Introduction to the General Trigonometry in Euclidian 2D-space

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Abstract: - The *General Trigonometry* is a new trend of trigonometry introduced by the author into the mathematical domain. It is introduced to replace the traditional trigonometry; it has huge advantages ahead the traditional one. It gives a general concept view of the trigonometry and forms an infinite number of trigonometry branches and each branch has its own characteristics and features. The concept of the *General Trigonometry* is completely different from the traditional one in which the study of angles will not be the relation between sides of a right triangle that describes a circle as the previous one, but the idea here is to use the relation between angles and sides of a geometrical form (e.g.: circle, elliptic, rectangle, quadrilateral …) with the internal and external circles formed by the intersection of the geometrical form and the positive parts of x'ox and y'oy axis in the Euclidian 2D space and their projections. This new concept of relations will open a huge gate in the mathematical domain and it can resolve many complicated problems that are difficult or almost impossible to solve with the traditional trigonometry, and it can describe a huge number of multi form periodic signals. The most remarkable trigonometry branches are the "Elliptical trigonometry" and the "Rectangular trigonometry" introduced by the author and published by WSEAS. The importance of these trigonometry branches is that with one function, we can produce multi signal forms by varying some parameters. In this paper, an original study is introduced and developed by the author and some few examples are discussed only to give an idea about the importance of the *General Trigonometry* and its huge application in all scientific domains especially in Mathematics, Power electronics, Signal theory and processing and in Energy Economic Systems.

Key-words: - Modern mathematics, trigonometry, angular function, multi form signal, power electronics.

1 Introduction

The traditional trigonometry is a branch of the mathematics that deals with relationships between sides and angles of a right triangle [7],[8],[9] [10]. Six principal functions (e.g. cosine, sine, tangent …) are used to produce signals that have an enormous variety of applications in all scientific domains [6],[12],[13],[15],[16],[17]. It can be considered as the basis and foundation of many domains as electronics, signal theory, astronomy, navigation, propagation of signals and many others… [17]. Now it is time to introduce a new concept of trigonometry in the Euclidean 2D-space to replace the previous one, and open new gates and new challenges by the reconstructing the science.

In this paper, the new concept of the general trigonometry is introduced and developed by the author and few examples are shown and discussed briefly. Figures are drawn and simulated using AutoCAD and Matlab.

In the second section, the general trigonometry is defined; the geometric closed curve which is the basis of this trigonometry and the relation between different elements and their intersections that form the general trigonometry are also presented. The angular functions are used to form the functions of these trigonometry branches [1],[2],[3],[4],[5]. The new functions created by this trigonometry are also defined. In the third section, three different trigonometry branches are presented and discussed briefly with some examples and finally in the fourth section, a survey on the applications in the engineering domain is presented and discussed briefly.

2 Definition of the General Trigonometry

The general trigonometry can be represented by a tree (Fig.1) in which the basic form and concept is the basis, and each branch has its own characteristics and features. Each branch has many sub-branches which are particular cases of the principal branch named "the General Trigonometry".

The General Trigonometry is based on five principal concepts:

- The geometric closed curve that defines the form of the specific trigonometry (e.g. elliptical, rectangular, Circular, Quadratic, closed parabolic …) (Fig.3.a).

-the Angular functions which are also introduced by the author, are widely used in all functions of this trigonometry [1],[2],[3],[4],[5].

- The intersection between the geometric closed curve and the relative axis that forms the internal and external circles (Fig.4).

- The intersection between the geometric closed curve, the relative axis, the internal and external circles and the half-line [od) of the first degree polynomial equation ($y = kx$) (Fig.4).

-the new concept of relationship between different segments created by the previous intersections that defines the functions of the general trigonometry.

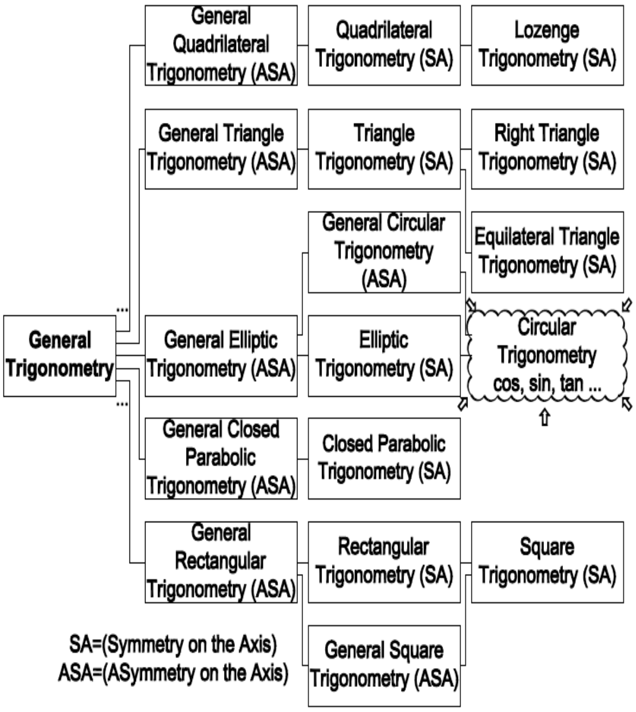
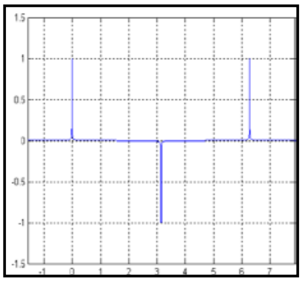
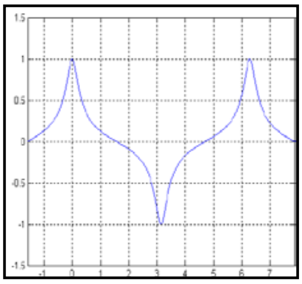


Fig.1: the General trigonometry and its Branches (the traditional trigonometry is a particular case that appears in the right part of the figure)

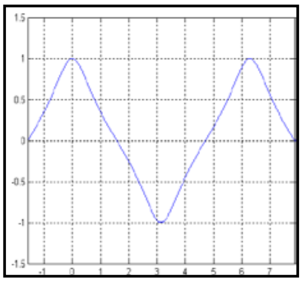
• The following example is a simple function of the "Elliptical Trigonometry" [4] in which a single function can produce more that 14 different signals by varying two parameters. The presented function is $\bar{E}jes_{i,b}(x)$. For the figures 2.a to 2.f the value of $i = 2$, for the figures 2.g to 2.l the value of $i = 1$.



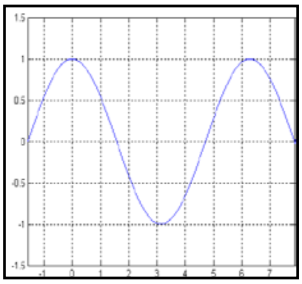
a) $i = 2, b = 0.001$



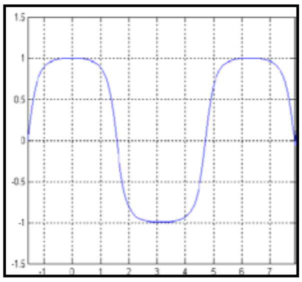
b) $i = 2, b = 0.2$



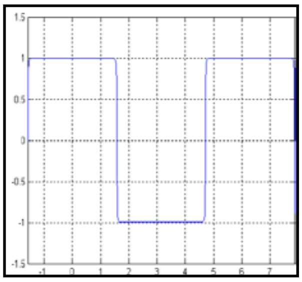
c) $i = 2, b = \sqrt{3}/3$



d) $i = 2, b = 1$

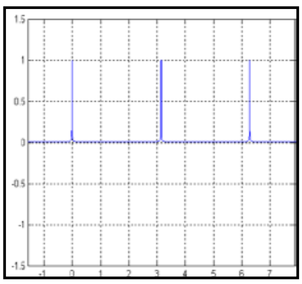


e) $i = 2, b = 3$

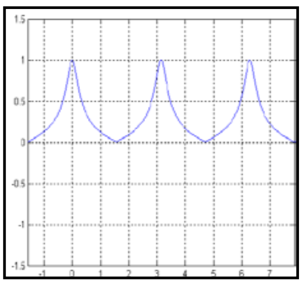


f) $i = 2, b = 90$

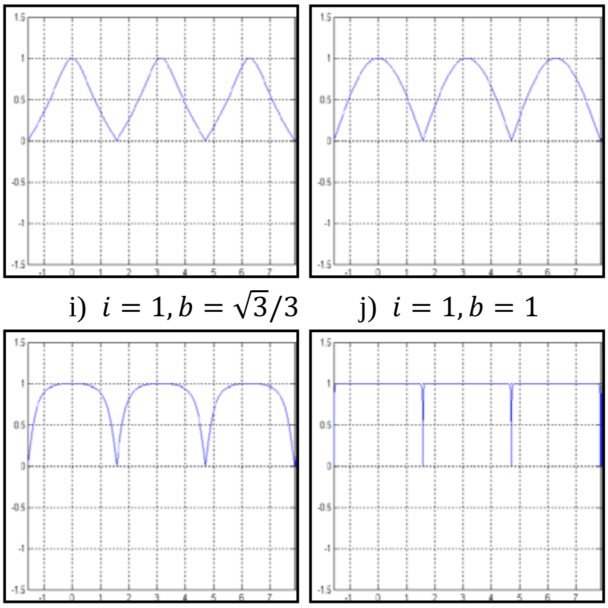
Fig. 2.a to 2.f: multi form signals of the function $\bar{E}jes_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



g) $i = 1, b = 0.001$



h) $i = 1, b = 0.2$



i) $i = 1, b = \sqrt{3}/3$ j) $i = 1, b = 1$

k) $i = 1, b = 3$ l) $i = 1, b = 90$

Fig. 2.g to 2.l: multi form signals of the function $\bar{E}jes_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using $\bar{E}jes_{i,b}(x)$:

Impulse train with positive and negative part, elliptic deflated, quasi-triangular, sinusoidal, elliptical swollen, square signal, rectangular signal, impulse train (positive part only), rectified elliptic deflated, saw signal, rectified elliptical swollen, continuous signal…

These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [17].

