Research on Delayed Complexity Based on Nonlinear Price Game of Insurance Market

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Abstract: - Based on the study of scholars, supposing that one of the two competitors in the market makes decision only with bounded rationality without delay, and the other competitor makes the delayed decision with one period and two periods, we established the dynamic price game models respectively. In this paper we mainly analyzed the stable points and their stabilities of the dynamic system with two-period delayed decision, and made computer simulations for the system stability under different decision rules and the complexity such as the bifurcations, chaos and so on. The numerical simulation results showed that, the delayed decision can not change the system's Nash equilibrium point, however it can improve the system's stability; the changes of delayed period and weights of delay variables will make the system's stability area change correspondingly; when the company make decision with delay, they should consider the introducing time. Because the proper delayed periods and weights of variables will obviously improve his competition advantages.

Key-Words: - duopoly, delayed decision, bifurcation, chaotic, nonlinear price game, insurance market

1 Introduction

In recent years, with the increasing of insurance companies in our country, the concentration ratio of the insurance market slightly decreased, however it is still difficult to get rid of the market structure characteristic of oligopoly in shorter time. There are many references which study on the oligarch competition in insurance market; however most of them concentrate on the study of static game among oligarchs. Zhu xiangjun and Liu mingdong [1] studied the oligarch's equilibrium price with considering the limitation of supply ability and without considering that based on Bertrand model respectively; Zheng wenzhe and Wu jilin [2] established the price competition model of the oligarchs with the difference of service quality, they assumed that the two companies played the static game with incomplete information; Han shufeng and Ding jiaxing [3] analyzed the market bidding from the view of game theory and put forward corresponding strategies. Agiza H N, Wang guodong and Yaohongxing [4~6] respectively studied the dynamic behavior based on the output adjustment model with bounded rationality. The concept of delay is put forward by Alderson in 1950, after that Jack and Stephan Biller [7~8] successively studied and applied the delay from different views of

supply chain. Pan yurong and Jia zhaoyong [9] studied the beingness of the system's Nash equilibrium point and its stability when one part used the delayed decision with bounded rationality and the other part used the decision rule of optimal reaction; Ma junhai and Pengjing [10] studied the system's stability, the bifurcation and chaos with different parameters assuming that all the oligarchs used the delayed decisions with bounded rationality. The studies about the delayed decision concentrates on the case of one-period delay, they can not yet cover the conditions of more periods, and they have studied little about the change of oligarch's profits and competitive status under different delay. We established the price competition game model of insurance market, assumed that one of the oligarchs used the delayed decision and the other part used the decision rule of expectation and bounded rationality, considered the two conditions of one-period delay and two-period delay. By numerical simulation, we made a comparative analysis to the simulation results to further uncover the influence which the delay has on the system's stability and the internal regulating action, thus it can serve for the economy better.

2 Model

Supposing that there are only two main competitors in the insurance market, they are respectively represented by 1 and 2. The two parties play games by making different prices and the pricing decisions are made in discrete time periods, the period is signified by $t, t = 0, 1, 2, 3, \cdots$. Their respective prices are p_1 and p_2 , and the demand function is $q_i = a_i - b_i p_i + d_i p_j$, i, j = 1, 2, $i \neq j$, $a_i, b_i > 0$, a_i represents the possible largest demand, b_i represents the influence which the price of product i has on its quantity demanded. $d_i > 0$, d_i represents the substitution rate which the products of i shows to the products of j. The two companies have no fixed cost and their variable cost of unit product is c_i , $c_i > 0$, so the cost function is $C_i(q_i) = c_i q_i$ and the profit function is $\pi_i(p_1, p_2) = p_i q_i - c_i q_i$, and their marginal profit is

$$\frac{\partial \pi_i(p_1, p_2)}{\partial p_i} = a_i - 2b_i p_i + d_i p_j + b_i c_i.$$

