In-situ Combustion Simulation for Heavy Oil Reservoirs.

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Abstract: - In this paper we study the continuity, momentum and coupled nonlinear energy and species convection-diffusion equations describing the in-situ combustion process in porous media. We assume the fuel depends on the space variable x. We examine the properties of solution under certain conditions. Using large activation energy asymptotics and shooting method we provide a numerical solution of the problem and obtained temperature and concentration profiles.

Key-Words: - In-situ, combustion, porous media, simulation, heavy oil, reservoirs.

1 Introduction

In-situ combustion is a method of thermal recovery in which fire is generated inside the reservoir by injecting a gas containing oxygen, such as air. A special heater in the well ignites the oil in the reservoir and starts a fire. The heat generated by burning the heavy hydrocarbons in place produces hydrocarbon cracking, vaporization of light hydrocarbons and reservoir water in addition to the deposition of heavier hydrocarbons known as coke. As the fire moves, the burning front pushes ahead a mixture of hot combustion gases, steam and hot water, which in turn reduces oil viscosity and displaces oil toward production wells.

Many scientists have studied the oxidation of crude oil with air injected in porous media. These include Davies (1990) who tracked an in-situ combustion front using thin flame technique. Marchesin and Schecter (2003) constructed a two-phase model for oxidation, involving air or oxygen and oil that include heat loss to the rock formation. De Souza et al. (2006) studied the Riemann problem with forward combustion due to injection of air into a porous medium containing solid fuel. Buckmaster and Ludford (1992) showed that a reacting Arrhenius exothermic reaction has two solutions. Olayiwola and Ayeni (2007, 2009) presented a mathematical model of in-situ combustion using high activation energy asymptotics. The effects of Frank-Kamenetskii parameter and flame velocity on the temperature and concentration field were discussed. Redl (2002) considered multi channel geometry to show the ability of the Lattice Boltzmann method to deal with fluid flow and heat transfer problems occurring in combustion processes.
In this paper we extend the model investigated by Redl in 2002 to include the continuity and momentum equations. We assume the fuel depends on the space variable x. We examine the properties

of solution. To simulate the flow, we assume that the other end of reservoir is at infinity.

2 Mathematical Model

We consider an underground reservoir contained heavy oil. We assume that air is injected at the leftmost part of the reservoir, so that all propagation is one dimensional. The fluid is assumed to be incompressible. One end of the reservoir is assumed kept at $x = 0$ while the other end is assumed far away (i.e at $x = \infty$). We also assume the fuel depends on space variable x . Then the primary dependent variables are the temperature, $T(x)$, the oxygen concentration, $C_{ox}(x)$, the solid fuel concentration, $C_{fuel}(x)$, and the gas product concentration, $C_p(x)$. Under these assumptions, the steady equations that describe the in-situ combustion process are

The continuity equation

$$\rho \frac{d(u)}{dx} = 0 \tag{1}$$

The momentum equation

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx} + \upsilon \frac{d^2 u}{dx^2} \tag{2}$$

The energy equation

$$u\frac{dT}{dx} = \frac{\lambda}{\rho c_p}\left(\frac{d^2 T}{dx^2}\right) + \frac{Q\omega}{\rho c_p} \tag{3}$$

The conservation equation for the oxidizer

$$u\frac{d(\rho C_{ox})}{dx} = D_{ox}\left(\frac{d^2(\rho C_{ox})}{dx^2}\right) - \omega s_{ox} \tag{4}$$

The conservation equation for the solid fuel

$$u\frac{d(\rho_{fuel} C_{fuel})}{dx} = D_f\left(\frac{d^2(\rho_{fuel} C_{fuel})}{dx^2}\right) - \omega\left(1 - s_{sf}\right) \tag{5}$$

The conservation equation for the gas product

$$u\frac{d(\rho C_p)}{dx} = D_p\left(\frac{d^2(\rho C_p)}{dx^2}\right) + \omega s_p \tag{6}$$

satisfying the boundary conditions

$$\left.\begin{array}{ll} T(0) = T_1, & T(\infty) = 0 \\ C_{ox}(0) = C_0, & C_{ox}(\infty) = 0 \\ C_{fuel}(0) = C_{f0}, & C_{fuel}(\infty) = 0 \\ C_p(0) = C_0, & C_p(\infty) = 0 \end{array}\right\} \tag{7}$$

