Manuscript received Feb. 2, 2007; revised May 9, 2007

Lumbar Spine: Parameter Estimation for Realistic Modeling

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Abstract: We have developed a parameter estimation routine that uses genetic algorithms to systematically identify stiffness and damping properties that can accurately predict spine segment motions associated with forces applied during chiropractic manipulation. Enhancements to the current computer simulation and visualization capabilities established are made. The ensuing computer simulation and modeling program is tested with experimental displacement-time and acceleration-time spine motion data. Experimentally-optimized stiffness and damping properties derived from the model are used to characterize the normal and pathologic spine. This study provides a tool for diagnosis of spinal disorders and will assist chiropractic clinicians and researchers in understanding the mechanical response of the human spine to mechanical forces.

Key-Words: Lumbar spine, Optimization, Genetic Algorithms, simulation, parameter estimation

1 Introduction

Without resorting to invasive spine measurements techniques, precise assessment of clinically relevant variables, such as vertebral and inter-vertebral displacements and stiffness, is very difficult to obtain [1]. Mathematical models are therefore often used to quantify the forces and moments acting on the spine. The mechanical response of the spine to externally applied static and dynamic forces is dependent on the complex interaction between the flexible joint structures (FJS) and rigid structures that comprise the function spinal unit. The inherent mechanical and geometric complexity of the spine makes the precise description of vertebral body movements a modeling challenge.

Previous research has demonstrated that the static and dynamic response of the lumbar spine to external forces can be studied using lumped parameter analytical models [2]. In these models, the vertebral body structures were represented as rigid masses and the FJS were represented elastic and viscous mechanical elements (springs and dampers). Using elastic stiffness coefficients derived from the literature and viscous damping defined by modal damping ratios, these authors found that their analytical model predictions showed general agreement with in vivo experimental studies of the posteroanterior (PA) motion response (natural frequency of vibration and peak-to-peak displacement behavior) of prone-lying subjects. However, their analytical models were limited to predicting the vertebral (segmental) and inter-vertebral (inter-segmental) motion responses with three or fewer displacement degrees of freedom, and did not accurately represent the multi-axial coupling behavior of the spine. Moreover, other than parametric variation of model stiffness and modal damping coefficients, these studies did not include a systematic optimization of these coefficients.

Lee & Evans (1994) [3] were perhaps the first investigators to develop a mathematical model to specifically study the lumbar spine's response to posterior-anteriorly directed forces. Lee et al. (1995) [4] developed a three-dimensional finite element model of the spine, ribcage, and pelvis, which was used to predict static segmental displacement responses of the lumbar vertebrae to PA forces. They validated their model by comparing predictions to low frequency (<1 Hz) PA oscillatory force-displacement data observed in human subjects and found generally good agreement with the mean responses.

Evans et al. (2005) [5] carried out right rotational mobilization. The initial starting position adopted was left side–lying with flexion of both hips and knees. Rotational mobilization was then performed by pushing the pelvis in an oscillatory manner. The spine was twisted further into the range by rotating the thorax to the left. Counter pressure was applied to the right shoulder by the shoulder fixation pad while the pelvis was pushed. The measured angular rotation and applied moments were used to predict angular stiffness of the current model. The results are matching the data to some extend due to the model linearity limitation.

Ralph et. al. (2005) [7] measured the dynamic force-displacement curves of an L2-3 cadaveric lumbar motion segment. A comparison of the resulting model with these data shows good agreement with the linear slope of the curve (least-squares fit) and small percentage error with the range of motion.

Although we have limited our test cases to the displacement response resulting from static and dynamic forces delivered to a single segment due to data shortage, the response to forces distributed over several segments can be modeled easily by directly applying forces and moments to specific vertebrae. Additional experimental data is needed to assess the model response to more complex loading conditions.

There are inherent limitations of the current model. First, in specifying a single linear set of elastic coefficients for vertebral segment, the response to larger deformations can't be guaranteed. The normal lumbar spine is nonlinear in both passive structures (ligaments, cartilage) and active structures (skeletal muscle) that contribute to the biomechanical behavior. As currently formulated, the model is limited to examination of the general linear, static and dynamic mechanical responses of the spine. Second, the model does not take into account the complex geometry of the spine, which can introduce other nonlinearities like contacting and sliding between vertebrae. In addition, in this model we have considered coupling of forces and moments between two axes when, in fact, forces and moments are coupled in all axes. All these limitations are considered in future work.

