Fuzzy Time Series Model Incorporating Predictor Variables and Interval Partition

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Abstract: - Prediction is a critical component in decision-making process for business management. Fuzzy Markov model is a common approach for dealing with the prediction of time series. However, not many studies devoted their attention to the effect of the parameters on model fitting for fuzzy Markov model. In the paper, we examine the prediction ability for fuzzy Markov model, based on the data of Taiwan's exports and foreign exchange rate. The empirical results indicate that fuzzy Markov model performs better for longer period forecasting; moreover, neither increment information nor increasing window basis would improve the performance for fuzzy Markov model. An advantage of the paper provides a beneficial knowledge when using Markov model for prediction.

Key-Words: - Fuzzy time series, Fuzzy Markov model, High order fuzzy relationship, Increment information, Interval partition, Taiwan exports.

1 Introduction

Prediction is a critical component in decision-making process for business management. Accurate predictions help decision-makers make correct judgments. In the previous study, when data is in the form of time series, predictions are often made using the traditional tools, such as ARIMA or VAR. However, problems arise where the time series represent just one point in a range of possible values. Stock markets are a case in point: should the data point for a given day be that day's opening or closing price? The daily high or low? Simply using closing prices for market prediction has been shown to lead to inaccurate analysis because of linguistic language. Fuzzy number are one of the step-ups of the uncertainly research. For the linguistic problem of data, fuzzy set theory defines a linguistic value is distributed between quantitative boundaries and how is the most certain points in the range. It is not only an adequate approach but widely used for dealing with the variability in the time series, especially in control system, intelligent system, and science prediction [36].

Since Zadeh [36] proposed fuzzy set theory as the tool to test uncertain membership, it has served as the theory framework in research of many fields and has solved 0-1 logic value limitation of traditional sets. The fuzzy theory has been successfully applied in decision analysis, artificial intelligence, economics, psychology, and control theory. The method of Song and Chissom [21] [22] named fuzzy time series is improved by several studies and papers on the model is increasing widely. Fuzzy models to forecast time series can be categorized into three groups [28], based on how they formulate the relationships between data points: fuzzy rule, fuzzy functions, or fuzzy relationships. Fuzzy rules can either be initialized or abstracted from real examples using neural networks [27] [33] or genetic algorithms [8]. Song and Chissom

[22] [23] proposed formulae for fuzzy relations for student enrollment forecasting. Based on their work [22] [23], a variety of fuzzy time series models have been developed. Most of these models have been shown to outperform their conventional counter- parts in forecasting accuracy.

For example, Chen [1] simplified fuzzy relation matrix construction procedure by Song and Chissom [21] [22] [23] used simple mathematical equation instead of complex Max-min equation, and suggested fuzzy logical relationship group. The empirical data are superior to the results from the model created by Song and Chissom [23].

Huarng [7][8] proposed two heuristic models to improve the model by Chen [1], constructed two fuzzy logical relationship groups of previous and next periods, and used a threshold value to construct three fuzzy logical relationship groups of previous and next periods. Meanwhile, distribution based and average based approaches were proposed to explain too long interval may cause fuzzy time series without fluctuation. The historical data variation trend cannot be known, and prediction accuracy lowers; too short interval may conflict with nature of fuzzy time series model.

Hwang et al [12] proposed a differential mode for stabilization of historical data to predict future variation not the value. The predicted value is equal to the variation plus previous-period value. A fuzzy correlation matrix is constructed to differentiate window basis and operation matrix. The fuzzy correlation matrix is more rational, operation time is more coefficient, and predicted value is superior to the model [21] [22] [23]. Cheng *et al* [3] used trapezoidal membership functions to fuzzify historical data and suggested minimize entropy principle approach to find out appropriate interval median points and form unequal interval segmen- tation. Trapezoid fuzzification approach is different from the triangular membership function approaches in the past literature. In the segmentation of various intervals, more than one membership is 1.

Yu [35] considered fuzzy logical relationship should be assigned with proper weights according to fuzzy logical relationship, reflecting the data information, and employed two interval segmentation methods by Huarng [7][8] to construct models. Huarng and Yu [9] proposed heuristic type-2 model to increase research variables to facilitate efficiency of prediction models. Huarng and Yu [10] introduced neural network nonlinear structure and employed back propagation for prediction. Huarng and Yu [11] suggested the unequal-length interval segmentation and firmly believed the same variation value has unequal validity in different intervals.

Chen et al [3] built fuzzy relation matrix based on Entropy concept and tested the model with data of secular trend. Li and Cheng [16] considered the fuzzy logical relationship group created by Chen [1] is the factor that leads to prediction uncertainty and thus suggested a backtracking system to establish a sole definite fuzzy logical relationship; the findings are inconsistent with the conclusion made by Huarng [7]. Cheng *et al* [4] extended the concept by Wang and Hsu [26] and Yu [34][35] and made linear correction with linear and non-linear concepts after nonlinear fuzzy logical relationship, and assigned suitable weight to the difference between predicted value and the actual value of previous period.

