The Effects of the Change of Bond Insurance Premium and Capital Regulatory Ratio on Loan and Deposit Rates: An Option-Pricing Model

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Abstract: We propose an option-based model that examines the relationships among municipal bonds with prepackaged insurance, capital insurance, and optimal bank interest margins. If the elasticity effect is positive (negative), then an increase in the bond insurance premium will increase the bank’s optimal loan rate (optimal deposit rate). If the elasticity effect is negative (positive), then an increase in the capital-to-deposit ratio will increase (decrease) the bank’s optimal loan rate. But an increase in the capital-to-deposit ratio increase the bank’s optimal deposit rate under the positive elasticity effect.

Key-Words: Municipal bond, Capital-to-deposit ratio, Interest margins, Bond insurance, Elasticity, option

1 Introduction

In recent decades, discussions about the successful issuance of municipal bonds or advantage of their existence have become an important topic.

The literature on the special insurance features of bonds, for example, is scare. To date, there have

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been relatively few defaults on insured bonds. However, default in the municipal bond market is not uncommon. Fons (1987) and Cohen (1989) provided models of bond default rate studies. Cirillo and Jessop (1993) showed that during the eighties, about 2% of the bonds were defaulted. Rather than emphasize the bond default rates, Angel (1994) proposed the potential benefits on bond insurance. Quigly and Rubinfeld (1991), and Nathans (1992) focused on determining the magnitude of the issuer’s saving from bond insurance.

General intuitional reflections always consider that the increase of the bond insurance premium will result in increase of operating costs of banks, therefore, banks will increase loan ratio and decrease deposit ratio, in order to shift these additionally increased costs to clients. However, in this study we found that when the bond insurance premium is increased, optimal loan ratio would be increased only under positive flexible effect condition (i.e. banks are in bad condition). Instead, under negative flexible effect condition (i.e. banks are in good condition), banks will increase rather than decrease optimal deposit ratio when the bond insurance premium is increased. Therefore, when the bond insurance premium increases, if banks adjust their optimal interest rate, it is purely to decrease quantity of municipal bond rather than to shift costs to clients. The increase of capital-to-deposit increase binding of banks, however, in this study we found that banks should decrease their optimal loan ratio and increase optimal deposit ratio under positive flexible effect condition (i.e. banks are in bad condition). Although adjustment of interest rate decrease unit profit, but business can be expanded and total revenues would therefore be increased.

The remaining parts of this paper are organized as follows. Section 2 lays out the basic model of a banking firm under the option-based valuation. Section 3 derives the solutions of the model. In Section 4, the effects of bond insurance and capital regulation on optimal loan and deposit rates, respectively, are investigated. The final section concludes the paper.

2 The Basic Model

We consider a single-period model for a banking firm. Our model is myopic in the sense that all economic decisions are made and values are determined with a one-period horizon only.

In our model, it is assumed that the bank acquires one kind of homogeneous loans ($L$) as the only form of earning asset. The bank holds three types of claims: deposits ($D$), tax-exempt municipal bonds ($B$), and equity capital ($K$). The balance sheet constraint can be written as:

$$L = D + B + K$$  \hspace{1cm} (1)

The model abstracts from legal reserve requirements and equity capital is assumed fixed throughout the decision period. We assume that the bank is a loan rate setter and loan demand faced by the bank is a downward-sloping function of loan rate, $R_L$. That is $L = L(R_L), \frac{\partial L}{\partial R_L} < 0$. The above assumption implies that the bank can exercise some monopoly power in the loan market.

The bank is also assumed to be a rate setter in the deposit market. Deposit supply is known upward-sloping function of deposit rate, $R_D$. The assumption of an upward-sloping deposit supply has been used in a number of models of the banking...
firm. Following Nanda and Singh (2004), the bank is assumed to operate in a perfectly competitive bond market so that the interest rate on bonds $R$ is given. The supply of deposits is assumed to be a negative function of bond market rate. Thus, the deposit supply function can be stated for the bank as $D = D(R_D, R)$, $\partial D / \partial R_D > 0$ and $\partial D / \partial R < 0$.