A general form of the reaction rate is

$$\omega = A\, k_{ov}\, C_{ox}^{\alpha}\, C_{fuel}^{\beta}\, e^{-\frac{E}{RT}}, \tag{8}$$

where A is a pre exponential factor, k_{ov} is the overall reaction coefficient, C are the concentrations, α and β are the orders of reaction, E is the activation energy, R is the ideal gas constant, T is the local gas temperature, ρ is the density, u is the velocity along x - axis, x is the position, p is the pressure, υ is the kinematic viscosity, λ is the thermal conductivity, c_p is the specific heat at constant pressure, Q is the heat of reaction, D are the diffusion coefficients, s_{ox} is the stoichiometric coefficient of oxidizer, s_{sf} is the stoichiometric coefficient of fuel, s_p is the stoichiometric coefficient of gas product.

The source term represents the consumption of fuel during the combustion process.

3 Method of Solution

Let $D_{ox} = \dfrac{\lambda}{\rho c_p}$, $D_f = D_p$, $\phi = \left(s_{ox} T + \dfrac{Q}{c_p} C_{ox}\right)$,

$\varphi = \left(s_p \rho_{fuel} C_{fuel} + (1 - s_{sf})\rho C_p\right)$

Then (3)-(6) become

$$u\frac{d\phi}{dx} - D_{ox}\frac{d^2\phi}{dx^2} = 0 \tag{9}$$

$$u\frac{d\varphi}{dx} - D_p\frac{d^2\varphi}{dx^2} = 0 \tag{10}$$

The boundary conditions are

$$\phi(0) = s_{ox}T_1 + \frac{Q}{c_p}C_0, \quad \phi(\infty) = 0 \tag{11}$$

$$\varphi(0) = s_p\rho_{fuel}C_{f0}, \quad \varphi(\infty) = 0 \tag{12}$$

From the continuity equation (1), we obtain

$$u = \text{constant} \tag{13}$$

Here, we let

$$u = -v_0, \qquad v_0 > 0$$

and we obtain solution for (9) as

$$\phi(x) = \left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x} \tag{14}$$

and solution for (10) as

$$\varphi(x) = \left(s_p\rho_{fuel}C_{f0}\right)e^{-\frac{v_0}{D_p}x} \tag{15}$$

Then

$$\phi(x) = s_{ox}T(x) + \frac{Q}{c_p}C_{ox}(x) = \left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x}$$

gives

$$T(x) = \frac{1}{s_{ox}}\left[\left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x} - \frac{Q}{c_p}C_{ox}(x)\right] \tag{16}$$

$$C_{ox}(x) = \frac{c_p}{Q}\left[\left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x} - s_{ox}T(x)\right] \tag{17}$$

Also

$$\varphi(x) = s_p\rho_{fuel}C_{fuel}(x) + (1 - s_{sf})\rho C_p(x) = \left(s_p\rho_{fuel}C_{f0}\right)e^{-\frac{v_0}{D_p}x}$$

gives

$$C_{fuel}(x) = \frac{1}{s_p\rho_{fuel}}\left[\begin{array}{l}\left(s_p\rho_{fuel}C_{f0}\right)e^{-\frac{v_0}{D_p}x} - \\ (1 - s_{sf})\rho C_p(x)\end{array}\right] \tag{18}$$

$$C_p(x) = \frac{1}{(1 - s_{sf})\rho}\left[\begin{array}{l}\left(s_p\rho_{fuel}C_{f0}\right)e^{-\frac{v_0}{D_p}x} - \\ s_p\rho_{fuel}C_{fuel}(x)\end{array}\right] \tag{19}$$

