Dynamic Behavior in a HIV Infection Model for the Delayed Immune Response*

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Abstract: -Considering full Logistic proliferation of CD4+ T-cells and retarded immune response, we analyze a HIV model in this paper. Global asymptotic stability of the infection-free equilibrium and immune-absent equilibrium is investigated, and some conditions for Hopf bifurcation around infected equilibrium to occur are also obtained by using the time delayed as a bifurcation parameter. Numerical simulating works are presented to illustrate the main results, and we can observe the effects of the proliferation rate of CD4+ T-cells for the dynamics of system. This result can be used to explain the complexity of the immune state of AIDS.

Key-Words: Global stability; Delayed immune response; Logistic proliferation

1 Introduction

Human Immunodeficiency Virus(HIV) has spread in successive waves in various regions around the globe and becoming a serious threat to public health. Over the last several years extensive research has been made in our understanding of the pathogenesis of HIV infection, many mathematical models [1-5] provide quantitative insights. The results of these models help to design treatment strategies which would more effectively bring the infection under control.

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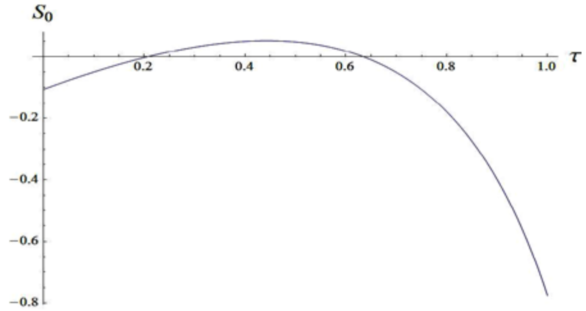


Fig.1. The graph of S_0 versus τ on U .

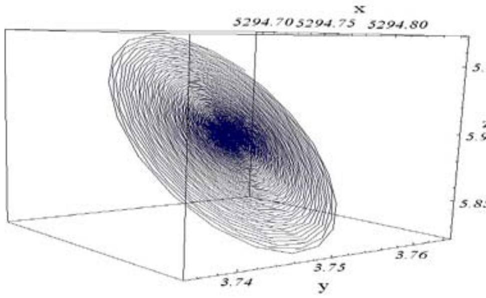
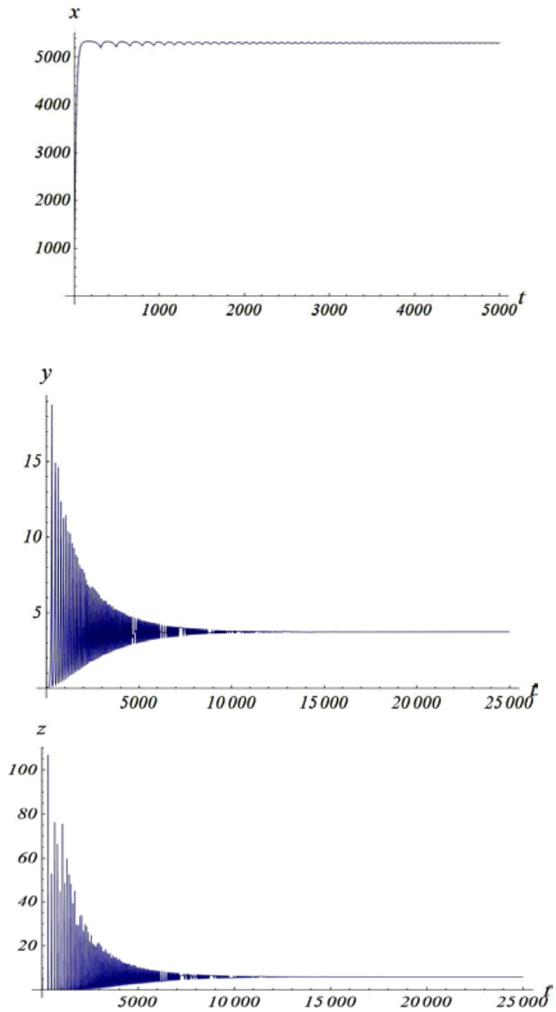
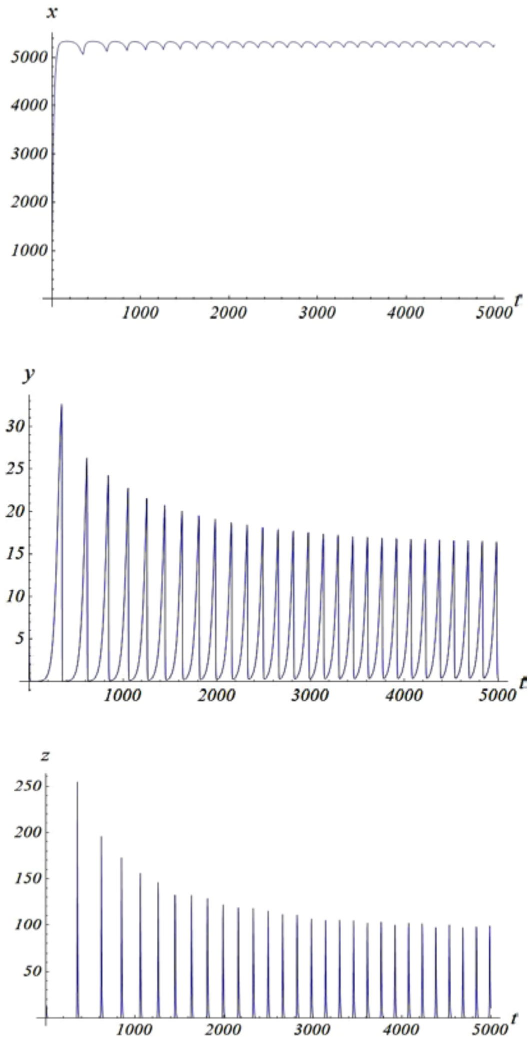


Fig. 2. The figure depicts the infected equilibrium E_2 locally asymptotically stable.





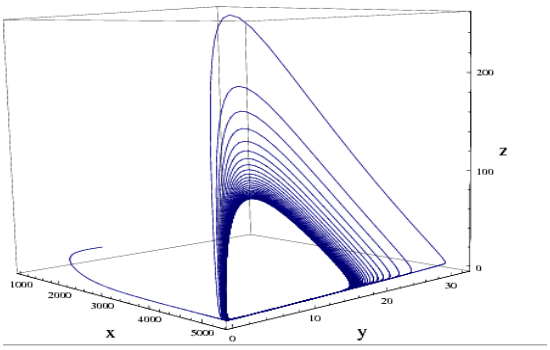


Fig. 3. The figure depicts the infected equilibrium E_2 unstable.

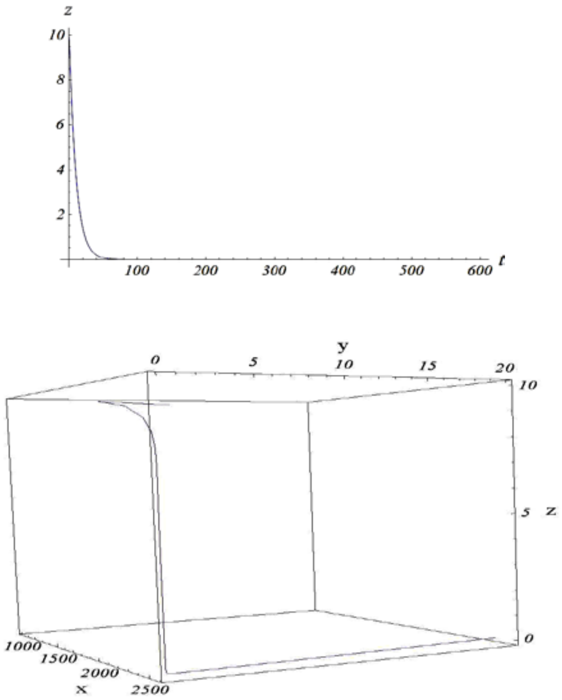


Fig.4. The figure depicts the trajectories converges to the immune-absent equilibrium $E_1 = (2500, 20, 0)$ with $r = 1$.

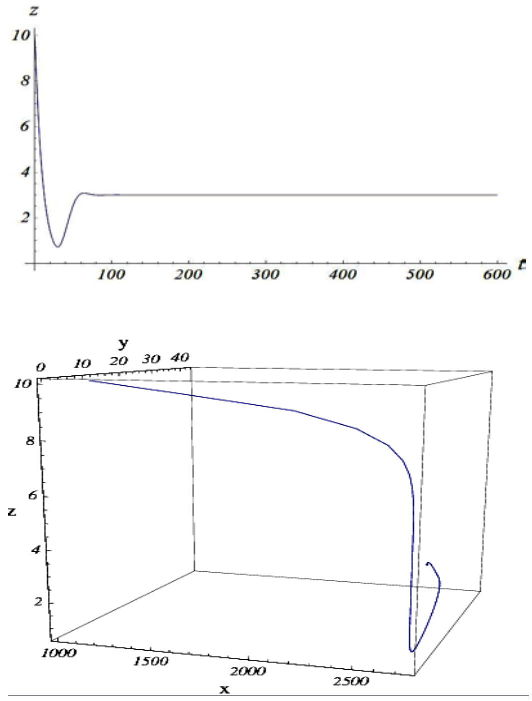


Fig.5. The figure depicts the trajectories converges to the equilibrium $E_2 = (264979, 21.2605, 299578)$ with $r = 0.7$.

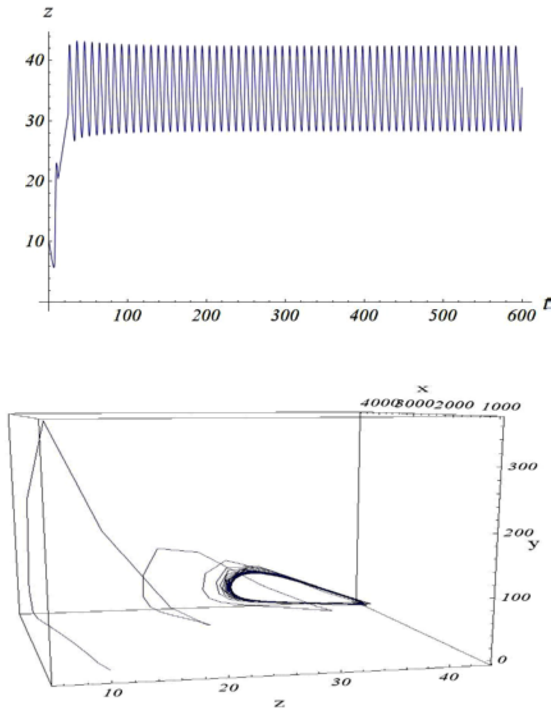


Fig.6. The figure depicts periodic oscillations of the trajectories emerge with $r = 0.08$.

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