At the start of the period, the bank issues $B$ dollars of risky municipal bond, which is packaged with the third-party insurance with an insurance premium of $P$ per dollars bonds. We assume that there is no private information that the bank is better than investors (bondholders) at monitoring or valuing the bonds to be insured. The bank provides bondholders with a rate of return equal to the market risk-free rate $R$. Equity capital held by the bank is tied by regulation to be a fixed proportion ($q$) of the bank’s deposits $K \geq qD$. The required regulatory ratio $q$ is assumed to be an increasing function of the number of loans contracted by the bank at the beginning of the period, $\partial q / \partial L > 0$ (Lin, Chang and Lin, 2009a; Lin, Lin and Jou, 2009). We assume that $q(L)$ is a linear function and $\varepsilon \equiv (\partial q / \partial L)(L / q) = 1$.

The initial loanable funds are invested in risky loans with an unspecified maturity greater than one period. At any time during the period horizon, the value of the bank’s risky assets is:

$$V(R_L) \begin{cases} = (1 + R_L)L(R_L) = V^0 & \text{if no loan losses,} \\ < V^0 & \text{if loan losses.} \end{cases}$$

(2)

The bank exposes itself to risk since it funds fixed-rate investments via variable-rate deposits. The bank is also insured by the Federal Deposit Insurance Corporation (FDIC), and for purpose of simplicity, it pays a zero insurance premium per dollar of deposit. It should be apparent in which follows that this abstraction does not affect the basic conclusions of our model. The bank’s total cost ($Z$) in our model is the deposit repayment, bond repayment, and bond insurance costs. That is:

$$Z = (1 + R_D)D(R_D, R) + (1 + R + P)B.$$  

At the end of the period, an audit takes place to determine the bank’s total lendings and costs, and assesses its current market value. The bank first pays its bondholders if its total assets are sufficient; otherwise, the third-party insurers pay the rest. In addition, if the value of the bank’s total assets after paying its municipal bonds is less than its total deposit payments, the FDIC pays out $[Z - (1 + R)B - V]$. Otherwise, the equity holders who retain any residual pay the deposits. The residual value of the bank after meeting all of its debts is the value of the bank’s equity capital at the end of the period. That is $S = \max \{0, V - Z\}$. We assume that the administrative costs and the fixed cost are omitted for simplicity. This assumption is frequently used in the literature.²

The bank’s objective is to set its loan and deposit rates to maximize the market value of the Black-Scholes’ (1973) function defined in terms of its profit or equity capital. Santomero (1984) noted that the choice of an appropriate goal in modeling the bank’s optimization problem remains a

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¹ For example, see Sealey (1980), Slovin and Sshka (1983), and Zarruk (1989).

² See, for example, Slovin and Sushka (1983).
controversial issue. In our model, we analyze the lending and borrowing decisions by setting loan and deposit rates for the bank. To highlight the role of bank investment and liquidity management with packing bond insurance, we assume that those decisions are based on the bank’s utility-free and market-value equity maximization.

The selection of our model’s objective function follows Merton (1974). It is derived in the spirit of Merton’s approach to evaluate the bank using a contingent claim analysis. The equity capital of the bank is viewed as a call option on the bank’s risky assets net of risky municipal bonds, $V_A$. The reason is that equity holders are residual claimants on bank’s assets net of bonds after all other obligations have been met. The strike price of the call option is the bank value of the bank’s deposit payments and bond insurance costs, $X$. When the value of the bank’s assets net of bonds is less than the strike price, the value of equity is zero. Thus, the market value of equity capital will be given by the Black-Scholes’ (1973) formula for call option:

$$S = V_A N(d_1) - X e^{-\delta} N(d_2)$$  \hspace{1cm} (3)

where

$$V_A = V(R_L) - (1 + R)[L(R_L) - (1 + q)D(R_D, R)]$$

$$X = (1 + R_D)D(R_D, R)$$

$$+ P[L(R_L) - (1 + q)D(R_D, R)],$$

$$d_1 = \frac{1}{\sigma} [\ln \frac{V_A}{X} + \delta + \frac{1}{2} \sigma^2],$$

$$d_2 = d_1 - \sigma,$$

$$\delta = R - R_D.$$

In this objective function, the cumulative standard normal distribution of $N(d_1)$ and $N(d_2)$ represent the risk-adjusted factors of $V_A$ and $X$, respectively. The volatility $\sigma^2$ is simply the variance of the risky loans only, and $\delta$ is the spread, the difference between the bond market rate and the promised deposit rate to the initial depositors.