2 Methods: The Simulator

The overall objective of this study was to develop a mathematical model capable of describing the three-dimensional static and dynamic motion response of the lumbar spine. A robust parameter estimation routine was developed to identify FJS stiffness and damping properties using available displacement-time data obtained from in vivo impulsive force experiments. Coefficients derived from this analysis were subsequently used to independently validate the model response to static and steady-state motion responses. The dynamic motion response is subsequently characterized using a three-dimensional visualization routine.

The lumbar spine vertebrae were modeled as five rigid bodies representing vertebral segments L1 to L5. Each vertebra was treated as a rigid-body mass possessing six displacement degrees of freedom: three components of translation and three components of rotation. All other flexible joint structures (i.e. ligaments, disc, muscles, tendons, and cartilage) or FJS were modeled as massless springs (elastic elements) and dampers (viscous elements) constraining the motion of the vertebrae in the six degrees of freedom. The flexible connections to the upper and lower part of the spine (thorax and pelvis) were also represented as spring-dashpot mechanical elements. To simulate the dynamic motion response of the lumbar spine a program was created that consisted of four custom Matlab (The Mathworks, Natick, MA) modules: Initialization, Ordinary Differential Equation Solver, Rigid-body Dynamics Routine, and Visualization. Each module is explained in detail in the following sections. The initialization module inputs each of the required vertebral mass, inertia and center of mass parameters. Vertebral masses are taken from reference [2] and moments of inertia are calculated using cad program from the 3D model found in (free 3D models) Input data are summarized in Table 1.

Table 1. Vertebrae mass and inertia properties

Property	L_1	L_2	L_3	L_4	L_5
Mass (kg)	0.17	0.17	0.114	0.114	0.114
I_{xx} (10^{-6}) kgm^2	26.7	24.5	16.5	14.8	22.5
I_{yy} (10^{-6}) kgm^2	34.2	31	17.4	20.4	31
I_{zz} (10^{-6}) kgm^2	36.8	36	22.2	26.5	40.3
I_{xz} (10^{-6}) kgm^2	-7.8	-6.9	-2.8	1.4	2.12

Two solvers are implemented in the simulator. For parameter estimations, a first order finite difference procedure is implemented to speed up calculations. For the simulator a fourth order Runge-Kutta procedure is implemented for high accuracy. The lumbar spine vertebrae were modeled as five rigid bodies (L1 to L5) with six degrees of freedom. Each degree of freedom was mathematically represented using a single, second order ordinary differential equation or two first ordinary differential equations [6]. Thus, each body was characterized by a total of 12 first order differential equations, and the complete system was comprised of 60 first order differential equations. The rigid body kinematic and dynamic equations are given in equations (1):

$$F_{x\,i} - m_i\, g\, sin(\theta_i) = m_i(\dot{u}_i + q_i w_i - r_i v_i)$$
$$F_{y\,i} + m_i\, g\, cos(\theta_i)\, sin(\phi_i) = m_i(\dot{v}_i + r_i u_i - p_i w_i)$$
$$F_{z\,i} + m_i\, g\, cos(\theta_i)\, sin(\phi_i) = m_i(\dot{w}_i + p_i v_i - q_i u_i)$$
$$M_{x\,i} = (I_x)_i\, \dot{p}_i - (I_{xz})_i\, \dot{r}_i + q_i r_i (I_z - I_y)_i - (I_{xz})_i\, p_i q_i$$
$$M_{y\,i} = (I_y)_i\, \dot{q}_i + q_i r_i (I_x - I_z)_i + (I_{xz})_i (p_i^2 - r_i^2)$$
$$M_{z\,i} = \square\, (I_{xz})_i\, \dot{p}_i + (I_z)_i\, \dot{r}_i + q_i p_i (I_y - I_z)_i - (I_{xz})_i\, r_i q_i$$

$$\dot{\theta}_i = q_i\, cos(\phi_i) - r_i\, sin(\phi_i)$$
$$\dot{\phi}_i = p_i + q_i\, sin(\phi_i)\, tan(\theta_i) - r_i\, cos(\phi_i)\, tan(\theta_i)$$
$$\dot{\psi}_i = (q_i\, sin(\phi_i) - r_i\, cos(\phi_i))\, sec(\theta_i)$$