In fact, each company has no the whole market information and they can not entirely forecast the future changes in the market, so they often made their decisions based on partial information. Supposing that the company's next-period decision is the bounded rationality adjustment based on the partial estimation to the marginal profit of current period, that is, if the marginal profit of current period is positive, the company will put up his price of next period; otherwise he will lower it. The adjustment process is as follows:

$$p_{i}(t+1) = p_{i}(t) + \alpha_{i}p_{i}(t)\frac{\partial\pi_{i}(p_{1}, p_{2})}{\partial p_{i}}$$
$$i = 1,2 \qquad (1)$$

 $p_i(t+1)$ represents the price of the t+1period of company i, $\alpha_i > 0$ and it represents

the price adjustment speed of company i, so we can get the price game model between the two companies:

$$p_{i}(t+1) = p_{i}(t) + \alpha_{i}p_{i}(t)(a_{i} - 2b_{i}p_{i} + d_{i}p_{j} + b_{i}c_{i})$$

$$i = 1,2$$
(2)

However being the bounded rational

economic body, when the company make the next-period price decision, he will make a comprehensive consideration about the profits of past continuous periods $(t, t-1, t-2, \dots, t-T)$, not just the profit of current period. This will make the decision more rational. Now the dynamic decision process can be adjusted as follows:

$$\begin{aligned} p_i(t+1) &= p_i(t) + \alpha_i p_i(t) \frac{\partial \pi_i(p^d)}{\partial p_i} \\ i &= 1,2 \quad (3) \\ p^d &= (p_1^d, p_2^d) \quad , \quad p_i^d &= \sum_{l=0}^T p_i(t-l) w_l \quad , \end{aligned}$$

 $w_l \ge 0$, $\sum_{l=0}^{I} w_l = 1$, w_l is the weight of the price

of the l th period.

Supposing that in the fierce competition, company 1 first considers making his price delayed decision to stabilize the market, and the delayed period is 1. That is, when the company makes his next-period price decision, he will make a comprehensive consideration about the marginal profit of the current period and the last period. Company 2 takes no consideration of making delayed decision. We can get the following discrete dynamic system.

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha_1 p_1(t)[a_1 - 2b_1 w_0 p_1(t) \\ -2b_1(1 - w_0) p_1(t-1) + d_1 p_2(t) + b_1 c_1] \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)[a_2 - 2b_2 p_2(t) \\ + d_2 w_0 p_1(t) + d_2(1 - w_0) p_1(t-1) + b_2 c_2] \end{cases}$$
(4)

Now we keep the other hypotheses changeless except the delayed period. Supposing that the delayed period is 2, that is, when the company makes his next-period price decision, he will make a comprehensive consideration about the marginal profit of the current period and the last two periods. Company 2 still follows the original process. So we can get the following system.

$$\begin{cases} p_{1}(t+1) = p_{1}(t) + \alpha_{1}p_{1}(t)[a_{1} - 2b_{1}w_{0}p_{1}(t) \\ -2b_{1}w_{1}p_{1}(t-1) - 2b_{1}(1 - w_{0} - w_{1})p_{1}(t-2) \\ +d_{1}p_{2}(t) + b_{1}c_{1}] \\ p_{2}(t+1) = p_{2}(t) + \alpha_{2}p_{2}(t)[a_{2} - 2b_{2}p_{2}(t) \\ +d_{2}w_{0}p_{1}(t) + d_{2}w_{1}p_{1}(t-1) \\ +d_{2}(1 - w_{0} - w_{1})p_{1}(t-2) + b_{2}c_{2}] \end{cases}$$
(5)

3 Model analysis

3.1 The equilibrium points

We emphatically discussed the case with T = 2. To study the stability of the system (5), we change system (5) to the following format.

$$p_{1}(t+1) = p_{1}(t) + \alpha_{1}p_{1}(t)[a_{1} - 2b_{1}w_{0}p_{1}(t) - 2b_{1}w_{1}x_{1}(t) - 2b_{1}(1 - w_{0} - w_{1})y_{1}(t) + d_{1}p_{2}(t) + b_{1}c_{1}]$$

$$p_{2}(t+1) = p_{2}(t) + \alpha_{2}p_{2}(t)[a_{2} - 2b_{2}p_{2}(t) + d_{2}w_{0}p_{1}(t) + d_{2}w_{1}x_{1}(t) + d_{2}(1 - w_{0} - w_{1})y_{1}(t) + b_{2}c_{2}]$$

$$x_{1}(t+1) = p_{1}(t)$$

$$y_{1}(t+1) = x_{1}(t)$$
(6)

