Stability analysis of periodic solutions for stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks with delays

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Abstract: In this paper, the mean square exponential stability of the periodic solution for stochastic reaction-diffusion high-order Cohen-Grossberg-Type BAM neural networks with time delays is investigated. By constructing suitable Lyapunov function, applying $It\hat{o}$ formula and Poincar$\acute{e}$ mapping, we give some sufficient conditions to guarantee the mean square exponential stability of the periodic solution. An illustrative example are also given in the end to show the effectiveness of our results.

Key–Words: stochastic reaction-diffusion Cohen-Grossberg-type BAM neural networks; $It\hat{o}$ formula; Poincar$\acute{e}$ mapping; periodic solution; mean square exponential stability

1 Introduction

The Cohen-Grossberg-type BAM neural networks model, which initially proposed by Cohen and Grossberg [1], have their promising potential for the tasks of parallel computation, associative memory and have great ability to solve difficult optimization problems. Such applications heavily depend on the dynamical behaviors. Thus, the analysis of the dynamical behaviors of bidirectional associative memory neural networks and Cohen- Grossberg neural networks are important and necessary. In recent years, many researchers have studied the global stability and other dynamical behaviors of the Cohen-Grossberg-type BAM neural networks, see [2-10]. For example, In [2], Cao and Song investigated the global exponential stability for Cohen-Grossberg-type BAM neural networks with time-varying delays by using Lyapunov function, M-matrix theory and inequality technique. In [6], by constructing a suitable Lyapunov functional, the asymptotic stability was investigated for Cohen-Grossberg-type BAM neural network. In [7], the authors have proposed a new Cohen-Grossberg-type BAM neural network model with time delays, and some new sufficient conditions ensuring the existence and global asymptotical stability of equilibrium point for this model have been derived. The authors in [8,9,10] have investigated the periodicity of Cohen-Grossberg-type BAM neural networks with variable coefficients.

In the factual operations, however, the diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a non-uniform electromagnetic field. Hence, it is essential to consider the state variables are varying with the time and space variables. The neural networks with diffusion terms can commonly be expressed by partial differential equations. The study on the stability of Cohen-Grossberg with delays and reaction-diffusion terms neural networks, for instance, see [11-14], and references therein.

In particular, the stability criteria for stochastic Cohen-Grossberg neural networks becomes an attractive research problem of importance. Some initial results have just appeared, for stochastic Cohen-Grossberg neural networks, respectively, where, in [15-20].

Higher-order neural networks have attracted considerable attention, this is due to the fact that high-order neural networks have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order neural networks. The authors in [21] have investigated the periodic solutions for a class of higher-order Cohen-Grossberg-type neural networks with delays. In [22], Huo, Li and Tang have investigated the dynamics of high-order BAM neural networks with and without impulses. The authors have investigated the exponential stability of high-order BAM neural networks with time delays [23].

Motivated by the above discussions, a class of stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks with time delay is considered in this paper, we will derive some sufficient conditions of existence, uniqueness and the mean square exponential stability of the periodic solution for stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks with time delays by constructing suitable Lyapunov functional, applying $It\hat{o}$ formula and Poincar$\acute{e}$ mapping. The rest of this paper is organized as follows: In Section 2 the model formulation and some preliminaries are given. The main results are stated in Section 3. Finally, an illustrative example is given to show the effectiveness of the proposed theory.

Consider the following stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks

$$\begin{cases} du_i(t,x) = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k})dt - a_i(u_i(t,x)) \\ \quad \cdot[b_i(u_i(t,x)) - \sum\limits_{j=1}^{m} a_{ij} f_j(v_j(t,x)) \\ \quad - \sum\limits_{j=1}^{m} c_{ij} f_j(v_j(t-\tau_{ij},x)) \\ \quad - \sum\limits_{j=1}^{m}\sum\limits_{k=1}^{m} T_{ijk} f_j(v_j(t)) f_k(v_k(t)) - I_i(t)]dt \\ \quad + \sum\limits_{j=1}^{m} k_{ij}(v_j(t,x))d\omega_{n+j}(t), \\ dv_j(t,x) = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(E_{jk}\frac{\partial v_j(t,x)}{\partial x_k})dt - d_j(v_j(t,x)) \\ \quad \cdot[e_j(v_j(t,x)) - \sum\limits_{i=1}^{n} b_{ji} g_i(u_i(t,x)) \\ \quad - \sum\limits_{i=1}^{n} h_{ji} g_i(u_i(t-\sigma_{ji},x)) \\ \quad - \sum\limits_{i=1}^{n}\sum\limits_{q=1}^{n} S_{jiq} g_i(u_i(t)) g_q(u_q(t)) - J_j(t)]dt \\ \quad + \sum\limits_{i=1}^{n} \rho_{ji}(u_i(t,x))d\omega_i(t), \end{cases} \tag{1}$$

where $i = 1,2,\cdots,n, j = 1,2,\cdots,m$, $x = (x_1,x_2,\cdots,x_l)^T \in \Omega_i \subset R^l$ and Ω_i is a bounded compact set with smooth boundary $\partial\Omega_i$ and mes $\Omega_i > 0$ in space R^l; $u(t,x) = (u_1(t,x),u_2(t,x),\cdots,u_n(t,x))^T \in R^n, v(t,x) = (v_1(t,x),v_2(t,x),\cdots,v_m(t,x))^T \in R^m$, $u_i(t,x)$ and $v_j(t,x)$ are the state of the ith neurons from the neural field F_U and the jth neurons from the neural field F_V at time t and in space x , respectively; $D_{ik} > 0$ and $E_{jk} > 0$ correspond to the transmission reaction-diffusion operator along the ith neurons and the jth neurons, respectively; f_j and g_i denote the activation function of the jth neurons and the ith neurons at time t and in space x, respectively ; a_{ij} and c_{ij} weights the strength of the ith neuron on the jth neuron at the time t and $t-\tau_{ij}$, respectively; b_{ji} and h_{ji} weights the strength of the jth neuron on the ith neuron at the time t and $t-\sigma_{ji}$, respectively; $\tau_{ij} \geq 0$ and $\sigma_{ji} \geq 0$ are nonnegative; $I_i(t), J_j(t)$ denote the external inputs on the ith neuron from F_U and the jth neuron from F_V at the time t, respectively; $a_i(u_i(t,x))$ and $d_j(v_j(t,x))$ represent amplification functions; $b_i(u_i(t,x))$ and $e_j(v_j(t,x))$ are appropriately behaved functions such that the solutions of model(1) remain bounded; T_{ijk} and S_{jiq} are constants, and denote the first- and second-order connection weights of the neural networks, respectively; $k(\cdot) = (k_{ij}(\cdot))_{n\times m})$ and $\rho(\cdot) = (\rho_{ji}(\cdot))_{m\times n})$ denote the diffusion coefficient; $\omega(t) = (\omega_1(t),\omega_2(t)\cdots,\omega_{n+m}(t))^T$ is an $n+m$ -dimensional Brownian motion defined on a complete probability space (Ω,Γ,P) with a natural filtration $\{\Gamma_t\}_{t\geq 0}$ generated by $\{\omega(s) : 0 \leq s \leq t\}$, where we associate Ω with the canonical space generated by all $\{\omega_i(t)\}$, and denote by Γ the associated $\sigma-$ algebra generated by $\{\omega(t)\}$ with the probability measure P.