Using (17) and (18) in equations (3)-(6). Then, the steady equation for temperature becomes

$$\begin{gathered}u\frac{dT}{dx} - D_{ox}\left(\frac{d^2T}{dx^2}\right) = \frac{QAk_{ov}}{\rho c_p} \\ \left[\frac{c_p}{Q}\left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x} - \frac{c_p s_{ox}}{Q}T(x)\right]^{\alpha} \\ \left[\frac{1}{s_p\rho}\left(s_p\rho C_{f0}\right)e^{-\frac{v_0}{D_p}x} - \frac{(1 - s_{sf})}{s_p}C_p(x)\right]^{\beta} e^{-\frac{E}{RT}}\end{gathered} \tag{20}$$

$$T(0) = T_1, \quad T(\infty) = 0$$

The steady equation for concentration of oxidizer becomes

$$\begin{gathered}u\frac{dC_{ox}}{dx} - D_{ox}\left(\frac{d^2C_{ox}}{dx^2}\right) = -\frac{s_{ox}Ak_{ov}}{\rho} \\ \left[\frac{c_p}{Q}\left(s_{ox}T_1 + \frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x} - \frac{c_p s_{ox}}{Q}T(x)\right]^{\alpha} \\ \left[\frac{1}{s_p\rho}\left(s_p\rho C_{f0}\right)e^{-\frac{v_0}{D_p}x} - \frac{(1 - s_{sf})}{s_p}C_p(x)\right]^{\beta} e^{-\frac{E}{RT}}\end{gathered} \tag{21}$$

$$C_{ox}(0) = C_0, \quad C_{ox}(\infty) = 0$$

The steady equation for concentration of solid fuel becomes

$$u\frac{dC_{fuel}}{dx}-D_p\left(\frac{d^2C_{fuel}}{dx^2}\right)=-\frac{(1-s_{sf})Ak_{ov}}{\rho}$$
$$\left[\frac{c_p}{Q}\left(s_{ox}T_1+\frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x}-\frac{c_ps_{ox}}{Q}T(x)\right]^{\alpha} \tag{22}$$
$$\left[\frac{1}{s_p\rho}(s_p\rho C_{f0})\,e^{-\frac{v_0}{D_p}x}-\frac{(1-s_{sf})}{s_p}C_p(x)\right]^{\beta}e^{-\frac{E}{RT}}$$

$$C_{fuel}(0)=C_{f0},\quad C_{fuel}(\infty)=0$$

The steady equation for concentration of gas product becomes

$$u\frac{dC_p}{dx}-D_p\left(\frac{d^2C_p}{dx^2}\right)=-\frac{s_pAk_{ov}}{\rho}$$
$$\left[\frac{c_p}{Q}\left(s_{ox}T_1+\frac{Q}{c_p}C_0\right)e^{-\frac{v_0}{D_{ox}}x}-\frac{c_ps_{ox}}{Q}T(x)\right]^{\alpha} \tag{23}$$
$$\left[\frac{1}{s_p\rho}(s_p\rho C_{f0})\,e^{-\frac{v_0}{D_p}x}-\frac{(1-s_{sf})}{s_p}C_p(x)\right]^{\beta}e^{-\frac{E}{RT}}$$

$$C_p(0)=0,\quad C_p(\infty)=0$$

We make the variable dimensionless by introducing

$$\left.\begin{array}{l}\theta=\frac{E}{RT_0^2}(T-T_0),\ \ x'=\frac{x}{L},\ \ v_o=\frac{D_{ox}}{L}=\frac{D_p}{L}\\ C'_{ox}=\frac{C_{ox}}{C_{ox}^0},\quad C'_f=\frac{C_f}{C_f^0},\quad C'_p=\frac{C_p}{C_p^0}\end{array}\right\} \tag{24}$$

θ is a non-dimensional temperature.

Then, equation (20) – (23) (after dropping prime) become

$$\frac{d^2\theta}{dx^2}+\frac{d\theta}{dx}+\delta\left[a\ e^{-x}-b\theta(x)-c\right]^{\alpha}$$
$$\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\frac{\theta}{1+\in\theta}}=0 \tag{25}$$

$$\frac{d^2C_{ox}}{dx^2}+\frac{dC_{ox}}{dx}-S\left[a\ e^{-x}-b\theta(x)-c\right]^{\alpha}$$
$$\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\frac{\theta}{1+\in\theta}}=0 \tag{26}$$

$$\frac{d^2C_{fuel}}{dx^2}+\frac{dC_{fuel}}{dx}-S_1\left[a\ e^{-x}-b\theta(x)-c\right]^{\alpha}$$
$$\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\frac{\theta}{1+\in\theta}}=0 \tag{27}$$