### 3 Solution to the Model

The first-order conditions are given by

$$\frac{\partial S}{\partial R_L} = \left[ \frac{\partial V}{\partial R_L} - (1 + R)(1 - D \frac{\partial q}{\partial L}) \frac{\partial L}{\partial R_L} \right] N(d_1)$$

$$- P(1 - D \frac{\partial q}{\partial L}) \frac{\partial L}{\partial R_L} e^{-\delta} N(d_2) = 0, \hspace{1cm} (4)$$

$$\frac{\partial S}{\partial R_D} = (1 + R)(1 + q) \frac{\partial D}{\partial R_D} N(d_1)$$

$$- \left[ \left( (1 + R_D) - P(1 + q) \frac{\partial D}{\partial R_D} + D \right) e^{-\delta} N(d_2) \right] = 0. \hspace{1cm} (5)$$

Equations (4) and (5) determine the optimal loan and deposit rates. In Eq. (4), the first term associated with $N(d_1)$ represents the bank’s risk-adjusted present value between the marginal loan repayments and the marginal bond payments from a change in the bank’s loan rate. From Eq. (4), we obtain

$$P(1 - D \frac{\partial q}{\partial L}) \frac{\partial L}{\partial R_L} e^{-\delta} =$$

$$\left[ \frac{\partial V}{\partial R_L} - (1 + R)(1 - D \frac{\partial q}{\partial L}) \frac{\partial L}{\partial R_L} \right] \frac{N(d_1)}{N(d_2)}. \hspace{1cm} (6)$$

Since $\varepsilon \equiv (\partial q / \partial L)(L / q) = 1$, therefore, we can derive the result as follows:

$$1 - D \frac{\partial q}{\partial L} = [1 - (K / L)\varepsilon] > 0.$$  \hspace{1cm} (7)

That is, $[1 - D \frac{\partial q}{\partial L}]\delta L / \delta R_L < 0$. Since $P$, $e^{-\delta}$, $N(d_1)$ and $N(d_2)$ all are positive, hence

$$\left[ \frac{\partial V}{\partial R_L} - (1 + R)(1 - D \frac{\partial q}{\partial L}) \frac{\partial L}{\partial R_L} \right] < 0. \hspace{1cm} (7)$$
That is, $\frac{\partial V}{\partial R_L} < 0$. It is reasonable to believe that the direct effect on the marginal loan repayments from a change in the bank’s loan rate is sufficient to offset the indirect effect on the marginal bond payments. Intuitively, the equilibrium condition in Eq. (4) shows that the marginal risk-adjusted present value for lending revenues is equal to the marginal risk-adjusted present value for costs of issuing bonds bundled with third-party insurance.

In Eq. (5), the first term associated with $N(d_1)$ represents the bank’s risk-adjusted present value for marginal bond payments from a change in its deposit rate. The marginal value is positive since the bank faces an upward-sloping deposit supply curve. The second term associated with $N(d_2)$ demonstrates the bank’s present value for marginal strike price is composed of the marginal deposit payments and the marginal bond insurance cost of deposit rate. As expected, the marginal strike price is positive. Thus, the equilibrium condition shows that the bank’s risk-adjusted present value for marginal bond payments equals that for marginal deposit payments net of marginal bond insurance cost from a change in its deposit rate-setting. The condition presents above can be given an alternative interpretation. That is, the risk-adjusted marginal bond payments bundled with marginal bond insurance cost equals the risk-adjusted marginal deposit payments.