The boundary conditions and initial conditions of system (1) are given by

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial n} = (\frac{\partial u_i(t,x)}{\partial x_1}, \frac{\partial u_i(t,x)}{\partial x_2}, \cdots, \frac{\partial u_i(t,x)}{\partial x_l})^T = 0, \\ \quad x \in \partial\Omega_i, \ \ i = 1,2,\cdots,n, \\ \frac{\partial v_j(t,x)}{\partial n} = (\frac{\partial v_j(t,x)}{\partial x_1}, \frac{\partial v_j(t,x)}{\partial x_2}, \cdots, \frac{\partial v_j(t,x)}{\partial x_l})^T = 0, \\ \quad x \in \partial\Omega_i, \ \ j = 1,2,\cdots,m. \end{cases} \tag{2}$$

and

$$\begin{cases} u_i(s,x) = \phi_{ui}(s,x), s \in [-\sigma,0], \\ v_j(s,x) = \phi_{vj}(s,x), s \in [-\tau,0], \end{cases} \tag{3}$$

for $x \in \Omega_i, i = 1,2,\cdots,n, j = 1,2,\cdots,m$, where $\sigma = \max\limits_{1\leq i\leq n, 1\leq j\leq m}\{\sigma_{ji}\}$, $\tau = \max\limits_{1\leq i\leq n, 1\leq j\leq m}\{\tau_{ij}\}$, $\phi_{ui}(s,x)$ and $\phi_{vj}(s,x)$ are bounded and continuous on $[-\delta,0]\times\Omega_i, \delta = \max\{\sigma,\tau\}$.

Consider a general stochastic system

$$\begin{cases} dx(t) = f(t,x(t))dt + g(t,x(t))dw(t), \\ x(t_0) = x_0, t \geq t_0 > 0, \end{cases} \tag{4}$$

where $f : R^+ \times R^n \longrightarrow R^n, g : R^+ \times R^n \longrightarrow R^n$.

Let $C^{1,2}(R^+ \times R^n; R^+)$ be the family of all nonnegative functions $V(t,x)$ on $R^+ \times R^n$ which are continuously twice differentiable in x and differentiable in t. If $V(t,x) \in C^{1,2}(R^+ \times R^n; R^+)$, an operator $LV(t,x)$ is defined from $R^+ \times R^n$ to R by

$$\begin{aligned} LV(t,x) &= V_t(t,x) + V_x(t,x)f(t,x) \\ &\quad + \frac{1}{2}trace[g^T(t,x)V_{xx}(t,x)g(t,x)], \end{aligned}$$

where

$V_t(t,x) = \frac{\partial V(t,x)}{\partial t}$,
$V_x(t,x) = (\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \cdots, \frac{\partial V(t,x)}{\partial x_n})$,
$V_{xx}(t,x) = (\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j})_{n\times n}$.
Applying Itó formula, we have

$$dV(t,x) = LV(t,x)dt + V_x(t,x)g(t,x)dw(t).$$

2 Preliminaries

In order to establish the stability conditions for system (1), we give some assumptions.

- $(H1)$: For each $i = 1,2,\ldots,n, j = 1,2,\ldots,m$, functions $a_i(t,z)), d_j(t,z)$ are differentiable and there exists $A_i > 0, D_j > 0$ such that $0 < a_i'(t,z)) \le A_i, 0 < d_j'(t,z) \le D_j$, and satisfy $a_i(t,z) > 0, d_j(t,z) > 0$, this is, there exist positive constants $\underline{a}_i, \bar{a}_i, \underline{d}_j, \bar{d}_j$, such that
 $0 < \underline{a}_i \le a_i(t,z) \le \bar{a}_i$,
 $0 < \underline{d}_j \le d_j(t,z) \le \bar{d}_j, \forall z \in R$.
- $(H2)$: For $a_i(t,z), d_j(t,z)$, $b_i(t,z)$ and $e_j(t,z)$, there exists $\beta_i > 0$ and $\gamma_j > 0$, $i = 1,2,\ldots,n, j = 1,2,\ldots,m$, for any $z, y \in R$, such that
 $(z-y)[a_i(t,z)b_i(t,z) - a_i(t,y)b_i(t,y)] \ge 0$,
 $|a_i(t,z)b_i(t,z) - a_i(t,y)b_i(t,y)| \ge \beta_i|z-y|$;
 $(z-y)[d_j(t,z)e_j(t,z) - d_j(t,y)e_j(t,y)] \ge 0$,
 $|d_j(t,z)e_j(t,z) - d_j(t,y)e_j(t,y)| \ge \gamma_j|z-y|$.
- $(H3)$: The activation functions f_j and g_i are continuously differentiable on $x \in R$, $i = 1,2,\ldots,n, j = 1,2,\ldots,m$, and there exists constant $\bar{f}_j > 0, \bar{g}_i > 0, F_j > 0$ and $G_i > 0$, such that
 $\sup_z |f_j(z)| \le \bar{f}_j, \sup_z |g_i(z)| \le \bar{g}_i$,
 $|f_j(\xi_1) - f_j(\xi_2)| \le F_j|\xi_1 - \xi_2|$,
 $|g_i(\xi_1) - g_i(\xi_2)| \le G_i|\xi_1 - \xi_2|$,
 for any $z, \xi_1, \xi_2 \in R$.
- $(H4)$: Functions $k_{ij}(\cdot)$ and $\rho_{ji}(\cdot)$ satisfy Lipschitz condition, that is, there exist constant $L_{ij} > 0$ and $T_{ji} > 0$, such that
 $|k_{ij}(\xi_1) - k_{ij}(\xi_2)| \le L_{ij}|\xi_1 - \xi_2|$,
 $|\rho_{ji}(\xi_1) - \rho_{ji}(\xi_2)| \le T_{ji}|\xi_1 - \xi_2|$,
 for any $\xi_1, \xi_2 \in R, i = 1,2,\ldots,n, j = 1,2,\ldots,m$.
- $(H5)$: $I_i(t), J_j(t), \omega_i(t)$ and $\omega_{n+j}(t)$ are continuously periodic functions defined on $t \in [0,\infty)$ with common period $\omega > 0$, and they are all bounded, denote
 $I_i^* = \sup_{0\le t<\infty} |I_i(t)|, \quad J_j^* = \sup_{0\le t<\infty} |J_j(t)|$,
 $i = 1,2,\cdots,n, j = 1,2,\cdots,m$.

Definition 1 *Let*
$Z^*(t,x) = (u_1^*(t,x), u_2^*(t,x), \cdots, u_n^*(t,x), v_1^*(t,x), v_2^*(t,x), \cdots, v_m^*(t,x))^T$ *be an $\omega-$ periodic solution of system (1) with initial value*
$\psi = (\psi_{u1}(t,x), \psi_{u2}(t,x), \cdots, \psi_{un}(t,x), \psi_{v1}(t,x), \psi_{v2}(t,x), \cdots, \psi_{vm}(t,x))^T$,
for every solution
$Z(t,x) = (u_1(t,x), u_2(t,x), \cdots, u_n(t,x), v_1(t,x), v_2(t,x), \cdots, v_m(t,x))^T$
of system (1) with initial value
$\phi = (\phi_{u1}(t,x), \phi_{u2}(t,x), \cdots, \phi_{un}(t,x), \phi_{v1}(t,x), \phi_{v2}(t,x), \cdots, \phi_{vm}(t,x))^T$,
if there exist constants $\alpha > 0$ and $M > 1$, such that

$$\sum_{i=1}^{n} E(\|u_i(t,x) - u_i^*(t,x)\|_2^2) + \sum_{j=1}^{m} E(\|v_j(t,x) - v_j^*(t,x)\|_2^2) \le Me^{-\alpha t}[E(\|\phi_u - \psi_u\|) + E(\|\phi_v - \psi_v\|)], \quad t > 0,$$

then $Z^*(t,x)$ is said to be exponentially stable in the mean square, where $E(.)$ denote mathematical expectation,

$$\|u_i(t,x) - u_i^*(t,x)\|_2^2 = \int_{\Omega_i} (u_i(t,x) - u_i^*(t,x))^2 dx,$$
$$\|v_j(t,x) - v_j^*(t,x)\|_2^2 = \int_{\Omega_i} (v_j(t,x) - v_j^*(t,x))^2 dx,$$
$$\|\phi_u - \psi_u\| = \sup_{-\sigma\le t\le 0} \sum_{i=1}^{n} \|\phi_{ui}(t,x) - \psi_{ui}(t,x)\|_2^2,$$
$$\|\phi_v - \psi_v\| = \sup_{-\tau\le t\le 0} \sum_{j=1}^{m} \|\phi_{vj}(t,x) - \psi_{vj}(t,x)\|_2^2.$$

Lemma 2 *For $(x_1(t), x_2(t), \cdots, x_m(t))^T, (x_1^*(t), x_2^*(t), \cdots, x_m^*(t))^T \in R^m$, if $h_j(x_j)$ are continuously differentiable on $x_j (j = 1,2,\ldots,m)$, then we have*

$$\sum_{j=1}^{m}\sum_{k=1}^{m} b_{ijk}[h_j(x_j)h_k(x_k) - h_j(x_j^*)h_k(x_k^*)] = \sum_{j=1}^{m}\sum_{k=1}^{m} (b_{ijk} + b_{ikj})[h_j(x_j) - h_j(x_j^*)]h_k(\xi_k),$$

where ξ_k lies between x_j and x_j^*, $k = 1,2,\ldots,m$.

