Effect of Variable Viscosity on Convective Heat and Mass Transfer by Natural Convection from Horizontal Surface in Porous Medium

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Abstract: - The aim of this paper is to investigate the effect of variable viscosity on free convective heat and mass transfer from a horizontal plate embedded in a saturated porous medium. The governing equations of continuity, momentum, energy and concentration are transformed into non linear ordinary differential equations using similarity transformations and then solved by using Runge – Kutta – Gill method along with shooting technique. Governing parameters for the problem under study are the variable viscosity (θ_c), the buoyancy ratio (N) and the Lewis number (Le). The velocity, temperature and concentration distributions are presented and discussed. The Nusselt and Sherwood number is also derived. The numerical values of local Nusselt and local Sherwood numbers have also been computed for a wide range of governing parameters θ_c, N and Le. The viscous and thermal boundary layer thicknesses are discussed.

Key-Words: - Free convection, Heat transfer, Mass transfer, Variable viscosity, Porous medium.

1 Introduction

In recent years the combined heat and mass transfer by natural convection in a fluid saturated porous medium has its own role in many engineering application problems such as nuclear reactor design, geothermal systems, petroleum engineering applications, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler. Kassoy [1] studied the effect of variable viscosity on the onset of convection in porous medium. Cheng and Chang [2] obtained similarity solutions for the buoyancy induced flows in a saturated porous medium adjacent to impermeable horizontal surface. Cheng and Minkowyz [3] studied the effect of free convection on a vertical plate embedded in a porous medium with application to heat transfer from a dike. Bejan and Khair [4] studied buoyancy induced heat and mass transfer from a vertical plate embedded in a saturated porous medium. Lai and Kulacki [6] studied the effect of variable viscosity on convection heat transfer along a vertical surface in a saturated porous medium. The coupled heat and mass transfer by natural convection from vertical surface in a porous medium was studied by Lai and kulacki [7]. Elbashbeshy [8] investigated the effect of steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Yih [9] analyzed the coupled heat and mass transfer in mixed convection about a wedge for variable wall temperature and concentration. Rami.Y.Jumah et al [10] studied the coupled heat and mass transfer for non-Newtonian fluids. Kumari [11] analyzed the effect of variable viscosity on free and mixed convection boundary layer flow from a horizontal surface in a saturated porous medium with variable heat flux. The effect of variable viscosity on non- Darcy free or mixed convection flow on a horizontal surface in a saturated porous medium was studied by Kumari [12]. Postelnicu et al [13] investigated the effect of variable viscosity on forced convection over a horizontal flat plate in a porous medium with internal heat generation. Seddeek et al [17, 18] studied the effects of chemical reaction, variable viscosity, and thermal diffusivity on mixed convection heat and mass transfer through porous media. Mohamed E- Ali [20] studied the effect of variable viscosity on mixed convection along a vertical plate. Alam et al [21] analyzed the study of the combined free –forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Pantokratoras [22] analyzed the effect of variable viscosity with constant wall temperature. Seddeek et al [23] studied the effects of chemical reaction and variable viscosity on hydro magnetic mixed convection heat and mass transfer through porous media. Recently Singh et al [25] used integral treatment to obtain the expressions for Nusselt number and Sherwood number. The aim of the present study is to investigate the effect of variable viscosity on heat and mass transfer along a horizontal surface embedded in a saturated porous medium.

2 Analysis

Consider a horizontal plate embedded in a saturated porous medium. The properties of the fluid and porous medium are isotropic and the viscosity of the fluid is assumed to be an inverse linear function of temperature. Under Darcy model, by using Boussinesq and boundary layer approximations, the governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=-\frac{\kappa}{\mu}\left(\frac{\partial p}{\partial x}\right) \tag{2}$$

$$v=-\frac{\kappa}{\mu}\left(\frac{\partial p}{\partial y}+\rho g\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \tag{5}$$

$$\rho=\rho_\infty\left\{1-\beta(T-T_\infty)-\beta^*(c-c_\infty)\right\} \tag{6}$$

The viscosity of the fluid is assumed to be an inverse linear function of temperature and can be expressed as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left\{1+\gamma\left(T-T_\infty\right)\right\} \tag{7}$$

which is reasonable for liquids such as water and oil. Here γ is a constant.