### 4 Comparative Static Effects

Having examined the solutions to the bank’s optimization problems, in this section we consider the effects on the bank’s optimal loan and deposit rate decisions from a change in the bond insurance premium and the capital regulatory ratio when the deposit rate is fixed. These nonsimultaneous results are obtained for the following reason. Banks frequently encounter situations where loan rates must be determined in the presence of fixed deposit rate. This behavioral mode has been modeled by Zarruk and Madura (1992), and Wong (1997) among others. To determine the effect of bond insurance premium on the bank’s optimal loan rate, implicitly differentiating Eq. (4) with respect to $P$ yields the following comparative static result:

$$\frac{\partial^2 S}{\partial R_L} \frac{\partial R_L}{\partial P} = (1 - D) \frac{\partial q}{\partial L} \frac{\partial L}{\partial R_L} e^{-\delta} N(d_2)$$

$$-\left[\frac{\partial V}{\partial R_L} - (1 + R)(1 - D) \frac{\partial q}{\partial L} \frac{\partial L}{\partial R_L} \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial P} \right]$$

$$+ P(1 - D) \frac{\partial q}{\partial L} \frac{\partial L}{\partial R_L} e^{-\delta} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial P}. \quad (8)$$

Since $\frac{\partial d_1}{\partial P} = \frac{\partial d_2}{\partial P}$, and using Eq. (6), then Eq. (8) can be rewritten as,

$$\frac{\partial R_L}{\partial P} = \left(\frac{\partial^2 S}{\partial R_L^2}\right)^{-1} \left(1 - D \frac{\partial q}{\partial L} \frac{\partial L}{\partial R_L} e^{-\delta} N(d_2) \right)$$

$$-\left[\frac{\partial V}{\partial R_L} - (1 + R)(1 - D) \frac{\partial q}{\partial L} \frac{\partial L}{\partial R_L} \right]$$

$$\cdot \left\{\frac{\partial N(d_1)}{\partial d_1} - \frac{\partial N(d_2)}{\partial d_2} \frac{N(d_1)}{N(d_2)} \frac{\partial d_1}{\partial P} \right\}. \quad (9)$$

We assume that the second-order condition in Eq. (9), $\frac{\partial^2 S}{\partial R_L^2} < 0$, is satisfied. Before proceeding with the analysis of the comparative static result of Eq. (9), we treat the term

$$\left[\frac{\partial N(d_1)}{\partial d_1} - \frac{\partial N(d_2)}{\partial d_2} \frac{N(d_1)}{N(d_2)} \right]$$

as the elasticity effect. And then we define that

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3 See Lin, Chang and Lin (2009b) for detail.
1. $[\partial N(d_1)/\partial d_1]/[N(d_1)/d_1]$ is the marginal ratio to the average cumulative standard distribution of $d_1$, which represents as $V_A$ the elasticity of the risk-adjusted factor $d_1$;

2. $[\partial N(d_2)/\partial d_2]/[N(d_2)/d_2]$ is the marginal ratio to the average cumulative standard distribution of $d_2$, which represents as $X$ the elasticity of the risk-adjusted factor $d_2$.

We know that both the elasticities of the risk-adjusted factors are positive. And

$$\frac{\partial N(d_1)}{\partial d_1} \cdot d_1 - \frac{\partial N(d_2)}{\partial d_2} \cdot d_2 = \frac{d_1}{N(d_1)} \cdot \frac{\partial N(d_1)}{\partial d_1} - \frac{d_2}{N(d_2)} \cdot \frac{\partial N(d_2)}{\partial d_2} \cdot N(d_1).$$

If the $V_A$ elasticity is less than the $X$ elasticity, then the elasticity effect is negative. But if the $V_A$ elasticity is greater than the $X$ elasticity, the elasticity effect is not necessarily positive. If the $V_A$ elasticity is less than the $X$ elasticity, the bank stays in a good state; if the $V_A$ elasticity is greater than the $X$ elasticity, the bank stays in a bad state (Lin and Yi, 2005). Without question, if the elasticity effect is positive, the bank stays in a bad state.

