Exponential $p-$stability of impulsive stochastic fuzzy cellular neural networks with mixed delays

QIANHONG ZHANG
Guizhou College of Finance and Economics
Guizhou Key Laboratory of Economics System Simulation, School of Mathematics and Statistics
Guiyang, Guizhou 550004
P.R.China
zqianhong68@163.com

LIHUI YANG
Hunan City University
Department of Mathematics
Yiyang, Hunan 413000
P.R.China
ll.hh.yang@gmail.com

Abstract: This paper deals with an impulsive stochastic fuzzy cellular neural networks with both time-varying and infinite distributed delays. Based on M-matrix theory and stochastic analysis technique, a sufficient condition is obtained to ensure the existence, uniqueness, and global exponential $p-$stability of the equilibrium point for the addressed impulsive stochastic fuzzy cellular neural network with mixed delays. Moreover a numerical example is given to illustrate the effectiveness of stability results.

Key–Words: Stochastic fuzzy cellular neural networks, Brownian motion, Global exponential $p-$stability, Mixed delays, Impulse

1 Introduction

Cellular neural networks (CNNs) introduced by Chua and Yang in [1, 2] have attracted considerable attention due to its potential applications in classification, parallel computing, associative memory, signal and image processing, especially in solving some difficult optimization problems. In such applications, it is of prime importance to ensure that the designed neural networks are stable. In practice, due to the finite speeds of the switching and transmission of signals, time delays do exist in a working network and thus should be incorporated into the model equation. Cellular neural networks with delay (DCCNs) introduced by Roska and Chua in [3], also proved to be important in practical applications specially in motion related applications, such as classification of pattern , processing of moving images objects. Both CNNs and DCNNs models have been studied by many authors (see, for example [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]).

Based on traditional CNNs, Yang and Yang [16, 17] proposed fuzzy cellular neural networks (FCNNs), which integrates fuzzy logic into the structure of cellular neural networks. Unlike CNNs structure, FCNNs has fuzzy logic between its template input and/or output besides the sum of product operations. Studies have shown that FCNNs has its potential in image processing and pattern recognition. Like the traditional CNNs, the stability of the system is very important in the design of the FCNNs. In recent years some results on stability for FCNNs have been derived(see, for example [16, 17, 18, 19, 20, 21, 22]). To a large extent, the existing literature on theoretical studies of DCNNs (or FCNNs) is predominantly concerned with deterministic differential equation.

However, the literature dealing with the inherent randomness associated with signal transmission seems to be scarce, such studies are, however, important for us to understand the dynamical characteristics of neuron behavior in random environments for two reasons: (i) in real nervous systems and in the implementation of artificial neural networks, synaptic transmission is noisy process brought on by random fluctuations from the release of neurotransmitters and probabilistic cause; hence, noise is unavoidable and should be taken into consideration in modeling. (ii) it has been realized that a neural network could be stabilized or destabilized by certain stochastic effects. To date, the stability analysis of stochastic cellular neural networks has been studied by some authors (see, for example, [23, 24, 25, 26, 27, 30, 31]).

Dynamical systems are often classified into two categories of either continuous-time or discrete-time systems. These two dynamic systems are widely studied in population models and neural networks, yet there is somewhat new category of dynamical systems, which is neither continuous-time nor purely discrete-time; these are called dynamical systems with impulses. A fundamental theory of impulsive differential equations has been developed in [28]. For instance, in the implementation of neural networks, the state of the networks is subject to instantaneous perturbations and experiences abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noise that it exhibits impulsive effects [28, 29, 30, 31, 32, 33]. Neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems. Therefore, it is necessary to consider both the impulsive effect and delay effect when investigating the stability of fuzzy cellular neural networks.

Motivated by the above discussion, this paper aims to develop the global exponential $p-$stability for stochastic impulsive fuzzy cellular neural networks with both time-varying and distributed delays. To the best of our knowledge, few authors investigated the stability of stochastic impulsive fuzzy cellular neural networks with mixed delays. Therefore, it is necessary to take both time delays and impulsive effects in to account on the dynamical behavior of fuzzy cellular neural networks.

2 Preliminaries

Consider the impulsive stochastic fuzzy cellular neural networks with time-varying and distributed delays as follows.

$$\begin{cases} dx_i(t) &= \Big[-c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij}g_j(x_j(t-\tau_{ij}(t))) \\ &+ \bigwedge_{j=1}^n \delta_{ij}\int_{-\infty}^t K_{ij}(t-s) \\ &\times h_j(x_j(s))ds+\bigvee_{j=1}^n \beta_{ij} \\ &\times g_j(x_j(t-\tau_{ij}(t)))+I_i \\ &+ \bigvee_{j=1}^n \eta_{ij}\int_{-\infty}^t K_{ij}(t-s) \\ &\times h_j(x_j(s))ds\Big]dt+\sum_{j=1}^n \\ &\times \sigma_{ij}(x_j(t),x_j(t-\tau_{ij}(t))d\omega_j(t), \\ & t\neq t_k \\ \Delta x_i(t_k) &= J_k(x_i(t_k^-)), t=t_k, k=1,2,\cdots \end{cases} \tag{1}$$