2.1 Definition of the Geometric Closed Curve

The Geometric closed curve has many features that describe the General trigonometry (Fig.3.a)

-The curve must be totally closed

-the curve must be uniquely identified, that means that for a specified angle from the center of the axis, a unique coordinate (x; y) and vice versa must be obtained.

-the center (0; 0) of the relative coordinate system related to the geometric closed curve must be inside the curved shape, even if the form of the shape changes.

-the closed curve must be presented in the four regions of the 2D axis, $(x \geq 0, y \geq 0)$, $(x \geq 0, y \leq 0)$, $(x \leq 0, y \geq 0)$, $(x \leq 0, y \leq 0)$.

-the closed curve must be continuous in all regions; it can be derivative or non derivative.

-the closed curve can be one function or many functions but it must respect all conditions.

-if the closed curve is Symmetric on both axis (ox) and (oy) (related to the relative coordinate system), the related trigonometry is called “form trigonometry” e.g.: Elliptic trigonometry, Square Trigonometry, Closed parabolic trigonometry… (Fig.3.b)

-if the closed curve is Asymmetric on axis (ox) or (oy) (related to the relative coordinate system), the related trigonometry is called “General form trigonometry” e.g.: General Elliptic trigonometry, General Rectangular Trigonometry… (Fig.3.c)

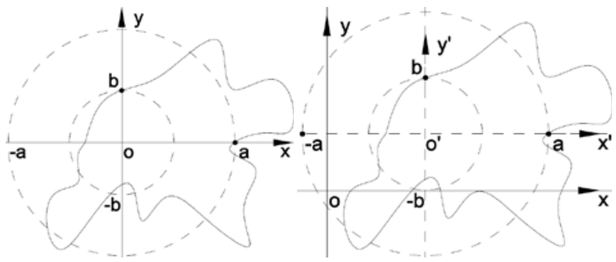


Fig.3.a: represents a Geometric Closed Curve in a relative coordinating system.

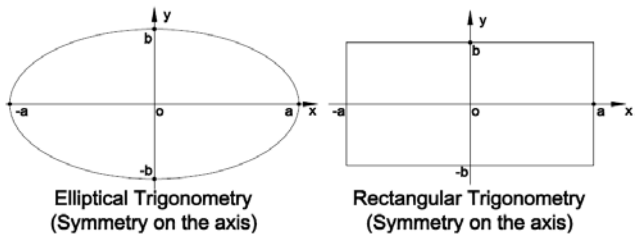


Fig.3.b: Particular cases of the general trigonometry (elliptical and rectangular) where the geometrical closed curve is symmetry on the axis.

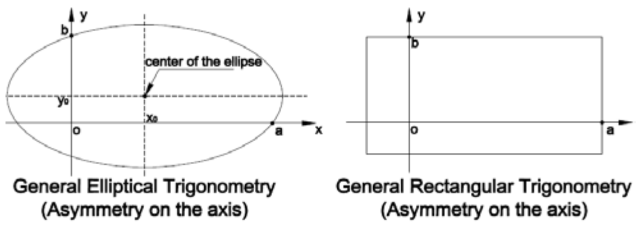


Fig.3.c: Particular cases of the general trigonometry (general elliptical and general rectangular) where the geometrical closed curve is asymmetry on the axis.

2.2 Intersections and projections of different elements of the General trigonometry on the relative axis

The intersections of the geometric closed curve with the axis (ox) and (oy) into their positive parts, create respectively two circles of radii $[oa]$ and $[ob]$, these radii can be variables or constants according to the form of the geometric closed curve, the importance of these circles is to determine the basic of the General Trigonometry (e.g.: Elliptical Trigonometry, General circular Trigonometry…).

The intersection of the half-line $[od)$ with the internal and external circles and with the geometric closed curve and their projections on axis (ox) and (oy) give many functions that have an extremely importance to create plenty of signals and forms that are very difficult to be created in the traditional trigonometry.

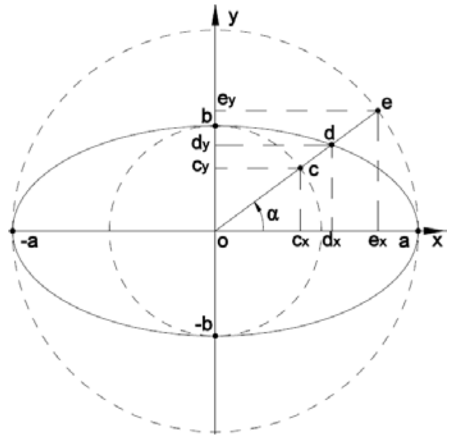


Fig.4: A branch of the General Trigonometry: "The Elliptic Trigonometry".

Definition of the letters in the Fig.4:

a: Is the intersection of the Geometric closed curve with the axis (ox) in the positive region which forms the relative circle on (ox), that can be variable.

b: Is the intersection of the Geometric closed curve with the axe (oy) in the positive region which forms the relative circle on (oy), that can be variable.

c: Is the intersection of the half-line $[od)$ with the circle of radius b.

d: Is the intersection of the half-line $[od)$ with the Geometric closed curve

e: Is the intersection of the half-line $[od)$ with the circle of radius a.

c_x: Is the projection of the point c on the ox axis.

d_x: Is the projection of the point d on the ox axis.

e_x: Is the projection of the point e on the ox axis.

c_y: Is the projection of the point c on the oy axis.

d_y: Is the projection of the point d on the oy axis.

e_y: Is the projection of the point e on the oy axis.

α: Is the angle between the (ox) axis and the half-line $[od)$.

o: Is the center (0, 0) of (ox, oy) axis.

2.3 Definition of the General Trigonometric functions $GTfun(\alpha)$

In epoch the traditional trigonometry contains only 6 principal functions: Cosine, Sine, Tangent, Cosec, Sec, Cotan. [7],[8],[9]. Now in the General Trigonometry, there are 32 principal functions and each function has its own characteristics according to the form of the geometric closed curve. E.g.: the function $(Gjes(\alpha))$ "General Jes" and the Elliptical trigonometry $(\mathrm{Ejes}(\alpha))$ is not the same [4], neither in the Rectangular Trigonometry [3] $(\mathrm{Rjes}(\alpha))$ nor in the General Triangular Trigonometry $(GTjes(\alpha))$. The same for others functions…

This variation gives the General Trigonometry a new world vision in all scientific domains and makes a new challenge in the reconstruction of the science especially when working on the economical side of the electrical power, power electronics, electrical transmission, Signal theory and many others domains [15],[16], [17].

The General trigonometric function in written using the following abbreviation "$GTfun(\alpha)$":

-the first letter "G" represents the type of the specific trigonometry if it is general case (Fig. 3.c), if not this letter is eliminated (Fig. 3.b), i.e. $GEfun(\alpha)$ is the General Elliptical function, $Efun(\alpha)$ is the Elliptical function.

-the second letter "T" represent the form of the specific trigonometry. "E" For Elliptical, "R" for rectangular, "C" for Circular, "P" for closed Parabolic… for the complicated forms, the second letter is written with an index that presents the specific form, i.e.: "T_E" for Equilateral Triangular trigonometry, "T_R" for Right Triangular

trigonometry… we can eliminate the second letter if we don't have a specific form.
-the third part " $fun(\alpha)$ " represent the specific function name that are defined in the next paragraph.