((1)

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}_i = \begin{bmatrix} C\theta_i C\psi_i & S\phi_i S\theta_i C\psi_i - C\phi_i S\psi_i & C\phi_i S\theta_i C\psi_i + S\phi_i S\psi_i \\ C\theta_i S\psi_i & S\phi_i S\theta_i S\psi_i + C\phi_i C\psi_i & C\phi_i S\theta_i S\psi_i - S\phi_i C\psi_i \\ -S\theta_i & S\phi_i C\theta_i & C\phi_i C\theta_i \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$where\quad \mathrm{Cx_i} \equiv cos(x_i),\ \ \mathrm{Sx_i} \equiv sin(x_i)$

L_1 constraints:

$$F_x = -k_x(x_1 - x_2) - k_u(u_1 - u_2) - k_{x1}x_1$$
$$F_y = -k_y(y_1 - y_2) - k_v(v_1 - v_2) - k_{y1}y_1$$
$$F_z = -k_z(z_1 - z_2) - k_w(w_1 - w_2) - k_{z1}z_1$$
$$M_x = -k_\phi(\phi_1 - \phi_2) - k_p(p_1 - p_2) - k_{\phi 1}\phi_1$$
$$M_y = -k_\theta(\theta_1 - \theta_2) - k_q(q_1 - q_2) - k_{\theta 1}\theta_1$$
$$M_z = -k_\psi(\psi_1 - \psi_2) - k_r(r_1 - r_2) - k_{\psi 1}\psi_1$$

L_2 to L_4 constraints (i=2, 3, 4):

$$F_x = -k_x(-x_{i-1} + 2x_i - x_{i+1}) - k_u(-u_{i-1} + 2u_i - u_{i+1})$$
$$F_y = -k_y(-y_{i-1} + 2y_i - y_{i+1}) - k_v(-v_{i-1} + 2v_i - v_{i+1})$$
$$F_z = -k_z(-z_{i-1} + 2z_i - z_{i+1}) - k_w(-w_{i-1} + 2w_i - w_{i+1})$$
$$M_x = -k_\phi(-\phi_{i-1} + 2\phi_i - \phi_{i+1}) - k_p(-p_{i-1} + 2p_i - p_{i+1})$$
$$M_y = -k_\theta(-\theta_{i-1} + 2\theta_i - \theta_{i+1}) - k_q(-q_{i-1} + 2q_i - q_{i+1})$$
$$M_z = -k_\psi(-\psi_{i-1} + 2\psi_i - \psi_{i+1}) - k_r(-r_{i-1} + 2r_i - r_{i+1})$$

L_5 constraints:

$$F_x = -k_x(x_5 - x_4) - k_u(u_5 - u_4) - k_{x2}x_5$$
$$F_y = -k_y(y_5 - y_4) - k_v(v_5 - v_4) - k_{y2}y_5$$
$$F_z = -k_z(z_5 - z_4) - k_w(w_5 - w_4) - k_{z2}z_5 \qquad (2)$$
$$M_x = -k_\phi(\phi_5 - \phi_4) - k_p(p_5 - p_4) - k_{\phi 2}\phi_5$$
$$M_y = -k_\theta(\theta_5 - \theta_4) - k_q(q_5 - q_4) - k_{\theta 2}\theta_5$$
$$M_z = -k_\psi(\psi_5 - \psi_4) - k_r(r_5 - r_4) - k_{\psi 2}\psi_5$$

$$\begin{aligned} F_{zc1} &= -k_{zx}F_{x2} & M_{c1} &= -k_{\theta x}F_{x2} \\ F_{zc2} &= k_{zx}F_{x1} - k_{zx}F_{x3} & M_{c2} &= k_{\theta x}F_{x1} - k_{\theta x}F_{x3} \\ F_{zc3} &= k_{zx}F_{x2} - k_{zx}F_{x4} & M_{c3} &= k_{\theta x}F_{x2} - k_{\theta x}F_{x4} \\ F_{zc4} &= k_{zx}F_{x3} - k_{zx}F_{x5} & M_{c4} &= k_{\theta x}F_{x3} - k_{\theta x}F_{x5} \\ F_{zc5} &= k_{zx}F_{x4} & M_{c5} &= k_{\theta x}F_{x4} \end{aligned} \qquad (3a)$$