We obtained the four fixed points (i.e. equilibrium points), they are as follows.

$$\begin{split} E_{0} &= (0,0,0,0) \\ E_{1} &= (0,\frac{a_{2}+b_{2}c_{2}}{2b_{2}},0,0) \\ E_{2} &= (\frac{a_{1}+b_{1}c_{1}}{2b_{1}},0,\frac{a_{1}+b_{1}c_{1}}{2b_{1}},\frac{a_{1}+b_{1}c_{1}}{2b_{1}}) \\ E_{3} &= (\frac{a_{2}d_{1}+2a_{1}b_{2}+b_{2}c_{2}d_{1}+2b_{1}b_{2}c_{1}}{4b_{1}b_{2}-d_{1}d_{2}}, \\ \frac{a_{1}d_{2}+2a_{2}b_{1}+b_{1}c_{1}d_{2}+2b_{1}b_{2}c_{2}}{4b_{1}b_{2}-d_{1}d_{2}}, \\ \frac{a_{2}d_{1}+2a_{1}b_{2}+b_{2}c_{2}d_{1}+2b_{1}b_{2}c_{1}}{4b_{1}b_{2}-d_{1}d_{2}}, \\ \frac{a_{2}d_{1}+2a_{1}b_{2}+b_{2}c_{2}d_{1}+2b_{1}b_{2}c_{1}}{4b_{1}b_{2}-d_{1}d_{2}}, \end{split}$$

It is obvious that the point E_0, E_1 and E_2 are bounded equilibrium points, point E_3 is the unique Nash equilibrium point^[11].

3.2 The stability of equilibrium points

Now we will analyze the stability of the preceding four market equilibrium points by using the stability of fixed points of the discrete dynamic system.

At point E_0 , its Jacobin matrix is:

 $J(E_0) = \begin{bmatrix} 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1 & 0 & 0 & 0 \\ 0 & 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ Its four characteristic roots are respectively $\lambda_1 = 1 + \alpha_1 a_1 + \alpha_1 b_1 c_1$, $\lambda_2 = 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2$, $\lambda_3 = 0$, $\lambda_4 = 0$. For $\alpha_i, a_i, b_i, c_i > 0$ (i = 1, 2), so $\lambda_1 > 1, \lambda_2 > 1$, and point E_0 is unstable.

At point E_1 , its Jacobin matrix is:

$$J(E_1) = \begin{bmatrix} s_1 & 0 & 0 & 0 \\ s_2 & s_3 & s_4 & s_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Among it,

$$s_{1} = 1 + \alpha_{1}a_{1} + \alpha_{1}b_{1}c_{1} + \alpha_{1}d_{1} \cdot \frac{a_{2} + b_{2}c_{2}}{2b_{2}}$$

$$s_{2} = \alpha_{2}d_{2}w_{0} \cdot \frac{a_{2} + b_{2}c_{2}}{2b_{2}}$$

$$s_{3} = 1 - \alpha_{2}a_{2} - \alpha_{2}b_{2}c_{2}$$

$$s_{4} = \alpha_{2}d_{2}w_{1} \cdot \frac{a_{2} + b_{2}c_{2}}{2b_{2}}$$

$$s_{5} = \alpha_{2}d_{2}(1 - w_{0} - w_{1}) \cdot \frac{a_{2} + b_{2}c_{2}}{2b_{2}}$$
Its characteristic roots are
$$\lambda_{1} = 1 + \alpha_{1}a_{1} + \alpha_{1}b_{1}c_{1} + \alpha_{1}d_{1} \cdot \frac{a_{2} + b_{2}c_{2}}{2b_{2}}$$

$$\lambda_{2} = 1 - \alpha_{2}a_{2} - \alpha_{2}b_{2} + \alpha_{2}a_{2} + \alpha_{2}b_{2} + \alpha_{2}b_{2}$$

 $\lambda_2 = 1 - \alpha_2 a_2 - \alpha_2 b_2 c_2$, $\lambda_3 = 0$, $\lambda_4 = 0$. It is obvious that $\lambda_1 > 1$, so point E_1 is also unstable.