Chen and Hwang [2] first introduced two-variable in fuzzy time series prediction to forecast main variables in combination with the concept by Hwang et al [12]. Cheng *et al* [5] applied novel fuzzy time series method to resolve the student enrollment in the University of Alabama and Taiwan stock TAIEX data. Recently, Wong *et al* [29] compared the three fuzzy prediction models in the literature and found that heuristic fuzzy time series model has the lowest prediction error. Further, a single-variable model is better than multi- variable model. This conclusion disproves the past literature [9]. Later, Wong *et al* [30] suggested that the decision of an optimal fuzzy model to the prediction is correlated with the type of time series data.

To conduct a fuzzy time series forecasting, the fuzzy intervals for data need to be first decided. To do so, the selection of intervals for data would affect the quality of prediction for the fuzzy models [9] [10]. In the past study, most researchers tried to use just 5 or 7 intervals for data for model prediction [22][23][1][12][2][6]; some authors tried different methodology for setting the intervals for time series data [7] [16][15][13][18][25].

As the original fuzzy time series models proposed were single variable, incorporating only the variable to be predicted itself [24] [30] [1]. Recently, for accuracy of model prediction, the multivariate models have been considered with increment information involved by quantities related to the variable for time series data [2][14][33][7][8] [32][20][31] [29].

However, for seeking the accuracy of fuzzy models, there are still some critical issues on fuzzy model. For example, first, is it proper conclusion that the increasing in number of intervals will improve the accuracy of forecasting? Second, how will fuzzy prediction models respond to the changes in the length of the time series data? Third, do multivariate fuzzy models outperform single variable models? These issues related to fuzzy model have not yet received much discussion in the previous studies.

In the paper we examine the sensitivity study for the fuzzy Markov model to the issues. The data of

Taiwan exports as used for the variable to be predicted, and spot exchange rates considered as increment information for model test. The study assesses if the forecasting performance of Markov model will be affected by the prediction parameters which includes (1) length of data period; (2) length of interval; (3) increment information.

The next section presents the concept of fuzzy set theory and multivariate Markov models. Section three gives the empirical results and discussion. Section four contains our conclusions and future recommendations.

2 Fuzzy Markov Model

The basic concept of fuzzy set theory is the function of membership degree. With this transformation for data, it can describe fuzzy set characteristics, quantify the fuzzy set and handle linguistic information by two-value logics. Fuzzy set can express a set of specific things with indistinct boundary, and membership function denotes the fuzzy set relation that elements belong to, and the fuzzy set is often denoted by [0,1].

If U_A denotes the membership function of fuzzy set A, the membership function value $U_A(x)$ expresses the degree of membership of element x of set A. The greater membership degree of element x of set A is, the closer is membership degree to 1, otherwise, it is closer to 0, see Figure (1). The membership function can be divided into discrete function and continuous function. Proper membership function can easily quantify interval data for numerical calculation. Fuzzy time series model applies fuzzy logic in the data series analysis to solve fuzzy problems in combination with linguistic degree, membership members and fuzzy relation.

Suppose that $\{Y(t) \in R, t = 1,2,...,n\}$ is a time series, U denotes its universe of discourse, and an ordered partition set is given to U, represented by $\{G_i, i = 1,2,...,m\}$, $\sum_{i=1}^{m} G_i = U$. Relatively, linguistic variable is $\{A_i, i = 1,2,...,m\}$, assume that time series $Y(t)$ experiences linguistic variable $\{A_i\}$ and corresponds to $F(t)$, which is called fuzzy set. The membership function of fuzzy set $F(t)$ is $\{u_1(Y(t)), u_2(Y(t)),...,u_m(Y(t))\}$ where $0 \le U_i(Y(t)) \le 1$, and the fuzzy set $F(t)$ can be expressed by discrete function:

$$F(t) = \sum_{i=1}^{m} \mu_i(Y(t)) \big/ A_i = \mu_1(Y(t)) \big/ A_1 + \mu_2(Y(t)) \big/ A_2 + ... + \mu_m(Y(t)) \big/ A_m \tag{1}$$

degree of original time series $Y(t)$ after experiencing linguistic where $\mu_i(Y(t)) \big/ A_i$ denotes the membership and membership variable $\{A_i\}$; "+" denotes a connecting symbol; $\mu_i : R \to [0,1]$, and $\sum_{i=1}^{n} u_i(y(t)) = 1$, $t=1,2,...,n$.