$$\begin{aligned} F_{zc1} &= -k_{zy}F_{y2} & L_{c1} &= -k_{\psi y}F_{y2} \\ F_{zc2} &= k_{zy}F_{y1} - k_{zy}F_{y3} & L_{c2} &= k_{\psi y}F_{y1} - k_{\psi y}F_{y3} \\ F_{zc3} &= k_{zy}F_{y2} - k_{zy}F_{y4} & L_{c3} &= k_{\psi y}F_{y2} - k_{\psi y}F_{y4} \\ F_{zc4} &= k_{zy}F_{y3} - k_{zy}F_{y5} & L_{c4} &= k_{\psi y}F_{y3} - k_{\psi y}F_{y5} \\ F_{zc5} &= k_{zy}F_{y4} & L_{c5} &= k_{\psi y}F_{y4} \end{aligned} \qquad (3b)$$

The FJS spring and coupling coefficients were computed using a parameter optimization routine described later.

A Matlab script was written to visualize the model simulation of the lumbar spine motion. The procedure is based on center of gravity and Euler angles. At time *t* each vertebra is defined by its center of gravity (c.g.) and orientation angles (Euler angles). For visualization purposes, a 3D lumbar spine model was adapted from a 3D studio model found on the internet (free 3D models). The model was edited and reformatted to work under Matlab. Each vertebra was modeled by surface triangular patches defined by node matrix (vertices) and connectivity matrix. The vertebral node coordinate (x,y,z) origin was located at the vertebral body center of mass in order to simplify the coordinate transformation. All vertebra surface coordinates are therefore known relatively to the center of gravity. At each time step these points were transformed using Euler angles and then translated to the new position using the new center of gravity. An MPEG encoder was used to produce moving picture experts group standard video coded compressed format (MPEG) movie files.

The cost function used in the optimization module was error based. Depending on the data available (i.e., flexion–extension (FE) rotation, axial (AX) cranial-caudal, and posteroanterior (PA) motions, etc.), the cost function is simply the integration over time of the difference error between the value the variable (displacement, rotation, etc.) from experiment and the value of the same variable from the proposed model. For a single variable, this can be written as

$$\text{cost function (f)} = \int |\mathrm{e(t)}|\, \mathrm{dt}$$
$$\text{where } \mathrm{e(t)} = \mathrm{V_{exp}(t)} - \mathrm{V_{model}(t)},$$

$V_{exp}(t)$ is the value of an experimentally-derived variable (or numerically-calculated from a high fidelity model) as a function of time and V_{model} (t) is the value of the lumbar spine model variable as a function of time.

For multiple variables, such as displacement-time histories for different axes, the total cost function is just the summation of cost function of each variable with weighting constants as follows:

$$\text{Total cost function (F)} = \mathrm{w_1 f_1 + w_2 f_2 + \ldots\ldots + w_n f_n} = \mathrm{w_1}\int |\mathrm{e_1(t)}|\, \mathrm{dt} + \mathrm{w_2}\int |\mathrm{e_2(t)}|\, \mathrm{dt} + \ldots. + \mathrm{w_n}\int |\mathrm{e_n(t)}|\, \mathrm{dt}$$

where w_i are weighting constants. In this work these weights are assumed unity.