The functions of the General Trigonometry are defined as the following: (refer to Fig.4)

General Jes:

General Jes: $Gjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe}$ (1)

General Jes-x: $Gjes_x(\alpha) = \frac{od_x}{oe_x} = \frac{Gjes\,(\alpha)}{Cjes\,(\alpha)}$ (2)

General Jes-y: $Gjes_y(\alpha) = \frac{od_x}{oe_y} = \frac{Gjes\,(\alpha)}{Cmar\,(\alpha)}$ (3)

General Mar:

General Mar: $Gmar(\alpha) = \frac{od_y}{ob} = \frac{od_y}{oc}$ (4)

General Mar-x: $Gmar_x(\alpha) = \frac{od_y}{oc_x} = \frac{Gmar\,(\alpha)}{Cjes\,(\alpha)}$ (5)

General Mar-y: $Gmar_y(\alpha) = \frac{od_y}{oc_y} = \frac{Gmar\,(\alpha)}{Cmar\,(\alpha)}$ (6)

General Ter:

General Ter: $Gter(\alpha) = \frac{Gmar\,(\alpha)}{Gjes\,(\alpha)}$ (7)

General Ter-x:

$Gter_x(\alpha) = \frac{Gmar_x(\alpha)}{Gjes_y(\alpha)} = Gter(\alpha) \cdot Cter(\alpha)$ (8)

General Ter-y:

$Gter_y(\alpha) = \frac{Gmar_y(\alpha)}{Gjes_x(\alpha)} = \frac{Gter\,(\alpha)}{Cter\,(\alpha)}$ (9)

General Rit:

General Rit: $Grit(\alpha) = \frac{od_x}{ob} = \frac{od_x}{oc} = \frac{Gmar\,(\alpha)}{Cter\,(\alpha)}$ (10)

General Rit-y: $Grit_y(\alpha) = \frac{od_x}{oc_y} = \frac{Grit\,(\alpha)}{Cmar\,(\alpha)}$ (11)

General Raf:

General Raf:

$Graf(\alpha) = \frac{od_y}{oa} = Cter(\alpha).Gjes(\alpha)$ (12)

General Raf-x: $Graf_x(\alpha) = \frac{od_y}{oe_x} = \frac{Graf(\alpha)}{Cjes\,(\alpha)}$ (13)

General Ber:

General Ber: $Gber(\alpha) = \frac{Graf\,(\alpha)}{Grit\,(\alpha)}$ (14)

General Ber-x:

$Gber_x(\alpha) = \frac{Graf_x(\alpha)}{Grit_y(\alpha)} = Gber(\alpha) \cdot Cter(\alpha)$ (15)

General Ber-y:

$Gber_y(\alpha) = \frac{Graf_y(\alpha)}{Grit_x(\alpha)} = \frac{Gber\,(\alpha)}{Cter\,(\alpha)}$ (16)

$GTfun^{-1}(\alpha)$ is the inverse function of $GTfun(\alpha)$. $(GTfun^{-1}(\alpha) = 1/GTfun(\alpha))$. In this way the reduced number of functions is equal to 32 principal functions.

E.g.: $Cjes^{-1}(\alpha) = \frac{1}{Cjes\,(\alpha)} = \frac{1}{cos\,(\alpha)} = sec(\alpha)$

$Gjes_x^{-1}(\alpha) = 1/Gjes_x(\alpha)$

Remark: $Cjes(\alpha) \Leftrightarrow cos(\alpha)$; $Cmar(\alpha) \Leftrightarrow sin(\alpha)$; $Cter(\alpha) \Leftrightarrow tan(\alpha)$; $Cjes_y(\alpha) \Leftrightarrow cotan(\alpha)$…

$Cjes$, $Cmar$ and $Cter$ are respectively equivalent to cosine, sine and tangent. These functions that appear in the previous formulae are particular cases of the "*Circular Trigonometry*", in the next section of this study; one will see the "*Circular trigonometry*". The names of the cosine, sine and tangent are replaced respectively by *Circular Jes*, *Circular Mar* and *Circular Ter*.

2.4 Definition of the Absolute General Trigonometric functions $\bar{G}\boldsymbol{Tfun}(\boldsymbol{\alpha})$

The *Absolute General Trigonometry* is introduced to create the absolute value of a function by varying only one parameter without using the traditional absolute value "| |" for example: $|\cos(x)|$ is absolute of $\cos(x)$ in which it gives only the positive part. The advantage is that we can change and control the sign of a General Trigonometric function without using the absolute value in an expression. We will show some functions to get an idea about its importance. To obtain the absolute General trigonometry for a specified function (e.g.: $Gjes(\alpha)$) we must multiply it by the corresponding Angular Function (e.g.: $\left(ang_x(\alpha)\right)^i$) in a way to obtain the original function if i is even, and to obtain the absolute value of the function if i is odd (e.g.: $|Gjes(\alpha)|$).

If the function doesn't have a negative part (not alternative), we multiply it by $\left(ang_x(\beta(\alpha-\gamma)\right)^i$ to obtain an alternating signal of which the form depends on the value of the frequency "β" and the translation value " γ ". By varying the last parameters, we can get a multi form signal.

Application on some important functions:

• Absolute General Jes:

$\bar{G}jes_i(\alpha) = \left(ang_x(\alpha)\right)^i \cdot Gjes(\alpha)$ (17)

$$= \begin{cases} \left(ang_x(\alpha)\right)^1 \cdot Gjes(\alpha) = |Gjes(\alpha)| & if\ i = 1 \\ \left(ang_x(\alpha)\right)^2 \cdot Gjes(\alpha) = Gjes(\alpha) & if\ i = 2 \end{cases}$$

- $\bar{G}jes_{i,x}(\alpha) = \left(ang_x(\alpha-\gamma)\right)^i \cdot Gjes_x(\alpha)$ (18)

$$= \begin{cases} ang_x(\alpha-\gamma) \cdot Gjes_x(\alpha) & if\ i = 1 \\ Gjes_x(\alpha) & if\ i = 2 \end{cases}$$

- $\bar{G}jes_{i,y}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Gjes_y(\alpha)$ (19)

$$= \begin{cases} ang_y(2\alpha) \cdot Gjes_y(\alpha) = |Gjes_y(\alpha)| & if\ i = 1 \\ Gjes_y(\alpha) & if\ i = 2 \end{cases}$$

- $\bar{G}mar_i(\alpha) = \left(ang_y(\alpha)\right)^i \cdot Gmar(\alpha)$ (20)

- $\bar{G}mar_{i,x}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Gmar_x(\alpha)$ (21)

- $\bar{G}mar_{i,y}(\alpha) = \left(ang_x(\alpha-\gamma)\right)^i \cdot Gmar_y(\alpha)$ (22)

- $\bar{G}rit_i(\alpha) = \left(ang_x(\alpha)\right)^i \cdot Grit(\alpha)$ (23)

And so on…

Examples:

- $y = \bar{G}jes_{\underline{x;>\pi;(0)}}(x) = \begin{cases} |Gjes(x)| & if\ x \le \pi \\ Gjes(x) & if\ x > \pi \end{cases}$
- $y = \sum_{i=0}^{n} i \cdot \bar{G}mar_{i,x}(x)$
$$= 0 + 1 \cdot |Gmar_x(x)| + 2 \cdot Gmar_x(x) + \cdots + n \cdot Gmar_{n,x}(x)$$

3 Particular cases of the General Trigonometry

In this section, a survey on three different branches of the general trigonometry is shown briefly, each branch with its own characteristics; some examples are given and discussed briefly.

3.1 The Elliptic Trigonometry

In this sub-section, three functions are given with examples [1],[2],[4],[5].

3.1.1 Absolute Elliptical Jes $\bar{E}jes_{i,b}(x)$

The Absolute Elliptic Jes is a powerful function that can produce more than 14 signals by varying only two parameters. Like the cosine function in the traditional trigonometry, the Absolute Elliptical Jes is more general than the precedent, more useful and more powerful. It can be used in all scientific domains especially in electrical and electronic engineering and many others.

a-The Expression of the Elliptical Jes:

$$Ejes_b(x) = \frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}(x)\right)^2}} \quad (24)$$

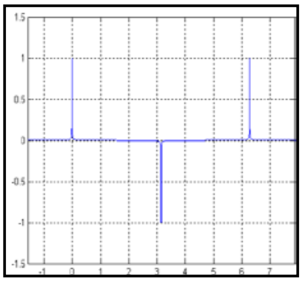
b- Absolute Elliptical Jes:

$$\bar{E}jes_{i,b}(x) = \frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}(x)\right)^2}} \cdot \left(ang_x(x)\right)^i \quad (25)$$

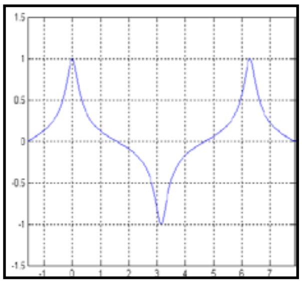
c- Multi form signals made by $\bar{E}jes_{i,b}(x)$:

The following figures represent multi form signals obtained by varying only two parameters (i and b).

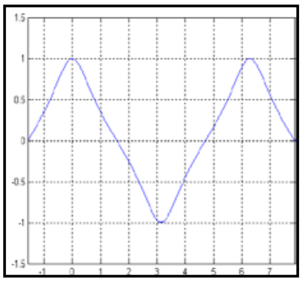
Figures 5 represent multi form signals obtained by varying two parameters (i and b). For the figures 5.a to 5.f the value of $i = 2$, for the figures 5.g to 5.l the value of $i = 1$.



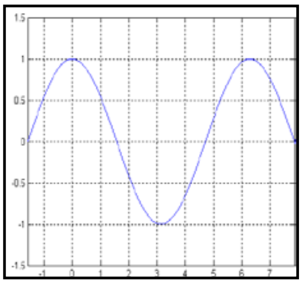
a) $i = 2, b = 0.001$



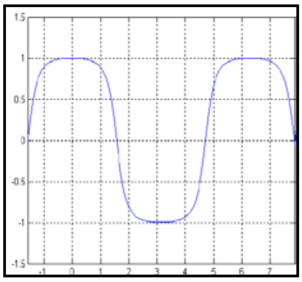
b) $i = 2, b = 0.2$



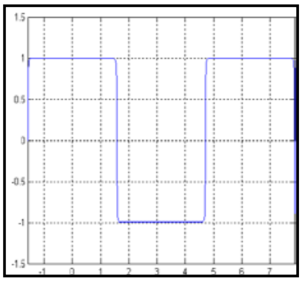
c) $i = 2, b = \sqrt{3}/3$



d) $i = 2, b = 1$

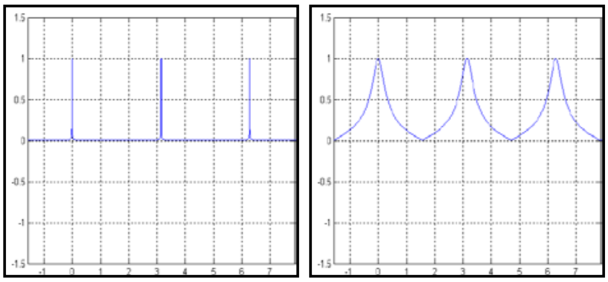


e) $i = 2, b = 3$

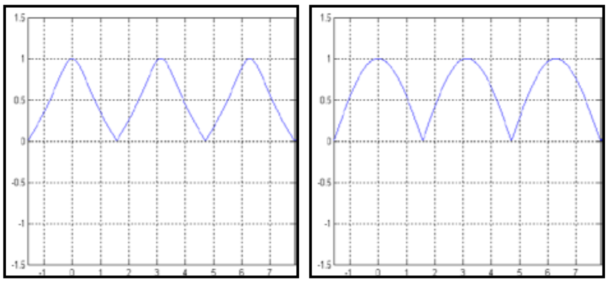


f) $i = 2, b = 90$

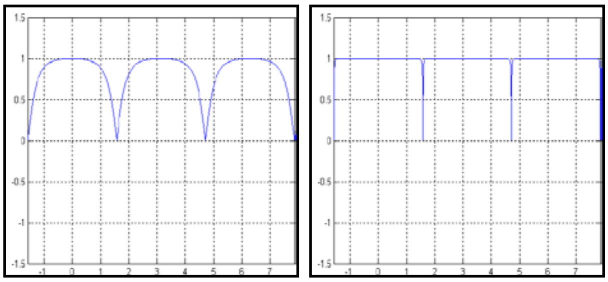
Fig. 5.a to 5.f: multi form signals of the function $\bar{E}jes_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



g) $i = 1, b = 0.001$ h) $i = 1, b = 0.2$



i) $i = 1, b = \sqrt{3}/3$ j) $i = 1, b = 1$



k) $i = 1, b = 3$ l) $i = 1, b = 90$

Fig. 5.g to 5.l: multi form signals of the function $\bar{E}jes_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function:
Impulse train with positive and negative part, elliptic deflated, quasi-triangular, sinusoidal, elliptical swollen, square signal, rectangular signal, impulse train (positive part only), rectified elliptic deflated, saw signal, rectified elliptical swollen, continuous signal…
These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [17].

3.1.1.1 Example using the Elliptical Jes
In this section, an example is treated in a goal to form many shapes by varying only one parameter.
Let's take the following equation:

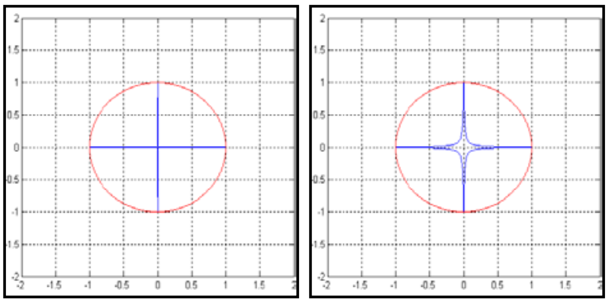
$$y^2 = \left(k.rect_T(x - x_0).Ejes_b(\alpha x)\right)^2 \quad (26)$$

with k, α and T positive values (> 0).
The rectangular function is defined as

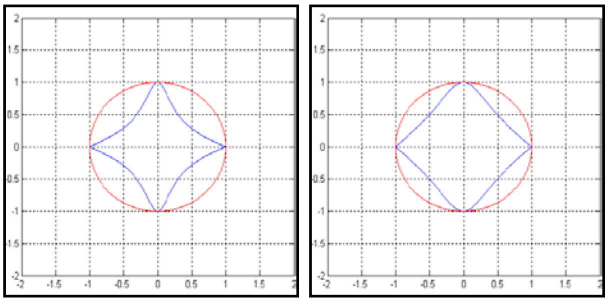
$$rect_T(x - x_0) = \begin{cases} 1 \;\; for \;\; x_0 - \frac{T}{2} \le x \le x_0 + \frac{T}{2} \\ 0 \qquad otherwise \end{cases}$$

$$\Rightarrow y = \pm\left(k.rect_T(x - x_0).Ejes_b(\alpha x)\right) \quad (27)$$

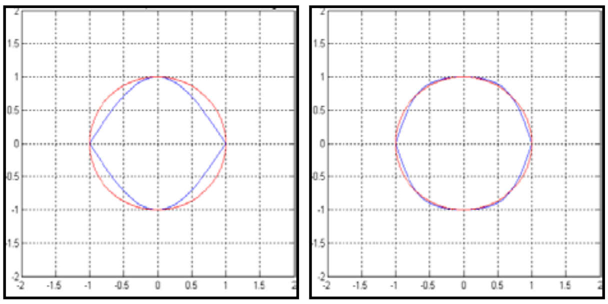
With k is the amplitude of the signal and α its frequency.
The period of $Ejes_b(\alpha x)$ is equal to $T_1 = 2\pi/\alpha$
In particular case we choose $T = 2$; $\alpha = \pi/2$; $x_0 = 0$ and $k = 1$.
By varying the parameter b in (26), the following remarkable shapes are illustrated in Fig 6.a to 6.g.



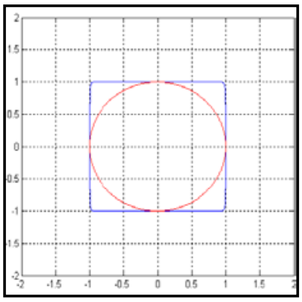
a) $b = 0.0001$ b) $b = 0.01$



c) $b = 0.3$ d) $b = \sqrt{3}/3$



e) $b = 1$ f) $b = 2$



g) $b = 120$

Fig. 6.a to 6.g: different shapes of the function (26) for different values of b (presented in blue color).

Important shapes obtained respectively using this expression:

Plus form, cross form, star form, Quasi-rhombus, eye form, quasi-circular, square. More signals can be obtained by varying more than one parameter.

3.1.2 Absolute Elliptical Jes-x $\bar{E}jes_{x_b^i}(x)$

The Absolute Elliptical Jes-x $\bar{E}jes_{x_b^i}(x)$ is also a function of the Elliptic trigonometry, as the previous one, it has its own characteristics and its own shapes formed by varying some parameters.

a-The Expression of the Elliptic Jes-x: $Ejes_{x_b}(x) =$

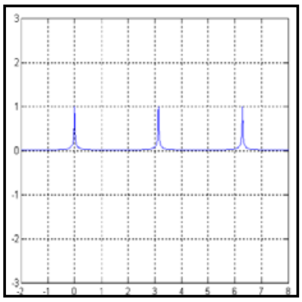
$$\frac{ang_x(x)}{Cjes(x)\cdot\sqrt{1+\left(\frac{a}{b}Cter(x)\right)^2}} \quad (28)$$

b- Absolute Elliptical Jes-x: $\bar{E}jes_{x_b^i}(x) =$

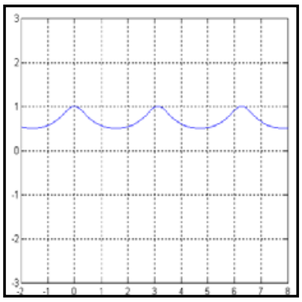
$$\frac{ang_x(x)}{Cjes(x)\cdot\sqrt{1+\left(\frac{a}{b}Cter(x)\right)^2}}\cdot\left(ang_x(x-\gamma)\right)^i \quad (29)$$

c- Multi form signals made by $\bar{E}jes_{x_b^i}(x)$:

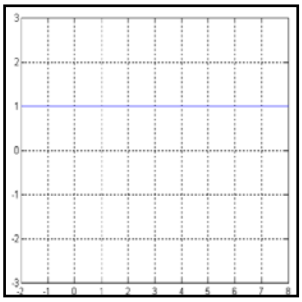
Taking $\gamma = 0$ for this example, figures 7.a to 7.k represent multi form signals obtained by varying two parameters (i and b). For the figures 7.a to 7.e the value of $i = 2$, for the figures 7.f to 7.k the value of $i = 1$.



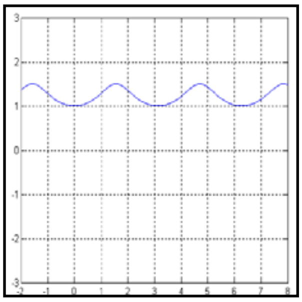
a) $i = 2; b = 0.01$



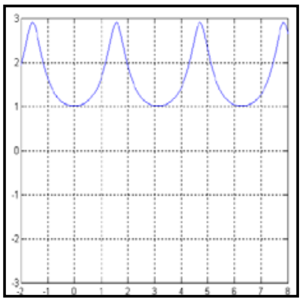
b) $i = 2; b = 0.5$



c) $i = 2; b = 1$

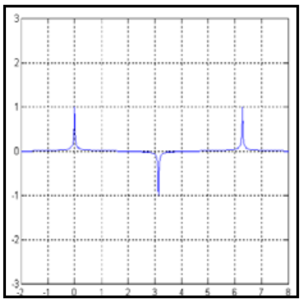


d) $i = 2; b = 1.5$

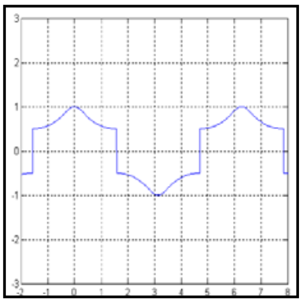


e) $i = 2; b = 2.9$

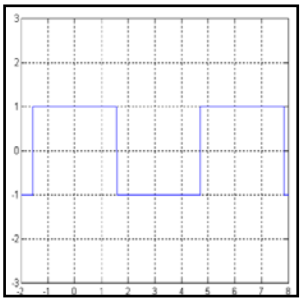
Fig. 7.a to 7.e: multi form signals of the function $\bar{E}jes_{x_b^i}(x)$ for $i = 2$ and for different values of $b > 0$.



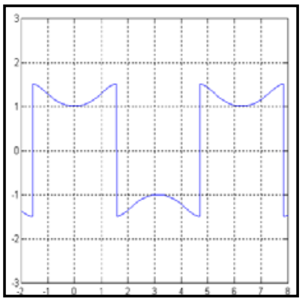
f) $i = 1; b = 0.01$



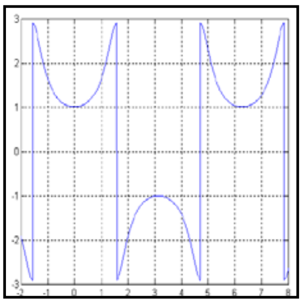
g) $i = 1; b = 0.5$



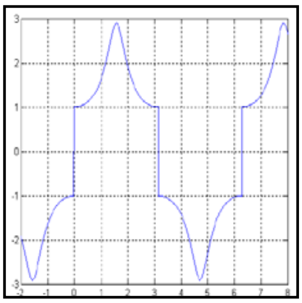
h) $i = 1; b = 1$



i) $i = 1; b = 1.5$



g) $i = 2; b = 2.9$



k) $i = 2; b = 2.9; \gamma = \pi/2$

Fig. 7.f to 7.k: multi form signals of the function $\bar{E}jes_{x_b^i}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function:
Impulse train with positive part only, sea waves, continuous signal, amplified sea waves, impulse train with positive and negative part, square, saw signal …

3.1.2.1 Example using the Elliptic Jes-x

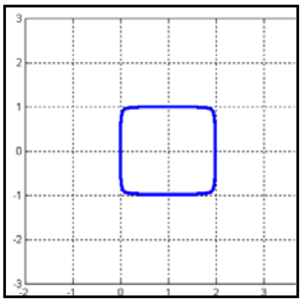
In this section, an example is treated in a goal to form many shapes by varying only one parameter.
Let's take the following equation:

$$y^2 = \left(k.\left(rect_T(x - x_0).Ejes_{x_b}(\alpha x) - 1\right)\right)^2 \quad (30)$$

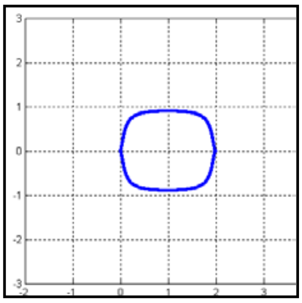
with k, α and T are positive values (> 0).

$$\Rightarrow y = \pm\left(k.\left(rect_T(x - x_0).Ejes_{x_b}(\alpha x) - 1\right)\right) (31)$$

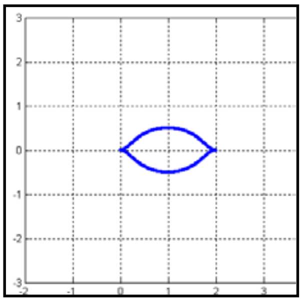
With k is the amplitude of the signal and α its frequency.
The period of $Ejes_{x_b}(\alpha x)$ is equal to $T_1 = \pi/\alpha$
In particular case we choose $T = 2$; $T_1 = 2$; $\alpha = \pi/2$; $x_0 = 1$ and $k = 1$.
By varying the parameter b in (30), the remarkable shapes illustrated in Fig 8.a to 8.h are obtained.



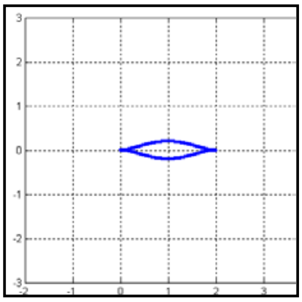
a) $b = 0.01$



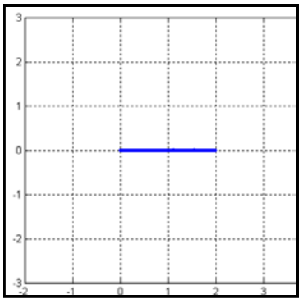
b) $b = 0.1$



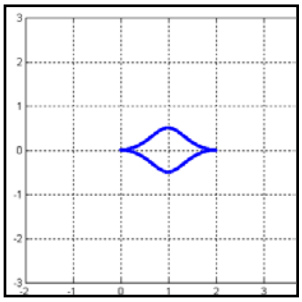
c) $b = 0.5$



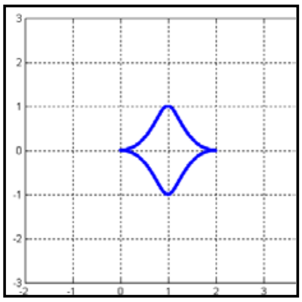
d) $b = 0.8$



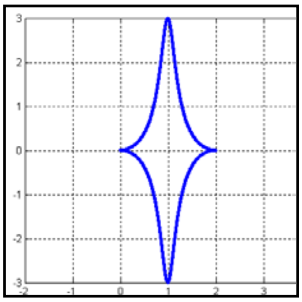
e) $b = 1$



f) $b = 1.5$



g) $b = 2$



h) $b = 4$

Fig. 8.a to 8.h: different shapes of the function (30) for different values of $b > 0$.

3.1.3 Absolute Elliptical Rit $\bar{E}rit_{x_b^i}(x)$

The Absolute Elliptical Rit $\bar{E}rit_{x_b^i}(x)$ is also a function of the Elliptic trigonometry, as the previous one, it has its own characteristics and its own shapes formed by varying some parameters.

a- The Expression of the Elliptical Rit:

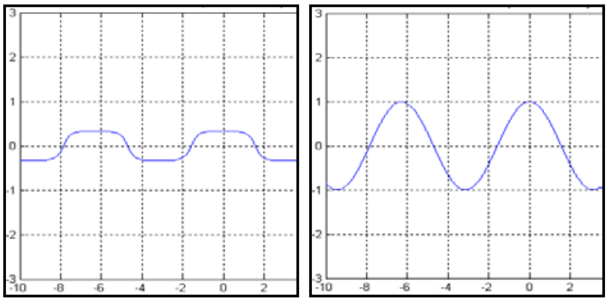
$$Erit_b(x) = \frac{Emar\,(x)}{\mathrm{Cter}(x)} = \frac{a}{b} \cdot \frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}(x)\right)^2}} \qquad (32)$$

b- Absolute Elliptical Rit:

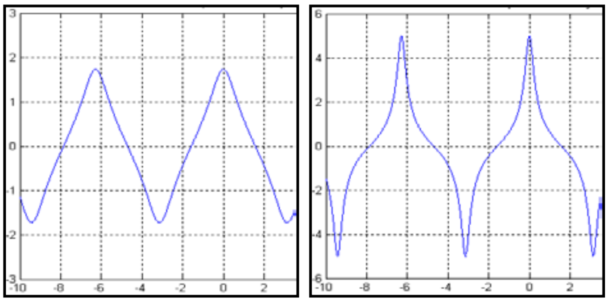
$$\bar{E}rit_{x_b^i}(x) = \frac{a}{b} \cdot \frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}(x)\right)^2}} \cdot \left(ang_x(x)\right)^i \qquad (33)$$

c- Multi form signals made by $\bar{E}rit_{x_b^i}(x)$:

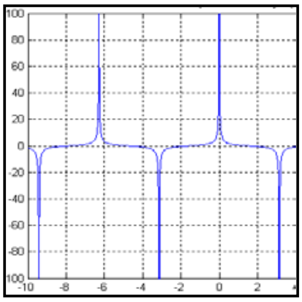
Figures 9.a to 9.j represent multi form signals obtained by varying two parameters (i and b). For the figures 9.a to 9.e the value of $i = 2$, for the figures 9.f to 9.j the value of $i = 1$.



a) $i = 2; b = 3$ b) $i = 2; b = 1$

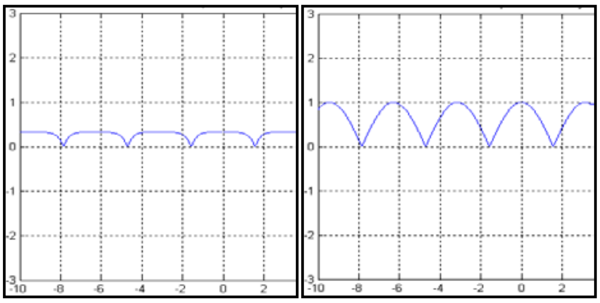


c) $i = 2; b = \sqrt{3}/3$ d) $i = 2; b = 0.2$

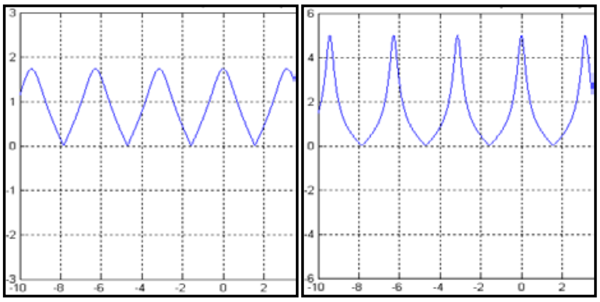


e) $i = 2; b = 0.001$

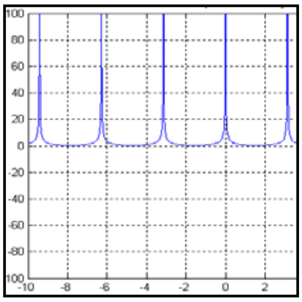
Fig. 9.a to 9.e: multi form signals of the function $\bar{E}rit_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



f) $i = 2; b = 3$ g) $i = 2; b = 1$



h) $i = 2; b = \sqrt{3}/3$ i) $i = 2; b = 0.2$



j) $i = 2; b = 0.001$

Fig. 9.f to 9.j: multi form signals of the function $\bar{E}rit_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Elliptical swollen compressed, sinusoidal, quasi-triangular, elliptical deflated amplified, impulse train (positive and negative part) with controlled amplitude, impulse train (positive part only) with controlled amplitude, saw signal, null signal, rectified sinusoidal …

3.1.3.1 Example using the Elliptical Rit

In this section, an example is treated using the Elliptical Rit in a goal to form many shapes by varying only one parameter.

Let's take the following equation:

$$y^2 = \left(k.rect_T(x - x_0).Erit_b(\alpha x)\right)^2 \quad (34)$$

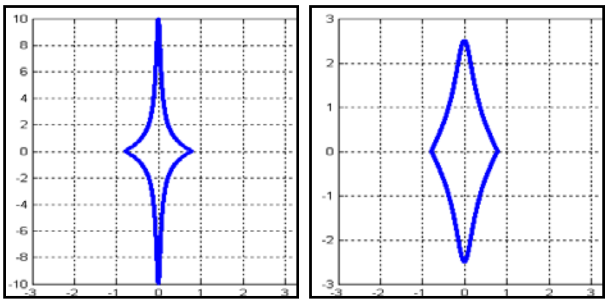
with k, α and T positive values (> 0).

$$\Rightarrow y = \pm\left(k.rect_T(x - x_0).Erit_b(\alpha x)\right) \quad (35)$$

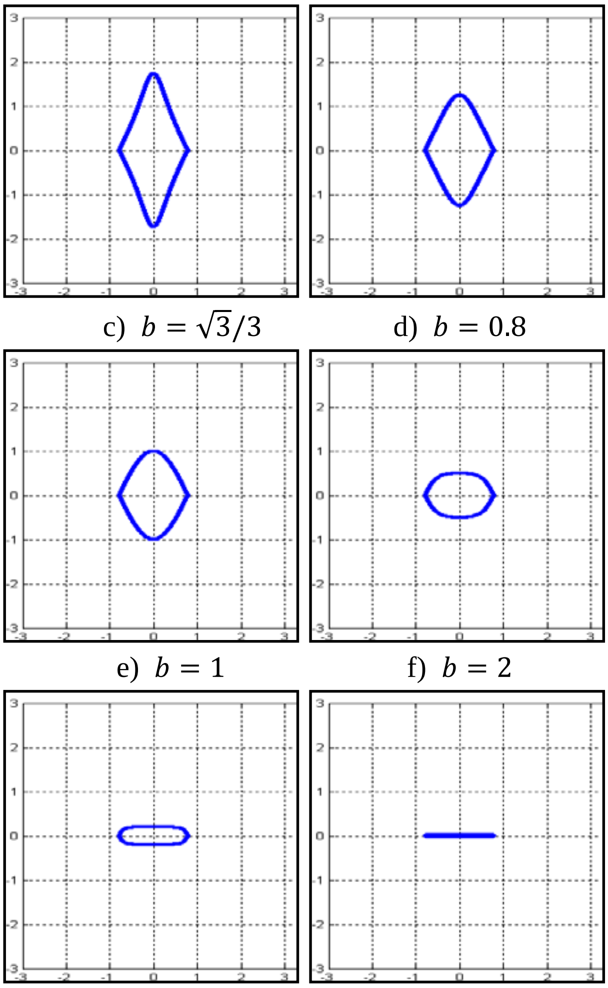
The period of $Erit_b(\alpha x)$ is equal to $T_1 = 2\pi/\alpha$

In particular case we choose $T_1 = \pi$; $T = \frac{T_1}{2}$; $\alpha = 2$; $x_0 = 0$ and $k = 1$.

By varying the parameter b in (34), the remarkable shapes illustrated in Fig 10.a to 10.h are obtained.



a) $b = 0.1$ b) $b = 0.4$



c) $b = \sqrt{3}/3$ d) $b = 0.8$

e) $b = 1$ f) $b = 2$

g) $b = 5$ h) $b = 50$

Fig. 10.a to 10.h: different shapes of the function (34) for different values of $b > 0$.

By using the same method we can obtain the others functions that are a part of the Elliptical trigonometry.

3.2 The Circular Trigonometry

The circular trigonometry known as traditional trigonometry or classical trigonometry is a particular case of the Elliptic trigonometry for $b = a = 1$.
The three points "c", "d" and "e" of the Fig.11 are confused unto a single point.

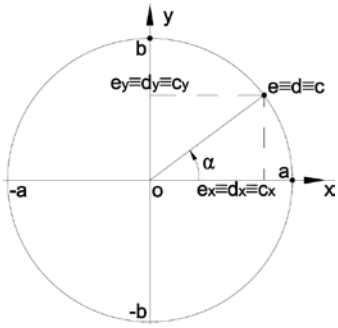


Fig.11: A *branch of the General Trigonometry "The Circular Trigonometry".*

In this case we obtain

$Cjes(\alpha) = Crit(\alpha) = \cos(\alpha)$
$Cmar(\alpha) = Craf(\alpha) = \sin(\alpha)$
$Cjes_y(\alpha) = Crit_y(\alpha) = cotan(\alpha)$
$Cter_x(\alpha) = Cber_x(\alpha) = tan(\alpha)^2$
$Cjes_x(\alpha) = Cmar_y(\alpha) = Cber_y(\alpha) = Cter_y(\alpha) = 1$
$Cter(\alpha) = Cmar_x(\alpha) = Cber(\alpha) = Craf_x(\alpha) = tan(\alpha)$

In this case we can consider the following functions as a basic of the Traditional Trigonometry:
$Cjes(\alpha) = \cos(\alpha)$; $Cmar(\alpha) = \sin(\alpha)$;
$Cter(\alpha) = tan(\alpha)$; $Cjes_y(\alpha) = cotan(\alpha)$;
$Cjes^{-1}(\alpha) = \sec(\alpha)$; $Cmar^{-1}(\alpha) = \mathrm{cosec}(\alpha)$.

3.3 The Rectangular Trigonometry

As previous sections, we will see a brief study on the Rectangular trigonometry and give some examples using this trigonometry to show multi form signals made using the characteristic of this trigonometry. Three functions are given with examples.

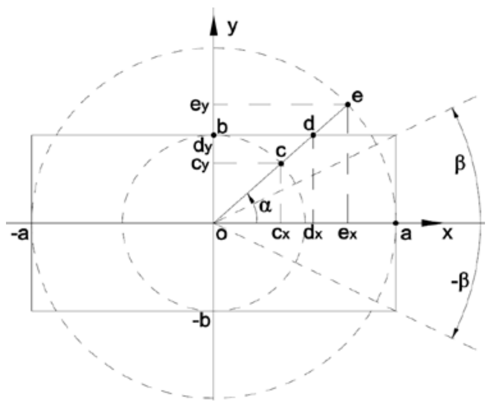


Fig.12: *A branch of the General Trigonometry "The Rectangular Trigonometry".*

With $b > 0$ the height of the rectangle from the center;
$\beta = \arctan\left(\frac{b}{a}\right) = \mathrm{arcCter}\left(\frac{b}{a}\right)$ and $\beta \in]0; \frac{\pi}{2}[$

3.3.1 Absolute Rectangular Jes $\bar{R}jes_{i,b}(x)$

a-The Expression of the Rectangular Jes:

$Rjes_b(x) =$

$$ang_y(x+\beta)\cdot\left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x)+\mathrm{ang}_\beta(x-\pi)}{2}} \quad (36)$$

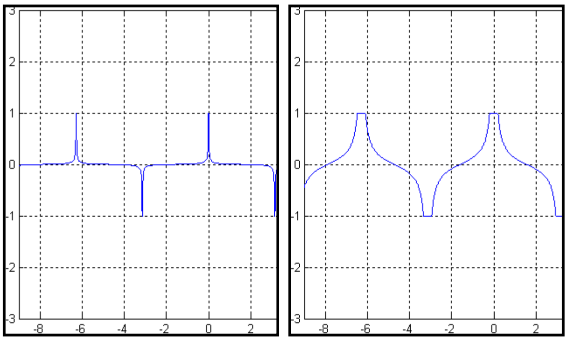
b- Absolute Rectangular Jes:

$\bar{R}jes_{i,b}(x) =$

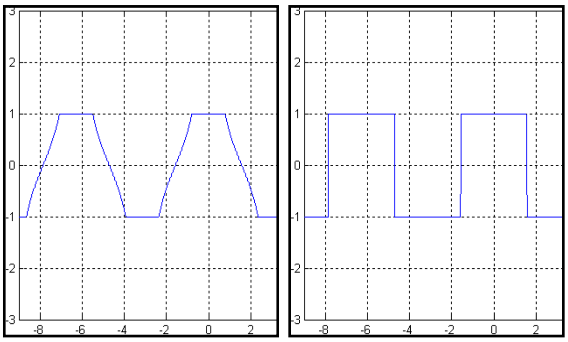
$$ang_y(x+\beta)\cdot\left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x)+\mathrm{ang}_\beta(x-\pi)}{2}}\cdot\left(ang_x(x)\right)^i \quad (37)$$

c- Multi form signals made by $\bar{R}jes_{i,b}(x)$:

Figures 13.a to 13.h represent multi form signals obtained by varying two parameters (i and b). For the figures 13.a to 13.d the value of $i = 2$, for the figures 13.e to 13.h the value of $i = 1$.

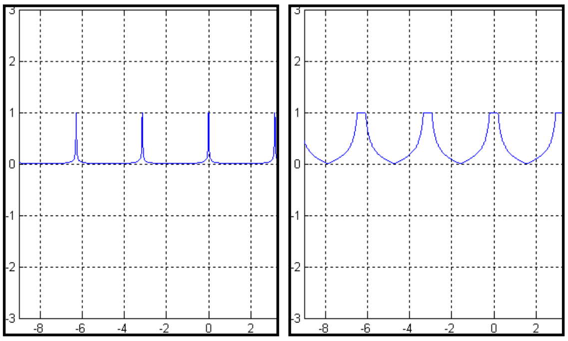


a) $i = 2, b = 0.01$ b) $i = 2, b = 0.2$

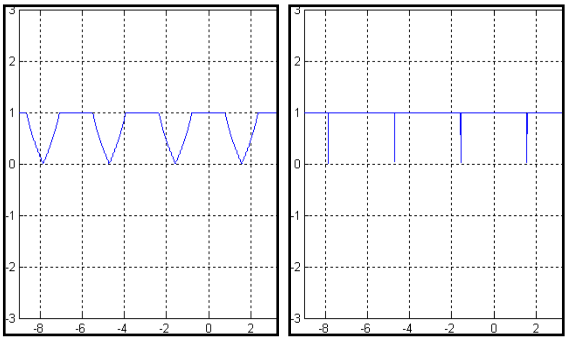


c) $i = 2, b = 1$ d) $i = 2, b = 100$

Fig. 13.a to 13.d: multi form signals of the function $\bar{R}jes_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



e) $i = 1, b = 0.01$ f) $i = 1, b = 0.2$



g) $i = 1, b = 1$ h) $i = 1, b = 100$

Fig. 13.e to 13.h: multi form signals of the function $\bar{R}jes_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Alternate impulse train (positive and negative part), Impulse train (positive part only), square, rectangle, saturated saw signal, saturated triangle signal, continuous signal …

These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [17].

3.3.2 Absolute Rectangular Mary $\bar{R}mar_{i,b}(x)$

a-The Expression of the Rectangular Mary:

$Rmar_b(x) =$

$$ang_y(x+\beta)\cdot\left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{1-\mathrm{ang}_\beta(x)\cdot\mathrm{ang}_\beta(x-\pi)}{2}} \quad (38)$$

b- Absolute Rectangular Mary:

$\bar{R}mar_{i,b}(x) =$

$$\left(ang_y(x)\right)^i ang_y(x+\beta)\cdot\left(\frac{a}{b}\tan(x)\right)^{\frac{1-\mathrm{ang}_\beta(x)\cdot\mathrm{ang}_\beta(x-\pi)}{2}} \quad (39)$$

For more information about these function, refer to the article [3] introduced by the author.

3.3.3 Absolute Rectangular Jes-x $\bar{R}jes_{x_b^i}(x)$

a-The Expression of the Rectangular Jes-x:

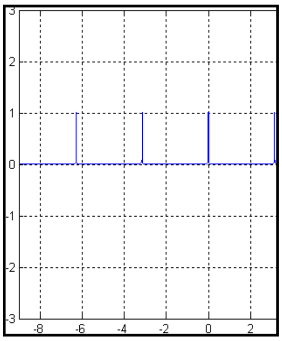
$$Rjes_{x_b}(x) = \frac{ang_y(x+\beta)}{Cjes(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x)+\mathrm{ang}_\beta(x-\pi)}{2}} \quad (40)$$

b- Absolute Rectangular Jes-x:

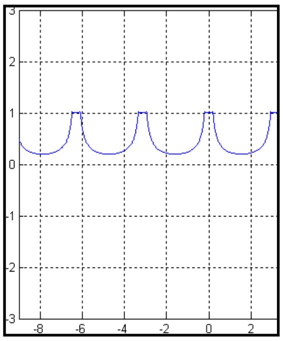
$$\bar{R}jes_{x_b^i}(x) = \frac{ang_y(x+\beta)}{Cjes(x)} \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x)+\mathrm{ang}_\beta(x-\pi)}{2}} \cdot \left(ang_x(x-\gamma)\right)^i \quad (41)$$

c- Multi form signals made by $\bar{R}jes_{x_b^i}(x)$:

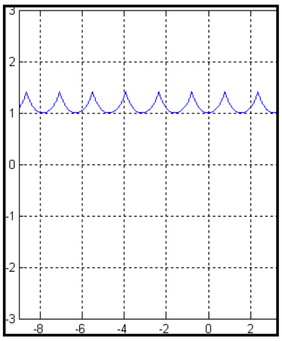
Figures 14.a to 14.i represent multi form signals obtained by varying two parameters (i and b). For the figures 14.a to 14.d the value of $i = 2$, for the figures 14.e to 10.i the value of $i = 1$.



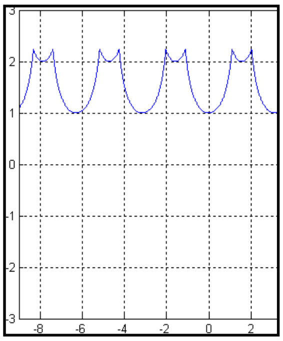
a) $i = 2, b = 0.01$



b) $i = 2, b = 0.2$

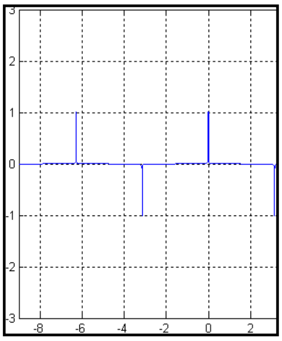


c) $i = 2, b = 1$

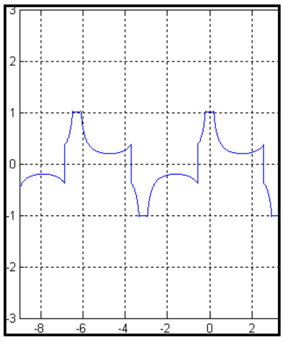


d) $i = 2, b = 2$

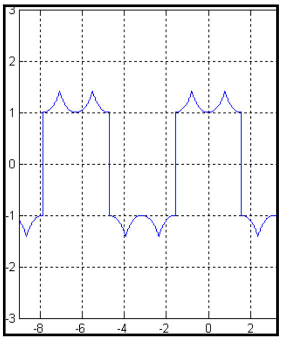
Fig. 14.a to 14.d: multi form signals of the function $\bar{R}jes_{x_b^i}(x)$ for $i = 2$ and for different values of b.



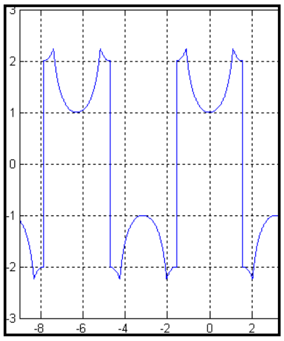
e) $b = 0.01,\ \gamma = 0$



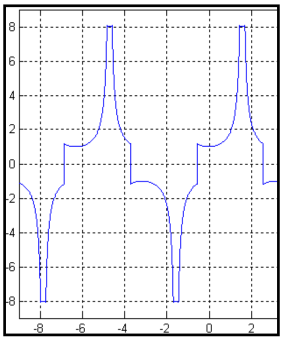
f) $b = 0.2, \gamma = 1$



g) $b = 1, \gamma = 0$



h) $b = 2, \gamma = 0$



i) $b = 8, \gamma = 1$

Fig. 14.e to 14.i: multi form signals of the function $\bar{R}jes_{x_{b;\gamma}^i}$ for $i = 1$ and for different values of b and γ.

In electronic application and in modeling of signals sources, this function is very important especially in the controlled part of an electrical unit as in motor drives, robotics, and many other industrial electronic applications. The main advantage is that by varying the 3 parameters, one can control easily the input and the output power of a controlled circuit especially when a low or high impulse signal is required. The other advantage is that the $\left(\int_t^{t+period} Rjes_{x_b}(x)dx\right)$ can takes values from 0 to ∞ by varying only one parameter "b".

3.3.4 Absolute Rectangular Rit $\bar{R}rit_{i,b}(x)$

a-The Expression of the Rectangular Rit:

$Rrit_b(x) =$

$$\frac{ang_y(x+\beta)}{Cter(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{1-\mathrm{ang}_\beta(\mathrm{x})\cdot\mathrm{ang}_\beta(\mathrm{x}-\pi)}{2}} \quad (42)$$

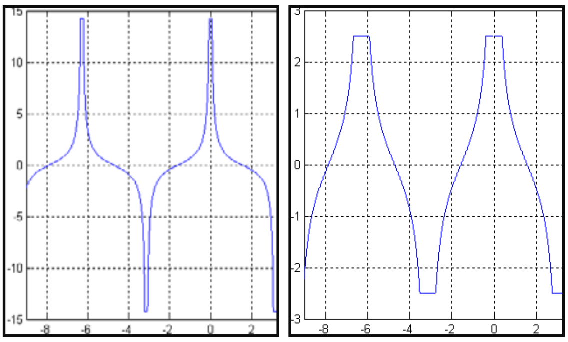
b-Absolute Rectangular Rit:

$\bar{R}rit_{i,b}(x) =$

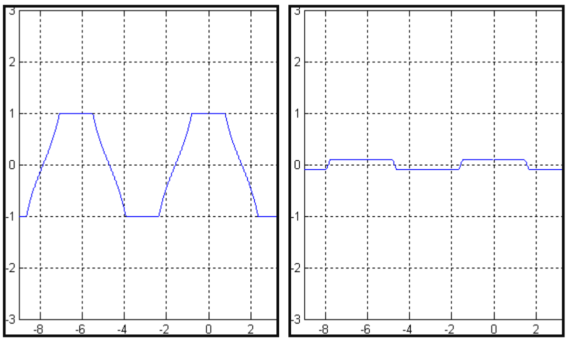
$$ang_y(x+\beta) \cdot \left(\frac{a}{b}\tan(x)\right)^{\frac{1-\mathrm{ang}_\beta(\mathrm{x})\cdot\mathrm{ang}_\beta(\mathrm{x}-\pi)}{2}} \cdot \left(ang_x(x)\right)^i \quad (43)$$

c- Multi form signals made by $\bar{R}rit_{i,b}(x)$

Figures 15.a to 15.h represent multi form signals obtained by varying two parameters (i and b). For the figures 15.a to 15.d the value of $i = 2$, for the figures 15.e to 15.h the value of $i = 1$.

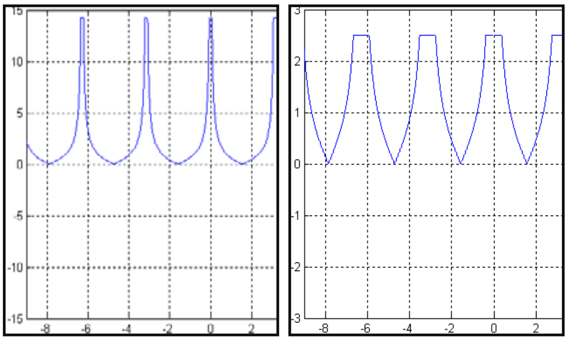


a) $i = 2, b = 0.07$ b) $i = 2, b = 0.4$

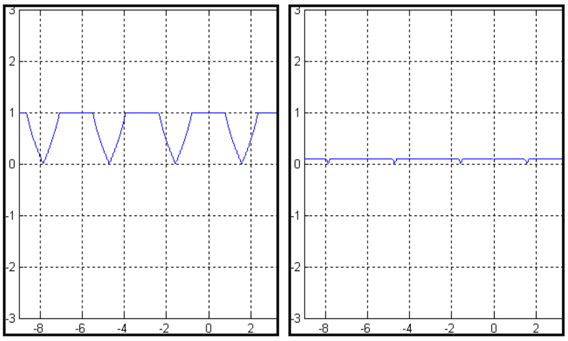


c) $i = 2, b = 1$ d) $i = 2, b = 10$

Fig. 15.a to 15.d: multi form signals of the function $\bar{R}rit_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



e) $i = 1, b = 0.07$ f) $i = 1, b = 0.4$



g) $i = 1, b = 1$ h) $i = 1, b = 10$

Fig. 15.e to 15.h: multi form signals of the function $\bar{R}rit_{i,b}(x)$ for $i = 1$ and for different values for $b > 0$.

4 Survey on the application of the general trigonometry in engineering domain

As we saw in the previous study, the main goal of the general trigonometry is to produce a huge number of multi form signals using a single function and varying some parameters of this function, for this reason we can imagine the importance of this trigonometry in all domains especially in telecommunication, signal theory, electrical and electronic engineering.

Particularly in electrical engineering, a special generator "machine" can be constructed to generate a function that produces multi form signals. E.g.: the active power $P = \sqrt{3}UIcos(\varphi)$ in three phase, is depending on the voltage U, current I, and the phase angle φ between U and I. The variable element is I and by varying I, the active power varies. We aren't interested by increasing the current for many reasons, but the case will be different in the general trigonometry, the active power

$P = \sqrt{3}UI.GTfun(\varphi)$ can be changed without varying any element that means for the same current, voltage and phase angle, we can obtain more active power from the same generator, without introducing another one in parallel in that way the efficiency increases, the cost and the heating decrease...

The functions of this trigonometry are easily programmed and simulated with software as Matlab and Labview, and many circuits can be formed to produce these functions producing multi form signals by varying some parameters, in another hand a new generation of modeling signals appears, and the most important thing is that we can deduce a function from another one, therefore we can create more multi form signals with a simple circuit.

5 Conclusion

The general trigonometry is an original study introduced by the author into the mathematical domain with new concepts; some functions are presented and discussed briefly in this paper. In fact more than 32 basic functions are obtained, and each form of a specific trigonometry contains also 32 different functions. E.g.: the number of functions in the closed parabolic trigonometry is 32, and for the Elliptic trigonometry is also 32, the same for the Triangular trigonometry… the importance of this trigonometry is by obtaining a multi form signals by varying some parameters of a single function. In the future, the new generation of scientific domains especially power electronics will be based on this trigonometry for its huge advantage ahead the traditional one. Thus, several studies will be developed after introducing the new functions of the general trigonometry. Some mathematical expressions and complex electronic circuits will be replaced by simplified expressions and reduced circuits. A new generation of Electrical Generators based on this trigonometry will change the concept of the transmission power and the feeding power and increase the efficiency. In order to illustrate the importance of this new trigonometry, some examples are treated. Consequently, with a single function, more than 14 signals are obtained.

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