Proof. Let
$F(x_j, x_k) = h_j(x_j)h_k(x_k) - h_j(x_j^*)h_k(x_k^*)$,
since $h_j(x_j)$ are continuously differentiable on $x_j (j = 1,2,\ldots,m)$, then $F(x_j, x_k)$ are differentiable on x_j, x_k, by Taylor formula, we can obtain

$$F(x_j, x_k) = F(x_j^*, x_k^*) + \frac{\partial F(\xi_j,\xi_k)}{\partial x_j}(x_j - x_j^*) + \frac{\partial F(\xi_j,\xi_k)}{\partial x_k}(x_k - x_k^*)$$
$$= \frac{\partial h_j(\xi_j)}{\partial x_j} h_k(\xi_k)(x_j - x_j^*) + \frac{\partial h_k(\xi_k)}{\partial x_k} h_j(\xi_j)(x_k - x_k^*),$$

where ξ_j, ξ_k lies between x_j and x_j^*, it follows that

$$\sum_{j=1}^{n}\sum_{k=1}^{m} b_{ijk}[h_j(x_j)h_k(x_k) - h_j(x_j^*)h_k(x_k^*)] = \sum_{j=1}^{m}\sum_{k=1}^{m} b_{ijk}[\frac{\partial h_j(\xi_j)}{\partial x_j} h_k(\xi_k)(x_j - x_j^*) + \frac{\partial h_k(\xi_k)}{\partial x_k} h_j(\xi_j)(x_k - x_k^*)] = \sum_{j=1}^{m}\sum_{k=1}^{m} (b_{ijk} + b_{ikj})\frac{\partial h_j(\xi_j)}{\partial x_j}(x_j - x_j^*)h_k(\xi_k).$$

By using differential mean value theorem, we can get from the above equation

$$\sum_{j=1}^{m}\sum_{k=1}^{m} b_{ijk}[h_j(x_j)h_k(x_k) - h_j(x_j^*)h_k(x_k^*)] = \sum_{j=1}^{m}\sum_{k=1}^{m}(b_{ijk}+b_{ikj})[h_j(x_j) - h_j(x_j^*)]h_k(\xi_k),$$

where ξ_j, ξ_k lies between x_j and x_j^*, $j = 1, 2, \ldots, m$.

Lemma 3 *Assume that*

$$\begin{aligned}&-2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2A_i\\ &+\sum_{j=1}^{m}[T_{ji}^2 + (\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|\\ &+(\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + |h_{ji}|\bar{d}_jG_i^2]\\ &+\bar{d}_j\sum_{j=1}^{m}\sum_{q=1}^{n}G_i|s_{jiq}+s_{jqi}| < 0,\end{aligned} \tag{5}$$

$$\begin{aligned}&-2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j\\ &+\sum_{i=1}^{n}[L_{ij}^2 + |a_{ij}|\bar{a}_iF_j + |c_{ij}|\bar{a}_iF_j^2\\ &+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]\\ &+\bar{a}_i\sum_{i=1}^{n}\sum_{k=1}^{m}F_j|T_{ijk}+T_{ikj}| < 0,\end{aligned} \tag{6}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, *then there exists* $\alpha > 0$, *such that*

$$\begin{aligned}&\alpha - 2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2A_i\\ &+\sum_{j=1}^{m}[T_{ji}^2 + (\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|\\ &+(\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_jG_i^2]\\ &+\bar{d}_j\sum_{j=1}^{m}\sum_{q=1}^{n}G_i|s_{jiq}+s_{jqi}| \le 0,\end{aligned}$$

$$\begin{aligned}&\alpha - 2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j\\ &+\sum_{i=1}^{n}[L_{ij}^2 + |a_{ij}|\bar{a}_iF_j + e^{\alpha\tau}|c_{ij}|\bar{a}_iF_j^2\\ &+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]\\ &+\bar{a}_i\sum_{i=1}^{n}\sum_{k=1}^{m}F_j|T_{ijk}+T_{ikj}| \le 0,\end{aligned}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$

Proof: Let

$$\begin{aligned}\varphi_i(\alpha) &= \alpha - 2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2A_i\\ &+\sum_{j=1}^{m}[T_{ji}^2 + (\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|\\ &+(\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_jG_i^2]\\ &+\bar{d}_j\sum_{j=1}^{m}\sum_{q=1}^{n}G_i|s_{jiq}+s_{jqi}|,\end{aligned}$$

$$\begin{aligned}\psi_j(\alpha) &= \alpha - 2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j\\ &+\sum_{i=1}^{n}[L_{ij}^2 + |a_{ij}|\bar{a}_iF_j + e^{\alpha\tau}|c_{ij}|\bar{a}_iF_j^2\\ &+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]\\ &+\bar{a}_i\sum_{i=1}^{n}\sum_{k=1}^{m}F_j|T_{ijk}+T_{ikj}|,\end{aligned}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$

Obviously,

$$\frac{d\varphi_i(\alpha)}{d\alpha} > 0, \quad \lim_{\alpha\to+\infty}\varphi_i(\alpha) = +\infty, \quad \varphi_i(0) < 0,$$

$$\frac{d\psi_j(\alpha)}{d\alpha} > 0, \quad \lim_{\alpha\to+\infty}\psi_j(\alpha) = +\infty, \quad \psi_j(0) < 0,$$

for $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m.$

Therefore, there exist constants α_i, $\alpha_j^* \in (0, +\infty)$, such that

$$\varphi_i(\alpha_i) = 0, \quad i = 1, 2, \ldots, n,$$
$$\psi_j(\alpha_j^*) = 0, \quad j = 1, 2, \ldots, m.$$

We choose

$$\alpha = min\{\alpha_1, \alpha_2, \ldots, \alpha_n, \alpha_1^*, \alpha_2^*, \ldots, \alpha_m^*\},$$

then $\alpha > 0$ and it satisfies that

$$\varphi_i(\alpha) \le 0, \quad i = 1, 2, \ldots, n,$$
$$\psi_j(\alpha) \le 0, \quad j = 1, 2, \ldots, m,$$

which means there exists constant $\alpha > 0$, such that

$$\begin{aligned}&\alpha - 2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2A_i\\ &+\sum_{j=1}^{m}[T_{ji}^2 + (\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|\\ &+(\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_jG_i^2]\\ &+\bar{d}_j\sum_{j=1}^{m}\sum_{q=1}^{n}G_i|s_{jiq}+s_{jqi}| \le 0,\end{aligned}$$

$$\begin{aligned}&\alpha - 2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j\\ &+\sum_{i=1}^{n}[L_{ij}^2 + |a_{ij}|\bar{a}_iF_j + e^{\alpha\tau}|c_{ij}|\bar{a}_iF_j^2\\ &+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]\\ &+\bar{a}_i\sum_{i=1}^{n}\sum_{k=1}^{m}F_j|T_{ijk}+T_{ikj}| \le 0,\end{aligned}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m.$

3 Main results

In this section, we will derive sufficient condition which ensure the existence, uniqueness and the mean square exponential stability of the periodic solution for system (1).