The degree of each data is found out according to their membership grade to fuzzy sets. The fuzzified data is expressed as $\tilde{A}_k$. Assume that there are k internals for time series expressed by u_1= [n_1, n_2], u_2 = [n_3, n_4], u_3= [n_5, n_6],…,u_k= [n_m,n_{m+1}], we define fuzzy sets by membership function as following:

$$\tilde{A}_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + ... + 0/u_m$$

$$\tilde{A}_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + ... + 0/u_m$$

$$\tilde{A}_3 = 0/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + ... + 0/u_m$$

$$\cdots$$

$$\tilde{A}_k = 0/u_1 + 0/u_2 + ... + 0/u_{m-2} + 0.5/u_{m-1} + 1/u_m$$

2.1 Fuzzy relation

Let U be the universe of discourse with $G=(u_1,u_2,...,u_r)$ and $H=(v_1,v_2,...,v_r)$, and $\{P_i, i=1,2,...,r\}$ defined as an ordered partition set of U, where u_i and v_i are the membership function on universe of the fuzzy set U, and the fuzzy relations between G and H is defined as

$$R = G^T \circ H = [R_{ij}]_{r \times r} \tag{2}$$

Where '∘' is the max-min 'operator ', 'T' is the

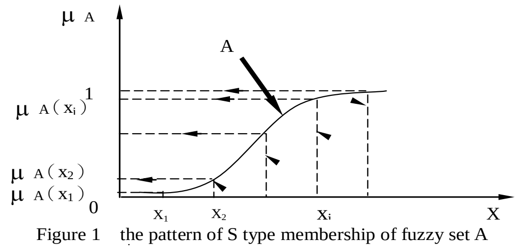

Figure 1 the pattern of S type membership of fuzzy set A

transpose, the R_{ij} defined as the membership function between G and H.

2.2 Fuzzy Markov relation matrix

Suppose that $\{F(X_t)\}$ is a fuzzy autoregressive process of order one, meaning that for any t, $F(X_t)$ can be determined by $F(X_{t-1})$ in time $t-1$. Let the membership function of $F(X_t)$ be $u_i(X_t)$, $i=1,2,...,r.$,then the Fuzzy Markov relation matrix (3) is expressed by

$$\Re^* = \left[\Re^*_i\right]_{r\times r} = \max_{2\le t\le n}\left[\min\left(\mu_i(X_{t-1}), \mu_j(X_t)\right)\right]_{r\times r} \qquad (3)$$

2.3 P-th order fuzzy Markov model

If the fuzzy Markov time series model (4) is expressed as below:

$$F(t) = F(t-1)\circ F(t-2)\circ \ldots \circ F(t-p)\times \Re^* \qquad (4)$$

then it is a *p*-th order fuzzy auto-regressive model, where $\Re^*$ is the fuzzy relation matrix between *F(t)* and previous various period of time *F(t-1), F(t-2),..., F(t-p)*.

If the fuzzy time series $(FX_{1,t}, FX_{2,t},..., FX_{k,t})$ in time point *t* is influenced by the variables $(FX_{1,t-1}, FX_{2,t-1},..., FX_{k,t-1})$ in *t-1*, it then is a first order multivariate fuzzy model expressed as below:

$$(FX_{1,t}, FX_{2,t},..., FX_{k,t}) = (FX_{1,t-1}, FX_{2,t-1},..., FX_{k,t-1})\begin{bmatrix} \Re_{11} & \cdots & \Re_{1k} \\ \vdots & \ddots & \vdots \\ \Re_{k1} & \cdots & \Re_{kk} \end{bmatrix}$$

$$i = 1,2,..., k \quad j = 1,2,..., k \qquad (5)$$

and *p*-order multivariate fuzzy model as below:

$$(FX_{1,t}(p), FX_{2,t}(p),..., FX_{k,t}(p)) = (FX_{1,t-1}, FX_{2,t-1},..., FX_{k,t-1})\begin{bmatrix} \Re_{11} & \cdots & \Re_{1k} \\ \vdots & \ddots & \vdots \\ \Re_{k1} & \cdots & \Re_{kk} \end{bmatrix}^p$$

$$i = 1,2,..., k \quad j = 1,2,..., k \qquad (6)$$

where $\Re_{ij}$ is the fuzzy Markov relation matrix. The two variable Markov model is expressed by using the square of the matrix $\Re_{ij}$.

The basic algorithm steps for a fuzzy Markov model are as follows:

Step1. Define the universe of discourse *U* for the historical data.
Step2. Partition *U* into several equal intervals.
Step3. Define fuzzy set on universal discourse *U*.
Step4. Maximize fuzzy membership function.
Step5. Decide the order of the fuzzy auto-regressive model.
Step6. Compute fuzzy Markov relation matrix.
Step7. Forecast and defuzzify.

2.4 Measurement of Prediction Accuracy

This study uses mean square error (MSE) to measure forecasting accuracy of Markov model. The equation is used to test the part of data which is not explained in the prediction model. The MSE value can be represented by:

$$MSE = \frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k) - \hat{x}^{(0)}(k))^2 \qquad (7)$$

where $X^{(0)}(K)$ denotes the actual value; $\hat{X}^{(0)}(K)$ denotes the predicted value of model; *n* denotes the number of data. In other words, $0 \le MSE \le \infty$. The smaller MSE value is, the closer is predicted value of the model to the historical data, meaning the high prediction capability of the model.