The boundary conditions are

$$y=0, v=0, T=T_W, c=c_W \tag{8}$$

$$y\to\infty, u=0, T=T_\infty, c=c_\infty \tag{9}$$

3 Method of solution

Introducing the stream function Ψ (x, y) such that

$$u=\psi_y, v=-\psi_x \tag{10}$$

where

$$\psi=\alpha f(Ra_x)^{1/3} \tag{11}$$

$$\eta=\frac{y}{x}(Ra_x)^{1/3} \tag{12}$$

$Ra_x=\left\{\frac{kg\beta\Delta Tx}{\nu\alpha}\right\}$ is the Rayleigh number

Define

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

$$\phi = \frac{c - c_\infty}{c_W - c_\infty} \tag{14}$$

and $$N = \frac{\beta^*(c_W - c_\infty)}{\beta(T_W - T_\infty)} \tag{15}$$

Substitution of these transformations (10) to (15) to equations (2) to (5) along with the equations (6) and (7), there results the non linear ordinary differential equations,

$$f'' = \frac{f'\theta'}{\theta - \theta_c} - \frac{2}{3}\left(\frac{\theta - \theta_c}{\theta_c}\right)(\eta\theta' + \phi'\eta N) \tag{16}$$

$$\theta'' = -\frac{1}{3} f\theta' \tag{17}$$

$$Le^{-1}\phi'' = -\frac{1}{3} f\phi' \tag{18}$$

together with the boundary conditions

$$\eta = 0,\ f = 0, \theta = 1, \phi = 1 \tag{19}$$

$$\eta \to \infty,\ f' = 0, \theta = 0, \phi = 0 \tag{20}$$

where $Le = \frac{\alpha}{D}$ is the Lewis number and $\theta_c = -\frac{1}{\gamma(T_W - T_\infty)}$ is the parameter characterizing the influence of viscosity. For a given temperature differential, large values of θ_c implies either γ or (T_W -T_∞) are small. In this case the effect of variable viscosity can be neglected. The effect of variable viscosity is important if θ_c is small. Since the viscosity of liquids decreases with increasing temperature while it increases for gases, θ_c is negative for liquids and positive for gases. The concept of this parameter θ_c was first introduced by Ling and Dybbs [5] in their study of forced convection flow in porous media. The parameter N measures the relative importance of mass and thermal diffusion in the buoyancy – driven flow. It is clear that N is zero for thermal- driven flow, infinite for mass driven flow, positive for aiding flow and negative for opposing flow. It may be noted that in the absence of mass transfer (N=0) for isothermal case (λ =0), the equations (16) and (17) with boundary conditions (19) and (20) of the present work are identical with those of equations (20) and (21) of Cheng and Chang [2] for the case of constant viscosity ($\theta_c \to \infty$). Equations (16), (17) and (18) are integrated numerically by using Runge – Kutta – Gill method along with the shooting technique.

The heat transfer coefficient in terms of the Nusselt number is given by

$$\frac{Nu_x}{(Ra_x)^{1/3}} = -\theta'(0) \tag{21}$$

The mass transfer coefficient in terms of the Sherwood number is given by

$$\frac{Sh_x}{(Ra_x)^{1/3}} = -\phi'(0) \tag{22}$$

4 Results and discussion

The velocity, temperature and concentration profiles are presented in Figs.1-2.

From Fig.1 it is seen that the velocity increases near the plate and decreases away from the plate as $\theta_c \to 0$ in the case of liquids (θ_c<0) and it decreases near the plate and increases away from the plate as $\theta_c \to 0$ in the case of gases (θ_c > 0) for N =1 and Le=1.

From Fig.2 below it is evident that the temperature and concentration increase as $\theta_c \to 0$ for θ_c > 0(i.e for gases) and decrease as $\theta_c \to 0$ for θ_c < 0(i.e for liquids) for N=1 and Le =1. It should be noticed that for large values of θ_c ,the velocity and temperature profiles are close to those of constant viscosity($\theta_c \to \infty$).This shows that the large values of θ_c implies the variable viscosity effect can be neglected

