A new study on the optimal prediction of partial transmission ratios of three-step helical gearboxes with second-step double gear-sets

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Abstract: - This paper presents a new study on the applications of the optimization and regression techniques for optimal prediction of partial ratios of three-step helical gearboxes with second-step double gear-sets for getting different objectives including the minimal gearbox length, the minimal gearbox cross section dimension and the minimal mass of gears. In the study, based on the condition of the moment equilibrium of a mechanic system including three gear units and their regular resistance condition, three optimization problems for getting minimal gearbox length, the minimal gearbox cross section dimension and minimal mass of gears were performed. In addition, explicit models for calculating the partial ratios of the gearboxes were found by using regression analysis technique. Using these models, the determination of the partial ratios becomes accurate and simple.

Key-Words: - Gearbox design; Optimal design; Helical gearbox; Transmission ratio.

1 Introduction

In optimal gearbox design, it is known that the optimal determination of partial transmission ratios is the most important task. This is because the partial ratios are main factors which affect the size, the mass, and the cost of the gearboxes.

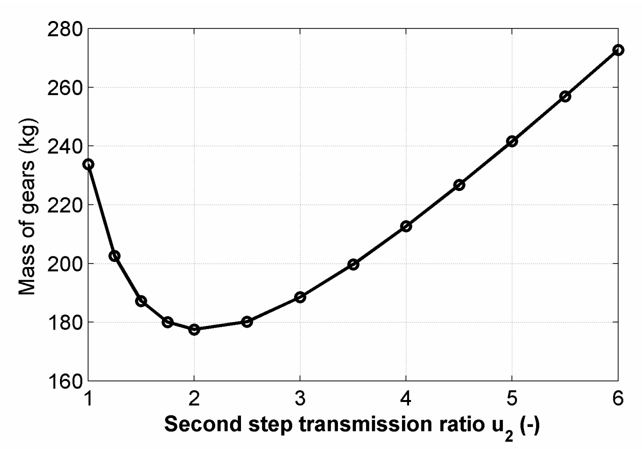

Fig. 1 Transmission ratio of second gear step versus mass of gears

Figure 1 shows the relation between the transmission ratio of the second gear step and the total mass of gears (calculated for a two step helical gearbox with the total transmission ratio is 10, the torque on the output shaft is 60.000 Nmm, the helix angle is 12^0). It follows the figure that there is an optimal value of the transmission ratio with which the total mass of gear is minimal. Therefore, optimal prediction of partial ratios of gearboxes has been subjected to many studies.

Until now, there have been many studies on the determination of the partial ratios of the three-step helical gearboxes. For the gearboxes, the partial ratios can be determined by two methods including graph method and modeling method.

In graph method, the partial ratios are predicted graphically. Using this method, V.N. Kudreavtev [1] gave a graph for the prediction of the partial ratios u_1 and u_2 of the first and the second gear steps (see Figure 2). The partial ratio of the third step then can be found by:

$$u_3 = \frac{u_h}{u_1 \cdot u_2} \tag{1}$$

Where, u_h is the total transmission ratio of the gearbox.

The graph method was also presented in other studies such as in the study of Trinh Chat [2] and in the study of A.N. Petrovski et al [3].

In modeling method, based on the results of optimization problems, models for the determination of the partial ratios are found in order to get different objectives. Using this method, Romhild I and Linke H. [4] proposed the following models for the prediction of the partial ratios of the first and the second step in order to get the minimal gearbox mass:

$$u_1 = 0.4643 \cdot u_h^{0.609} \tag{2}$$

$$u_2 = 1.205 \cdot u_h^{0.262} \tag{3}$$

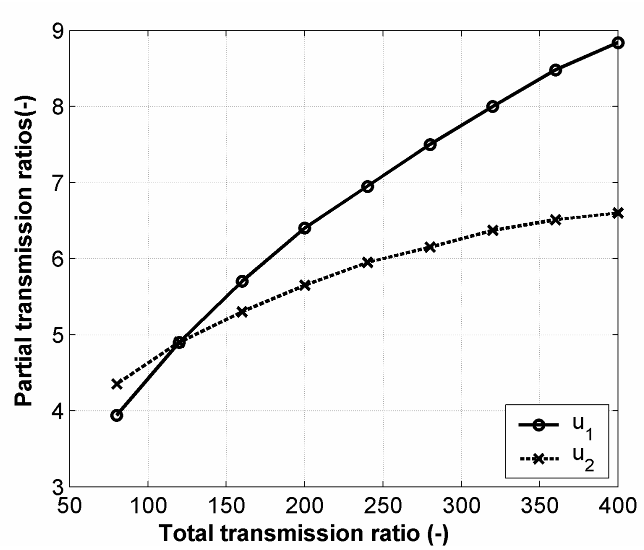

Fig. 2 Determination of partial ratios of three-step helical gearboxes [1]

To find the minimal mass of gears of the gearboxes, Vu Ngoc Pi et al. [5] carried out a study on optimal determination the partial ratios for getting the minimal mass of gears. In the study, the partial ratios of gear steps 2 and 3 are calculated as follows:

$$u_1 = 0.314 \cdot \sqrt[3]{u_h^2} \tag{4}$$

$$u_2 = 1.33 \cdot \sqrt[4]{u_h} \tag{5}$$

In both studies [4] and [5], after calculating the partial ratios u_1 and u_2, the partial ratio of the third gear unit u_3 is determined by Equation 1.