3 Empirical Results and Discussion

In the section, we test the changes in prediction parameters to see how fuzzy Markov model respond. For the stability test of Markov model, the data are randomly divided into three different periods which include (I) January 1995 to March 2002 with data set of 87; (II) January 1998 to March 2002 with data 51; (III) January 2000 to March 2002 with data 27. As high order would increase computation complexity, most of past studies just used one order for Markov model test. In the paper, the window basis is considered by the order 2~4 for the data assessment. And, the length of interval is divided by 16 sections. The model factor is Taiwan exports used as the prediction varialbe, as shown in Fig 2a, and foreign exchange rate as the auxiliary variable shown in Fig 2b.

The data for the fuzzy Markov model test are from AEROM, Taiwan. As both are different data used for test, it is necessary to check if they are adequate for model analysis [21]. After the computation of the data correlation, the value of coefficient of two data is obtained to be 0.80331, *p*=0.0001 (<0.05), meaning that

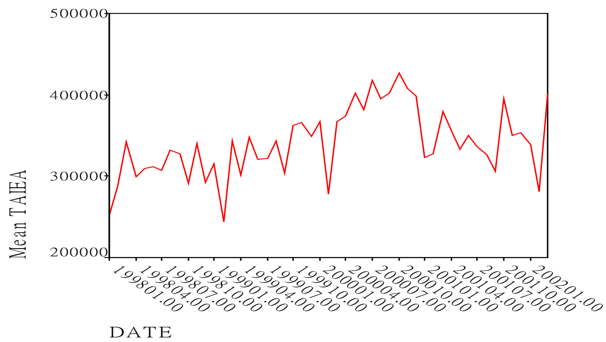

Figure.2a The pattern of Taiwan exports

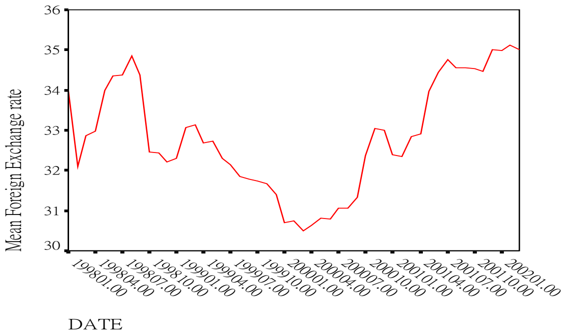

Figure. 2b The pattern of foreign exchange rate

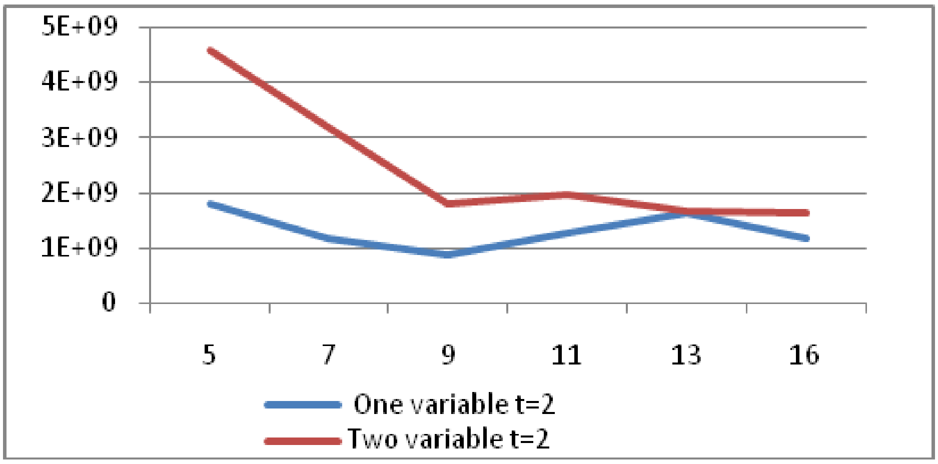

Figure. 3 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=2, n=87(199501~200203)

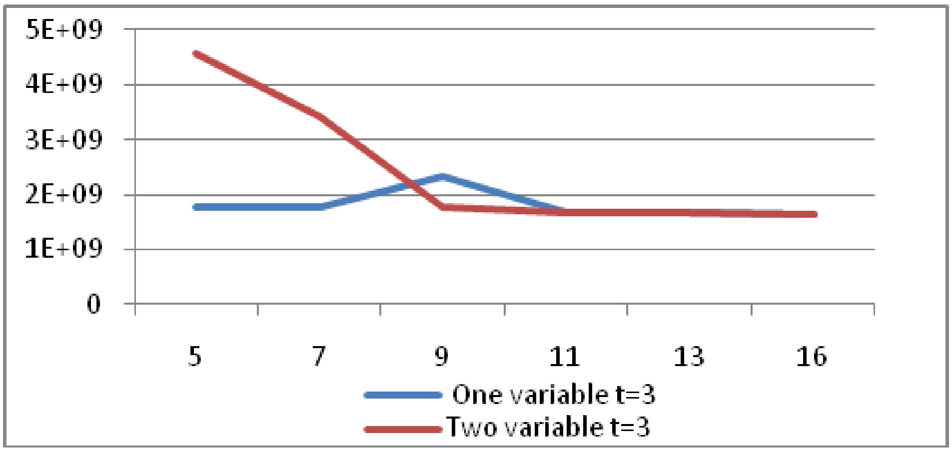

Figure. 4 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=3, n=87(199501~200203)