$$\frac{d^2C_p}{dx^2}+\frac{dC_p}{dx}+S_2\left[a\ e^{-x}-b\theta(x)-c\right]^{\alpha}$$
$$\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\frac{\theta}{1+\in\theta}}=0 \tag{28}$$

$$\left.\begin{array}{ll}\theta(0)=\theta_*, & \theta(\infty)=0\\ C_{ox}(0)=C_0, & C_{ox}(\infty)=0\\ C_{fuel}(0)=C_{f0}, & C_{ox}(\infty)=0\\ C_p(0)=0, & C_p(\infty)=0\end{array}\right\}, \tag{29}$$

where

$$\delta=\frac{QAk_{ov}Le^{-\frac{E}{RT_0}}}{\rho c_p\in T_0v_0}\ ,\quad a=\frac{c_p}{Q}\left(s_{ox}T_1+\frac{Q}{c_p}C_0\right)\ ,$$
$$b=\frac{c_ps_{ox}}{Q}\in\ ,\quad c=\frac{c_ps_{ox}}{Q}T_0\ ,\quad d=\frac{(1-s_{sf})C_p^0}{s_p}\ ,$$
$$S=\frac{s_{ox}Ak_{ov}Le^{-\frac{E}{RT_0}}}{\rho v_0C_{ox}^0},\quad S_1=\frac{(1-s_{sf})Ak_{ov}Le^{-\frac{E}{RT_0}}}{\rho v_0C_{fuel}^0},$$
$$S_2=\frac{s_pAk_{ov}Le^{-\frac{E}{RT_0}}}{\rho v_0C_p^0}$$

When $\in\to 0$, equation (25) – (28) become

$$\frac{d^2\theta}{dx^2}+\frac{d\theta}{dx}+\delta\left[a\ e^{-x}-b\theta(x)-c\right]^{\alpha}$$
$$\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\theta}=0 \tag{30}$$

$$\theta(0)=\theta_*,\qquad \theta(\infty)=0$$

$$\frac{d^2C_{ox}}{dx^2}+\frac{dC_{ox}}{dx}-S\left[a\,e^{-x}-b\theta(x)-c\right]^{\alpha}\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\theta}=0 \quad (31)$$

$$C_{ox}(0)=C_0,\qquad C_{ox}(\infty)=0$$

$$\frac{d^2C_{fuel}}{dx^2}+\frac{dC_{fuel}}{dx}-S_1\left[a\,e^{-x}-b\theta(x)-c\right]^{\alpha}\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\theta}=0 \quad (32)$$

$$C_{fuel}(0)=C_{f0},\qquad C_{ox}(\infty)=0$$

$$\frac{d^2C_p}{dx^2}+\frac{dC_p}{dx}+S_2\left[a\,e^{-x}-b\theta(x)-c\right]^{\alpha}\left[C_{f0}\,e^{-x}-dC_p(x)\right]^{\beta}e^{\theta}=0 \quad (33)$$

$$C_p(0)=0,\qquad C_p(\infty)=0$$

3.1 Properties of Solution

In this section, we consider equation (30) when $\theta_*=0$ and transform the equation from infinite domain to finite domain, using the transformation

$$y=e^{-x}$$

We obtain

$$\frac{d^2\theta}{dy^2}+\frac{\delta}{y^2}(a\,y-b\theta-c)^{\alpha}\left(C_{f0}y-dC_p\right)^{\beta}e^{\theta}=0 \quad (34)$$

$$\theta(0)=0,\qquad \theta(1)=0$$

Theorem 1 *Let* $b=c=d=0$ *and* $\alpha=\beta=1$ *in (34). Then* $\theta(y)$ *is symmetric about* $y=\frac{1}{2}$.

Proof: Let $b=c=d=0$ and $\alpha=\beta=1$ in (34). We obtain

$$\frac{d^2\theta(y)}{dy^2}+\delta_1e^{\theta(y)}=0$$

$$\theta(0)=0,\qquad \theta(1)=0,$$

Let $z=2y-1$

Then

$$\frac{d^2}{dy^2}=4\frac{d^2}{dz^2}$$

So the problem becomes

$$\frac{d^2\theta(z)}{dz^2}+\frac{\delta_1}{4}e^{\theta(z)}=0$$

$$\theta(-1)=\theta(1)=0$$

It suffices to show that $\theta(-z)=\theta(z)$.