Test Case 1

Six displacement-time histories were used for parameter estimation [2]. These data represent the posteroanterior (PA or X-axis) displacement, axial (AX or Z-axis) displacement, and flexion extension (FE) rotation (about Y-axis) for the L3 and L3-L4 vertebral segments subjected to an impulsive force of 100 N applied to the L3 vertebra of a 36 year old, 185 cm, 82 kg male volunteer. Displacement-time histories, with equal time steps (0.2 msec.) were obtained from Keller's model [2]. The cost function is given by

$$\text{Total cost function (F)} = \sum_{i=1}^{6} \varepsilon_i$$

where

$$\varepsilon_1 = \sum \left|(axial\ \text{displacement}_{\text{Keller}} - axial\ \text{displacement}_{\text{Model}})\right| \text{ for } L_3$$

$$\varepsilon_2 = \sum \left|(axial\ \text{displacement}_{\text{Keller}} - axial\ \text{displacement}_{\text{Model}})\right| \text{ for } L_4$$

$$\varepsilon_3 = \sum \left|(PA\ \text{displacement}_{\text{Keller}} - PA\ \text{displacement}_{\text{Model}})\right| \text{ for } L_3$$

$$\varepsilon_4 = \sum \left|(PA\ \text{displacement}_{\text{Keller}} - PA\ \text{displacement}_{\text{Model}})\right| \text{ for } L_4$$

$$\varepsilon_5 = \sum \left|(FE\ \text{displacement}_{\text{Keller}} - FE\ \text{displacement}_{\text{Model}})\right| \text{ for } L_3$$

$$\varepsilon_6 = \sum \left|(FE\ \text{displacement}_{\text{Keller}} - FE\ \text{displacement}_{\text{Model}})\right| \text{ for } L_4$$

Test Case 2

Three displacement-time histories were used for parameter estimation [4]. These data represent typical moment and movement pattern of the lumbar spine
in the 3 anatomical planes during right rotational mobilization of a 30-year-old woman.. The cost function is given by

$$\text{Total cost function (F)} = \sum_{i=1}^{3} \varepsilon_i$$

where

$$\varepsilon_1 = \sum \left|(\,axial\ \ rotation_{\text{Tsung}} - axial\ \ rotation_{\text{Model}})\right| \text{ for } L_3$$

$$\varepsilon_2 = \sum \left|(\,lateral\ \ bending_{\text{Tsung}} - lateral\ \ bending_{\text{Model}})\right| \text{ for } L_3$$

$$\varepsilon_3 = \sum \left|(\,flextion\ \ extension_{\text{Tsung}} - flextion\ \ extension_{\text{Model}})\right| \text{ for } L_3$$

The FJS spring and dashpot coefficients were derived from displacement-time data using a parameter estimation procedure comprised of two modules: a simulator with variable parameters and an optimizer. The parameter estimation algorithm proceeds as follows:

1- Initialize parameters (spring and dashpot coefficients).

2- Run the simulator with known input-output data pairs.

3- Calculate the simulator output and compare it with the actual spine output and calculate the error function.

4- Repeat steps 1-3 through the optimizer to minimize the error function till reaching minimum.

5- End.

The optimizer uses a robust optimization procedure based on a Monte Carlo procedure followed by genetic algorithm (GA).Parameter estimates for the spring, damper and coupling coefficients are not generally available, or are highly variable. Hence, a Monte Carlo technique was employed to obtain candidate solutions to start a genetic algorithm (GA). The Monte Carlo technique implemented in the optimizer is as follows:

1- Generate the unknown parameters randomly.

2- Substitute into the cost function and calculate the error.

3- Repeat step 1 and 2 (n) times.

4- Keep the best m trails with the minimum error.

5- Pass these trials to the GA algorithm.

The GA proceeds with the initial population taken from the Monte Carlo procedure. Alternatively, if initial spring, damper and coupling parameters estimates are available, then an optimization technique such as steepest decent is employed. The cost function used in the optimization module was error based. Depending on the data available (i.e., flexion–extension (FE) rotation, axial (AX) cranial-caudal, and posteroanterior (PA) motions, etc.), the cost function is simply the integration over time of the difference error between the value the variable (displacement, rotation, etc.) from experiment and the value of the same variable from the proposed model.

3 Results

Figure 1 graphically illustrates the experimental [2] and parameter optimized axial (AX), transverse (PA), and FE rotation displacement-time histories. Coefficients derived from the impulsive force optimization procedure were used to simulate the PA static and oscillatory force response (100 N peak amplitude). The model predictions compare favorably with experimental results.

Figure 2 shows comparison between experimental work of [5] and current model for movement pattern of the lumbar spine in the 3 anatomical planes (axial rotation, lateral bending and flextion-extension) during right rotational mobilization movement with respect to the initial starting positions.

