]}$$

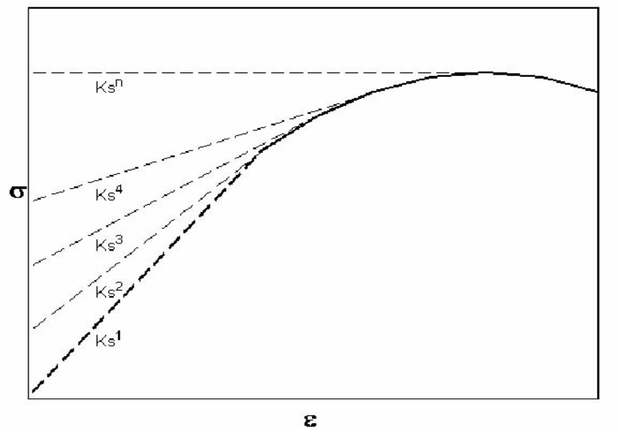

Fig. 6: Diagram of stress-strain state. The velocities of waves are decreasing by growing of pressure. On figure we can see the less and less velocities by growing pressure in rocks.

5 Conclusions

5.1 Common conclusions

5.1.1 Structured continuum set by *P* operator describe as sound waves as perturbation moving with exclusively small velocities that from the bottom nothing limited. Small oscillations of structured continuum can rise beyond all bounds at parametric resonance approach. Possible catastrophes can arise even at small oscillations.

5.1.2. The low dispersion of average size structure lead to media stabilization while (on the contrary) the high one lead to media destabilization.

5.1.3. Unexpectedly huge amplitude size multiple harmonics appears at nonlinear perturbations. In that case the structured medium(s) become nonlinear even at relatively low amplitude oscillations. Decrease of rift media specific surface occurs at nonlinear oscillations (in other words appears crack generalization).

5.2 Some opportunities for future and some important applications

5.2.1 The catastrophes by small vibrations, which can cause fatigue fracture in aviation.

5.2.2 The explanation of tidal waves effect from the Moon for earthquakes, which observed for classes 6-9. This effect is negligibly small, if we use usual continuous mechanics.

5.2.3 The registration of unusual small waves may be some instrument for prediction of stress-strain instability in coal mines.

5.2.4 The small random vibrations stabilize the stress-strain state in micro inhomogeneous media. Hence, there are different scenarios of catastrophes, which manage by artificial equipments. These scenarios are an interesting field for application of GA technology.

References:

[1] F.A. Usmanov, *Foundation of mathematical analysis of geological structures*, FAS UzSSR. Tashkent (1977).

[2] V. P. Maslov, *Operator Methods* (Nauka, Moscow, 1973) [in Russian]; English transl.: *Operational Methods* (Mir, Moscow, 1976).

[3] I.S. Gradshteyn, I.M. Ryzhik; Alan Jeffrey, Daniel Zwillinger, editors. *Table of Integrals, Series, and Products*, seventh edition. Academic Press, 2007.

[4] George Й. Tsamasphyros, Energy Theorems in the Framework of the Strain Gradient Theories, *Abstract of Plenary Lecture 3 of the 3rd IASME / WSEAS International Conference on CONTINUUM MECHANICS (CM'08)*, Cambridge, UK, February 23-25, 2008.

[5] Yuan-Ming, Y. Bing, Z. Chuan-Yao, C. Tian-Xia, S., Dynamics Modeling Analysis of the Mechanism System Based on Rigid Body Motion and Elastic Motion, *WSEAS TRANSACTIONS ON SYSTEMS AND CONTROL*, 2006, VOL 1; ISSU 1, pages 43-48.

[6] Klyatskin, V.I.: *Stochastic equations and waves in random media* (in Russian, Nauka, 1980).

[7] Sibiriakov B.P., Podberezhny M.Y., Instability of structured media and some scenarios of catastrophes. *Russian Geology and Geophysics*, 2006, v. 47, N 5, p.648-654.