From the definition about the variables in Eq. (3), we can determine

$$\frac{\partial d_1}{\partial P} = \frac{1}{\sigma} \left( \frac{1}{X} \right) \left[ L - (1 + q)D \right] < 0.$$

We define $\Delta$ as a change. For example, $\Delta P > 0$ represents an increase in the bond insurance premium. From the above condition, we know that $\Delta P > 0 \Rightarrow \Delta d_1 < 0 \Rightarrow \Delta N(d_1) < 0 \Rightarrow$ the hedge ratio decreases $\Rightarrow$ the possibility of fulfilling the call option decreases.

Therefore, if this bank is in good condition at the beginning, increase of the bond insurance premium could result in decrease of hedging rate, but this does not mean that the former good condition would become bad condition, therefore, it will not necessarily to increase optimal loan ratio in order to decrease loan amount, but also because of decrease of hedging rate it will not decrease optimal loan ratio in order to increase loan amount. However, if the bank is in bad condition at the beginning, when hedging rate decreases, it will certainly increase optimal loan ratio in order to decrease loan amount of risky nature. Therefore, if in the initial state the bank stays in a good state, no matter whether the $V_A$ elasticity is less or greater than the $X$ elasticity, then the sign of $\partial R_L / \partial P$ in Eq. (9) is indeterminate. Only when we directly assume the elasticity effect is positive, then, we can decide $\partial R_L / \partial P > 0$. To summarize, we have the following proposition.

**Proposition 1.** If the elasticity effect is positive, then an increase in the bond insurance premium will increase the bank’s optimal loan rate.

Wong (1997) pointed out that bank interest margins are positively related to operating (insurance) cost. But proposition 1 reveals that the bank passes the burden of rising insurance express to borrowers by widening the bank interest margin only when the bank stays in the bad state. In other words, if the bank stays in the good state, it does not certainly increase its optimal loan rate when the bond insurance increases. Since $B = L - (1 + q)D$, therefore,
\[
\frac{\partial B}{\partial P} = \frac{\partial L}{\partial R_L} \frac{\partial R_L}{\partial P} < 0.
\]

That is, the amount of municipal bonds decreases when the bond insurance premium increases.

A related question is to consider the impact of an increase in the required capital regulatory ratio (capital-to-deposit) on the bank’s optimal loan rate decisions. Implicit differentiation of Eq. (4) with respect to \(q\) yields:

\[
\frac{\partial R_L}{\partial q} = \left(\frac{\partial^2 S}{\partial R_L^2}\right)^{-1} \left\{[(1 + R)(1 - D\frac{\partial q}{\partial L})] \frac{\partial L}{\partial R_L} - \frac{\partial V}{\partial R_L}\right\}
\]

\[
\cdot \left\{\frac{\partial N(d_1)}{\partial d_1} - \frac{\partial N(d_2)}{\partial d_2} \frac{N(d_1)}{N(d_2)}\right\} \frac{\partial q}{\partial q}. \tag{10}
\]

Similarly, from the definition about the variables in Eq. (3), we can determine

\[
\frac{\partial d_1}{\partial q} = \frac{1}{\sigma} \left[\frac{(1 + R)D}{V_A} + \frac{PD}{X}\right] > 0.
\]

That is, \(\Delta q > 0 \Rightarrow \Delta d_1 > 0 \Rightarrow \Delta N(d_1) > 0 \Rightarrow\) the hedge ratio increases \(\Rightarrow\) the possibility of fulfilling the call option increases.