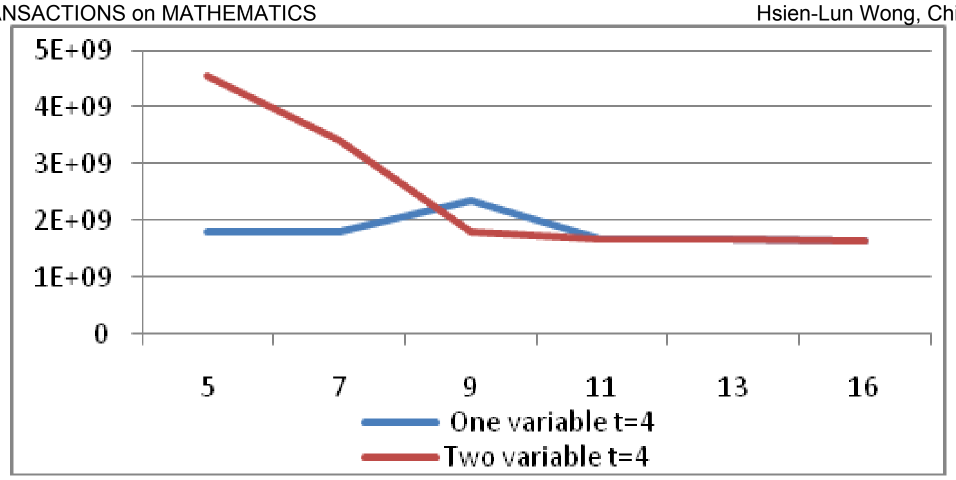

Figure. 5 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=4, n=87(199501~200203)

Table.1 The MSE of fuzzy Markov model to forecast Taiwan exports (n=87)

Month	Period	Interval	Fuzzy Markov model					
			One-variable			Two-variable		
			t=2	t=3	t=4	t=2	t=3	t=4
n=87	199501 ~ 200203	5	1.79×10^9	1.79×10^9	1.79×10^9	4.56×10^9	4.56×10^9	4.56×10^9
		6	1.58×10^9	1.67×10^9	1.67×10^9	1.67×10^9	1.67×10^9	1.67×10^9
		7	1.16×10^9	1.79×10^9	1.79×10^9	3.19×10^9	3.41×10^9	3.41×10^9
		8	1.37×10^9	1.64×10^9	1.64×10^9	1.64×10^9	1.64×10^9	1.64×10^9
		9	8.87×10^8	2.35×10^9	2.35×10^9	1.79×10^9	1.79×10^9	1.79×10^9
		10	1.43×10^9	1.64×10^9	1.64×10^9	1.65×10^9	1.65×10^9	1.65×10^9
		11	1.29×10^9	1.69×10^9	1.69×10^9	1.96×10^9	1.96×10^9	1.96×10^9
		12	1.65×10^9	1.65×10^9	1.65×10^9	1.65×10^9	1.65×10^9	1.65×10^9
		13	1.65×10^9	1.66×10^9	1.66×10^9	1.66×10^9	1.66×10^9	1.66×10^9
		14	1.59×10^9	1.74×10^9	1.74×10^9	1.74×10^9	1.74×10^9	1.74×10^9
		15	1.18×10^9	1.64×10^9	1.64×10^9	1.64×10^9	1.64×10^9	1.64×10^9
		16	1.31×10^9	1.67×10^9	1.67×10^9	1.64×10^9	1.64×10^9	1.64×10^9

3.1 January 1995 to March 2002

For the data period, the MSE of fuzzy Markov model, based on different parameters to forecast Taiwan export, is shown in Fig3, Fig4, and Fig5, respectively. Whatever window basis and length of interval are, from Fig 3, it is interesting to note that the MSE, averagely, of single variable Markov model lowers than that of the two variable model. Meanwhile, the result is the same as Fig4 and Fig 5 expect in the window basis of order 9 where one variable is higher than two variable model.

Moreover, from Fig3, Fig4, and Fig 5, it is noted that the model with low window basis (t=2) performs better than that of high window basis model (t=3, t=4). However, the change in the number of interval does not produce significant improvements in prediction accuracy. For two variable Markov model, increasing the number of interval will decrease the MSE, but not influence the MSE for one variable model, especially in low window basis. The final result is shown in Table 1.

3.2 January 1998 to March 2002

The results of using 51 month time series are shown in Fig6, Fig7, and Fig8. If controlling for number of intervals, predictions are largely similar in quality to those obtained using the 87 month series, so decreasing the length of the data period does not seem to improve predictions significantly.