Theorem 4 *For the system (1), under the hypotheses* $(H1) - (H5)$, *there exists one* ω*-periodic solution of system (1), and all other solutions of (1) exponentially converge to it as* $t \to +\infty$ *in the mean square, if (5) and (6) in Lemma 2 hold.*

Proof: Let

$$\Phi = \{\phi|\phi = \begin{pmatrix}\phi_u\\ \phi_v\end{pmatrix}, \phi_u = (\phi_{u1}, \phi_{u2}, \cdots, \phi_{un})^T,$$
$$\phi_v = (\phi_{v1}, \phi_{v2}, \cdots, \phi_{vn})^T.\}$$

For any $\phi = (\phi_u^T, \phi_v^T)^T \in \Phi$, we define the norm of

ϕ: $\|\phi\| = \|\phi_u\| + \|\phi_v\|$, in which

$$\|\phi_u\| = \sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}\|\phi_{ui}(s)\|_2^2,$$
$$\|\phi_v\| = \sup_{-\tau\le s\le 0}\sum_{j=1}^{m}\|\phi_{vj}(s)\|_2^2,$$

then Φ is the Banach space of continuous functions which map $([-\sigma,0],[-\tau,0])^T$ into R^{n+m} with the topology of uniform convergence.

For any $(\phi_u^T,\phi_v^T)^T,(\psi_u^T,\psi_v^T)^T,\in\ \Phi$, we denote the solutions of system (1) in the initial conditions

$$\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\phi_u\\\phi_v\end{pmatrix}\right),\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\psi_u\\\psi_v\end{pmatrix}\right),$$

as
$u(t,x,\phi_u) = (u_1(t,x,\phi_u),\cdots,u_n(t,x,\phi_u))^T$,
$v(t,x,\phi_v) = (v_1(t,x,\phi_v),\cdots,v_m(t,x,\phi_v))^T$,
and
$u(t,x,\psi_u) = (u_1(t,x,\psi_u),\cdots,u_n(t,x,\psi_u))^T$,
$v(t,x,\psi_v) = (v_1(t,x,\psi_v),\cdots,v_m(t,x,\psi_v))^T$,
respectively. Defining
$u_t(\phi_u) = u(t+\rho,x,\phi_u), \rho\in[-\sigma,0]$,
$v_t(\phi_v) = v(t+\rho,x,\phi_v), \rho\in[-\tau,0], t\ge 0, x\in\Omega_i$,
then $(u_t(\phi_u),v_t(\phi_v))^T\in\Phi$, for all $t\ge 0$.
Let
$y_i(t,x) = u_i(t,x,\phi_u) - u_i(t,x,\psi_u)$,
$z_j(t,x) = v_j(t,x,\phi_v) - v_j(t,x,\psi_v)$,
for $i = 1,2,\cdots,n, j = 1,2,\cdots,m$.
From (1), we derive

$$\begin{aligned}dy_i(t,x) &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt\\ &- [a_i(u_i(t,x,\phi_u))b_i(u_i(t,x,\phi_u))\\ &- a_i(u_i(t,x,\psi_u))b_i(u_i(t,x,\psi_u))]dt\\ &- a_i(u_i(t,x,\phi_u))\{\sum_{j=1}^{m}a_{ij}[f_j(v_j(t,x,\phi_v))\\ &- f_j(v_j(t,x,\psi_v))] + \sum_{j=1}^{m}c_{ij}[f_j(v_j(t-\tau_{ij},x,\phi_v))\\ &- f_j(v_j(t-\tau_{ij},x,\psi_v))]\\ &+ \sum_{j=1}^{m}\sum_{k=1}^{m}T_{ijk}[f_j(v_j(t,x,\phi_v))f_k(v_k(t,x,\phi_v))\\ &- f_j(v_j(t,x,\psi_v))f_k(v_k(t,x,\psi_v))]\}dt\\ &+[a_i(u_i(t,x,\phi_u)) - a_i(u_i(t,x,\psi_u))]\cdot\\ &[\sum_{j=1}^{m}a_{ij}f_j(v_j(t,x,\psi_v)) + \sum_{j=1}^{m}c_{ij}f_j(v_j(t-\tau_{ij},x,\psi_v))\\ &+ \sum_{j=1}^{m}\sum_{k=1}^{m}T_{ijk}f_j(v_j(t,x,\psi_v))f_k(v_k(t,x,\psi_v))\\ &+ I_i(t)]dt + \sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v))\\ &-k_{ij}(v_j(t,x,\psi_v))]d\omega_{n+j}(t),\end{aligned}\tag{7}$$

$$\begin{aligned}dz_j(t,x) &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})dt\\ &- [d_j(v_j(t,x,\phi_v))e_j(v_j(t,x,\phi_v))\\ &- d_j(v_j(t,x,\psi_v))e_j(v_j(t,x,\psi_v))]dt\\ &- d_j(v_j(t,x,\phi_v))\{\sum_{i=1}^{n}b_{ji}[g_i(u_i(t,x,\phi_u))\\ &- g_i(u_i(t,x,\psi_u))] + \sum_{i=1}^{n}h_{ji}[g_i(u_i(t-\sigma_{ji},x,\phi_u))\\ &- g_i(v_i(t-\sigma_{ji},x,\psi_u))]\\ &+ \sum_{i=1}^{n}\sum_{q=1}^{n}S_{jiq}[g_i(u_i(t,x,\phi_u))g_q(u_q(t,x,\phi_u))\\ &- g_i(u_i(t,x,\psi_u))g_q(u_q(t,x,\psi_u))]\}dt\\ &+[d_j(v_j(t,x,\phi_v)) - d_j(v_j(t,x,\psi_v))].\\ &[\sum_{i=1}^{n}b_{ji}g_i(u_i(t,x,\psi_u)) + \sum_{i=1}^{n}h_{ji}g_i(v_i(t-\sigma_{ji},x,\psi_u))\\ &+ \sum_{i=1}^{n}\sum_{q=1}^{n}S_{jiq}g_i(u_i(t,x,\psi_u))g_q(u_q(t,x,\psi_u))\\ &+ J_j(t)]dt + \sum_{i=1}^{n}[\rho_{ji}(u_i(t,x,\phi_u))\\ &-\rho_{ji}(u_i(t,x,\psi_u))]d\omega_i(t),\end{aligned}\tag{8}$$

for $i = 1,2,\cdots,n, j = 1,2,\cdots,m$.
We consider the following Lyapunov functions

$$\begin{aligned}V_1(t,y(t,x)) &= e^{\alpha t}\sum_{i=1}^{n}y_i^2(t,x)\\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i\int_{t-\tau_{ij}}^{t}e^{\alpha(s+\tau_{ij})}|f_j(v_j(s,x,\phi_v))\\ &-f_j(v_j(s,x,\psi_v))|^2ds,\end{aligned}\tag{9}$$

$$\begin{aligned}V_2(t,z(t,x)) &= e^{\alpha t}\sum_{j=1}^{m}z_j^2(t,x)\\ &+ \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j\int_{t-\sigma_{ji}}^{t}e^{\alpha(s+\sigma_{ji})}|g_i(u_i(s,x,\phi_u))\\ &-g_i(u_i(s,x,\psi_u))|^2ds,\end{aligned}\tag{10}$$

where α is given by Lemma 3.