At point E_2 , its Jacobin matrix is:

$$J(E_2) = \begin{bmatrix} s_1^* & s_2^* & s_3^* & s_4^* \\ 0 & s_5^* & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Among it,

$$s_{1}^{*} = 1 - \alpha_{1}w_{0}a_{1} - \alpha_{1}w_{0}b_{1}c_{1}$$

$$s_{2}^{*} = \alpha_{1}d_{1}\frac{a_{1} + b_{1}c_{1}}{2b_{1}}$$

$$s_{3}^{*} = -\alpha_{1}w_{1}a_{1} - \alpha_{1}w_{1}b_{1}c_{1}$$

$$s_{4}^{*} = -\alpha_{1}(1 - w_{0} - w_{1})(a_{1} + b_{1}c_{1})$$

$$\begin{split} s_5^* &= 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2 + \alpha_2 d_2 \frac{a_1 + b_1 c_1}{2b_1} \\ & \text{One of its characteristic roots is} \\ \lambda_2 &= 1 + \alpha_2 a_2 + \alpha_2 b_2 c_2 + \alpha_2 d_2 \frac{a_1 + b_1 c_1}{2b_1}, \text{ the other} \\ \text{three roots are worked out} \\ \lambda^3 - (1 - \alpha_1 w_0 a_1 - \alpha_1 w_0 b_1 c_1) \lambda^2 \\ &+ (\alpha_1 w_1 a_1 + \alpha_1 w_1 b_1 c_1) \lambda + \alpha_1 (1 - w_0 - w_1) (a_1 + b_1 c_1) = 0 \end{split}$$

It is obvious that $\lambda_2 > 1$, and point E_2 is also unstable.

At point E_3 , its Jacobin matrix is:

$$J(E_3) = \begin{bmatrix} s_1^{**} & s_2^{**} & s_3^{**} & s_4^{**} \\ s_1^{**} & s_2^{**} & s_3^{**} & s_4^{**} \\ s_5^{**} & s_6^{**} & s_7^{**} & s_8^{**} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Among it,

$$\begin{split} s_{1}^{**} &= 1 + \alpha_{1}a_{1} + \alpha_{1}b_{1}c_{1} - \\ &2\alpha_{1}b_{1}(1+w_{0})\frac{a_{2}d_{1} + 2a_{1}b_{2} + b_{2}c_{2}d_{1} + 2b_{1}b_{2}c_{1}}{4b_{1}b_{2} - d_{1}d_{2}} \\ &+ \alpha_{1}d_{1}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{2}^{**} &= \alpha_{1}d_{1}\frac{a_{2}d_{1} + 2a_{1}b_{2} + b_{2}c_{2}d_{1} + 2b_{1}b_{2}c_{1}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{3}^{**} &= -2\alpha_{1}b_{1}w_{1}\frac{a_{2}d_{1} + 2a_{1}b_{2} + b_{2}c_{2}d_{1} + 2b_{1}b_{2}c_{1}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{4}^{**} &= -2\alpha_{1}b_{1}(1-w_{0}-w_{1})\frac{a_{2}d_{1} + 2a_{1}b_{2} + b_{2}c_{2}d_{1} + 2b_{1}b_{2}c_{1}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{5}^{**} &= \alpha_{2}d_{2}w_{0}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{5}^{**} &= \alpha_{2}d_{2}w_{0}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ + \alpha_{2}d_{2}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ + \alpha_{2}d_{2}\frac{a_{2}d_{1} + 2a_{1}b_{2} + b_{2}c_{2}d_{1} + 2b_{1}b_{2}c_{1}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{7}^{**} &= \alpha_{2}d_{2}w_{1}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{7}^{**} &= \alpha_{2}d_{2}w_{1}\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{7}^{**} &= \alpha_{2}d_{2}(1-w_{0} - w_{1})\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{8}^{**} &= \alpha_{2}d_{2}(1-w_{0} - w_{1})\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{8}^{**} &= \alpha_{2}d_{2}(1-w_{0} - w_{1})\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{8}^{**} &= \alpha_{2}d_{2}(1-w_{0} - w_{1})\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{8}^{**} &= \alpha_{2}d_{2}(1-w_{0} - w_{1})\frac{a_{1}d_{2} + 2a_{2}b_{1} + b_{1}c_{1}d_{2} + 2b_{1}b_{2}c_{2}}{4b_{1}b_{2} - d_{1}d_{2}} \\ s_{8}^{**} &= \alpha_{2}d_{2}(1-w$$

And its characteristic polynomial is

$$\lambda^{4} - (s_{1} + s_{6})\lambda^{3} + (s_{1}s_{6} - s_{5}s_{2} - s_{3})\lambda^{2} + (s_{3}s_{6} - s_{2}s_{7} - s_{4})\lambda + (s_{4}s_{6} - s_{2}s_{8}) = 0$$

According to Jury conditions, the necessary and sufficient conditions that Nash equilibrium point E_3 is stable are,

(1)

$$\begin{array}{l}
1 - s_1 - s_6 + s_1 s_6 - s_5 s_2 - s_3 \\
+ s_3 s_6 - s_2 s_7 - s_4 + s_4 s_6 - s_2 s_8 > 0 \\
(2) \quad 1 + s_1 + s_6 + s_1 s_6 - s_5 s_2 - s_3 \\
- s_3 s_6 + s_2 s_7 + s_4 + s_4 s_6 - s_2 s_8 > 0 \\
(3) \quad |s_1 s_4 - s_5 s_6| \le 1
\end{array}$$

(4)
$$\frac{|(s_1 + s_6)(s_4 s_6 - s_2 s_8) - (s_3 s_6 - s_2 s_7 - s_4)|}{<1 - (s_4 s_6 - s_2 s_8)^2}$$

(5)

$$\begin{vmatrix}
(s_1s_6 - s_5s_2 - s_3)(1 - s_4s_6 + s_2s_8)[1 - (s_4s_6 - s_2s_8)^2] \\
+[(s_1 + s_6)(s_4s_6 - s_2s_8) + (s_3s_6 - s_2s_7 - s_4)][(s_1 + s_6) + (s_4s_6 - s_2s_8)(s_3s_6 - s_2s_7 - s_4)] \\
+(s_4s_6 - s_2s_8)(s_3s_6 - s_2s_7 - s_4)] \\
<[1 - (s_4s_6 - s_2s_8)^2]^2 - [(s_1 + s_6)(s_4s_6 - s_2s_8) + (s_3s_6 - s_2s_7 - s_4)]^2
\end{vmatrix}$$

4 The numerical simulation and analysis

To know better about the dynamic behavior of the systems, we make numerical simulation and analysis for them. First, when the company makes decision without delay, we determine the values of different parameters,

$$a_1 = 4, a_2 = 4.1, b_1 = 4, b_2 = 4.1, d_1 = 0.8, d_2 = 0.7,$$

 $c_1 = 0.0015, c_2 = 0.0010, \alpha_2 = 0.4$

so we can get the path diagrams about the prices of company 1 company 2 with the change of α_1 , see Fig.1. It can be seen that, with gradually increasing of the price adjustment speed α_1 of company 1, the prices of company 1 and company 2 begin to appear stable periodic, period-doubling bifurcation, even chaotic phenomena. When company 1 makes decision with one-period delay, we keep the preceding parameters changeless and make value of the delay coefficient (i.e. w_0), it is $w_0 = 0.6$ and $w_0 = 0.3$ respectively, then we can get the two bifurcation diagrams about the prices of two companies with the change of α_1 , see Fig.2 and Fig.3. Then consider the case with two-period delayed decision, we still keep the preceding parameters changeless and make $w_0 = 0.8, w_1 = 0.1$, and we can get the bifurcation diagram about the prices of two companies with the change of α_1 , see Fig.4; Keep the value of w_0 changeless and making $w_1 = 0.15$, the bifurcation diagram is seen in Fig.5.