Therefore, if this bank is in bad condition at the beginning, when capital-to-deposit ratio decreases by \(N(d_1)\), this means original positive expected profit would increase, while increase of expected profit implies increase of expected risk (in other words, increase of call option premium it assumes). Therefore, it will increase optimal loan ratio in order to decrease loan amount, resulting in decrease of total revenue from loan (because \(\frac{\partial V}{\partial R_L} < 0\)); i.e. the pressure of increase of expected risk it assumes can be decreased (in other words, the range of increase of call option premium it assumes can be decreased). If this bank is in bad condition at the beginning, when capital-to-deposit ratio increases by \(N(d_1)\), this means the increase of probability that its future expected profit would become positive. But this rage of increase of positive expected profit is less, i.e. expected risk it assumes (or call option premium it assumes) is less, it will decrease optimal loan ratio in order to increase loan amount, resulting in increase of total revenue from its loan. Therefore, if the \(V_A\) elasticity is less than the \(X\) elasticity, that is, the elasticity effect is negative, then the sign of \(\frac{\partial R_L}{\partial q}\) in Eq. (10) is positive. But if the \(V_A\) elasticity is greater than the \(X\) elasticity, then the sign of \(\frac{\partial R_L}{\partial q}\) in Eq. (10) is indeterminate. Of course, if we assume directly that the elasticity effect is positive, then we can conclude that an increase in the capital-to-deposit ratio will decrease the bank’s optimal loan rate.

Similarly, since \(B = L - (1 + q)D\), so we obtain

\[
\frac{\partial B}{\partial q} = \frac{\partial L}{\partial R_L} \frac{\partial R_L}{\partial q} - D.
\]

That’s, if the elasticity effect is negative, then \(\frac{\partial B}{\partial q} < 0\), but if the elasticity effect is positive, then the sign of \(\frac{\partial B}{\partial q}\) is indeterminate.

Thus, we established the following proposition.

**Proposition 2.** If the elasticity effect is negative (positive), then an increase in the capital-to-deposit ratio will increase (decrease) the bank’s optimal loan rate.

Next, the effects of bond insurance premium and capital-to-deposit ratio on optimal deposit rates are investigated when loan rates are fixed. Although this behavioral mode is less likely to be observed than that on optimal loan rates, there may be instances where banks have fixed loan rates and must set deposit rates. Sealey (1980), for example,
considered this behavioral mode for local-oriented savings and loan associations.

The implicit differentiation of Eq. (5) with respect to bond insurance premium $P$ yields:

$$\frac{\partial R_D}{\partial P} = -\left(\frac{\partial^2 S}{\partial R_D^2}\right)^{-1}\{(1+q)\frac{\partial D}{\partial R_D}e^{-s}N(d_2) + (1+R)(1+q)\frac{\partial D}{\partial R_D}
\left[\frac{\partial N(d_2)}{\partial d_2} - \frac{\partial N(d_1)}{\partial d_1} \frac{N(d_1)}{N(d_2)}\right]\}. \tag{11}$$

We assume that the second-order condition in the above equation, $\partial^2 S / \partial R_D^2 < 0$, is satisfied. We establish the following proposition.

**Proposition 3.** If the elasticity effect is negative, then an increase in the bond insurance premium will increase the bank’s optimal deposit rate.

If the $V_A$ elasticity is less than the $X$ elasticity (negative elasticity effect), then $\partial R_D / \partial P > 0$. But if the $V_A$ elasticity is greater than the $X$ elasticity, then the sign of $\partial R_D / \partial P$ in Eq. (11) is indeterminate. We can not decide the sign of $\partial R_D / \partial P$ even under the assumption that the elasticity effect is positive.

Again, from the equation, $B = L - (1+q)D$, so we can derive

$$\frac{\partial B}{\partial P} = -(1+q)\frac{\partial D}{\partial R_D} \frac{\partial R_D}{\partial P} < 0.$$ 

Therefore, we know that if the $V_A$ elasticity is less than the $X$ elasticity, a rise in the optimal loan rate and a decrease in the amount of municipal bonds will follow an increase in the bond insurance premium.