From (7), applying $It\hat{o}$'s formula to $V_1(t,y(t,x))$, using Lemma 2, we have

$$\begin{aligned}dV_1(t,y(t,x)) &= \alpha e^{\alpha t}\sum_{i=1}^{n}y_i^2(t,x)dt\\ &+ 2e^{\alpha t}\sum_{i=1}^{n}y_i(t,x)\{\sum_{k=1}^{l}\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt\\ &- [a_i(u_i(t,x,\phi_u))b_i(u_i(t,x,\phi_u))\\ &- a_i(u_i(t,x,\psi_u))b_i(u_i(t,x,\psi_u))]dt\\ &- a_i(u_i(t,x,\phi_u))\{\sum_{j=1}^{m}a_{ij}[f_j(v_j(t,x,\phi_v))\\ &- f_j(v_j(t,x,\psi_v))] + \sum_{j=1}^{m}c_{ij}[f_j(v_j(t-\tau_{ij},x,\phi_v))\\ &- f_j(v_j(t-\tau_{ij},x,\psi_v))]\\ &+ \sum_{j=1}^{m}\sum_{k=1}^{m}T_{ijk}[f_j(v_j(t,x,\phi_v))f_k(v_k(t,x,\phi_v))\\ &- f_j(v_j(t,x,\psi_v))f_k(v_k(t,x,\psi_v))]\}dt\\ &+[a_i(u_i(t,x,\phi_u)) - a_i(u_i(t,x,\psi_u))]\cdot\end{aligned}$$

$[\sum_{j=1}^{m} a_{ij}f_j(v_j(t,x,\psi_v)) + \sum_{j=1}^{m} c_{ij}f_j(v_j(t-\tau_{ij},x,\psi_v))$
$+ \sum_{j=1}^{m}\sum_{k=1}^{m} T_{ijk}f_j(v_j(t,x,\psi_v))f_k(v_k(t,x,\psi_v))$
$+ I_i(t)]dt + \sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v)) - k_{ij}(v_j(t,x,\psi_v))]$
$\cdot d\omega_{n+j}(t)\} + e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v))$
$- k_{ij}(v_j(t,x,\psi_v))]^2 dt$
$+ \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha(t+\tau_{ij})}|f_j(v_j(t,x,\phi_v))$
$- f_j(v_j(t,x,\psi_v))|^2 - e^{\alpha t}|f_j(v_j(t-\tau_{ij},x,\phi_v))$
$- f_j(v_j(t-\tau_{ij},x,\psi_v))|^2]dt$
$\le \alpha e^{\alpha t}\sum_{i=1}^{n} y_i^2(t,x)dt$
$+ 2e^{\alpha t}\sum_{i=1}^{n}\{\sum_{k=1}^{l} y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt$
$- \beta_i y_i^2(t,x)dt + \bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j|z_j(t,x)||y_i(t,x)|dt$
$+ \bar{a}_i\sum_{j=1}^{m}|c_{ij}||f_j(v_j(t-\tau_{ij},x,\phi_v))$
$- f_j(v_j(t-\tau_{ij},x,\psi_v))||y_i(t,x)|dt$
$+\bar{a}_i\sum_{j=1}^{m}\sum_{k=1}^{m} F_j|T_{ijk}+T_{ikj}||z_j(t,x)||y_i(t,x)|dt$
$+[\sum_{j=1}^{m}|a_{ij}|\bar{f}_j + \sum_{j=1}^{m}|c_{ij}|\bar{f}_j + \sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2$
$+ I_i^*]A_i y_i^2(t,x)dt + \sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v))$
$- k_{ij}(v_j(t,x,\psi_v))]y_i(t,x)d\omega_{n+j}(t)\}$
$+e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m} L_{ij}^2 z_j^2(t,x)dt + \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha(t+\tau_{ij})}$
$|f_j(v_j(t,x,\phi_v)) - f_j(v_j(t,x,\psi_v))|^2$
$-e^{\alpha t}|f_j(v_j(t-\tau_{ij},x,\phi_v)) - f_j(v_j(t-\tau_{ij},x,\psi_v))|^2]dt$
$\le \alpha e^{\alpha t}\sum_{i=1}^{n} y_i^2(t,x)dt$
$+ e^{\alpha t}\sum_{i=1}^{n}\{\sum_{k=1}^{l} 2y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt$
$- 2\beta_i y_i^2(t,x)dt + \bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j[z_j^2(t,x)$
$+ y_i^2(t,x)]dt + \bar{a}_i\sum_{j=1}^{m}|c_{ij}|[|f_j(v_j(t-\tau_{ij},x,\phi_v))$
$- f_j(v_j(t-\tau_{ij},x,\psi_v))|^2 + y_i^2(t,x)]dt$
$+\bar{a}_i\sum_{j=1}^{m}\sum_{k=1}^{m} F_j|T_{ijk}+T_{ikj}|[z_j^2(t,x)$
$+ y_i^2(t,x)]dt + 2[\sum_{j=1}^{m}|a_{ij}|\bar{f}_j + \sum_{j=1}^{m}|c_{ij}|\bar{f}_j$
$+ \sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2 + I_i^*]A_i y_i^2(t,x)dt$
$+ 2\sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v)) - k_{ij}(v_j(t,x,\psi_v))]$
$\cdot y_i(t,x)d\omega_{n+j}(t)\} + e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m} L_{ij}^2 z_j^2(t,x)dt$
$+ \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i[e^{\alpha(t+\tau_{ij})}|f_j(v_j(t,x,\phi_v))$
$- f_j(v_j(t,x,\psi_v))|^2 - e^{\alpha t}|f_j(v_j(t-\tau_{ij},x,\phi_v))$
$- f_j(v_j(t-\tau_{ij},x,\psi_v))|^2]dt$
$\le 2e^{\alpha t}\sum_{i=1}^{n}\sum_{k=1}^{l} y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt$
$+ e^{\alpha t}\sum_{i=1}^{n}[\alpha - 2\beta_i + 2(\sum_{j=1}^{m}|a_{ij}|\bar{f}_j + \sum_{j=1}^{m}|c_{ij}|\bar{f}_j$
$+ \sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2 + I_i^*)A_i + \bar{a}_i\sum_{j=1}^{m}|a_{ij}|F_j$
$+ \bar{a}_i\sum_{j=1}^{m}|c_{ij}| + \bar{a}_i\sum_{j=1}^{m}\sum_{k=1}^{m} F_j|T_{ijk}+T_{ikj}|]y_i^2(t,x)dt$
$+ e^{\alpha t}\sum_{j=1}^{m}[\sum_{i=1}^{n}\bar{a}_i|a_{ij}|F_j + \bar{a}_i\sum_{j=1}^{m}\sum_{k=1}^{m} F_j|T_{ijk}+T_{ikj}|$
$+ \sum_{i=1}^{n}|c_{ij}|\bar{a}_i F_j^2 e^{\alpha\tau} + \sum_{i=1}^{n} L_{ij}^2]z_j^2(t,x)dt$
$+ 2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v))$

$$-k_{ij}(v_j(t,x,\psi_v))]y_i(t,x)d\omega_{n+j}(t). \qquad (11)$$

From (8), applying $It\hat{o}'s$ formula to $V_2(t,z(t,x))$, similarly, we also get

$dV_2(t,z(t,x)) \le 2e^{\alpha t}\sum_{j=1}^{m}\sum_{k=1}^{l} z_j(t,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})dt$
$+ e^{\alpha t}\sum_{j=1}^{m}[\alpha - 2\gamma_j + 2(\sum_{i=1}^{n}|b_{ji}|\bar{g}_i + \sum_{i=1}^{n}|h_{ji}|\bar{g}_i$
$+ \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}| + J_i^*)D_j + \bar{d}_j\sum_{i=1}^{n}|b_{ji}|G_i$
$+ \bar{d}_j\sum_{i=1}^{n}|h_{ji}|]z_j^2(t,x)dt + e^{\alpha t}\sum_{i=1}^{n}[\sum_{j=1}^{m}\bar{d}_j|b_{ji}|G_i$
$+ \sum_{j=1}^{m}|h_{ji}|\bar{d}_j G_i^2 e^{\alpha\sigma} + \bar{d}_j\sum_{i=1}^{n}\sum_{q=1}^{n} G_i|s_{jiq}+s_{jqi}|$
$+ \sum_{j=1}^{m} T_{ji}^2]y_i^2(t,x)dt + 2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[\rho_{ji}(u_i(t,x,\phi_u))$

$$-\rho_{ji}(u_i(t,x,\psi_u))]z_j(t,x)d\omega_i(t). \qquad (12)$$

Let $V(t,y(t,x),z(t,x)) = V_1(t,y(t,x)) + V_2(t,z(t,x))$. Applying $It\hat{o}'s$ formula to $V(t,y(t,x),z(t,x))$, from (11)-(12), we can obtain

$dV(t,y(t,x),z(t,x))$
$\le 2e^{\alpha t}\sum_{i=1}^{n}\sum_{k=1}^{l} y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt$
$+ 2e^{\alpha t}\sum_{j=1}^{m}\sum_{k=1}^{l} z_j(t,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})dt$
$+e^{\alpha t}\sum_{i=1}^{n}\{\alpha - 2\beta_i + 2A_i I_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2 A_i$
$+ \sum_{j=1}^{m}[T_{ji}^2 + (\bar{a}_i F_j + 2\bar{f}_j A_i)|a_{ij}| + (\bar{a}_i + 2\bar{f}_j A_i)|c_{ij}|$