Recently, Vu Ngoc Pi [6] introduced a new study on optimal prediction of the partial transmission ratios in order to get the minimal cross section dimension of the gearbox. The partial ratios of the second and the third gear steps are determined as follows:

$$u_2 \approx 1.0134 \cdot \frac{K_{c2}^{0.4291}}{K_{c3}^{0.144}} \cdot \frac{\psi_{ba2}^{0.4294}}{\psi_{ba3}^{0.1442}} \cdot \frac{u_h^{0.2848}}{\psi_{ba1}^{0.2853}} \tag{6}$$

$$u_3 \approx 1.0269 \cdot \frac{K_{c3}^{0.4279}}{K_{c2}^{0.2854}} \cdot \frac{\psi_{ba3}^{0.428}}{\psi_{ba2}^{0.2854}} \cdot \frac{u_h^{0.1423}}{\psi_{ba1}^{0.1426}} \tag{7}$$

In which, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are coefficients of helical gear face width of the first, the second and the third gear units, respectively; k_{c2} and k_{c3} are coefficients.

The partial ratio of the first gear step u_1 then is calculated by the following equation:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \tag{8}$$

It is clear that the modeling method is more accurate and easier to determine the partial ratios than the graph method. Especially, the method can be usefully applied in computer programming or other applications. In addition, it is allowed to have different objectives when using the modeling method for determining the partial transmission ratios.

From previous studies, it is apparent that there have been many researches on the splitting the total transmission ratio for three-step helical gearboxes. However, the optimal determination of the partial ratios of the helical gearbox with second-step double gear-sets has not been investigated. In this paper, a study on optimal calculation of partial ratios for the gearboxes for getting the minimal gear length, the minimal gearbox cross section dimension and the minimal mass of gears are presented.

2 Determination of the gearbox length

In practice, the length of a three-step helical gearbox with second-step double gear-sets is decided by the dimension of L which can be determined as follows (see Figure 3):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + a_{w3} + \frac{d_{w23}}{2} \tag{9}$$

The center distance of the first gear step can be calculated by the following equation:

$$a_{w1} = \frac{d_{w11}}{2} + \frac{d_{w21}}{2} = \frac{d_{w21}}{2} \cdot \left(\frac{d_{w11}}{d_{w21}} + 1 \right)$$

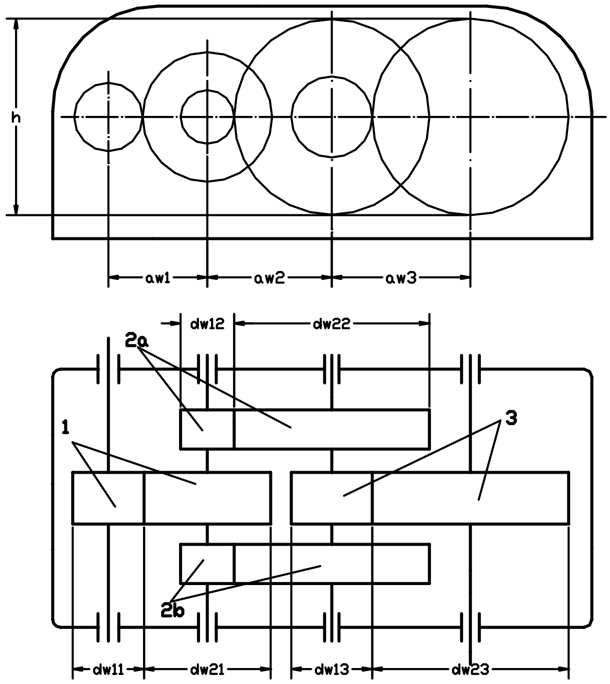

Fig. 3 Calculating schema for three-step helical gearboxes with second-step double gear-sets

Or we have

$$a_{w1} = \frac{d_{w21}}{2} \left(\frac{1}{u_1} + 1 \right) \tag{10}$$

Using the same way, the center distances of the second and the third steps are calculated by:

$$a_{w2} = \frac{d_{w22}}{2} \left(\frac{1}{u_2} + 1 \right) \tag{11}$$

$$a_{w3} = \frac{d_{w23}}{2} \left(\frac{1}{u_3} + 1 \right) \tag{12}$$

Where,

u_1, u_2 and u_3 –the transmission ratios of the first, the second and the third gear units, respectively;

d_{w11}, d_{w12} and d_{w13} are pitch diameters (mm) of driver gears of the first, the second and the third gear units, respectively;

d_{w21}, d_{w22} and d_{w23} are pitch diameters (mm) of driven gears of the first, the second and the third gear units, respectively;

ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are coefficients of helical gear face width of the first, the second and the third gear units, respectively;

a_{w1}, a_{w2} and a_{w3} are center distances (mm) of the first, the second and the third gear units, respectively.

From Equations 10, 11 and 12 and with the note that $d_{w11} = d_{w21}/u_1$, Equation 9 can be rewritten as follows:

$$L = \frac{d_{w21}}{2} \cdot \left(\frac{2}{u_1} + 1 \right) + \frac{d_{w22}}{2} \cdot \left(\frac{1}{u_2} + 1 \right) + \frac{d_{w23}}{2} \cdot \left(\frac{1}{u_3} + 2 \right) \tag{13}$$

For the first helical gear unit, the design equation for pitting resistance is calculated by the following equation [7]:

$$\sigma_{H1} = Z_{M1} Z_{H1} Z_{\varepsilon 1} \sqrt{\frac{2T_{11} K_{H1} \sqrt{u_1 + 1}}{b_{w1} d_{w11}^2 u_1}} \le \left[\sigma_{H1} \right] \tag{14}$$

From (14) we have:

$$\left[T_{11} \right] = \frac{b_{w1} \cdot d_{w11}^2 \cdot u_1}{2 \cdot (u_1 + 1)} \frac{\left[\sigma_{H1} \right]^2}{K_{H1} \cdot \left(Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1} \right)^2} \tag{15}$$

Where, b_{w1} and d_{w11} are calculated by the following equations:

$$b_{w1} = \psi_{ba1} \cdot a_{w1} = \frac{\psi_{ba1} \cdot d_{w11} \cdot (u_1 + 1)}{2} \tag{16}$$

$$d_{w11} = \frac{d_{w21}}{u_1} \tag{17}$$

Substituting Equation 16 and 17 into Equation 15 we get

$$\left[T_{11} \right] = \frac{\psi_{ba1} \cdot d_{w21}^3 \cdot \left[K_{01} \right]}{4 \cdot u_1^2} \tag{18}$$

Where

$$\left[K_{01}\right]=\frac{\left[\sigma_{H1}\right]^2}{K_{H1}\cdot\left(Z_{M1}\cdot Z_{H1}\cdot Z_{\varepsilon1}\right)^2} \tag{19}$$

From (18) the pitch diameter d_{w21} can be calculated by

$$d_{w21}=\left(\frac{4\left[T_{11}\right]u_1^2}{\psi_{ba1}\left[K_{01}\right]}\right)^{1/3} \tag{20}$$

Calculating in the same manner, we have the following equations for the second and the third gear units:

$$d_{w22}=\left(\frac{4\left[T_{12}\right]u_2^2}{\psi_{ba2}\left[K_{02}\right]}\right)^{1/3} \tag{21}$$

$$d_{w23}=\left(\frac{4\left[T_{13}\right]u_3^2}{\psi_{ba3}\left[K_{03}\right]}\right)^{1/3} \tag{22}$$

In the above equations:

$$\left[K_{02}\right]=\frac{\left[\sigma_{H2}\right]^2}{K_{H2}\cdot\left(Z_{M2}\cdot Z_{H2}\cdot Z_{\varepsilon2}\right)^2} \tag{23}$$

$$\left[K_{03}\right]=\frac{\left[\sigma_{H3}\right]^2}{K_{H3}\cdot\left(Z_{M3}\cdot Z_{H3}\cdot Z_{\varepsilon3}\right)^2} \tag{24}$$

Where,

Z_{M1}, Z_{M2}, Z_{M3} are coefficients which consider the effects of the gear material when calculate the pitting resistance of the first, the second and the third gear units, respectively;

Z_{H1}, Z_{H2}, Z_{H3} are coefficients which consider the effects contact surface shape when calculate the pitting resistance of the first, the second and the third gear units, respectively;

$Z_{\varepsilon1}, Z_{\varepsilon2}$, $Z_{\varepsilon3}$ are coefficients which consider the effects of the contact ratio when calculate the pitting resistance of the first, the second and the third gear units, respectively;

σ_{H1}- contact stresses of the first gear unit (N/mm^2);

$\left[\sigma_{H1}\right], \left[\sigma_{H2}\right]$ and $\left[\sigma_{H3}\right]$ are allowable contact stresses (N/mm^2) of the first, the second and the third gear units, respectively.

From the condition of the moment equilibrium of the first gear step and the regular resistance condition of the system we have:

$$\frac{T_r}{T_{11}}=\frac{\left[T_r\right]}{\left[T_{11}\right]}=u_1\cdot u_2\cdot u_3\cdot\eta_{brt}^3\cdot\eta_o^3 \tag{25}$$

In which,

η_{brt} -helical gear transmission efficiency; η_{brt} is from 0.96 to 0.98 [7];

η_o -transmission efficiency of a pair of rolling bearing ; η_o is from 0.99 to 0.995 [7].

Choosing $\eta_{brt}=0.97$, $\eta_o=0.992$ and substituting them into Equation 24 we get

$$\left[T_{11}\right]=\frac{\left[T_r\right]}{0.8909\cdot u_1\cdot u_2\cdot u_3} \tag{26}$$

Substituting (26) into (20) we have

$$d_{w21}=\left(\frac{4.4898\cdot\left[T_r\right]\cdot u_1}{\psi_{ba1}\cdot\left[K_{01}\right]\cdot u_2\cdot u_3}\right)^{1/3} \tag{27}$$

For the second step, the following equation can be given

$$\frac{T_r}{2\cdot T_{12}}=\frac{\left[T_r\right]}{2\cdot\left[T_{12}\right]}=u_2\cdot u_3\cdot\eta_{brt}^2\cdot\eta_o^2 \tag{28}$$

As it was done with the first step, by choosing $\eta_{brt}=0.97$ and $\eta_o=0.992$ Equation 28 can be rewritten as follows:

$$\left[T_{12}\right]=\frac{\left[T_r\right]}{1.8518\cdot u_2\cdot u_3} \tag{29}$$

Where,

$\left[T_{11}\right]$, $\left[T_{12}\right]$ -torque (Nmm) on the driver shaft of the first and the second gear units, respectively;

$\left[T_r\right]$ –torque (Nmm) on the out put shaft.

Substituting (29) into (21) we can have d_{w22} as follows:

$$d_{w22}=\left(\frac{2.1601\cdot\left[T_r\right]\cdot u_2}{\psi_{ba2}\cdot\left[K_{02}\right]\cdot u_3}\right)^{1/3} \tag{30}$$

Using the same way, the pitch diameter for the third step d_{w23} is determined by the following equation:

$$d_{w23}=\left(\frac{4.1571\cdot\left[T_r\right]\cdot u_3}{\psi_{ba3}\cdot\left[K_{03}\right]}\right)^{1/3} \tag{31}$$

Substituting (27), (30) and (31) into Equation 13 with the note that $u_1=u_h/\left(u_2\cdot u_3\right)$, the length of the gearbox can be rewritten as follows:

$$L=\frac{1}{2}\left(\frac{\left[T_r\right]}{\left[K_{01}\right]}\right)^{1/3}\left[\left(\frac{4.4898\cdot u_h}{\psi_{ba1}\cdot u_2^2\cdot u_3^2}\right)^{1/3}\cdot\left(\frac{2\cdot u_2\cdot u_3}{u_h}+1\right)+\right.$$
$$\left.+\left(\frac{2.1601\cdot u_2}{\psi_{ba2}\cdot K_{c2}\cdot u_3}\right)^{1/3}\cdot\left(\frac{1}{u_2}+1\right)+\left(\frac{4.1571\cdot u_3}{\psi_{ba3}\cdot K_{c3}}\right)^{1/3}\cdot\left(\frac{1}{u_3}+2\right)\right] \tag{32}$$

Where, $K_{c2}=\frac{\left[K_{02}\right]}{\left[K_{01}\right]}$ and $K_{c3}=\frac{\left[K_{03}\right]}{\left[K_{01}\right]}$ are coefficients.

3 Determination of the dimension of the gearbox cross section

The cross section of a three-step helical gearbox with second-step double gear-sets is decided by the dimension of A which is determined as follows (see Figure 2):

$$A=L\cdot h \tag{33}$$

Where

L-the length of the gearbox (mm) which is determined by Equation 32.

h - the height of the gearbox (mm); h is determined by the maximal value among pitch diameters d_{w21}, d_{w22} and d_{w23} or it can be predicted by the following equation:

$$h=\max\left(d_{w21},d_{w22},d_{w23}\right) \tag{34}$$

From (27), (30) and (31), Equation 33 can be rewritten as follows:

$$h=\max\left(\frac{4.4898\cdot u_h}{\psi_{ba1}\cdot u_2^2\cdot u_3^2};\frac{2.1601\cdot u_2}{\psi_{ba2}\cdot k_{c2}\cdot u_3};\frac{4.1571\cdot u_3}{\psi_{ba3}\cdot k_{c3}}\right) \tag{35}$$

4 Determination of the mass of gears

For a three-step helical gearbox with second-step double gear-sets, the mass of gears can be determined as follows:

$$G=G_1+2\cdot G_{2a}+G_3 \tag{36}$$

Where, G_1, G_{2a} and G_3 are the mass of gears of unit 1, unit 2a and unit 3, respectively (see Figure 2). G_1 can be determined as follows:

$$G_1=\frac{\pi\cdot\rho\cdot d_{w11}^2\cdot b_{w1}\cdot(e_1+e_2\cdot u_1^2)}{4} \tag{37}$$

In which,

ρ -the density of the gear material (kg/m^3);

e_1, e_2 -volume coefficients of gear 1 and 2, respectively; the volume coefficient of a gear is ratio of the actual volume to the theoretical volume of the gear. For a helical gear unit, we can have e_1=1; e_2=0.6 [2].

From Equation 37, with $\psi_{bd1}=b_{w1}/d_{w11}$ is diameter coefficient of the first helical gear step, the mass of gears of unit 1 can be calculated as follows:

$$G_1=\frac{\pi\cdot\rho\cdot\psi_{bd1}\cdot d_{w11}^3\cdot(1+0.6\cdot u_1^2)}{4} \tag{38}$$

For the first helical gear unit, the allowable torque on the driving shaft 1 can be predicted by the following equation [7]:

$$\left[T_{11}\right]=\frac{\psi_{bd1}\cdot d_{w11}^3\cdot u_1\cdot[K_{01}]}{2\cdot(u_1+1)} \tag{39}$$

It follows Equation 39 that:

$$\psi_{bd1}\cdot d_{w11}^3=\frac{2\cdot(u_1+1)\cdot[T_{11}]}{u_1\cdot[K_{01}]} \tag{40}$$

Substituting (40) into (38) the mass of gears of the first step is determined as follows

$$G_1 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{11}] \cdot (u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{4 \cdot [K_{01}] \cdot u_1} \quad (41)$$

By calculating in the same way, the following equations were found for the calculation of the mass of gears of the second gear unit G_{2a} and the third gear unit G_3:

$$G_{2a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_{12}] \cdot (u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{4 \cdot [K_{02}] \cdot u_2} \quad (42)$$

$$G_3 = \frac{2 \cdot \pi \cdot \rho \cdot [T_{13}] \cdot (u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{4 \cdot [K_{03}] \cdot u_3} \quad (43)$$

In the above equations, $[K_{01}]$, $[K_{02}]$ and $[K_{03}]$ are determined by Equation 19, 23 and 24, respectively.

Based on the condition of the moment equilibrium of the mechanic system which includes the gear units and the regular resistance condition of the system we have:

$$\frac{T_r}{T_{11}} = \frac{[T_r]}{[T_{11}]} = u_1 \cdot u_2 \cdot u_3 \cdot \eta_{brt}^3 \cdot \eta_o^3 \quad (44)$$

As it was done in Section 3, by choosing $\eta_{brt} = 0.97$, $\eta_o = 0.992$ and substituting them into Equation 44 we have

$$[T_{11}] = \frac{[T_r]}{0.8909 \cdot u_1 \cdot u_2 \cdot u_3} \quad (45)$$

Substituting (45) into (41) the following equation can be obtained:

$$G_1 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \cdot \frac{(u_1 + 1) \cdot (1 + 0.6 \cdot u_1^2)}{0.8909 \cdot u_1^2 \cdot u_2 \cdot u_3} \quad (46)$$

For the second helical gear unit it can be written as:

$$\frac{T_r}{2 \cdot T_{12}} = \frac{[T_r]}{2 \cdot [T_{12}]} = u_2 \cdot u_3 \cdot \eta_{brt}^2 \cdot \eta_o^2 \quad (47)$$

With $\eta_{brt} = 0.97$ and $\eta_o = 0.992$ Equation 47 becomes

$$[T_{12}] = \frac{[T_r]}{1.8518 \cdot u_2 \cdot u_3} \quad (48)$$

From Equations 48 and 42 we get the mass of gear of the second unit:

$$G_{2a} = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{02}]} \cdot \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{1.8518 \cdot u_2^2 \cdot u_3} \quad (49)$$

In exactly similar manner, we can derive the mass of gears of unit 3 as follows:

$$G_3 = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{03}]} \cdot \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{0.9622 \cdot u_3^2} \quad (50)$$

Substituting (46), (49) and (50) into (28) with the note that $u_1 = u_h / (u_2 \cdot u_3)$, the mass of gears of the gearbox can be rewritten as follows:

$$G = \frac{2 \cdot \pi \cdot \rho \cdot [T_r]}{4 \cdot [K_{01}]} \left\{ \frac{(u_h + u_2 \cdot u_3) \cdot \left[1 + 0.6 \cdot u_h^2 / \left(u_2^2 \cdot u_3^2\right)\right]}{0.8909 \cdot u_h^2} + \frac{(u_2 + 1) \cdot (1 + 0.6 \cdot u_2^2)}{1.8518 \cdot k_{C2} \cdot u_2^2 \cdot u_3} + \frac{(u_3 + 1) \cdot (1 + 0.6 \cdot u_3^2)}{0.9622 \cdot k_{C3} \cdot u_3^2} \right\} \quad (51)$$

where, $k_{C2} = [K_{02}]/[K_{01}]$ and $k_{C3} = [K_{03}]/[K_{01}]$.

5 Optimization problems and results

5.1 For finding the optimal partial ratios for getting the minimal gearbox length

5.1.1 Optimization problem

From Equations 32, the optimal problem for determining the minimal gearbox length can be expressed as follows:

The objective function is:

$$\min L = f(u_h; u_2; u_3) \quad (52)$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max}$$

$$K_{c2\min} \le K_{c2} \le K_{c2\max} \quad (53)$$

$$K_{c3\min} \le K_{c3} \le K_{c3\max}$$

$$\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}$$

$$\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}$$

$$\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}$$

A computer program was built in order to solve the above optimization problem. The used data in the program were: K_{c2} and K_{c3} are from 1 to 1.3, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are from 0.25 to 0.4 [7], u_2 and u_3 are from 1 to 9 [1]; u_h is from 30 to 120.

5.1.2 Results and discussions

Figure 4 describes the relation between the partial transmission ratios and the total transmission ratio when calculating with the following values of coefficients: K_{c2}=1.2, K_{c3}=1.3, ψ_{ba1}=0.25, ψ_{ba2}=0.3 and ψ_{ba3}=0.35.

It is observed that with the increase of the total ratio u_h the partial ratios increase. In addition, it is found that with the increase of the total ratio, the increase of partial ratio of the first step u_1 is much larger than that of the third step u_3. It can be explained that with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driver shaft of the first gear unit T_{11}. For that reason, the increase of the partial ratio u_3 should be much smaller than that of u_1 in order to reduce the gearbox length.

To find the models for determination of the partial ratios, regression analysis was carried out based on the results of the optimization program. The following regression models then were found for finding the optimal values of partial ratios u_2 and u_3 of the second and the third step, respectively:

$$u_2 \approx 1.8452 \cdot \frac{\left(K_{C2} \cdot \psi_{ba2}\right)^{0.4386}}{\left(K_{C3} \cdot \psi_{ba3}\right)^{0.1328}} \cdot \frac{u_h^{0.2649}}{\psi_{ba1}^{\;\;0.3061}} \quad (54)$$

$$u_3 \approx 1.0689 \cdot \frac{\left(K_{C3} \cdot \psi_{ba3}\right)^{0.4386}}{\left(K_{C2} \cdot \psi_{ba2}\right)^{0.2974}} \cdot \frac{u_h^{0.1219}}{\psi_{ba1}^{\;\;0.1412}} \quad (55)$$

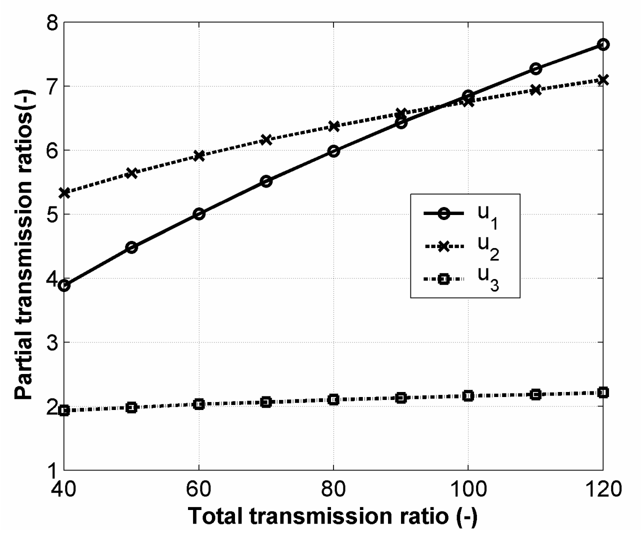

Fig. 4: Partial transmission ratios versus the total transmission ratio

The above regression models fit quite well with the calculated data. The coefficients of determination for the models for the second step and the third step were $R^2 = 0.9999$ and $R^2 = 0.9998$, respectively.

Equations 54 and 55 can be used for the prediction of the partial transmission ratios u_2 and u_3 of gear steps 2 and 3, respectively. After finding u_2 and u_3, the partial transmission ratio of the first gear step u_1 then can be calculated by the following equation:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \quad (56)$$

5.2 For finding the optimal partial ratios for getting the minimal cross section dimension of the gearbox

5.2.1 Optimization problem

From Equations 33, 32 and 35, the optimal problem for finding the minimal cross section dimension of the gearbox can be expressed as follows:

The objective function is:

$$\min A = f(u_h; u_2; u_3) \tag{57}$$

With the following constraints:

$$\begin{aligned}
&u_{h\min} \le u_h \le u_{h\max}\\
&u_{2\min} \le u_2 \le u_{2\max}\\
&u_{3\min} \le u_3 \le u_{3\max}\\
&K_{c2\min} \le K_{c2} \le K_{c2\max}\\
&K_{c3\min} \le K_{c3} \le K_{c3\max}\\
&\psi_{ba1\min} \le \psi_{ba1} \le \psi_{ba1\max}\\
&\psi_{ba2\min} \le \psi_{ba2} \le \psi_{ba2\max}\\
&\psi_{ba3\min} \le \psi_{ba3} \le \psi_{ba3\max}
\end{aligned} \tag{58}$$

For solving the above optimization problem a computer program was built. The following data were used in the program: K_{c2} and K_{c3} are from 1 to 1.3, ψ_{ba1}, ψ_{ba2} and ψ_{ba3} are from 0.25 to 0.4 [7], u_2 and u_3 are from 1 to 9 [1]; u_h is from 30 to 120.

5.2.2 Results and discussions

The relation between the partial transmission ratios and the total transmission ratio is described in Figure 5. The following data were used in the calculation: K_{c2}=1.2, K_{c3}=1.3, ψ_{ba1}=0.25, ψ_{ba2}=0.3 and ψ_{ba3}=0.35.

As in the optimization problem for getting the minimal gearbox length (see Subsection 5.1.2), with the increase of the total transmission ratio u_h the partial transmission ratios increase. Also, while the partial transmission ratio of the first step u_1 increases very much, the partial transmission ratio of the third step u_3 nearly does not change when the total ratio increases. This is because with the increase of the total ratio u_h, the torque on the output shaft T_r is much larger than that on the driver shaft of the first gear unit T_{11}. Therefore, the partial transmission ratio u_3 should be much smaller than the partial transmission ratio u_1 in order to reduce the gearbox size as well as the gearbox cross section dimension.

It is observed that when the total transmission ratio is beyond about 90, the partial transmission ratio of the first gear unit is larger than 9 (see Figure 5) - the limit of the transmission ratio of a helical gear set [7]. Consequently, the objective of getting the minimal gearbox cross section dimension should be used in cases that the total transmission ratio is less than 90.

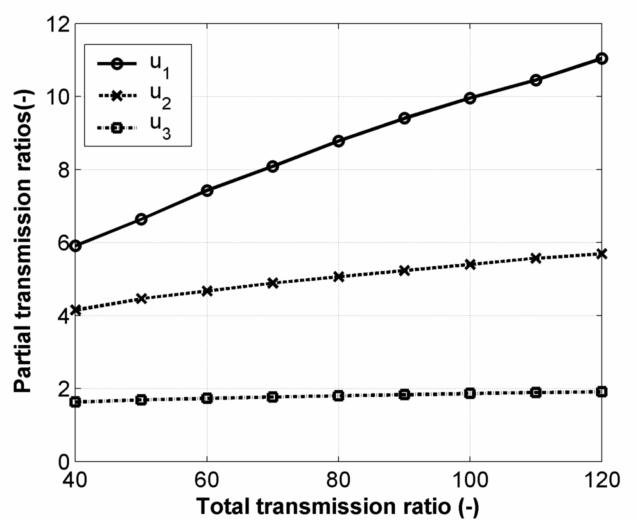

Fig. 5: Partial transmission ratios versus the total transmission ratio

From the calculated results of the optimization program, the following regression models were found to determine the optimal values of partial ratios u_2 and u_3 of the second and the third step, respectively:

$$u_2 \approx 1.3655 \cdot \frac{\left(K_{C2} \cdot \psi_{ba2}\right)^{0.43}}{\left(K_{C3} \cdot \psi_{ba3}\right)^{0.1443}} \cdot \left(\frac{u_h}{\psi_{ba1}}\right)^{0.285} \tag{59}$$

$$u_3 \approx 0.8409 \cdot \frac{(K_{C3} \cdot \psi_{ba3})^{0.428}}{(K_{C2} \cdot \psi_{ba2})^{0.2852}} \cdot \left(\frac{u_h}{\psi_{ba1}}\right)^{0.1424} \tag{60}$$

The above models fit very well with the calculated data. The coefficients of determination for the models for the second step and the third step were $R^2 = 0.9995$ and $R^2 = 0.9998$, respectively.