Replace z by $-z$, we obtain

$$\frac{d^2\theta(-z)}{d(-z)^2}+\frac{\delta_1}{4}e^{\theta(-z)}=0$$

Hence θ is symmetric about $z=0$ i.e. θ is symmetric about $y=\frac{1}{2}$. This completes the proof.

Theorem 2 *Let* $b=c=d=0$ *and* $\alpha=\beta=1$ *in (34). Then* $\theta'\left(\frac{1}{2}\right)=0$.

Proof: Let $b=c=d=0$ and $\alpha=\beta=1$ in (34). We obtain

$$\frac{d^2\theta(y)}{dy^2}+\delta_1e^{\theta(y)}=0$$

$$\theta(0)=0,\qquad \theta(1)=0,$$

Since $\theta(y)$ is symmetric about $y=\frac{1}{2}$. Then $\theta'\left(\frac{1}{2}\right)=0$. This completes the proof.

Theorem 3 *Let* $b=c=d=0$ *and* $\alpha=\beta=1$ *in (34). Then* $\theta'(y)>0$ *for* $y\in\left(0,\frac{1}{2}\right)$.

Proof: Let $b=c=d=0$ and $\alpha=\beta=1$ in (34). We obtain

$$\frac{d^2\theta(y)}{dy^2} = -\delta_1 e^{\theta(y)}$$

$$\theta(0) = 0, \qquad \theta(1) = 0,$$

Using Ayeni (1978), we obtain

$$\theta(y) = \delta_1 \int_0^{\frac{1}{2}} k(y,t) e^{\theta(t)} dt,$$

where

$$k(y,t) = \begin{cases} y, & 0 \le y \le t \\ t, & t \le y \le \frac{1}{2} \end{cases}$$

So

$$\theta'(y) = \delta_1 \left[y e^{\theta(y)} + \int_y^{\frac{1}{2}} e^{\theta(t)} dt - y e^{\theta(y)} \right]$$

$$= \delta_1 \int_y^{\frac{1}{2}} e^{\theta(t)} dt$$

Hence, $\theta(y)$ is strictly monotonically increasing for $y \in \left(0, \frac{1}{2}\right)$. This completes the proof.

Theorem 4 *Let* $\alpha = 1,\ a = 0,\ c = 0,\ d = 0,\ \beta = 2$ *in (34). Then* $\theta(y) = 0$ *is the only non-negative solution.*

Proof: Let $\alpha = 1,\ a = 0,\ c = 0,\ d = 0,\ \beta = 2$. Then (34) becomes

$$\frac{d^2\theta}{dy^2} - \delta_1 \theta e^{\theta} = 0$$

$$\theta(0) = 0, \qquad \theta(1) = 0$$

Clearly $\theta(y) = 0$ satisfies the equation and the boundary conditions.

Next, we shall show that $\theta(y) = 0$ is the only solution.

Suppose $\theta(y)$ is another solution and $\theta(y) \ne 0,\ \theta(y) > 0$ for $y \in (0,1)$. Then

$$\frac{d^2\theta}{dy^2} - \delta_1 \theta e^{\theta} = 0$$

and $\theta(y)$ has no maximum in $(0,1)$ i.e. $\theta(y)$ cannot be greater than zero. Hence $\theta(y) = 0$ i.e. $\theta(y) \equiv 0$ is the only solution. This completes the proof.

Theorem 5 *Let* $b = 0$ *in (34) and assume* α *and* β *are even natural numbers. Then there exists* $y_0 \in (0,1)$ *such that* $\theta'(y_0) = 0$.

Proof: By theorem (3)

$$\theta(y) = \delta_1 \int_0^{y_0} k(y,t)(at - c)^{\alpha} \left(C_{f0} t - dC_p(t)\right)^{\beta} e^{\theta(t)} dt$$

where

$$k(y,t) = \begin{cases} y, & 0 \le y \le t \\ t, & t \le y \le y_0 \end{cases}.$$

So

$$\theta'(y) = \delta_1 \int_y^{y_0} (at - c)^{\alpha} \left(C_{f0} t - dC_p(t)\right)^{\beta} e^{\theta(t)} dt$$

Hence, $\theta(y)$ is strictly monotonically increasing for $y \in (0, y_0)$. But $\theta(1) = 0$. So there exists $y_0 \in (0,1)$ such that $\theta'(y_0) = 0$ and $\theta'(y_0 + \in) < 0$ for $\in > 0$. This completes the proof.

3.2 Numerical Solution

Due to the nonlinear nature of the governing equations (30) – (33), it is convenient to solve the equations by the use of shooting method technique. Here we replace ∞ by 2 because the software used does not recognized ∞ and we need to see the effect of parameters on the solutions.

We transform the differential equations (30) – (33) into a system of nine equations. To achieve the nine equations, let

$$x_1 = x,\ x_2 = \theta,\ x_3 = C_{ox},\ x_4 = C_{fuel},\ x_5 = C_p,$$
$$x_6 = \theta',\ x_7 = C'_{ox},\ x_8 = C'_{fuel},\ x_9 = C'_p$$

Then equations (30) – (33) become

$$x'_1 = 1 \tag{35}$$

$$x'_2 = \theta' = x_6 \tag{36}$$

$$x'_3 = C'_{ox} = x_7 \tag{37}$$

$$x'_4 = C'_{fuel} = x_8 \tag{38}$$

$$x'_5 = C'_p = x_9 \tag{39}$$

$$x'_6 = \theta'' = -\begin{bmatrix} x_6 + \delta\left(ae^{-x_1} - bx_2 - c\right)^{\alpha} \\ \left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2} \end{bmatrix} \tag{40}$$

$$x'_7 = C''_{ox} = -\begin{bmatrix} x_7 - S\left(ae^{-x_1} - bx_2 - c\right)^{\alpha} \\ \left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2} \end{bmatrix} \tag{41}$$

$$x'_8 = C''_{fuel} = -\begin{bmatrix} x_8 - S_1\left(ae^{-x_1} - bx_2 - c\right)^{\alpha} \\ \left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2} \end{bmatrix} \tag{42}$$

$$x'_9 = C''_p = -\begin{bmatrix} x_9 - S_2\left(ae^{-x_1} - bx_2 - c\right)^{\alpha} \\ \left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2} \end{bmatrix} \tag{43}$$

The initial conditions are

$$\left.\begin{array}{l} x_1(0) = 0,\ x_2(0) = \theta_*,\ x_3(0) = C_0, \\ x_4(0) = C_{f0},\ x_5(0) = 0,\ x_6(0) = a_1, \\ x_7(0) = a_2,\ x_8(0) = a_3,\ x_9(0) = a_4 \end{array}\right\} \tag{44}$$

Here $x_1 = x$ and when $x = 0,\ x_1 = 0$. Also $x_6(0) = a_1,\ x_7(0) = a_2,\ x_8(0) = a_3,\ x_9(0) = a_4$ are guessed and it is changed until $x_2(2) = 0,\ x_3(2) = 0,\ x_4(2) = 0,\ x_5(2) = 0$ as given by equation (29).

By Runge Kutta of four- order, we have

$$z_{n+1} = z_n + \frac{1}{6}\left(F_1 + 2F_2 + 2F_3 + F_4\right), \tag{45}$$

where

$$F_1 = hF(z_n),\ F_2 = hF\left(z_n + \frac{1}{2}F_1\right),$$
$$F_3 = hF\left(z_n + \frac{1}{2}F_2\right),\ F_4 = hF(z_n + F_3)$$

$$z = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix},$$

$$F(z) = \begin{pmatrix} 1 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ -\left(x_6 + \delta\left(ae^{-x_1} - bx_2 - c\right)^{\alpha}\left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2}\right) \\ -\left(x_7 - S\left(ae^{-x_1} - bx_2 - c\right)^{\alpha}\left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2}\right) \\ -\left(x_8 - S_1\left(ae^{-x_1} - bx_2 - c\right)^{\alpha}\left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2}\right) \\ -\left(x_9 - S_2\left(ae^{-x_1} - bx_2 - c\right)^{\alpha}\left(C_{f0}e^{-x_1} - dx_5\right)^{\beta} e^{x_2}\right) \end{pmatrix}$$

A computer program in Pascal codes was written to perform the iterative computations.