$+ |b_{ji}|\bar{d}_j G_i + e^{\alpha\sigma}|h_{ji}|\bar{d}_j G_i^2]$
$+ \bar{d}_j \sum_{j=1}^{m}\sum_{q=1}^{n} G_i|s_{jiq} + s_{jqi}|\}y_i^2(t,x)dt$
$+e^{\alpha t}\sum_{j=1}^{m}\{\alpha - 2\gamma_j + 2D_j J_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j$
$+ \sum_{i=1}^{n}[L_{ij}^2 + |a_{ij}|\bar{a}_i F_j + e^{\alpha\tau}|c_{ij}|\bar{a}_i F_j^2$
$+ (\bar{d}_j G_i + 2\bar{g}_i D_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_i D_j)|h_{ji}|]$
$+ \bar{a}_i \sum_{i=1}^{n}\sum_{k=1}^{m} F_j|T_{ijk} + T_{ikj}|\}z_j^2(t,x)dt$
$+2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v)) - k_{ij}(v_j(t,x,\psi_v))]$
$\cdot y_i(t,x)d\omega_{n+j}(t) + 2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[\rho_{ji}(u_i(t,x,\phi_u))$
$- \rho_{ji}(u_i(t,x,\psi_u))]z_j(t,x)d\omega_i(t).$

By Lemma 3, we obtain

$$\begin{aligned} &dV(t,y(t,x),z(t,x)) \\ &\le 2e^{\alpha t}\sum_{i=1}^{n}\sum_{k=1}^{l} y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dt \\ &+ 2e^{\alpha t}\sum_{j=1}^{m}\sum_{k=1}^{l} z_j(t,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})dt \\ &+2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[k_{ij}(v_j(t,x,\phi_v)) - k_{ij}(v_j(t,x,\psi_v))] \\ &\cdot y_i(t,x)d\omega_{n+j}(t) +2e^{\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}[\rho_{ji}(u_i(t,x,\phi_u)) \\ &-\rho_{ji}(u_i(t,x,\psi_u))]z_j(t,x)d\omega_i(t). \end{aligned} \tag{13}$$

From (13), we get

$$\begin{aligned} &V(t,y(t,x),z(t,x)) \le V(0,y(0,x),z(0,x)) \\ &+ 2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{k=1}^{l} y_i(s,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(s,x)}{\partial x_k})ds \\ &+ 2\int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{k=1}^{l} z_j(s,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(s,x)}{\partial x_k})ds \\ &+2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}[k_{ij}(v_j(s,x,\phi_v)) \\ &- k_{ij}(v_j(s,x,\psi_v))]y_i(s,x)d\omega_{n+j}(s) \\ &+ 2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}[\rho_{ji}(u_i(s,x,\phi_u)) \\ &-\rho_{ji}(u_i(s,x,\psi_u))]z_j(s,x)d\omega_i(s). \end{aligned} \tag{14}$$

From (14), we have

$$\begin{aligned} &\int_{\Omega_i} V(t,y(t,x),z(t,x))dx \\ &\le \int_{\Omega_i} V(0,y(0,x),z(0,x))dx \\ &+ 2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{k=1}^{l}\int_{\Omega_i} y_i(s,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(s,x)}{\partial x_k})dxds \\ &+ 2\int_0^t e^{\alpha s}\sum_{j=1}^{m}\sum_{k=1}^{l}\int_{\Omega_i} z_j(s,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(s,x)}{\partial x_k})dxds \\ &+2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}\int_{\Omega_i}[k_{ij}(v_j(s,x,\phi_v)) \\ &- k_{ij}(v_j(s,x,\psi_v))]y_i(s,x)dxd\omega_{n+j}(s) \\ &+2\int_0^t e^{\alpha s}\sum_{i=1}^{n}\sum_{j=1}^{m}\int_{\Omega_i}[\rho_{ji}(u_i(s,x,\phi_u)) \\ &-\rho_{ji}(u_i(s,x,\psi_u))]z_j(s,x)dxd\omega_i(s). \end{aligned} \tag{15}$$

From (9) and (10), we have

$$V(t,y(t,x),z(t,x)) \ge e^{\alpha t}[\sum_{i=1}^{n} y_i^2(t,x) + \sum_{j=1}^{m} z_j^2(t,x)], \quad t \ge 0. \tag{16}$$

$$\begin{aligned} &V(0,y(0,x),z(0,x)) \\ &= \sum_{i=1}^{n} y_i^2(0,x) + \sum_{j=1}^{m} z_j^2(0,x) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_i\int_{-\tau_{ij}}^{0} e^{\alpha(s+\tau_{ij})}[f_j(v_j(s,x,\phi_v)) \\ &- f_j(v_j(s,x,\psi_v))]^2 ds + \sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_j\int_{-\sigma_{ji}}^{0} e^{\alpha(s+\sigma_{ji})} \\ &\cdot[g_i(u_i(s,x,\phi_u)) - g_i(u_i(s,x,\psi_u))]^2 ds. \end{aligned} \tag{17}$$

It follows from the boundary condition that

$$\begin{aligned} &\sum_{k=1}^{l}\int_{\Omega_i} y_i(t,x)\frac{\partial}{\partial x_k}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})dx \\ &= \int_{\Omega_i} y_i(t,x)\nabla(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}dx \\ &= \int_{\Omega_i}\nabla\cdot y_i(t,x)(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}dx \\ &- \int_{\Omega_i}(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}\cdot\nabla y_i(t,x)dx \\ &= \int_{\partial\Omega_i} y_i(t,x)D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l}ds \\ &- \sum_{k=1}^{l}\int_{\Omega_i} D_{ik}(\frac{\partial y_i(t,x)}{\partial x_k})^2 dx \\ &= -\sum_{k=1}^{l}\int_{\Omega_i} D_{ik}(\frac{\partial y_i(t,x)}{\partial x_k})^2 dx, \end{aligned} \tag{18}$$

where $\nabla = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial)}{\partial x_l})^T$,
$(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k})_{k=1}^{l} = (D_{i1}\frac{\partial y_i(t,x)}{\partial x_1}, \ldots, D_{il}\frac{\partial y_i(t,x)}{\partial x_l})^T$.

$$\begin{aligned} &\sum_{k=1}^{l}\int_{\Omega_i} z_j(t,x)\frac{\partial}{\partial x_k}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})dx \\ &= \int_{\Omega_i} z_j(t,x)\nabla(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})_{k=1}^{l}dx \\ &= \int_{\Omega_i}\nabla\cdot z_j(t,x)(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})_{k=1}^{l}dx \\ &- \int_{\Omega_i}(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})_{k=1}^{l}\cdot\nabla z_j(t,x)dx \\ &= \int_{\partial\Omega_i} z_j(t,x)D_{ik}\frac{\partial z_j(t,x)}{\partial x_k})_{k=1}^{l}ds \end{aligned}$$

$$-\sum_{k=1}^{l}\int_{\Omega_i} E_{jk}(\frac{\partial z_j(t,x)}{\partial x_k})^2dx$$

$$=-\sum_{k=1}^{l}\int_{\Omega_i} E_{jk}(\frac{\partial z_j(t,x)}{\partial x_k})^2dx, \quad (19)$$

where

$(E_{jk}\frac{\partial z_j(t,x)}{\partial x_k})_{k=1}^{l}=(E_{j1}\frac{\partial z_j(t,x)}{\partial x_1},\ldots,E_{jl}\frac{\partial z_j(t,x)}{\partial x_l})^T$.