Assessing the range of fuzzy intervals, the result shows that the fuzzy Markov single variable model (16 intervals) is best performance with the lowest MSE. Low order (t=2) would improve the forecasting performance for fuzzy Markov model. However, it is no trend for the Markov model that increasing the number of intervals for data would be linked to predictive accuracy. The final result is shown in Table 2.

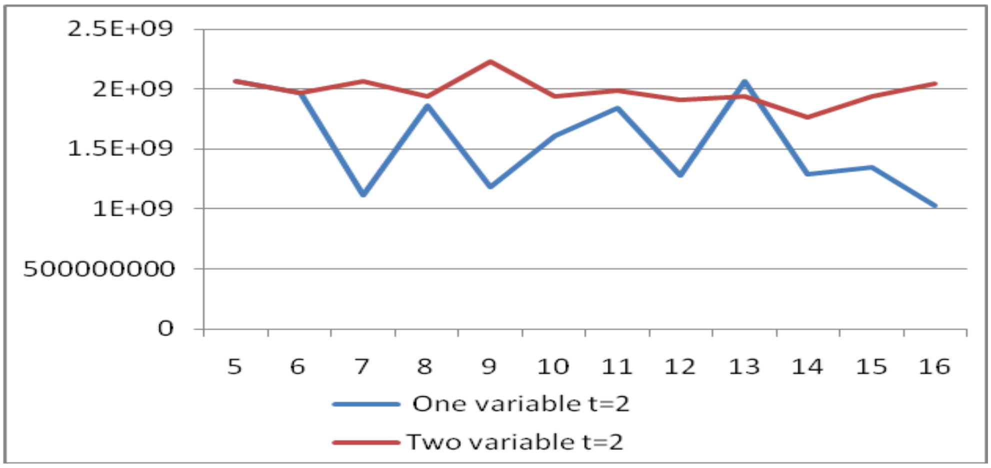

Figure. 6 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=2, n=51 (199801~200203)

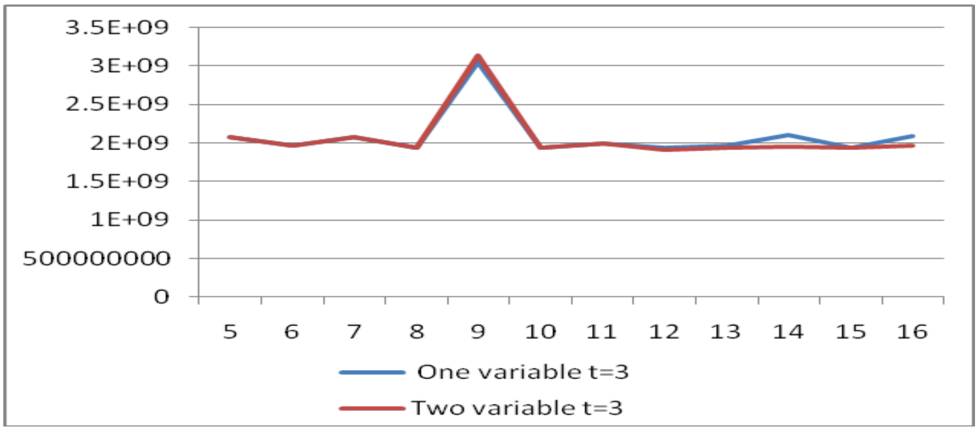

Figure. 7 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=3, n=51 (199801~200203)

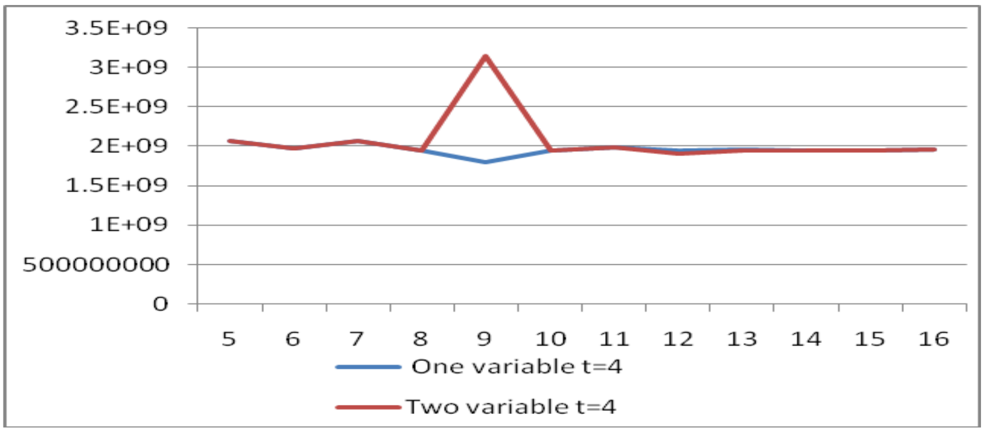

Figure. 8 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=4, n=51 (199801~200203)

Table.2 The MSE of fuzzy Markov model to forecast Taiwan exports (n=51)