3.3 Results and Discussion

We have shown, under certain conditions, that (i) $\theta(y)$ is symmetric about $y = \frac{1}{2}$. (ii) $\theta'\left(\frac{1}{2}\right) = 0$. (iii) $\theta(y)$ is strictly monotonically increasing for $y \in \left(0, \frac{1}{2}\right)$. (iv) $\theta(y) = 0$ is the only non-negative solution. (v) $\theta'(y_0) = 0$.

In Figures 1, 2, 3 and 4 we display the graphs of $\theta(x)$, $C_{ox}(x)$, $C_{fuel}(x)$ and $C_p(x)$ versus x for

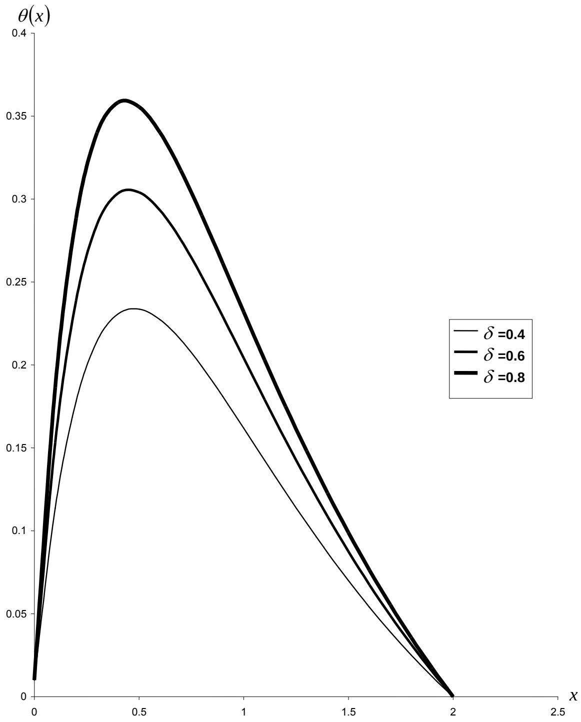

Fig. 1: Plots of $\theta(x)$ against x for equations (30) and (31) at various values of δ when α =1, β =1.

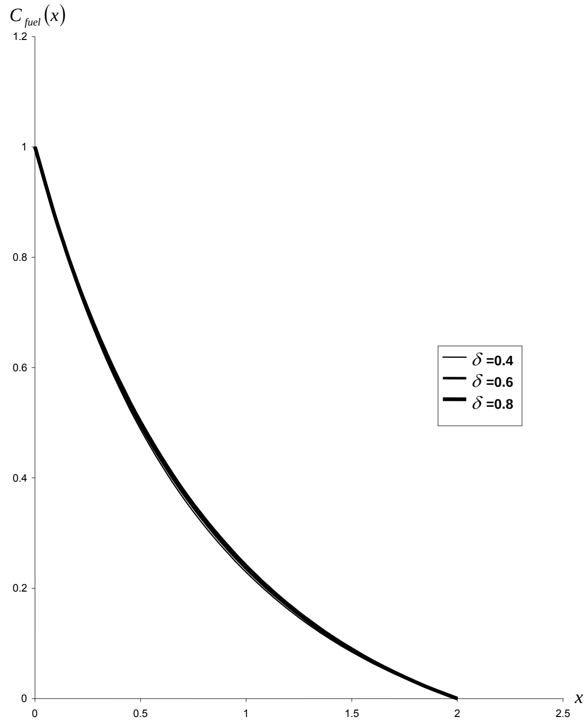

Fig. 3: Plots of $C_{fuel}(x)$ against x for equations (30), (32) and (33) at Various values of δ when α =1, β =1.

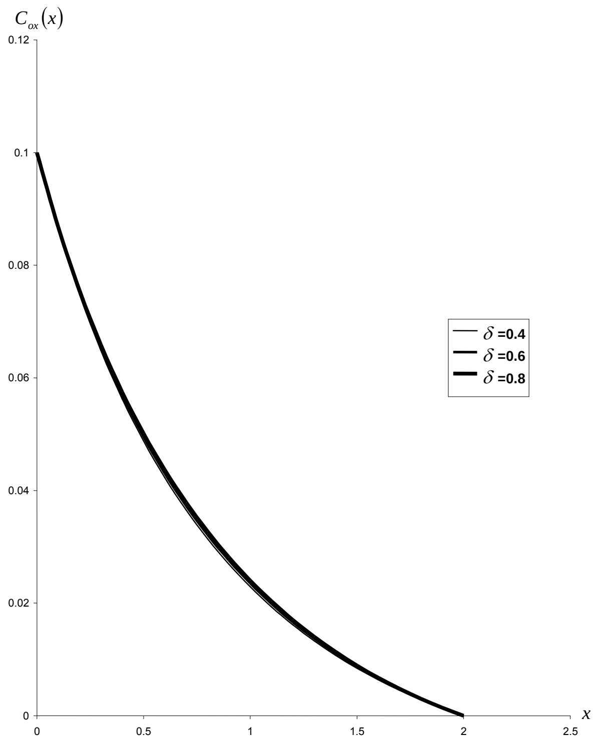

Fig. 2: Plots of $C_{ox}(x)$ against x for equations (30), (31) and (33) at various values of δ when α =1, β =1.

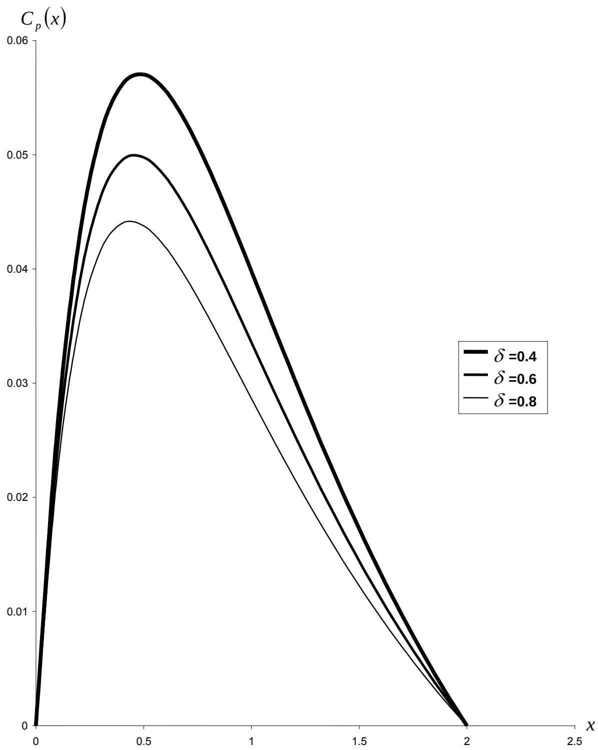

Fig. 4: Plots of $C_p(x)$ against x for equations (30) and (33) at various values of δ when α =1, β =1.

various values of δ. It is easy to see that $\theta(x)$ increases as δ increases, $C_{ox}(x)$ and $C_{fuel}(x)$ does not change much with increase of δ and $C_p(x)$ increases with decrease of δ.

It is worth pointing out that the effect of δ as seen in Figure 1 indicating that there is increase in heat of reaction Q. When the heat of reaction is high, the rate of conversion of heavy oils into light oils, water and gas is high and consequently, the recovery rate is boosted. This is of great economic importance.

4 Conclusion

The main goal of this simulation is, on one hand, to identify the condition under which a high temperature combustion front can propagate in a fractured system and, on the other hand, to determine the oil production mechanism(s) under a sustained combustion process. The equations of in-situ combustion in porous media have been presented by mathematical point of view. The equations have been solved by shooting method techniques and the results and discussion for that have been described. The graphical summaries of the system responses were provided.

It can be concluded from the simulations that chemical reactions occurring due to injection of air into a reservoir have considerable effects on the phenomena of flow in the medium. The presented analysis has also shown that the parameters involved in the in-situ combustion model have significant effects on temperature and concentration field of the system.

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