$i=1,2,\cdots,n$. where n is the number of the neurons in the neural networks, $c_i>0$ represents the passive decay rates to the state of i-th unit at time t. $f_j(\cdot)$, $g_j(\cdot)$, and $h_j(\cdot)$ are the activation function. $I_i=\sum_{j=1}^n b_{ij}u_j+\tilde{I}_i+\bigwedge_{j=1}^n T_{ij}u_j+\bigvee_{j=1}^n H_{ij}u_j$. a_{ij} and b_{ij} are elements of feedback and feed forward template. α_{ij},δ_{ij} denote connection weights of delay fuzzy feedback MIN template. β_{ij},η_{ij} denote connection weights of delay fuzzy feedback MAX template, T_{ij} and H_{ij} are elements of fuzzy feed forward MIN template and fuzzy feed forward MAX template. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. x_i,u_j and $\tilde{I}_i$ denote state, input and bias of the ith neurons, respectively. $\tau_{ij}(t)$ represents the transmission delay with $0\le\tau_{ij}(t)\le\tau_{ij}$($\tau_{ij}$ is a constant). $K_{ij}(\cdot)$ is the delay kernel function; $\omega(t)=(\omega_1(t),\omega_2(t),\cdots,\omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space (Ω,F,P) with a natural filtration $\{F_t\}_{t\ge0}$(i. e. $F_t=\sigma\{\omega(s):0\le s\le t\}$). t_k is called impulsive moment, and satisfies $0<t_1<t_2<\cdots,\lim_{k\to+\infty}t_k=+\infty$. and $x_i(t_k^+),x_i(t_k^-)$ denote the left-hand and right-hand limts at t_k. Suppose that system (1) has initial condition with $x_i(t_0+s)=\phi_i(s),s\le0$ for all $i\in\{1,2,\cdots,n\}$.

Remark 1 If $\sigma_{ij}(x_i(t))=0$, then system (1) may reduce to the following model:

$$\begin{cases} \frac{dx_i(t)}{dt} &= -c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t)) \\ &+ \bigwedge_{j=1}^n \alpha_{ij}g_j(x_j(t-\tau_{ij}(t)))+I_i \\ &+ \bigwedge_{j=1}^n \delta_{ij}\int_{-\infty}^t K_{ij}(t-s)h_j(x_j(s))ds \\ &+ \bigvee_{j=1}^n \beta_{ij}g_j(x_j(t-\tau_{ij}(t))) \\ &+ \bigvee_{j=1}^n \eta_{ij}\int_{-\infty}^t K_{ij}(t-s)h_j(x_j(s))ds, \\ & t\neq t_k \\ \Delta x_i(t_k) &= J_k(x_i(t_k^-)), t=t_k, k=1,2,\cdots \end{cases} \tag{2}$$

Since the solution $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T$ of model (2) is discontinuous at the point t_k , by theory of impulsive differential equations, we assume that $x(t_k)=(x_1(t_k),x_2(t_k),\cdots,x_n(t_k))^T=(x_1(t_k+0),x_2(t_k+0),\cdots,x_n(t_k+0))^T$. It is clear that, in general, the derivatives $\frac{dx_i(t_k)}{dt}$ don't exist. On the other hand, we can see from the first equation of model (2) that the limits $\frac{dx_i(t_k\mp0)}{dt}$ exist. According to the above convention, we assume that $\frac{dx_i(t_k)}{dt}=\frac{dx_i(t_k+0)}{dt}$.

Remark 2 If $\sigma_{ij}(x_i(t))=0,I_{ik}(x_i(t))=0$, then system (1) may reduce to the following model:

$$\frac{dx_i(t)}{dt} = -c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t))$$

$$+\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j(t-\tau_{ij}(t)))+I_i$$
$$+\bigwedge_{j=1}^{n}\delta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(x_j(s))ds$$
$$+\bigvee_{j=1}^{n}\eta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(x_j(s))ds$$
$$+\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j(t-\tau_{ij}(t))) \qquad (3)$$

For convenience, we introduce several notations. $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ denotes a column vector. $\|x\|$ denotes a vector norm defined by $\|x\| = (\sum_{i=1}^{n} |x_i|^p)^{1/p}$. $C[X, Y]$ denotes the space of continuous mappings from topological space X to topological space Y. Denoted by $C^b_{F_0}[(-\infty, 0), R^n]$ the family of all bounded F_0-measurable, $C[(-\infty, 0), R^n]-$valued random variables ϕ, satisfying $\|\phi\|_{L^P} = \sup_{-\infty\le\theta\le 0} E\|\phi(\theta)\| < +\infty$, where $E(\cdot)$ denotes the expectation of stochastic process. The initial condition $\phi \in C^b_{F_0}[(-\infty, 0), R^n]$. $PC[I, R] = \{\psi : I \to R^n|\psi(t^+) = \psi(t), t \in I, \psi(t^-)$ exist for $t \in (t_0, +\infty), \psi(t^-) = \psi(t)$ for all but points $t_k \in (t_0, +\infty)\}$, where $I \subset R$ is an interval, $\psi(t^+)$ and $\psi(t^-)$ denote the left-hand limit and right-hand limit of the scalar function $\psi(t)$, respectively.

Throughout this paper, we make the following assumptions:

$(A1)$ $f_j(\cdot)$ and $g_j(\cdot)(j = 1, 2, \dots, n)$ are globally Lipschitz continuous, i. e., there exist positive constant μ_j and ν_j such that

$$|f_j(x)-f_j(y)| \le \mu_j|x-y|, |g_j(x)-g_j(y)| \le \nu_j|x-y|,$$
$$|h_j(x) - h_j(y)| \le \vartheta_j|x - y|$$

and $f_j(0) = g_j(0) = h_j(0) = 0$ for any $x, y \in R$ and $j = 1, 2, \dots, n$.

$(A2)$ The delay kernel $K_{ij} : [0, +\infty) \to [0, +\infty)$ is a real-valued non-negative continuous function and satisfies

$$\int_{0}^{+\infty} e^{\rho s}K_{ij}(s)ds = r_{ij}(\rho).$$

where $r_{ij}(\rho)$ is continuous function in $[0, \theta), \theta > 0$, and $r_{ij}(0) = 1, i, j = 1, 2, \cdots, n$.