From (15)-(19), we obtain

$$\sum_{i=1}^{n}E(\|u_i(t,x,\phi_u)-u_i(t,x,\psi_u)\|_2^2)$$
$$+\sum_{j=1}^{m}E(\|v_j(t,x,\phi_v)-v_j(t,x,\psi_v)\|_2^2)$$
$$\le e^{-\alpha t}[\sum_{i=1}^{n}E(\|\phi_{ui}(0,x)-\psi_{ui}(0,x)\|_2^2)$$
$$+\sum_{j=1}^{m}E(\|\phi_{vi}(0,x)-\psi_{vj}(0,x)\|_2^2)]$$
$$+e^{-\alpha t}\sum_{i=1}^{n}\sum_{j=1}^{m}|c_{ij}|\bar{a}_iF_j^2\int_{-\tau_{ij}}^{0}e^{\alpha(s+\tau_{ij})}E(\|\phi_{vj}(s,x)$$
$$-\psi_{vj}(s,x)\|_2^2)ds+e^{-\alpha t}\sum_{j=1}^{m}\sum_{i=1}^{n}|h_{ji}|\bar{d}_jG_i^2$$
$$\cdot\int_{-\sigma_{ji}}^{0}e^{\alpha(s+\sigma_{ji})}E(\|\phi_{ui}(s,x)-\psi_{ui}(s,x)\|_2^2)ds$$
$$\le e^{-\alpha t}[1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j]E(\|\phi_u-\psi_u\|)$$
$$+e^{-\alpha t}[1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i]E(\|\phi_v-\psi_v\|)$$
$$=Me^{-\alpha t}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],\quad t>0, \quad (20)$$

where $M=\max\{1+\frac{e^{\alpha\sigma}}{\alpha}\sum_{j=1}^{m}\max_{1\le i\le n}(|h_{ji}|G_i^2)\bar{d}_j,$

$1+\frac{e^{\alpha\tau}}{\alpha}\sum_{i=1}^{n}\max_{1\le j\le m}(|c_{ij}|F_j^2)\bar{a}_i\}>1.$

Let

$$E(\|u(\phi_u)-u(\psi_u)\|)=\sum_{i=1}^{n}E(\|u_i(t,x,\phi_u)-u_i(t,x,\psi_u)\|_2^2),$$
$$E(\|u(\phi_v)-u(\psi_v)\|)=\sum_{j=1}^{m}E(\|v_j(t,x,\phi_v)-v_j(t,x,\psi_v)\|_2^2).$$

From (20), we have

$$E(\|u(\phi_u)-u(\psi_u)\|)\le Me^{-\alpha t}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],t>0,$$
$$E(\|u(\phi_v)-u(\psi_v)\|))\le Me^{-\alpha t}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],t>0.$$

We can choose a positive integer N, such that $Me^{-\alpha(N\omega+\rho)}\le\frac{1}{3},\rho\in[-\delta,0]$. Now we define a Poincaré mapping F: $\rho\to\rho$ by

$$F(\phi_u,\phi_v)^T=(u_\omega(\phi_u),v_\omega(\phi_v))^T,$$

then

$$F^N(\phi_u,\phi_v)^T=(u_{N\omega}(\phi_u),v_{N\omega}(\phi_v))^T.$$

Let $t=N\omega$, then have

$$E(\|F^N\phi_u-F^N\psi_u\|)\le\frac{1}{3}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],$$
$$E(\|F^N\phi_v-F^N\psi_v\|)\le\frac{1}{3}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)].$$

By the integral property of measurable functions, we can obtain

$$\|F^N\phi_u-F^N\psi_u\|\le\frac{1}{3}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],\ \ a.e.,$$
$$\|F^N\phi_v-F^N\psi_v\|\le\frac{1}{3}[E(\|\phi_u-\psi_u\|)+E(\|\phi_v-\psi_v\|)],\ \ a.e..$$

This implies that F^N is a contraction mapping, hence there exist a unique fixed point $(\phi_u^*,\phi_v^*)^T\in\Phi$, such that $F^N(\phi_u^*,\phi_v^*)^T=(\phi_u^*,\phi_v^*)^T$. Since

$$F^N\left(F\begin{pmatrix}\phi_u^*\\\phi_v^*\end{pmatrix}\right)=F\left(F^N\begin{pmatrix}\phi_u^*\\\phi_v^*\end{pmatrix}\right)=F\begin{pmatrix}\phi_u^*\\\phi_v^*\end{pmatrix}$$

, then $F(\phi_u^*,\phi_v^*)^T\in\Phi$ is also a fixed point of F^N, and so

$$F(\phi_u^*,\phi_v^*)^T=(\phi_u^*,\phi_v^*)^T,$$

i.e.,

$$(u_\omega(\phi_u^*),v_\omega(\phi_v^*))^T=(\phi_u^*,\phi_v^*)^T.$$

Let $(u(t,x,\phi_u^*),v(t,x,\phi_v^*))^T$ be the solution of system (1) through

$$\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\phi_u^*\\\phi_v^*\end{pmatrix}\right),$$

then $(u(t+\omega,x,\phi_u^*),v(t+\omega,x,\phi_v^*))^T$ is also a solution of (1). Obviously

$$\begin{pmatrix}u_{t+\omega}(\phi_u^*)\\v_{t+\omega}(\phi_v^*)\end{pmatrix}=\begin{pmatrix}u_t(u_\omega(\phi_u^*))\\v_t(v_\omega(\phi_v^*))\end{pmatrix}=\begin{pmatrix}u_t(\phi_u^*)\\v_t(\phi_v^*)\end{pmatrix},$$

for all $t\ge0$. Hence

$$\begin{pmatrix}u(t+\omega,x,\phi_u^*)\\v(t+\omega,x,\phi_v^*)\end{pmatrix}=\begin{pmatrix}u(t,x,\phi_u^*)\\v(t,x,\phi_v^*)\end{pmatrix},$$

for all $t\ge0$.

This shows that system (1) exists one $\omega-$ periodic solution, and other solutions of system (1) exponentially converge to it as $t\to+\infty$ in the mean square. This completes the proof.

Remark 1: In system (1), if $k_{ij}(\cdot)=0,\rho_{ji}(\cdot)=0,i=1,2,\cdots,n,j=1,2,\cdots,m$, we obtain the following reaction-diffusion high-order Cohen-

Grossberg-type BAM neural networks

$$\begin{cases} \frac{du_i(t,x)}{dt} = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}) - a_i(u_i(t,x)) \\ \quad \cdot[b_i(u_i(t,x)) - \sum\limits_{j=1}^{m} a_{ij} f_j(v_j(t,x)) \\ \quad - \sum\limits_{j=1}^{m} c_{ij} f_j(v_j(t-\tau_{ij},x)) \\ - \sum\limits_{j=1}^{m}\sum\limits_{k=1}^{m} T_{ijk} f_j(v_j(t)) f_k(v_k(t)) - I_i(t)], \\ \frac{dv_j(t,x)}{dt} = \sum\limits_{k=1}^{l} \frac{\partial}{\partial x_k}(E_{jk}\frac{\partial v_j(t,x)}{\partial x_k}) - d_j(v_j(t,x)) \\ \quad \cdot[e_j(v_j(t,x)) - \sum\limits_{i=1}^{n} b_{ji} g_i(u_i(t,x)) \\ \quad - \sum\limits_{i=1}^{n} h_{ji} g_i(u_i(t-\sigma_{ji},x)) \\ - \sum\limits_{i=1}^{2}\sum\limits_{q=1}^{2} S_{jiq} g_i(u_i(t)) g_q(u_q(t)) - J_j(t)], \end{cases} \tag{21}$$

where $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$.

Using same proof methods of Theorem 4, we may obtain the following Theorem 5.

Theorem 5 *For the system (21), under the hypotheses* $(H1), (H2), (H3), H(5)$, *if*

$$-2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{m}\sum_{k=1}^{m}|T_{ijk}|\bar{f}_j^2A_i$$
$$+ \sum_{j=1}^{m}[(\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|$$
$$+ (\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + |h_{ji}|\bar{d}_jG_i^2]$$
$$+\bar{d}_j\sum_{j=1}^{m}\sum_{q=1}^{n}G_i|s_{jiq} + s_{jqi}| < 0,$$
$$-2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{n}\sum_{q=1}^{n}\bar{g}_j^2|s_{jiq}|D_j$$
$$+ \sum_{i=1}^{n}[|a_{ij}|\bar{a}_iF_j + |c_{ij}|\bar{a}_iF_j^2$$
$$+ (\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]$$
$$+\bar{a}_i\sum_{i=1}^{n}\sum_{k=1}^{m}F_j|T_{ijk} + T_{ikj}| < 0,$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$, *there exists one* ω*-periodic solution of system (21), and all other solutions of (21) exponentially converge to it as* $t \to +\infty$ *in the mean square.*

4 Example

In this section, we give an example for showing our results.