Month	Period	Interval	Fuzzy Markov model					
			One-variable			Two-variable		
			t=2	t=3	t=4	t=2	t=3	t=4
n=51	199801 ~ 200203	5	2.07×10^9	2.07×10^9	2.07×10^9	2.07×10^9	2.07×10^9	2.07×10^9
		6	1.97×10^9	1.97×10^9	1.97×10^9	1.97×10^9	1.97×10^9	1.97×10^9
		7	1.12×10^9	2.07×10^9	2.07×10^9	2.07×10^9	2.07×10^9	2.07×10^9
		8	1.86×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9
		9	1.18×10^9	3.05×10^9	1.80×10^9	2.23×10^9	3.14×10^9	3.14×10^9
		10	1.61×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9
		11	1.84×10^9	1.99×10^9	1.99×10^9	1.99×10^9	1.99×10^9	1.99×10^9
		12	1.28×10^9	1.94×10^9	1.94×10^9	1.91×10^9	1.91×10^9	1.91×10^9
		13	2.07×10^9	1.96×10^9	1.96×10^9	1.94×10^9	1.94×10^9	1.94×10^9
		14	1.29×10^9	2.10×10^9	1.95×10^9	1.77×10^9	1.95×10^9	1.95×10^9
		15	1.35×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9	1.94×10^9
		16	1.03×10^9	2.09×10^9	1.96×10^9	2.05×10^9	1.96×10^9	1.96×10^9

3.3 January 2000 to March 2002

The results of using just a 27 month time series are shown in Fig9, Fig10, and Fig11. From table3, we know that the performance of the model is not better than those based on 87 and 51 data period, so the length of data period does affect predictive accuracy. When the number of intervals is varied, predictions would not improve with the greater numbers of intervals. Moreover, for short 27 month data, there are no significant differences between using one or two variable model to predict in low length of interval. Moreover, the Markov model with low window basis has better performance than model with high window basis does, especially in one variable model. The final result is shown in Table 3.

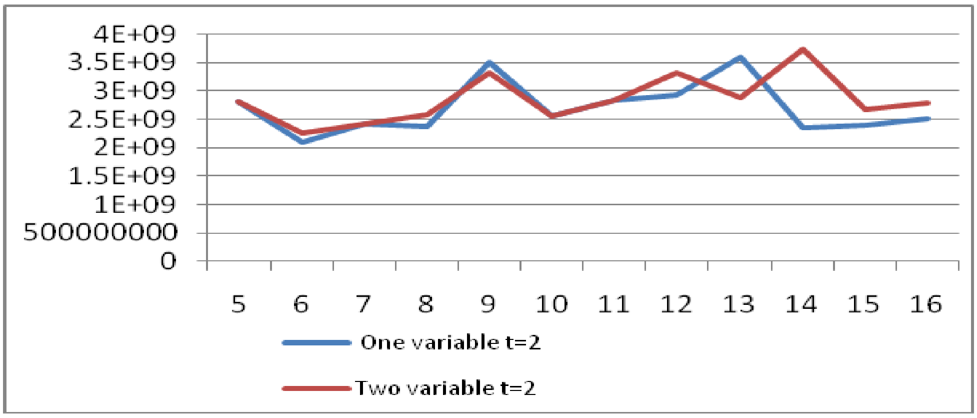

Figure. 9 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=2, n=27 (200001~200203)

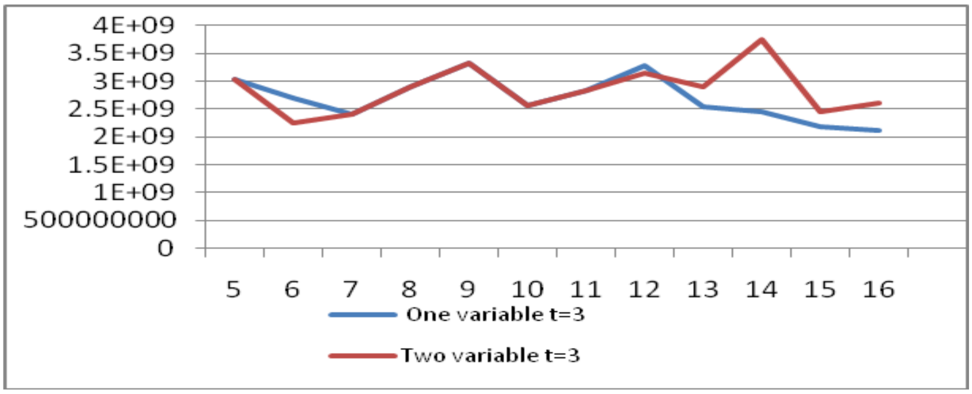

Figure. 10 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=3, n=27 (200001~200203)