$(A3)$ There exist non-negative number s_{ij}, w_{ij} such that

$$\sigma_i(u, v)\sigma_i^T(u, v) \le \sum_{j=1}^{n} s_{ij}u^2 + \sum_{j=1}^{n} w_{ij}v^2$$

for all $u = (u_1, u_2, \cdots, u_n)^T \in R^n, v = (v_1, v_2, \cdots, v_n)^T \in R^n, i = 1, 2, \cdots, n$.

Definition 1 *The equilibrium point $x^* = (x^*_1, x^*_2, \cdots, x^*_n)^T$ of system (1) is said to be globally exponentially p-stable($p \ge 2$), if there are constants $\lambda > 0$ and $M \ge 1$ such that*

$$E(\|x(t) - x^*\|^p) \le ME\|\phi - x^*\|^p e^{-\lambda(t-t_0)}$$

for any $t \ge 0$, where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is any solution of system (1) with initial value $x_i(t_0 + s) = \phi_i(s) \in PC((-\infty, 0], R), i = 1, 2, \cdots, n$.

Definition 2 *A real matrix $A = (a_{ij})_{n\times n}$ is said to be an $M-$matrix if $a_{ij} \le 0(i, j = 1, 2, \cdots, n; i \ne j)$ and the diagonal entries a_{ii} of A are positive.*

Lemma 3 *Let Q be an $n \times n$ matrix with non-positive off-diagonal elements, then Q is an M-matrix if and only if one of the following conditions holds:*
(i) There exists a vector $\xi > 0$ such that $\xi^T Q > 0$;
(ii) There exists a vector $\xi > 0$ such that $Q\xi > 0$.

Lemma 4 *[16] Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j) - \bigwedge_{j=1}^{n}\alpha_{ij}g_j(y_j)\right| \le \sum_{j=1}^{n}|\alpha_{ij}||g_j(x_j) - g_j(y_j)| \qquad (4)$$

and

$$\left|\bigvee_{j=1}^{n}\beta_{ij}g_j(x_j) - \bigvee_{j=1}^{n}\beta_{ij}g_j(y_j)\right| \le \sum_{j=1}^{n}|\beta_{ij}||g_j(x_j) - g_j(y_j)| \qquad (5)$$

Lemma 5 *If $H(x) \in C^0(R^n, R^n)$ satisfies the following conditions:*
(i) $H(x)$ is injective on R^n;
(ii) $\|H(x)\| \to +\infty$ as $n \to +\infty$,
then $H(x)$ is homeomorphism of R^n onto itself.

Lemma 6 *Let $a, b \ge 0, p \ge i > 0$, then*

$$a^{p-i}b \le \frac{p-i}{p}a^p + \frac{i}{p}b^p.$$

3 Global exponential $p-$stability

In this section, we will discuss global exponential $p-$stability of impulsive stochastic fuzzy cellular neural networks with time-varying delays and distributed delays.

Theorem 7 *Assume that $(A1)-(A3)$ hold, if there exist a positive constant $p \geq 2$ such that $-(Q+L)$ is an $M-$matrix ,where*

$$\begin{aligned} Q &= (q_{ij})_{n\times n}, \\ q_{ij} &= |a_{ij}|\mu_j + (p-1)s_{ij},\ i \neq j, \\ q_{ii} &= -pc_i + (p-1)\Bigg(\sum_{j=1}^{n}(|a_{ij}|\mu_j + (|\alpha_{ij}| \\ &\quad +|\beta_{ij}|)\nu_j + (|\delta_{ij}| + |\eta_{ij}|)\vartheta_j) \\ &\quad +\frac{p-2}{2}\sum_{j=1}^{n}(s_{ij}+w_{ij})\Bigg) \\ &\quad +|a_{ii}|\mu_i + (p-1)s_{ii}, \end{aligned}$$

and

$$\begin{aligned} L &= (l_{ij})_{n\times n}, \\ l_{ij} &= (|\alpha_{ij}| + |\beta_{ij}|)\nu_j + (p-1)w_{ij} \\ &\quad +(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j \end{aligned}$$

Then system (3) has a unique equilibrium point $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$.

Proof: Let $H(x) = (H_1(x), H_2(x), \cdots, H_n(x))^T$, where

$$\begin{aligned} H_i(x) &= -c_ix_i + \sum_{j=1}^{n} a_{ij}f_j(x_j) + \bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j) \\ &\quad + \bigvee_{j=1}^{n}\beta_{ij}g_j(x_j) + \bigwedge_{j=1}^{n}\delta_{ij}h_j(x_j) \\ &\quad + \bigvee_{j=1}^{n}\eta_{ij}h_j(x_j) + I_i, i = 1, 2, \cdots, n. \end{aligned}$$

In the following we will prove $H(x)$ is a homeomorphism of R^n onto itself.