Equations 59 and 60 are used to calculate the transmission ratios u_2 and u_3 of steps 2 and 3, respectively. From u_2 and u_3, the transmission ratio of the first step u_1 then can be determined as follows:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \tag{61}$$

5.3 For finding the optimal partial ratios for getting the minimal mass of gears

5.3.2 Optimization problem

From Equation 51, the optimal problem for determining the partial transmission ratio in order to get the minimal mass of the gears is described as follows:

Objective function:

$$\min G = f(u_h; u_2; u_3) \tag{62}$$

With the following constraints:

$$u_{h\min} \le u_h \le u_{h\max}$$

$$u_{2\min} \le u_2 \le u_{2\max}$$

$$u_{3\min} \le u_3 \le u_{3\max} \tag{63}$$

$$k_{C2\min} \le k_{C2} \le k_{C2\max}$$

$$k_{C3\min} \le k_{C3} \le k_{C3\max}$$

For performing the above optimization problem, a computer program was built. The data used in the program were follows: u_2 and u_3 were from 1 to 9 [1], k_{C2} and k_{C3} are from 1 to 1.3 [7] and u_h was from 30 to 180.

5.3.2 Results and discussions

Figure 6 shows the relation between the partial transmission ratios and the total transmission ratio when the coefficients K_{c2} and K_{c3} equal 1.1.

As in Sections 5.1 and 5.2, with the increase of the total ratio u_h the partial ratio of the first step u_1 increases significantly. In contrast, the partial transmission ratio of the third step u_3 increases very slowly. The reason of that can be explained as in the above sections.

From the results of the optimization program, regression analysis was carried out and the following models were found for determining the optimal values of the partial ratios of the second and the third step:

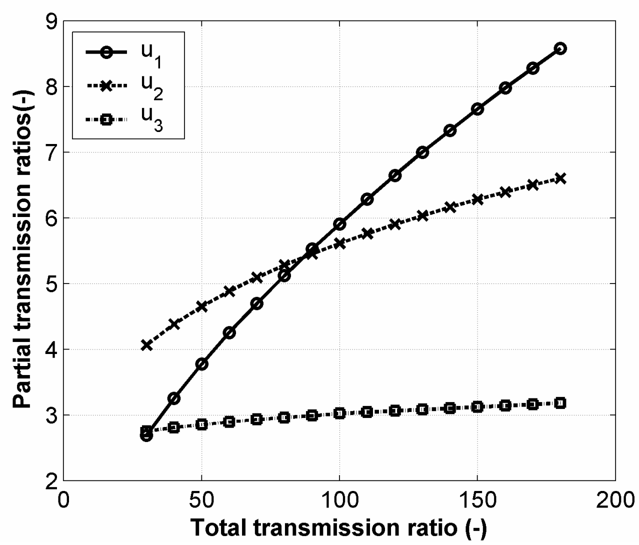

Fig. 6: Partial transmission ratios versus the total transmission ratio

$$u_2 \approx 1.5598 \cdot \frac{k_{C2}^{0.3827}}{k_{C3}^{0.089}} \cdot u_h^{0.2721} \tag{64}$$

$$u_3 \approx 2.0604 \cdot \frac{k_{C3}^{0.2984}}{k_{C2}^{0.2107}} \cdot u_h^{0.0814} \tag{65}$$

As in Subsection 5.1.2 and 5.2.2, the regression models (64) and (65) fit quite well with the calculated data. The coefficients of determination for the above models were $R^2 = 0.9998$ and $R^2 = 0.9978$, respectively.

Equations 19 and 20 are used to determine the transmission partial ratios u_2 and u_3 of the second and the third helical gear units. After finding u_2 and u_3, the transmission ratio of the first gear unit u_1 is calculated by the following equation:

$$u_1 = \frac{u_h}{u_2 \cdot u_3} \tag{66}$$

6 Conclusion

It can be concluded that the minimal gearbox length, the minimal gearbox cross section dimension and the minimal mass of gears of a three-step helical gearbox with second-step double gear-sets can be obtained by optimal splitting the total transmission ratio of the gearbox.

The objective of finding the optimal partial ratios in order to get the minimal gearbox cross section dimension must be used for the gearboxes with the total transmission ratios is less than 90.

Models for prediction of the optimal partial ratios of the gearboxes in order to get the minimal gearbox length, the minimal gearbox cross section dimension and the minimal mass of gears have been found.

Using explicit models, the partial ratios of the gearboxes can be calculated accurately and simply by.

Further researches on the optimal determination of the partial transmission ratios of helical gearboxes with four and more steps should be carried out.

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