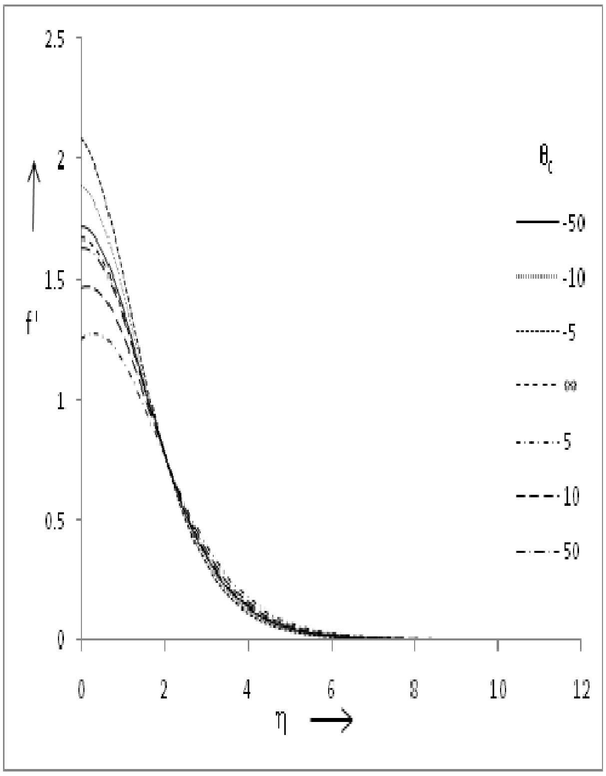

Fig.1.Velocity profiles for different values of θ_c for N =1 and Le=1

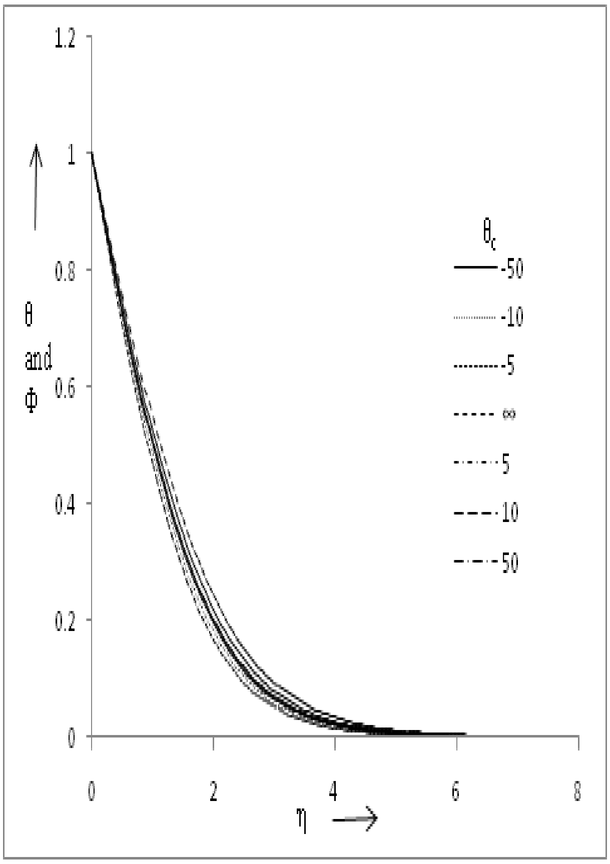

Fig.2.Temperature and concentration profiles for different values of θ_c for N =1 and Le=1