First, we prove that $H(x)$ is an injective map on R^n. In fact, if there exist $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ and $y = (y_1, y_2, \cdots, y_n)^T \in R^n, x \neq y$, such that $H(x) = H(y)$, then

$$\begin{aligned} c_ix_i - c_iy_i &= \sum_{j=1}^{n} a_{ij}(f_j(x_j) - f_j(y_j)) + \bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j) \\ &- \bigwedge_{j=1}^{n}\alpha_{ij}g_j(y_j) + \bigvee_{j=1}^{n}\beta_{ij}g_j(x_j) - \bigvee_{j=1}^{n}\beta_{ij}g_j(y_j) \\ &+ \bigwedge_{j=1}^{n}\delta_{ij}h_j(x_j) - \bigwedge_{j=1}^{n}\delta_{ij}h_j(y_j) \\ &+ \bigvee_{j=1}^{n}\eta_{ij}h_j(x_j) - \bigvee_{j=1}^{n}\eta_{ij}h_j(y_j) \end{aligned} \tag{6}$$

Multiply both side of (6) by $|x_i - y_i|^{p-1}$, it follows from assumptions $(A1)$, Lemma 5 and Lemma 6 that

$$\begin{aligned} (pc_i &- (p-1)\sum_{j=1}^{n}[|a_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &+ (|\delta_{ij}| + |\eta_{ij}|)\vartheta_j]) |x_i - y_i|^p \\ &\leq \sum_{j=1}^{n}(|a_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &\quad +(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j) |x_j - y_j|^p \end{aligned} \tag{7}$$

Let $\Upsilon = (\zeta_{ij})_{n\times n}$, where

$$\begin{aligned} \zeta_{ii} &= pc_i - (p-1)\sum_{j=1}^{n}(|a_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &\quad +(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j) - |a_{ii}|\mu_i \\ &\quad -(|\alpha_{ij}| + |\beta_{ij}|)\nu_i - (|\delta_{ii}| + |\eta_{ii}|)\vartheta_i \end{aligned}$$

$$\begin{aligned} \zeta_{ij} &= -|a_{ij}|\mu_j - (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &\quad -(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j, i \neq j,\ i, j = 1, 2, \cdots, n \end{aligned}$$

Then (7) transforms into the following inequality

$$\Upsilon(|x_1 - y_1|^p, |x_2 - y_2|^p, \cdots, |x_n - y_n|^p)^T \leq 0 \tag{8}$$

Set $-(Q+L) = (\kappa_{ij})_{n\times n}$, noting that $s_{ij} > 0, w_{ij} > 0$, we have

$$\kappa_{ij} \leq \zeta_{ij},\ \ i, j = 1, 2, \cdots, n.$$

Since $-(Q+L)$ is an $M-$matrix, Hence Υ is also an $M-$matrix. It follow from (8) that $x_i = y_i, i = 1, 2, \cdots, n$. which is a contradiction. So $H(x)$ is an injective on R^n.

Next we prove that $\|H(x)\| \to +\infty$ as $n \to +\infty$. Since Υ is an $M-$matrix. From Lemma 3, there exists a positive vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in R^n$ such that

$$\begin{aligned} &\xi_i\Bigg(pc_i - (p-1)\sum_{j=1}^{n}[|a_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &+(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j]) - \sum_{j=1}^{n}\xi_j[|a_{ji}|\mu_i \\ &+(|\alpha_{ji}| + |\beta_{ji}|)\nu_i + (|\delta_{ji}| + |\eta_{ji}|)\vartheta_i] > 0 \end{aligned}$$

for $i = 1, 2, \cdots, n$. We can choose a small $\varrho > 0$ such that

$$\begin{aligned} &\xi_i\Bigg(pc_i - (p-1)\sum_{j=1}^{n}[|a_{ij}|\mu_j + (|\alpha_{ij}| + |\beta_{ij}|)\nu_j \\ &+(|\delta_{ij}| + |\eta_{ij}|)\vartheta_j]) - \sum_{j=1}^{n}\xi_j[|a_{ji}|\mu_i \\ &+(|\alpha_{ji}| + |\beta_{ji}|)\nu_i + (|\delta_{ji}| + |\eta_{ji}|)\vartheta_i] \\ &\geq \varrho > 0 \end{aligned} \tag{9}$$

for $i = 1, 2, \cdots, n$. Let $\widetilde{H}(x) = (\widetilde{H_1}(x), \widetilde{H_2}(x), \cdots, \widetilde{H_n}(x))^T$, where

$$\begin{aligned}\widetilde{H_i}(x) &= -c_i x_i + \sum_{j=1}^n a_{ij}(f_j(x_j) - f_j(0)) \\ &\quad + \bigwedge_{j=1}^n \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^n \alpha_{ij} g_j(0) \\ &\quad + \bigvee_{j=1}^n \beta_{ij} g_j(x_j) - \bigvee_{j=1}^n \beta_{ij} g_j(0) \\ &\quad + \bigwedge_{j=1}^n \delta_{ij} h_j(x_j) - \bigwedge_{j=1}^n \delta_{ij} h_j(0) \\ &\quad + \bigvee_{j=1}^n \eta_{ij} h_j(x_j) - \bigvee_{j=1}^n \eta_{ij} h_j(0) \quad (10)\end{aligned}$$

From assumptions $(A1)$ and Lemma 6, we can get

$$\begin{aligned}&\sum_{i=1}^n p\xi_i |x_i|^{p-1} \mathrm{sgn}(x_i) \widetilde{H_i}(x) \\ \leq &\sum_{i=1}^n \left[\xi_i \left(-pc_i + (p-1)\sum_{j=1}^n (|a_{ij}|\mu_j \right.\right. \\ &+(|\alpha_{ij}| + |\beta_{ij}|)\nu_j + (|\delta_{ij}| + |\eta_{ij}|)\vartheta_j)) \\ &+\sum_{j=1}^n \xi_j (|a_{ji}|\mu_i + (|\alpha_{ji}| + |\beta_{ji}|)\nu_i \\ &\left.+(|\delta_{ji}| + |\eta_{ji}|)\vartheta_i)\right] |x_i|^p \leq -\varrho \|x\|^p\end{aligned}$$

Hence

$$\begin{aligned}\varrho\|x\|^p &\leq \sum_{i=1}^n 2\xi_i |x_i|^{p-1} |\widetilde{H_i}(x)| \\ &\leq p \max_{1\leq i\leq n}\{\xi_i\} \sum_{i=1}^n |x_i|^{p-1} |\widetilde{H_i}(x)| \\ &\leq p \max_{1\leq i\leq n}\{\xi_i\} \|x_i\|^{p-1} \|\widetilde{H_i}(x)\|\end{aligned}$$

That is

$$\varrho\|x\| \leq p \max_{1\leq i\leq n}\{\xi_i\} \|\widetilde{H_i}(x)\|$$

Therefore $\|\widetilde{H}(x)\| \to +\infty$ as $\|x\| \to +\infty$, which directly implies that $\|H(x)\| \to +\infty$ as $\|x\| \to +\infty$.

By Lemma 5, we know that $H(x)$ is a homeomorphism on R^n, hence $H(x) = 0$ has a unique equilibrium point $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T \in R^n$. i. e., Model (3) has a unique equilibrium point x^*. □

Theorem 8 *Assume that all conditions of Theorem 7 hold. Furthermore, suppose that*
(i) $\sigma_{ij}(x_j^*, x_j^*) = 0, i, j = 1, 2, \cdots, n;$
(ii) $J_k(x_i(t_k)) = -\gamma_{ik}(x_i(t_k^-) - x_i^*), 0 < \gamma_{ik} < 2, i = 1, 2, \cdots, n; k = 1, 2, \cdots.$

Then $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ *is a unique equilibrium of system (1), which is globally exponentially* $p-$*stable.*

Proof: By virtue of Theorem 7, system (3) has a unique equilibrium point x^*. From conditions (i) and (ii), we know that x^* is also a unique equilibrium point of model (1).

Set $y_i(t) = x_i(t) - x_i^*$, $\widetilde{\sigma_{ij}}(y_j(t)) = \sigma_{ij}(y_j(t) + x_j^*) - \sigma_{ij}(x_j^*)$, then the first equation of system (1) can be transformed into the following equation

$$\begin{aligned}&dy_i(t) \\ = &[-(c_i(y_i(t) + x_i^*) - c_i x_i^*) \\ + &\sum_{j=1}^n a_{ij}(f_j(y_j(t) + x_j^*) - f_j(x_j^*)) \\ + &\left(\bigwedge_{j=1}^n \alpha_{ij} g_j(y_j(t - \tau_{ij}(t)) + x_j^*)\right. \\ - &\left.\bigwedge_{j=1}^n \alpha_{ij} g_j(x_j^*) ds\right) \\ + &\left(\bigvee_{j=1}^n \beta_{ij} g_j(y_j(t - \tau_{ij}(t)) + x_j^*)\right. \\ - &\left.\bigvee_{j=1}^n \beta_{ij} g_j(x_j^*)\right) \\ + &\left(\bigwedge_{j=1}^n \delta_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(y_j(s) + x_j^*) ds\right. \\ - &\left.\bigwedge_{j=1}^n \delta_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j^*) ds\right) \\ + &\left(\bigvee_{j=1}^n \eta_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(y_j(s) + x_j^*) ds\right. \\ - &\left.\left.\bigvee_{j=1}^n \eta_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j^*) ds\right)\right] dt \\ + &\sum_{j=1}^n \widetilde{\sigma_{ij}}(y_j(t), y_j(t - \tau_{ij}(t))) d\omega_j(t), \\ &t \neq t_k, i = 1, 2, \cdots, n; k = 1, 2, \cdots \quad (11)\end{aligned}$$

Since $-(Q + L)$ is an M-matrix, there exists $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T > 0$ such that $0 < -(Q + L)\xi$, that is

$$0 < \left[pc_i - (p-1)\left(\sum_{j=1}^n |a_{ij}|\mu_j + \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|)\nu_j\right.\right.$$

$$
\begin{aligned}
+ & \sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j+\frac{p-2}{2}\sum_{j=1}^{n}(s_{ij}+w_{ij})\Bigg)\Bigg]\xi_i \\
- & \sum_{j=1}^{n}[(|a_{ij}|\mu_j+(p-1)s_{ij})+(|\alpha_{ij}|+|\beta_{ij}|)\nu_j \\
+ & (p-1)w_{ij}+(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j]\,\xi_j, i=1,2,\cdots,n.
\end{aligned}
$$

We can choose a small positive number $\varepsilon > 0$ such that, for $i = 1, 2, \cdots, n$

$$
\begin{aligned}
0 \ < \ & \Bigg[pc_i-\varepsilon-(p-1)\Bigg(\sum_{j=1}^{n}|a_{ij}|\mu_j \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j \\
& +\frac{p-2}{2}\sum_{j=1}^{n}(s_{ij}+w_{ij})\Bigg)\Bigg]\xi_i \\
& -\sum_{j=1}^{n}[(|a_{ij}|\mu_j+(p-1)s_{ij}) \\
& +e^{\varepsilon\tau}((|\alpha_{ij}|+|\beta_{ij}|)\nu_j+(p-1)w_{ij}) \\
& +(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j r_{ij}(\varepsilon)]\,\xi_j \qquad (12)
\end{aligned}
$$

Let

$$u_i(t)=e^{\varepsilon(t-t_0)}|y_i(t)|^p,\ p\ge 2,\ i=1,2,\cdots,n.$$

By the Ito differential formula, the stochastic derivative of $u_i(t)$ along (11) can be obtained as follows:

$$
\begin{aligned}
L \ & u_i(t) \\
= \ & \varepsilon e^{\varepsilon(t-t_0)}|y_i(t)|^p \\
& +pe^{\varepsilon(t-t_0)}|y_i(t)|^{p-1}\mathrm{sgn}(y_i(t)) \\
& \times\{[-c_i(y_i(t)+x_i^*)-c_ix_i^* \\
& +\sum_{j=1}^{n}a_{ij}(f_j(y_j(t)+x_j^*)-f_j(x_j^*)) \\
& +\Bigg(\bigwedge_{j=1}^{n}\alpha_{ij}g_j(y_j(t-\tau_{ij}(t))+x_j^*) \\
& -\bigwedge_{j=1}^{n}\alpha_{ij}g_j(x_j^*)\Bigg) \\
& +\Bigg(\bigvee_{j=1}^{n}\beta_{ji}g_j(y_j(t-\tau_{ij}(t))+x_j^*) \\
& -\bigvee_{j=1}^{n}\beta_{ji}g_j(x_j^*)\Bigg) \\
& +\Bigg(\bigwedge_{j=1}^{n}\delta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(y_j(s)+x_j^*)ds \\
& -\bigwedge_{j=1}^{n}\delta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(x_j^*)ds\Bigg) \\
& +\Bigg(\bigvee_{j=1}^{n}\eta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(y_j(s)+x_j^*)ds \\
& -\bigvee_{j=1}^{n}\eta_{ij}\int_{-\infty}^{t}K_{ij}(t-s)h_j(x_j^*)ds\Bigg)\Bigg]\Bigg\} \\
& +e^{\varepsilon(t-t_0)}\widetilde{\sigma}_i\widetilde{\sigma}_i^{\,T}
\end{aligned}
$$

for $i = 1, 2, \cdots, n; t_{k-1} < t < t_k, k = 1, 2, \cdots$.

Applying assumptions $(A1)$, $(A3)$ and Lemma 4, we can get

$$
\begin{aligned}
L \ & u_i(t) \qquad (13) \\
\le \ & \varepsilon e^{\varepsilon(t-t_0)}|y_i(t)|^p \\
& +pe^{\varepsilon(t-t_0)}|y_i(t)|^{p-1}\mathrm{sgn}(y_i(t))\,[-c_i|y_i(t)| \\
& +\sum_{j=1}^{n}|a_{ij}|\mu_j|y_j(t)| \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j|y_j(t-\tau_{ij}(t))|+\overline{a_i} \\
& \times\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\int_{-\infty}^{t}K_{ij}(t-s)|y_j(s)|\vartheta_j ds\Bigg] \\
& +\frac{1}{2}p(p-1)e^{\varepsilon(t-t_0)}|y_i(t)|^{p-2} \\
& \times\left[\sum_{j=1}^{n}s_{ij}y_j^2(t)+\sum_{j=1}^{n}w_{ij}y_j^2(t-\tau_{ij}(t))\right] \qquad (14)
\end{aligned}
$$

By applying Lemma 6, it follows that

$$
\begin{aligned}
L \ & u_i(t)\le\varepsilon u_i(t)-pc_iu_i(t) \\
& +\Bigg[(p-1)\sum_{j=1}^{n}|a_{ij}|\mu_ju_i(t)+\sum_{j=1}^{n}|a_{ij}|\mu_ju_j \\
& +(p-1)\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_ju_i(t) \\
& +\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_je^{\varepsilon\tau_{ij}}u_j(t-\tau_{ij}(t)) \\
& +(p-1)\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_ju_i(t) \\
& +\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j \\
& \times\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)u_j(s)ds\Bigg] \\
& +\frac{1}{2}(p-1)(p-2)\sum_{j=1}^{n}s_{ij}u_i(t)
\end{aligned}
$$

$$
\begin{aligned}
&+(p-1)\sum_{j=1}^{n} s_{ij}u_j(t)\\
&+\frac{1}{2}(p-1)(p-2)\sum_{j=1}^{n} w_{ij}u_i(t)\\
&+(p-1)\sum_{j=1}^{n} w_{ij}e^{\varepsilon\tau_{ij}}u_j(t-\tau_{ij}(t))\\
\leq\ &\Bigg\{\Bigg[-pc_i+\varepsilon+(p-1)\Bigg(\sum_{j=1}^{n}|a_{ij}|\mu_j\\
&+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j\\
&+\frac{p-2}{2}\sum_{j=1}^{n}(s_{ij}+w_{ij})\Bigg)\Bigg]u_i(t)\\
&+\sum_{j=1}^{n}(|a_{ij}|\mu_j+(p-1)s_{ij})u_j(t)\\
&+e^{\varepsilon\tau}\sum_{j=1}^{n}\left((|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right.\\
&\left.+(p-1)w_{ij}\right)u_j(t-\tau_{ij}(t))\\
&+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j\\
&\times\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)u_j(s)ds\Bigg\}
\end{aligned}
$$

for $i=1,2,\cdots,n; t_{k-1}<t<t_k, k=1,2,\cdots$. Furthermore, we have

$$
\begin{aligned}
D^{+}\ &(Eu_i(t))\\
\leq\ &\Bigg\{\Bigg[-pc_i+\varepsilon+(p-1)\Bigg(\sum_{j=1}^{n}|a_{ij}|\mu_j\\
&+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j\\
&+\frac{p-2}{2}\sum_{j=1}^{n}(s_{ij}+w_{ij})\Bigg)\Bigg]Eu_i(t)\\
&+\sum_{j=1}^{n}(|a_{ij}|\mu_j+(p-1)s_{ij})Eu_j(t)\\
&+e^{\varepsilon\tau}\sum_{j=1}^{n}\left((|\alpha_{ij}|+|\beta_{ij}|)\nu_j\right.\\
&\left.+(p-1)w_{ij}\right)Eu_j(t-\tau_{ij}(t))\\
&+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j\\
&\times\int_{-\infty}^{t}e^{\varepsilon(t-s)}K_{ij}(t-s)Eu_j(s)ds\Bigg\} \qquad (15)
\end{aligned}
$$