Example 4.1. Consider the following stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks($n = m = l = 2$)

$$\begin{cases} du_i(t,x) = \sum\limits_{k=1}^{2} \frac{\partial}{\partial x_k}(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k})dt - a_i(u_i(t,x)) \\ \quad \cdot[b_i(u_i(t,x)) - \sum\limits_{j=1}^{2} a_{ij} f_j(v_j(t,x)) \\ \quad - \sum\limits_{j=1}^{2} c_{ij} f_j(v_j(t-\tau_{ij},x)) \\ - \sum\limits_{j=1}^{2}\sum\limits_{k=1}^{2} T_{ijk} f_j(v_j(t)) f_k(v_k(t)) - I_i(t)]dt \\ \quad + \sum\limits_{j=1}^{2} k_{ij}(v_j(t,x))d\omega_{n+j}(t), \\ dv_j(t,x) = \sum\limits_{k=1}^{2} \frac{\partial}{\partial x_k}(E_{jk}\frac{\partial v_j(t,x)}{\partial x_k})dt - d_j(v_j(t,x)) \\ \quad \cdot[e_j(v_j(t,x)) - \sum\limits_{i=1}^{2} b_{ji} g_i(u_i(t,x)) \\ \quad - \sum\limits_{i=1}^{2} h_{ji} g_i(u_i(t-\sigma_{ji},x)) \\ - \sum\limits_{i=1}^{2}\sum\limits_{q=1}^{2} S_{jiq} g_i(u_i(t)) g_q(u_q(t)) - J_j(t)]dt \\ + \sum\limits_{i=1}^{2} \rho_{ji}(u_i(t,x))d\omega_i(t), \end{cases} \tag{22}$$

where
$a_{11} = \frac{1}{42},\ a_{12} = \frac{1}{35},\ a_{21} = -\frac{1}{42},\ a_{22} = -\frac{1}{35},$
$c_{11} = \frac{1}{44},\ c_{12} = \frac{1}{88},\ c_{21} = -\frac{1}{44},\ c_{22} = -\frac{1}{88},$
$b_{11} = \frac{1}{50},\ b_{12} = \frac{1}{50},\ b_{21} = \frac{1}{100},\ b_{22} = -\frac{1}{100},$
$h_{11} = -\frac{1}{50},\ h_{12} = \frac{1}{50},\ h_{21} = -\frac{1}{100},\ h_{22} = \frac{1}{100},$
$T_{ijk} = \frac{1}{1280}, S_{jiq} = -\frac{1}{1000}, i, j, k, q = 1, 2.$
$a_1(t,x) = a_2(t,x) = 1 + \frac{1}{\pi}arctanx,$
$d_1(t,x) = d_2(t,x) = 2 + \frac{1}{\pi}arctanx,$
$b_1(t,x) = b_2(t,x) = x,$
$e_1(t,x) = e_2(t,x) = \frac{x}{2},$
$f_1(t,x) = f_2(t,x) = 2sinx,$
$g_1(t,x) = g_2(t,x) = cosx,$
$k_{ij}(t,x) = \frac{1}{4}sinx,\ \rho_{ij}(t,x) = \frac{1}{8}cosx,$
$I_i(t) = \frac{\pi}{24}(2 + sint),\ J_i(t) = \frac{\pi}{16}(1 + sint),$
$\omega_i(t) = \omega_{2+i}(t) = 1 + sint,\ i = 1, 2, j = 1, 2.$

It is easy to show that we can choose the constants in the conditions $(H_1) - (H_5)$ as follows.
$F_i = 2,\ G_i = 1,\ \bar{f}_i = 2,\ \bar{g}_i = 1,\ \beta_i = \frac{1}{2},$
$\gamma_i = \frac{3}{4},\ \bar{a}_i = \frac{3}{2},\ \underline{a}_i = \frac{1}{2},\ \bar{d}_i = \frac{5}{2},\ \underline{d}_i = \frac{3}{2},$
$A_i = \frac{1}{\pi},\ D_i = \frac{1}{\pi},\ L_{ij} = \frac{1}{4},\ T_{ji} = \frac{1}{8},$
$I_i^* = \frac{\pi}{8},\ J_j^* = \frac{\pi}{8},\quad i, j = 1, 2.$

We have the following results by simple calculation

$$-2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{2}\sum_{k=1}^{2}|T_{ijk}|\bar{f}_j^2A_i$$
$$+ \sum_{j=1}^{2}[T_{ji}^2 + (\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|$$
$$+ (\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + |h_{ji}|\bar{d}_jG_i^2]$$
$$+\bar{d}_j\sum_{j=1}^{2}\sum_{q=1}^{2}G_i|s_{jiq} + s_{jqi}| < 0,$$

$$-2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{2}\sum_{q=1}^{2}\bar{g}_j^2|s_{jiq}|D_j$$
$$+\sum_{i=1}^{2}[L_{ij}^2 + |a_{ij}|\bar{a}_iF_j + |c_{ij}|\bar{a}_iF_j^2$$
$$+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]$$
$$+\bar{a}_i\sum_{i=1}^{2}\sum_{k=1}^{2}F_j|T_{ijk} + T_{ikj}| < 0,$$

for $i = 1, 2, j = 1, 2.$

It is straightforward to check that all conditions needed in Theorem 4 are satisfied. Therefore, by Theorem 4, system (22) has exactly one $2\pi-$ periodic solution, and the $2\pi-$ periodic solution of system (22) is mean square exponentially stable.

Remark 2: In system (1), if $k_{ij}(\cdot) = 0, \rho_{ji}(\cdot) = 0, i, j = 1, 2$, the other parameters are the same as that in Example 4.1. We have the following results by simple calculation

$$-2\beta_i + 2A_iI_i^* + 2\sum_{j=1}^{2}\sum_{k=1}^{2}|T_{ijk}|\bar{f}_j^2A_i$$
$$+\sum_{j=1}^{2}[(\bar{a}_iF_j + 2\bar{f}_jA_i)|a_{ij}|$$
$$+(\bar{a}_i + 2\bar{f}_jA_i)|c_{ij}| + |b_{ji}|\bar{d}_jG_i + |h_{ji}|\bar{d}_jG_i^2]$$
$$+\bar{d}_j\sum_{j=1}^{2}\sum_{q=1}^{2}G_i|s_{jiq} + s_{jqi}| < 0,$$

$$-2\gamma_j + 2D_jJ_j^* + \sum_{i=1}^{2}\sum_{q=1}^{2}\bar{g}_j^2|s_{jiq}|D_j$$
$$+\sum_{i=1}^{2}[|a_{ij}|\bar{a}_iF_j + |c_{ij}|\bar{a}_iF_j^2$$
$$+(\bar{d}_jG_i + 2\bar{g}_iD_j)|b_{ji}| + (\bar{d}_j + 2\bar{g}_iD_j)|h_{ji}|]$$
$$+\bar{a}_i\sum_{i=1}^{2}\sum_{k=1}^{2}F_j|T_{ijk} + T_{ikj}| < 0,$$

for $i, j = 1, 2.$

It is straightforward to check that all conditions needed in Theorem 5 are satisfied. Therefore, by Theorem 5, system (21) has exactly one $2\pi-$ periodic solution, and the $2\pi-$ periodic solution of system (21) is mean square exponentially stable.

5 Conclusion

In this paper, some sufficient conditions have been obtained ensuring the existence, uniqueness and the mean square exponential stability of the periodic solution for a stochastic reaction-diffusion high-order Cohen-Grossberg-type BAM neural networks with time delays. An examples is given to show the effectiveness of the results. The given algebra conditions are easily verifiable and useful in theories and applications.

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