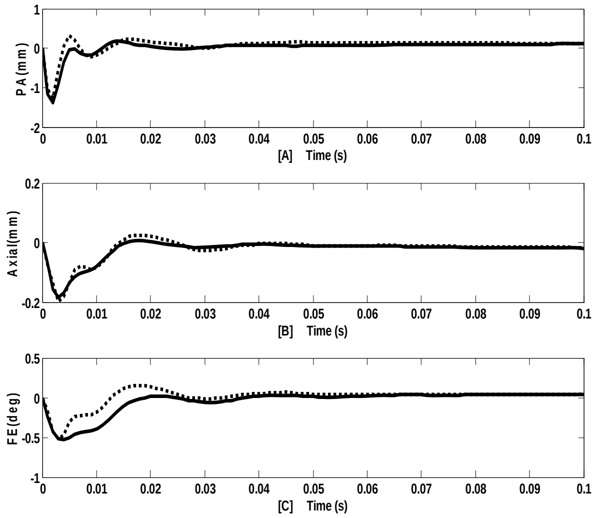

Figure 1: L_3-L_4 displacement-time responses (PA, Axial, and FE rot) to a 100 N impulsive force delivered to the L_3 segment. Solid and dashed lines illustrate the model predictions and Keller's response, respectively

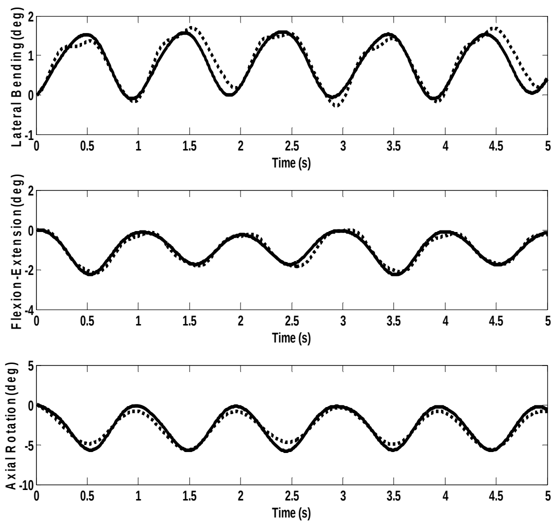

Figure 2: Typical movement pattern of the lumbar spine in the 3 anatomical planes during right rotational mobilization movement with respect to the initial starting positions [Evans et al. experimental work [5] (dotted line), Model (continuous line)]

Figure 3 compares between Ralph et. al. (2005) [7] work and the current model for the FE response for cyclic excitation with magnitude 5N.m . A least-squares linear fit is also ploted which agrees with the model predection.

A number of improvements to the existing lumbar spine mathematical model will be made, including:

• Addition of rigid masses and FJS representative of the pelvis and thorax. In the prone-lying position used to treat patients, the thorax and pelvis support the lumbar spine and play an important role in load transmission and damping. Additional degrees of freedom will be added to the constraint and coupling equations consistent with a recently published lumbar spine lumped parameter model.

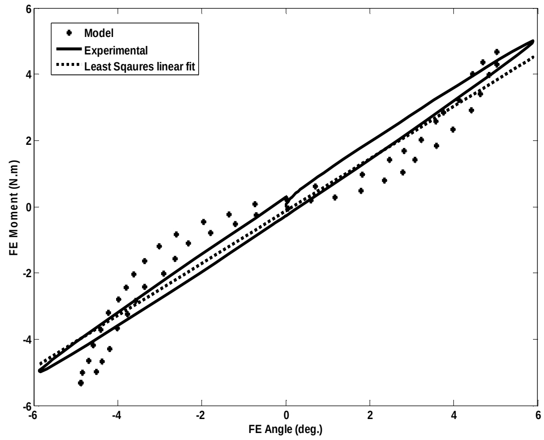

Figure 3: Flexion-Extension angular response due to sinusoidal input

• Generalization of the model to include both anterior and posterior elements of the vertebrae. The existing model lumps the anterior (centrum) and posterior (facet joints) columns of the spine into a single structure and therefore does not accurately simulate the inherently non-linear axial, transverse (PA) and rotational load-displacement behavior of the vertebral column. Additional degrees of freedom will be added to the constraint and coupling equations.