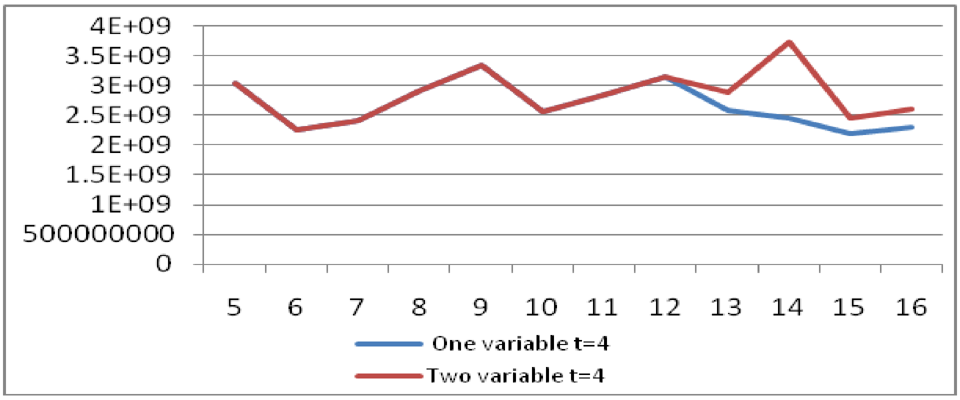

Figure. 11 The MSE and optimal interval for fuzzy Markov model to forecast Taiwan exports, t=4, n=27 (200001~200203)

Table.3 The MSE of fuzzy Markov model to forecast Taiwan exports (n=27)

Month	Period	Interval	Fuzzy Markov model					
			One-variable			Two-variable		
			t=2	t=3	t=4	t=2	t=3	t=4
n=27	200001 ~ 200203	5	2.81×10^9	3.04×10^9	3.04×10^9	2.80×10^9	3.04×10^9	3.04×10^9
		6	2.10×10^9	2.70×10^9	2.25×10^9	2.25×10^9	2.25×10^9	2.25×10^9
		7	2.41×10^9	2.41×10^9	2.41×10^9	2.41×10^9	2.41×10^9	2.41×10^9
		8	2.38×10^9	2.90×10^9	2.90×10^9	2.57×10^9	2.90×10^9	2.90×10^9
		9	3.50×10^9	3.33×10^9	3.33×10^9	3.33×10^9	3.33×10^9	3.33×10^9
		10	2.56×10^9	2.56×10^9	2.56×10^9	2.56×10^9	2.56×10^9	2.56×10^9
		11	2.84×10^9	2.84×10^9	2.84×10^9	2.84×10^9	2.84×10^9	2.84×10^9
		12	2.92×10^9	3.27×10^9	3.14×10^9	3.33×10^9	3.14×10^9	3.14×10^9
		13	3.59×10^9	2.55×10^9	2.59×10^9	2.89×10^9	2.89×10^9	2.89×10^9
		14	2.36×10^9	2.45×10^9	2.45×10^9	3.74×10^9	3.74×10^9	3.74×10^9
		15	2.39×10^9	2.19×10^9	2.19×10^9	2.67×10^9	2.45×10^9	2.45×10^9
		16	2.51×10^9	2.11×10^9	2.30×10^9	2.78×10^9	2.61×10^9	2.61×10^9

From the comparison of MSE value shown in three tables (table2~ table4), it is noted that fuzzy Markov model has the tendency to longer data forecasting. Averagely, the MSE of model with long data 87 is the smallest; MSE with data 51 is second; however, MSE of model with data 27 is the greatest. For model variable involved, from the figures (figure3~ figure11), we know that fuzzy Markov one variable model outperforms two variable model. For the window basis, fuzzy Markov model with the order t=2 has better forecasting performance than higher order. When order is higher than 3, the MSE value is getting stable. For the length of interval, partitioning 9 intervals for data is optimal for the longer series, 6~7 internals is optimal for the shortest series. In practice, the paper suggests at least 7 intervals for rules. The conclusion, which has been suggested in previous research, that performance should improve with more intervals, is only partially supported, for some models but not all. These empirical results suggest that there is no single optimum number of intervals for fuzzy Markov model.

4 Conclusions

In the paper, we examine the important parameters related to prediction performance for fuzzy Markov model. The data of Taiwan exports and exchange rates used are adequate for model calibration. The empirical results indict that for the window basis, the Markov model produces best results with a chain of order 2. With an order 2, one variable input generates better predictions than two variables for model. That is, the increasing in increment information by more variables does not reduce errors in prediction. Although fuzzy Markov model is widely applied to business forecasting, the high-order relation matrix is very complex for computation. Therefore, the finding that low order is suitable to Markov model has a beneficial for saving computation time when using Markov model for prediction.

For the data period, it is noted that fuzzy Markov model would perform better for longer period. For the length of intervals, with only a short time series, 6 or more intervals generates accurate predictions. This contradicts the claim made in previous research that higher numbers of intervals result in more accurate predictions. Rather, the number of intervals must be selected on a case-by-case basis. For the future study, we are to continue the calibration of other fuzzy prediction models, such as Heuristic and Two-factor model. The conclusion of the paper can satisfy our prediction purpose with choosing interval partition when considering MSE, cost and rational explanations.

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