Fig.1 The bifurcation diagram about prices of company 1 and company 2 with change of α_1 when company 1 does not use the delayed decision



Fig.2 The bifurcation diagram about prices of company 1 and company 2 with change of α_1 when company 1 makes the delayed decision with one period and $w_0 = 0.6$



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Fig.3 The bifurcation diagram about prices of company 1 and company 2 with change of α_1 when company 1 makes the delayed decision with one period and $w_0 = 0.3$



Fig.4 The bifurcation diagram about prices of company 1 and company 2 with change of α_1 when company 1 makes the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.1$



Fig.5 The bifurcation diagram about prices of company 1 and company 2 with change of α_1 when company 1 makes the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.15$

By comparing Fig.1, Fig.2, Fig.3 and Fig.4, Fig.5, we can find that whether company 1 uses the delayed decision or not, the Nash equilibrium point of the system does not change, it is point (0.5555, 0.5479), that is, the delayed decision does not lead the system's stable point to change; Compare Fig.2, Fig.3 and Fig.4, Fig.5, we can see that the different value of the delay coefficient weight of company 1 can bring large influence to the system's dynamic behavior. In Fig.3, when $\alpha_1 = 0.33$, the two companies' prices have entered into the bifurcation area, however they are still in the stable area in Fig.2. That is, the weight of the delay coefficient can influence the size of the system's stable area.

Consider company 1 uses the delayed decision of price, and the delayed phase is 2, T = 2, $w_0 = 0.8$, $w_1 = 0.1$, when we keep the other parameters changeless, the changes of the system's maximum Lyapunov exponent with the changing of price adjustment parameter α_1 of company 1 are seen in Fig.6. The value of Lyapunov exponent marks the mean convergence or divergence exponential rate of similar tracks in the phase space of the system motion, any system which has at least one positive Lyapunov exponent is chaotic, and the bigger the Lyapunov exponent, the stronger the chaos. We can see from Fig.6 that different values of α_1 are corresponding to different Lyapunov exponent. Compare Fig.6 with Fig.4 we can find that they are basically coincident, when α_1 is smaller, the system is the process of period-doubling bifurcation and it has regular periodic motion, so the maximal Lyapunov exponent is always negative and only at the bifurcation point it is equal to zero because of the uncertain critical state; When α_1 is bigger, the maximal Lyapunov exponent is positive for the most part, this shows the system is in chaos area this moment. This also further verifies the motion state after the system is added a delay variable.



Lyapunov







company 2 when company 1 makes the delayed decision with one period and $w_0 = 0.6$ in the first time



company 2 when company 1 makes the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.1$ in the first time

In the first time of the two companies' price competition, we determine values of parameters. $a_1 = 4, a_2 = 4.1, b_1 = 4, b_2 = 4.1, d_1 = 0.8, d_2 = 0.7,$ $c_1 = 0.0015, c_2 = 0.0010, \alpha_1 = 0.3, \alpha_2 = 0.4, t = 15$ Fig.7, Fig.8 and Fig.9 are respectively the profit diagrams of company 1 and company 2 in their early competition when company 1 makes decision without delay, with one-period delay ($w_0 = 0.6$) and two-period delay ($w_0 = 0.8, w_1 = 0.1$). By simulation we find that when company 1 does not use the delayed decision the accumulated profits of company 1 and company 2 are $\Pi_1 = 12.5536, \Pi_2 = 13.4921$, it is apparent that company 2 has the obvious competition advantages than company 1; When company 1 uses the delayed decision with one period and $w_0 = 0.6$, the accumulated profits of company 1 and company 2 are $\Pi_1 = 12.5595, \Pi_2 = 13.5474$, company 1 does not change the competitive disadvantages because of the use of delayed decision. We still consider the case of one-period delay and make the value of $w_0 = 0.3$, now the two companies' accumulated profits are $\Pi_1 = 11.8855, \Pi_2 = 13.5616$, the competition is same as the case of $w_0 = 0.6$ and company 1 is still at a disadvantage; When company 1 uses the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.1$, the accumulated profits of company and company 1 2 are $\Pi_1 = 12.5818, \Pi_2 = 13.5411$, it is apparent that the inferior position of company 1 does not change

after the period of delay lengthens. Making $w_0 = 0.8, w_1 = 0.15$, their accumulated profits are $\Pi_1 = 12.5850, \Pi_2 = 13.5323$, the result is same as case of $w_0 = 0.8, w_1 = 0.1$.