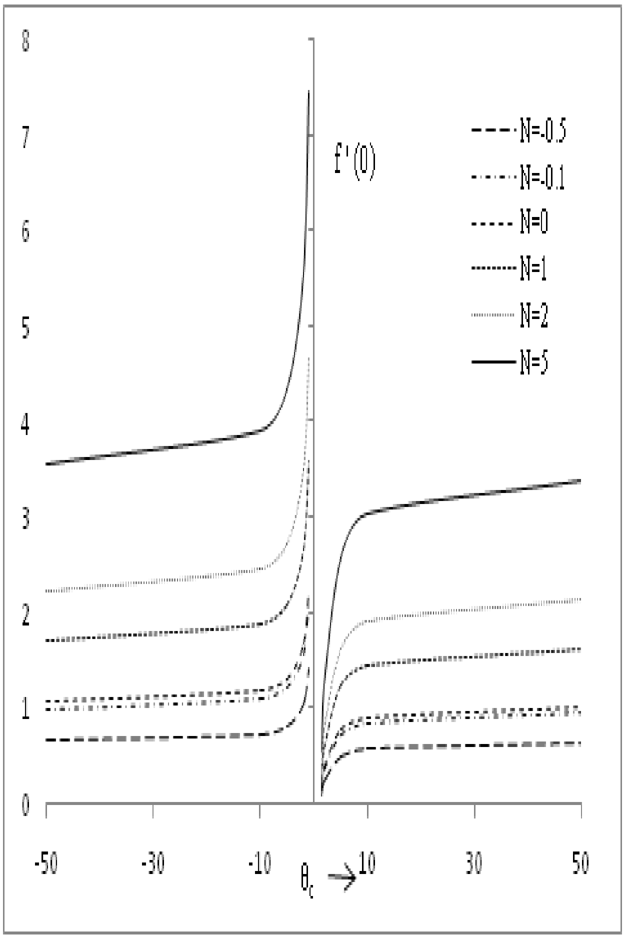

Fig.3.Effect of variable viscosity θ_c on the slip velocity for different values of N and Le =1

Fig.3 illustrates the effect of variable viscosity θ_c on the slip velocity f '(0) for different values of the buoyancy ratio N for Le =1. It is observed that the slip velocity decreases as $\theta_C \to 0$ in the case of gases($\theta_c > 0$) but in the cases of liquids ($\theta_c < 0$) the slip velocity increases as $\theta_C \to 0$ for aiding (N > 0), thermal-driven (N=0) and opposing (N < 0) flows.

Fig.4 displays the effect of variable viscosity θ_c on the local Nusselt and Sherwood numbers for aiding (N > 0), thermal- driven (N= 0) and opposing (N < 0) flows for Le =1. It is evident that the heat and mass transfer rates decrease as $\theta_c \to 0$ for the case of gases ($\theta_c > 0$), but in the case of liquids ($\theta_c < 0$) the heat and mass transfer rates increase as $\theta_c \to 0$ for aiding, thermal- driven and opposing flows. Similar behavior has also been observed for the case of vertical surface by Lai and kulacki [6].

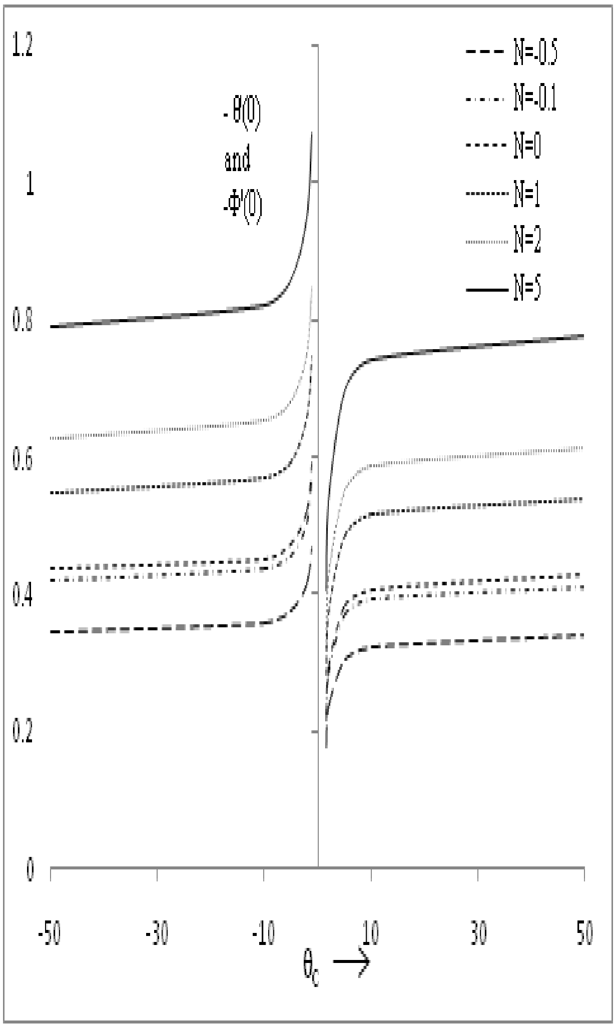

Fig.4. Effect of variable viscosity θ_c on the rate of heat transfer and rate of mass transfer for different values of N and Le =1

Fig.5 gives the effect of Lewis number Le on the slip velocity f '(0) for different values of variable viscosity θ_c for given values of N. It is observed that as Lewis number increases the slip velocity decreases for the case of liquids ($\theta_c < 0$)and gases ($\theta_c > 0$).

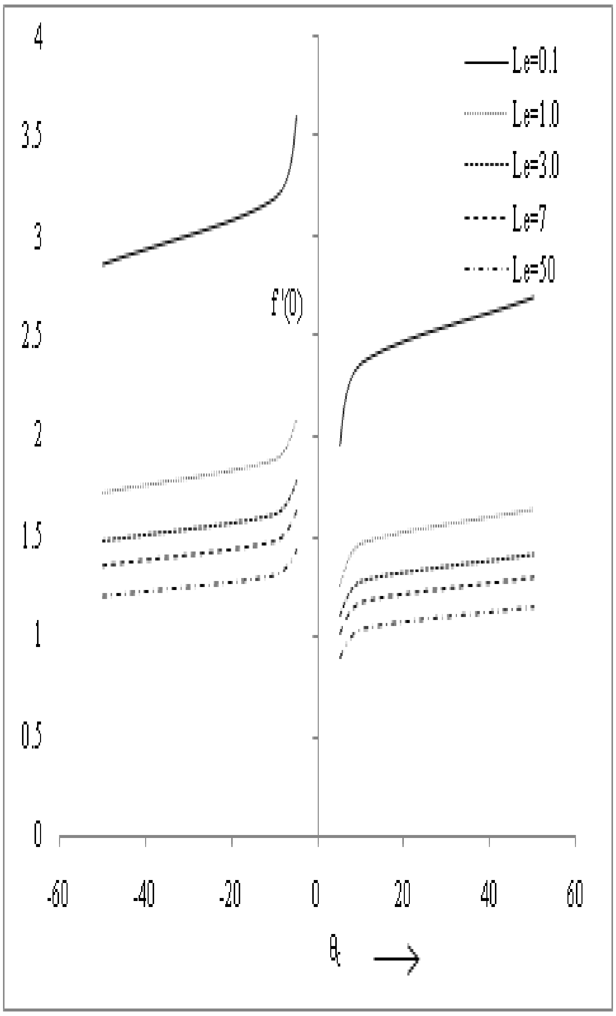

Fig.5. Effect of Lewis number Le on the slip velocity for different values of θ_c and N=1

The effect of Lewis number Le on local Nusselt and Sherwood numbers are shown in Figs.6 and 7 for different values of variable viscosity θ_c for N= 1. It is seen that as Lewis number increasesthe heat transfer decreases and mass transfer increases for the case of gases ($\theta_c > 0$) and liquids ($\theta_c < 0$).

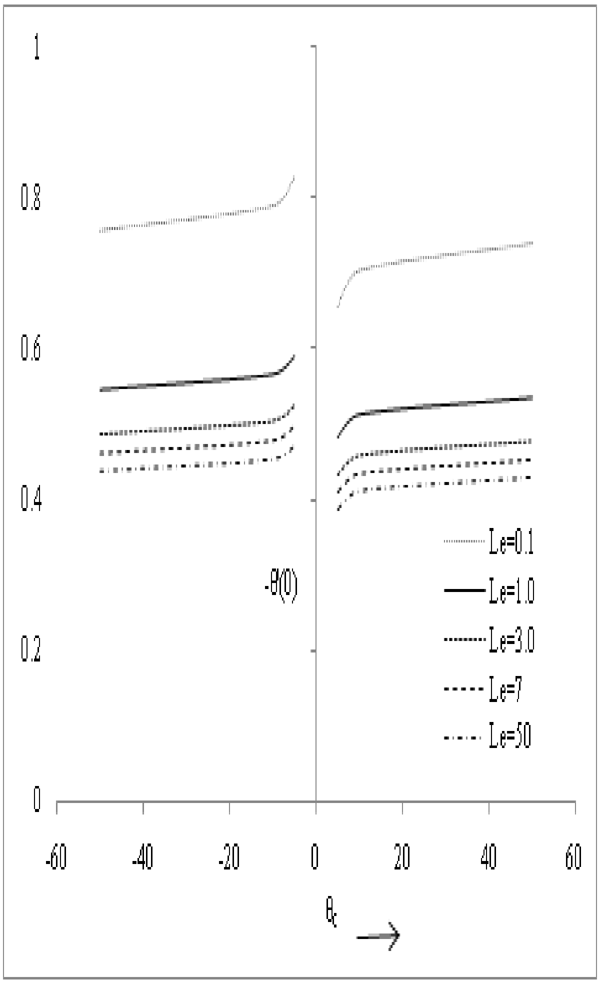

Fig.6. Effect of Lewis number Le on the rate of heat transfer for different values of θ_c and N=1

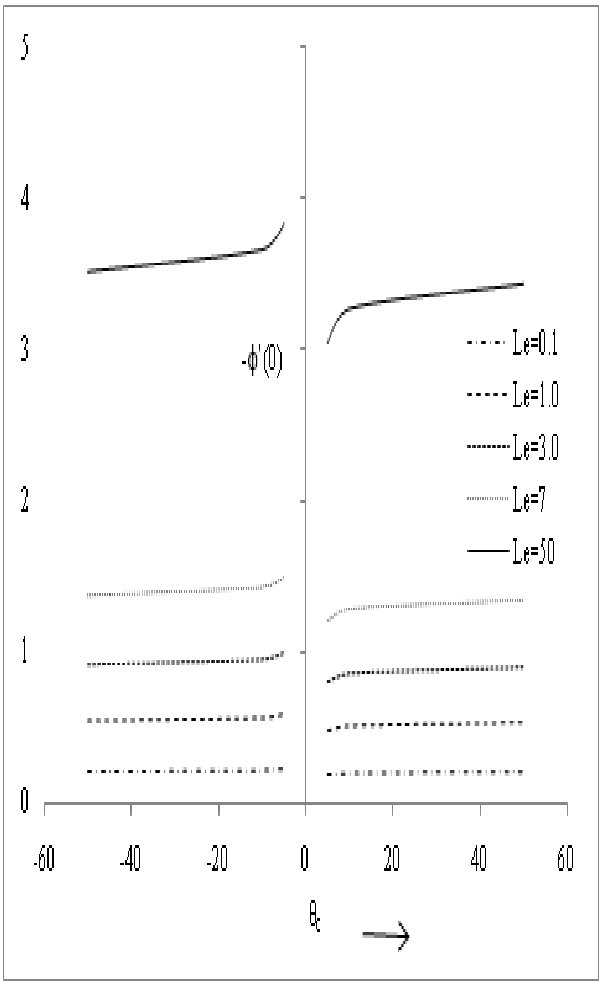

Fig.7. Effect of Lewis number Le on the rate of mass transfer for different values of θ_c and N=1

For applications in geothermal engineering, the practical interest is the thermal boundary thickness. The viscous and thermal boundary layer thicknesses are presented in Table 1 & 2 for different values of θ_c, N and for fixed value of Le. It is observed that the viscous and thermal boundary layer thickness increase as $\theta_c \to 0$ for the case of gases ($\theta_c > 0$) and decrease as $\theta_c \to 0$ for the case of liquids($\theta_c < 0$) for aiding ($N > 0$), thermal- driven ($N= 0$) and opposing ($N < 0$) flows.

5 Conclusion

For coupled heat and mass transfer by natural convection in porous media, solutions have been presented for the case of horizontal plate with linear temperature and concentration distribution. The governing parameters of the problem are the variable viscosity (θ_c), Lewis number (Le) and buoyancy ratio (N). The heat transfer and mass transfer increase as the buoyancy ratio increases for the case of liquids ($\theta_c < 0$) and gases ($\theta_c > 0$). The heat transfer decreases and mass transfer increases as Lewis number increases for the case of liquids ($\theta_c < 0$) and gases ($\theta_c > 0$).

N θ_c	-0.5	-0.1	0	1	2	5
5	9.75	8.56	8.37	7.12	6.43	5.437
10	9.37	8.25	8.00	6.81	6.18	5.25
50	9.12	8.00	7.81	6.62	6.00	5.06
-5	8.56	7.50	7.31	6.18	5.62	4.75
-10	8.81	7.68	7.43	6.37	5.81	4.87
-50	9.0	7.87	7.62	6.56	5.93	5.00

Table.1 Values of Viscous boundary layer thicknesses for different values of N and θ_c for Le=1.

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N θ_c	-0.5	-0.1	0	1	2	5
5	8.12	6.68	6.43	5.12	4.44	3.56
10	7.68	6.37	6.12	4.87	4.25	3.37
50	7.44	6.12	5.87	4.69	4.06	3.25
-5	6.81	5.62	5.56	4.31	3.75	2.93
-10	7.06	5.81	5.62	4.43	3.87	3.06
-50	7.31	6.00	5.81	4.56	4.00	3.18

Table.2 Values of Thermal boundary layer thicknesses for different values of N and θ_c for Le=1.

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