• Ability to model forces distributed over more than one vertebral segment. During manual manipulation, forces are often applied in a manner that involves direct loading of more than one spine segment. The input force and moment constraint equations will therefore be generalized to accommodate more complex loading conditions consistent with chiropractic practice.

The suggested changes present an efficient multi-resolution approach to model the Lumbosacral section of the spine for clinical evaluation. It poses two algorithms. The first algorithm (see figure 4) models the Lumbosacral section globally, with low level of details, as three-dimensional, beam–column model [1]. This algorithm studies the overall motion response of the whole lumbar spine (L1-sacrum) under the influence of gravitational and active muscle loads. Such a continuum spine model can simulate the overall motion response of the human spine provided the model utilizes appropriate flexural rigidity values. Such modeling methodology has been used successfully in the past [2.3.4]. We will compare this to the modified version of our rigid body (static equilibrium and add the sacrum vertebra) above.

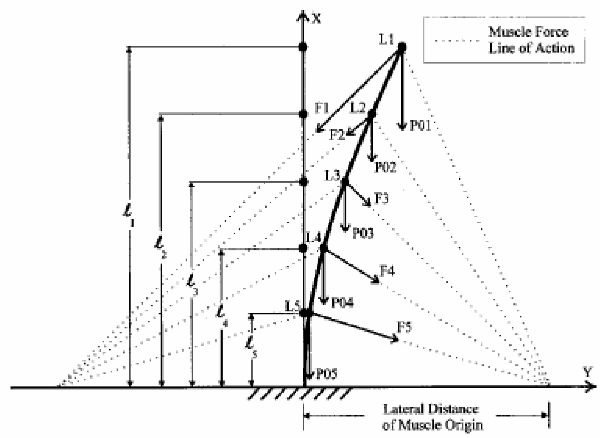

Figure 4:This is a figure for two dimensions spine model as a beam (we will have similar view in the other plane)

The second algorithm models the Lumbosacral section locally, with high level of details (see figure 5). It uses mass-spring-damper system or finite elements to model a spinal motion segment (two vertebrae and a disc). The algorithm uses the position and orientation results from algorithm one and calculate the stresses and deflection in the disc, and spinal cord compression.

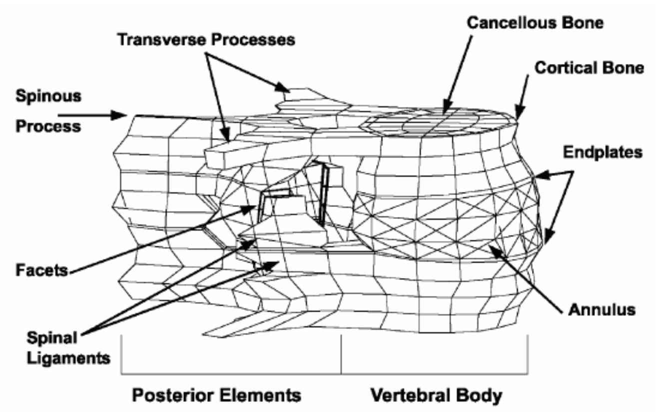

Figure 5: Finite element model of a motion segment (we will use a model for L5-S1)

4 Conclusion

This research provides quantitative tools for diagnosis of spinal disorders and will assist chiropractic clinicians and researchers in understanding the mechanical response of the human spine to mechanical forces. Parameter optimized model simulations showed good agreement with the test data, and subsequent independent validation of the static and oscillatory displacement response demonstrated that impulsive force and rotational mobilization test data can be used to predict the lumbar spine motion response during other types of loading conditions. Lumped parameter models, therefore, provide an efficient and effective method to determine the vertebral and/or inter-vertebral displacement-time history response of the lumbar spine to static, dynamic and impact forces. The ability to characterize spine motion and to systematically identify spine stiffness and damping characteristics will provide clinicians (Chiropractors, Orthopaedic Surgeons) and researchers with important biomechanical information and visualization tools that can be used to assist in the diagnosis and treatment of spinal disorders, including low back pain.

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