Keep the other parameters changeless and continue to increase the value of α_1 , when $\alpha_1 = 0.6, \alpha_2 = 0.4$, the system entered into chaotic state. Fig.10, Fig.11 and Fig.12 are respectively the profit diagrams of company 1 and company 2 in the chaos when company 1 makes decision without delay, with one-period delay two-period $w_0 = 0.6$) and delay $(w_0 = 0.8, w_1 = 0.1)$. When company 1 does not use the delayed decision the accumulated profits of company and company 1 2 are $\Pi_1 = 13.1851, \Pi_2 = 13.3767$, company 2 has the competitive advantages than company 1; When company 1 uses the delayed decision with one period and $w_0 = 0.6$, the accumulated profits of company 1 and company 2 are $\Pi_1 = 13.2130, \Pi_2 = 13.5977$, the state that company 1 is at a competitive disadvantage is not changed; When company 1 uses the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.1$, the accumulated profits of company 1 and company 2 are $\Pi_1 = 14.3585, \Pi_2 = 13.7786$, now company 1 begins to have obvious advantages than company 2. Then we change the values of $w_0 = 0.8, w_1 = 0.15$, the two companies' accumulated profits are $\Pi_1 = 14.6028, \Pi_2 = 13.8012$, company 1 still has obvious advantages.



Fig.10 The profit diagram of company 1 and company 2 when company 1 does not use the delayed decision in the chaos



Fig.11 The profit diagram of company 1 and company 2 when company 1 makes the delayed decision with one period and $w_0 = 0.6$ in the chaos



Fig.12 The profit diagram of company 1 and company 2 when company 1 makes the delayed decision with two periods and $w_0 = 0.8, w_1 = 0.1$ in the chaos

So when the company thinks of using the delayed decision, he should consider the time of using. In the first time the using of the delayed decision can not change the competitive state, on the contrary, it may weaken the company's competitive power; In the chaotic period if the company uses the delayed decisions properly, it can obviously strengthen the company's competitive advantages. Therefore the accurate judgment to time when the companies enter into the chaotic state is of key importance.

5 Conclusions

We established the duopoly game model about price competition and considered three different race conditions, they are both the two companies do not use the delayed decision, one of them uses the delayed decision with one period, and one of them uses the delayed decision with two periods, in theory we emphatically studied the stable points and their stabilities of the discrete dynamic system with two-period delay. By the numerical simulation and analysis to the discrete dynamic system with three different game conditions, we have drawn the following conclusions.

(1) The use of the delayed decision not only has effect on the company himself, but also can have effect on his competitor. The use of the delayed decision can not change the system's Nash equilibrium point(this conclusion has been verified both in the theoretical analysis and in the system simulation), but it can improve the system's stability, the different delayed periods or different weights of delay coefficient both can change correspondingly the system's stable area.

(2) When the company thinks of using the delayed decision, he should consider the time of using. If the company uses the delayed decision in the first time, his competition can not be changed, on the contrary, it is maybe weakened; However in the chaos if the company can make proper delayed decision, that is, the delayed period and the weight of delay coefficient are all proper, his stability and competition advantages will be improved immensely.

It is worth pointing out we have only studied the price competition behavior between two companies in the insurance market and the influences which the using of delayed decision has on the game result in theory, the demonstration to the market is expected to be further studied. In addition, other problems such as the competition among three oligarchs or even more, if more competitors all use the delayed decisions what influence they will have effect on the company's future competition advantages and so on are all not considered. So it has still certain limitations to be solved in the future study work.

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