Implicit differentiation of Eq. (5) with respect to the capital-to-deposits ratio yields:

$$\frac{\partial R_D}{\partial q} = -\left(\frac{\partial^2 S}{\partial R_D^2}\right)^{-1}\bullet\left\{[(1+R)N(d_1) + Pe^{-s}N(d_2)]\frac{\partial D}{\partial R_D} + (1+R)(1+q)\frac{\partial D}{\partial R_D}\right.$$

$$\bullet\left[\frac{\partial N(d_1)}{\partial d_1} - \frac{\partial N(d_2)}{\partial d_2} \frac{N(d_2)}{N(d_1)}\right]\}$$. \tag{12}

We find that no matter the $V_A$ elasticity is less or greater than the $X$ elasticity, the sign of $\partial R_D / \partial q$ in Eq. (12) is indeterminate. Only if the condition that the elasticity effect is positive, then $\partial R_D / \partial q > 0$. As the bank is forced to increase its capital relative to its deposit level, it must set up a larger equity or decrease its deposit amount by decreasing its deposit rate. The result of Eq. (12) conforms to proposition 2. If this bank is in good condition at the beginning, when capital-to-deposit ratio increases, we cannot determine if this bank will increase or decrease optimal loan ration, therefore we also cannot determine if this bank will increase or decrease optimal deposit ration. But if this bank is in bad condition at the beginning, once capital-to-deposit ratio, it will increase optimal deposit ratio. As stated in proposition 2, this bank decrease optimal loan ration in order to increase loan amount and increase revenue. If, in addition to adjusting optimal loan ratio, this bank desires to re-adjust optimal deposit ratio, this bank, for the purpose of financing funds, will increase optimal deposit ratio to attract more deposits. Further from $B = L - (1+q)D$, we can derive

$$\frac{\partial B}{\partial q} = -D - (1+q)\frac{\partial D}{\partial R_D} \frac{\partial R_D}{\partial q} < 0.$$
Therefore, when a bank in bad condition at the beginning faces increase of capital-to-deposit ratio, it will decrease quantity of municipal bond. If neglecting the change of optimal loan ration, then we can easily derive from Eq. (1)

\[ 0 = \frac{\partial D}{\partial R_D} \frac{\partial R_D}{\partial P} + \frac{\partial B}{\partial q} + \frac{\partial K}{\partial q} , \]

Therefore: \( \partial K / \partial q > 0 \).

**Proposition 4.** An increase in the capital-to-deposit ratio increase the bank’s optimal deposit rate under the positive elasticity effect.

### 4 Conclusion

In this paper, we developed a simple-theoretic model to study the optimal bank interest margin (i.e., the spread between the optimal loan rate and the optimal deposit rate) of a bank under the option-based valuation. We utilize the model to show how cost, regulation and risk condition jointly determine the optimal spread decisions. The results imply that changes in bond insurance premium and the capital-to-deposit ratio have a direct effect on the bank’s optimal spread decisions.

We find that the optimal loan rate is positively related to the cost of bond insurance under the positive elasticity effect but the positive relationship between the optimal deposit rate and the cost of bond insurance is under the negative elasticity effect. Our study indicated that when the bond insurance premium increases, operating cost of will increase, but it will increase optimal ratio only when the bank is in bad condition, i.e. only in bad condition, the bank will reflect the cost of increasing the bond insurance premium to loan ratio. Instead, if the bank is in good condition at the beginning, when the bond insurance premium increases, the bank will increase rather than decrease optimal deposit ratio. Therefore, in the opinion of this thesis, because the bond insurance premium increases, the bank adjust it optimal loan ration or optimal deposit ratio. The main purpose is to decrease the reliance on the bond insurance premium, rather than intend to shift additionally increased cost to clients.

Increase of the bond insurance premium will result in decrease of hedging ratio. On the contrary, increase of capital-to-deposit will result in increase of hedging ratio. In other word, increase of capital-to-deposit is favorable to the future prospect of the bank. If the bank is in good condition at the beginning, then favorable future prospect undoubtedly represents higher expected profit. Higher future expected profit means higher risk to be assume. Therefore, if the bank is in good condition at the beginning, we cannot determine if the bank will increase or decrease its optimal loan ratio and optimal deposit ratio when capital-to-deposit increase. But if the bank is in bad condition at the beginning, even its future prospect is favorable, its expected profit will be relatively low; i.e. risk assumed by it will also be relatively low. Therefore, when capital-to-deposit increases, the bank should increase optimal deposit ratio and decrease optimal loan ratio to expand business volume.

### References:


