3.1 The equilibrium points

We emphatically discussed the case with $T = 2$. To study the stability of the system (5), we change system (5) to the following format.

$$\begin{aligned}
&p_1(t+1) = p_1(t) + \alpha_1 p_1(t)[a_1 - 2b_1 w_0 p_1(t) \\
&- 2b_1 w_1 x_1(t) - 2b_1(1 - w_0 - w_1) y_1(t) \\
&+ d_1 p_2(t) + b_1 c_1] \\
&p_2(t+1) = p_2(t) + \alpha_2 p_2(t)[a_2 - 2b_2 p_2(t) \\
&+ d_2 w_0 p_1(t) + d_2 w_1 x_1(t) + d_2(1 - w_0 - w_1) y_1(t) \\
&+ b_2 c_2] \\
&x_1(t+1) = p_1(t) \\
&y_1(t+1) = x_1(t)
\end{aligned} \tag{6}$$

We obtained the four fixed points (i.e. equilibrium points), they are as follows.

$$E_0 = (0,0,0,0)$$

$$E_1 = (0, \frac{a_2 + b_2 c_2}{2b_2}, 0, 0)$$

$$E_2 = (\frac{a_1 + b_1 c_1}{2b_1}, 0, \frac{a_1 + b_1 c_1}{2b_1}, \frac{a_1 + b_1 c_1}{2b_1})$$

$$E_3 = (\frac{a_2 d_1 + 2a_1 b_2 + b_2 c_2 d_1 + 2b_1 b_2 c_1}{4b_1 b_2 - d_1 d_2}, \frac{a_1 d_2 + 2a_2 b_1 + b_1 c_1 d_2 + 2b_1 b_2 c_2}{4b_1 b_2 - d_1 d_2}, \frac{a_2 d_1 + 2a_1 b_2 + b_2 c_2 d_1 + 2b_1 b_2 c_1}{4b_1 b_2 - d_1 d_2}, \frac{a_2 d_1 + 2a_1 b_2 + b_2 c_2 d_1 + 2b_1 b_2 c_1}{4b_1 b_2 - d_1 d_2})$$

It is obvious that the point E_0, E_1 and E_2 are bounded equilibrium points, point E_3 is the unique Nash equilibrium point[11].

3.2 The stability of equilibrium points

Now we will analyze the stability of the preceding four market equilibrium points by using the stability of fixed points of the discrete dynamic system.

At point E_0, its Jacobin matrix is:

$$J(E_0) = \begin{bmatrix} 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1 & 0 & 0 & 0 \\ 0 & 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Its four characteristic roots are respectively $\lambda_1 = 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1$, $\lambda_2 = 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2$, $\lambda_3 = 0$, $\lambda_4 = 0$. For $\alpha_i, a_i, b_i, c_i > 0$ ($i = 1,2$), so $\lambda_1 > 1, \lambda_2 > 1$, and point E_0 is unstable.

At point E_1, its Jacobin matrix is:

$$J(E_1) = \begin{bmatrix} s_1 & 0 & 0 & 0 \\ s_2 & s_3 & s_4 & s_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Among it,

$$s_1 = 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1 + \alpha_1 d_1 \cdot \frac{a_2 + b_2 c_2}{2b_2}$$

$$s_2 = \alpha_2 d_2 w_0 \cdot \frac{a_2 + b_2 c_2}{2b_2}$$

$$s_3 = 1 - \alpha_2 a_2 - \alpha_2 b_2 c_2$$

$$s_4 = \alpha_2 d_2 w_1 \cdot \frac{a_2 + b_2 c_2}{2b_2}$$

$$s_5 = \alpha_2 d_2 (1 - w_0 - w_1) \cdot \frac{a_2 + b_2 c_2}{2b_2}$$

Its characteristic roots are $\lambda_1 = 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1 + \alpha_1 d_1 \cdot \frac{a_2 + b_2 c_2}{2b_2}$, $\lambda_2 = 1 - \alpha_2 a_2 - \alpha_2 b_2 c_2$, $\lambda_3 = 0$, $\lambda_4 = 0$. It is obvious that $\lambda_1 > 1$, so point E_1 is also unstable.

At point E_2, its Jacobin matrix is:

$$J(E_2) = \begin{bmatrix} s_1^* & s_2^* & s_3^* & s_4^* \\ 0 & s_5^* & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Among it,

$$s_1^* = 1 - \alpha_1 w_0 a_1 - \alpha_1 w_0 b_1 c_1$$

$$s_2^* = \alpha_1 d_1 \frac{a_1 + b_1 c_1}{2b_1}$$

$$s_3^* = -\alpha_1 w_1 a_1 - \alpha_1 w_1 b_1 c_1$$

$$s_4^* = -\alpha_1 (1 - w_0 - w_1)(a_1 + b_1 c_1)$$