Set

$$h_0=\frac{\|\phi-x^*\|_{L^p}^p}{\min\limits_{1\leq i\leq n}\{\xi_i\}}.$$

then $s\in(-\infty,t_0]$, we have

$$
\begin{aligned}
Eu_i(s)\ &=\ e^{\varepsilon(s-t_0)}E|y_i(s)|^p\leq E|y_i(s)|^p\\
&=\ E|\phi_i(s-t_0)-x_i^*|^p\leq\|\phi-x^*\|_{L^2}^p\\
&\leq\ \xi_i h_0. \qquad (16)
\end{aligned}
$$

In the following, we will use the mathematical induction to prove that, for $i=1,2,\cdots,n; k=1,2,\cdots$,

$$Eu_i(t)\leq\xi_i h_0,\ t_{k-1}\leq t<t_k. \qquad (17)$$

When $k=1$, let us prove that

$$Eu_i(t)\leq\xi_i h_0,\ t_0\leq t<t_1,\ i=1,2,\cdots,n. \qquad (18)$$

In fact, if (17) is not true, then there exist some i_0 and $t^*\in[t_0,t_1)$ such that, for $t\in(-\infty,t^*), j=1,2,\cdots,n$.

$$Eu_{i_0}(t^*)=\xi_{i_0}h_0,\ D^+Ex_{i_0}(t^*)\geq 0,\ Eu_j(t)\leq\xi_j h_0. \qquad (19)$$

From (14) and (18), we can get

$$
\begin{aligned}
D^{+}\ &(Eu_{i_0}(t^*))\\
\leq\ &\Bigg\{\Bigg[-pc_{i_0}+\varepsilon+(p-1)\Bigg(\sum_{j=1}^{n}|a_{i_0j}|\mu_j\\
&+\sum_{j=1}^{n}(|\alpha_{i_0j}|+|\beta_{i_0j}|)\nu_j\\
&+\sum_{j=1}^{n}(|\delta_{ij}|+|\eta_{ij}|)\vartheta_j\\
&+\frac{p-2}{2}\sum_{j=1}^{n}(s_{i_0j}+w_{i_0j})\Bigg)\Bigg]\xi_0\\
&+\sum_{j=1}^{n}\left[(|a_{i_0j}|\mu_j+(p-1)s_{i_0j})\right.\\
&+e^{\varepsilon\tau}\left((|\alpha_{i_0j}|+|\beta_{i_0j}|)\nu_j+(p-1)w_{i_0j}\right)\\
&\left.+(|\delta_{i_0j}|+|\eta_{ij}|)\vartheta_j r_{i_0j}(\varepsilon)\right]\xi_j\Big\}h_0 \qquad (20)
\end{aligned}
$$

It follows from (12) and (19) that

$$D^+(Eu_{i_0}(t^*))<0,$$

which is a contradiction. So (17) is true.

Suppose that the inequalities, for $i=1,2,\cdots,n$,

$$Eu_i(t)\leq\xi_i h_0,\ t_{k-1}\leq t<t_k, k=1,2,\cdots, \qquad (21)$$

hold for $k = 1, 2, \cdots, m$. From condition (ii) of this theorem, we have

$$\begin{aligned} |x_i(t_k) - x_i^*| &= |x_i(t_k^-) + J_k(x_i(t_k^-)) - x_i^*| \\ &= |1 - \gamma_{ik}||x_i(t_k^-) - x_i^*| \\ &\leq |x_i(t_k^-) - x_i^*| \end{aligned}$$

for $i = 1, 2, \cdots, n; k = 1, 2, \cdots$. Therefore

$$u_i(t_k) \leq u_i(t_k^-),\ i = 1, 2, \cdots, n; k = 1, 2, \cdots.$$

Furthermore, we can get

$$Eu_i(t_k) \leq Eu_i(t_k^-),\ i = 1, 2, \cdots, n; k = 1, 2, \cdots. \tag{22}$$

It follows from (20) and (21) that

$$Eu_i(t_m) \leq Eu_i(t_m^-) < \xi_i h_0,\ i = 1, 2, \cdots, n. \tag{23}$$

This, together with both (15), (20) and (22), leads to

$$Eu_i(t) \leq \xi_i h_0,\ t \in (-\infty, t_m],\ i = 1, 2, \cdots, n. \tag{24}$$

Similar to the proof of (16), we can prove that

$$Eu_i(t) \leq \xi_i h_0,\ t \in [t_m, t_{m+1}),\ i = 1, 2, \cdots, n. \tag{25}$$

By mathematical induction, we can conclude that (16) holds. Hence

$$E|x_i(t) - x_i^*|^p \leq \xi_i h_0 e^{-\varepsilon(t-t_0)}, t \geq t_0,\ i = 1, 2, \cdots, n.$$

So $E\|x(t) - x^*\|^p \leq M\|\phi - x^*\|_{L^p}^p e^{-\varepsilon(t-t_0)},\ t \geq t_0$. This means that the unique equilibrium point x^* of model (1) is globally exponentially $p-$stable. The proof is completed. □

Remark 3 In Theorem 8, if we don't consider fuzzy AND and OR operation, it becomes traditional cellular neural networks. The results in [30] are the corollary of theorem 2. Therefore the results of this paper extend the previous known publication.

Remark 4 In this paper, we don't assume that activation function is differentiable, bounded and monotonically increasing. Clearly, these functions are more general. For example, the Gaussian and inverse Gaussian functions have been used in the circuit designs and applications of cellular neural networks.

4 An example

In this section, we give an example to illustrate effectiveness of our results.

Example 4.1 Consider the following impulsive stochastic fuzzy neural networks with time-varying delays and distributed delays

$$\left\{ \begin{aligned} d\quad & x_1(t) \\ =\quad & [-12x_1(t) + 0.2f_1(x_1(t)) + 0.6f_2(x_2(t)) \\ & + \textstyle\bigwedge_{j=1}^2 \alpha_{1j} g_j(t - \tau_{1j}(t)) + I_1 \\ & + \textstyle\bigvee_{j=1}^2 \beta_{1j} g_j(t - \tau_{1j}(t)) \\ & + \textstyle\bigwedge_{j=1}^2 \delta_{1j} \int_{-\infty}^t K_{1j}(t-s) h_j(x_j(s)) ds \\ & + \textstyle\bigvee_{j=1}^2 \eta_{1j} \int_{-\infty}^t K_{1j}(t-s) h_j(x_j(s)) ds \\ & + \textstyle\bigwedge_{j=1}^2 T_{1j} u_j + \bigvee_{j=1}^2 H_{1j} u_j \Big] dt \\ & + \sigma_{11}(x_1(t), x_1(t - \tau_{11}(t))) d\omega_1 \\ & + \sigma_{12}(x_2(t), x_2(t - \tau_{12}(t))) d\omega_2, t \neq t_k \\ d\quad & x_2(t) \\ =\quad & [-16x_2(t) + 0.3f_2(x_1(t)) + 0.4f_2(x_2(t)) \\ & + \textstyle\bigwedge_{j=1}^2 \alpha_{2j} g_j(x_j(t - \tau_{ij}(t))) + I_2 \\ & + \textstyle\bigvee_{j=1}^2 \beta_{2j} g_j(x_j(t - \tau_{ij}(t))) \\ & + \textstyle\bigwedge_{j=1}^2 \delta_{2j} \int_{-\infty}^t K_{2j}(t-s) h_j(x_j(s)) ds \\ & + \textstyle\bigvee_{j=1}^2 \eta_{2j} \int_{-\infty}^t K_{2j}(t-s) h_j(x_j(s)) ds \\ & + \textstyle\bigwedge_{j=1}^2 T_{2j} u_j + \bigvee_{j=1}^2 H_{2j} u_j \Big] dt \\ & + \sigma_{21}(x_1(t), x_1(t - \tau_{21}(t))) d\omega_1 \\ & + \sigma_{22}(x_2(t), x_2(t - \tau_{22}(t))) d\omega_2, t \neq t_k \\ \Delta\quad & x_1(t_k)) = -(1 + 0.2\sin(1 + k^2)) x_1(t_k^-) \\ \Delta\quad & x_2(t_k)) = -(1 + 0.3\sin(1 + k)) x_2(t_k^-) \end{aligned} \right. \tag{26}$$

where $t_0 = 0, t_k = t_{k-1} + 0.2k, k = 1, 2, \cdots, I_i = \bigwedge_{j=1}^2 T_{ij} u_j + \bigvee_{j=1}^2 H_{ij} u_j + \tilde{I}_i (i = 1, 2)$, and

$$\alpha = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix},\quad \beta = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{pmatrix},$$
$$\delta = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix},\quad \eta = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$f_i(x) = g_i(x) = h_i(x) = -|x|, i = 1, 2, \tilde{I}_1 = \tilde{I}_2 =$

$4, u_1 = u_2 = 1, \tau_{ij}(t) = 0.3|\sin t| + 0.1, K_{ij}(t) = te^{-t}(i, j = 1, 2)$ and the matrices $T = (T_{ij})_{2\times2}, H = (H_{ij})_{2\times2}$ are identity matrices. Clearly f_i, g_i, h_i are unbounded and Lipschitz continuous with the Lipschitz constants $\mu_i = \nu_i = \vartheta_i = 1$.

Obviously, model (26) satisfies assumptions $(A1) - (A2)$.

Set

$$\sigma_{11}(x, y) = 0.1x + 0.3y, \quad \sigma_{12}(x, y) = 0.2x + 0.1y,$$
$$\sigma_{21}(x, y) = 0.1x + 0.2y, \quad \sigma_{22}(x, y) = 0.2x + 0.3y.$$

It can be easily checked that the assumption $(A3)$ is satisfied with

$$s_{11} = 0.06, s_{12} = 0.09, s_{21} = 0.09, s_{22} = 0.1,$$

$$w_{11} = 0.08, w_{12} = 0.12, w_{21} = 0.1, w_{22} = 0.15.$$

Taking $p = 3$. It is easy to compute

$$Q = \begin{pmatrix} -28.571 & 0.78 \\ 0.48 & -43.823 \end{pmatrix},$$

$$L = \begin{pmatrix} 1.493 & 1.573 \\ 1.533 & 1.633 \end{pmatrix}$$

and

$$-(Q + L) = \begin{pmatrix} 27.078 & -2.353 \\ -2.013 & 42.19 \end{pmatrix}$$

is an $M-$matrix. Clearly, all conditions of Theorem 7 are satisfied. Thus model (26) has a unique equilibrium point x^* which is globally exponentially 3-stable.

5 Conclusion

In this paper, the problem on stability analysis has been investigated for a class of impulsive stochastic fuzzy cellular neural networks with both time-varying delays and infinite distributed delays. A sufficient condition to ensure the existence, uniqueness, and exponential p-stability of equilibrium point for the addressed neural network has been obtained by employing a combination of the M-matrix theory and stochastic analysis technique. The proposed method has been shown to be simple and effective for analyzing the stability of impulsive or stochastic fuzzy cellular neural networks with variable and/or distributed delays. The obtained criteria can be applied to design globally exponential $p-$stable fuzzy cellular neural networks.

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