Introduction to the Rectangular Trigonometry in Euclidian 2D-space

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Abstract: - Trigonometry is a branch of mathematics that deals with relations between sides and angles of triangles. It has some relationship to geometry, though there is disagreement on exactly what that relationship is. For some, trigonometry is just a subtopic of geometry. The trigonometric functions are very important in technical subjects like Astronomy, Relativity, science, engineering, architecture, and even medicine. In this paper, the rectangular trigonometry is introduced in order to be in the future a part of the General trigonometry topic. Thus, the definition of this original part is presented. The rectangular trigonometric functions are also defined. The importance of these functions is by producing multi signal forms by varying some parameters of a single function. Different signals and forms are analyzed and discussed.

The concept of the rectangular Trigonometry is completely different from the traditional trigonometry in which the study of angles is not the relation between sides of a right triangle that describes a circle as the previous one, but the idea here is to use the relation between angles and sides of a rectangular form with the internal and external circles formed by the intersection of the rectangular form and the positive parts of x'ox and y'oy axis in the Euclidian 2D space and their projections. This new concept of relations will open a huge gate in the mathematical domain and it can resolve many complicated problems that are difficult or almost impossible to solve with the traditional trigonometry, and it can describe a huge number of multi form periodic signals.

Key-words: - Mathematics, geometry, trigonometry, angular function, multi form signal, power electronics.

1 Introduction

Trigonometry is a branch of mathematics that deals with triangles, particularly those plane triangles in which one angle has 90 degrees (right triangles) [1]. Trigonometry deals with relationships between the sides and the angles of triangles and with the trigonometric functions, which describe those relationships [2],[3]. Trigonometry has an enormous variety of applications. The ones mentioned explicitly in textbooks and courses on trigonometry are its uses in practical endeavors such as navigation, land surveying, building, and the like [4], [5]. It is also used extensively in a number of academic fields, primarily mathematics, science and engineering [6],[7],[8]. The mathematical topics of Fourier series and Fourier transforms rely heavily on knowledge of trigonometric functions and find application in a number of areas, including statistics [9],[10],[11],[12].

The Rectangular Trigonometry, which is introduced in this paper, is an original study. A new concept of trigonometry in the Euclidean 2D-space is introduced to define this trigonometry. Few examples are shown and discussed briefly. Figures are drawn and simulated using AutoCAD and Matlab. In fact, the rectangular trigonometry is based on the angular functions which are defined in the second section. The rectangular trigonometry and its functions are defined in the third and fourth sections. An example is treated in the fifth section. A survey on the application in the engineering domain is presented in the sixth section. Finally, a conclusion about the rectangular trigonometry is presented in section 7.

2 The angular functions

Angular functions are new mathematical functions that produce a rectangular signal, in which period is function of angles. Similar to trigonometric functions, the angular functions have the same properties as the precedent, but the difference is that a rectangular signal is obtained instead of a sinusoidal signal [13],[14],[15] and moreover, one can change the width of each positive and negative alternate in the same period. This is not the case of any other trigonometric function. In other hand, one

can change the frequency, the amplitude and the width of any period of the signal by using the general form of the angular function.
In this section three types of angular functions are presented, they are used in this trigonometry; of course there are more than three types, but in this paper the study is limited to three functions.

2.1 Angular function $ang_x(x)$

The expression of the angular function related to the (ox) axis is defined, for $K \in \mathbb{Z}$, as:

$$ang_x(\beta(x+\gamma)) = \begin{cases} +1 \ for \ (4K-1)\frac{\pi}{2\beta} - \gamma \le x \le (4K+1)\frac{\pi}{2\beta} - \gamma \\ -1 \ for \ (4K+1)\frac{\pi}{2\beta} - \gamma < x < (4K+3)\frac{\pi}{2\beta} - \gamma \end{cases} \quad (1)$$

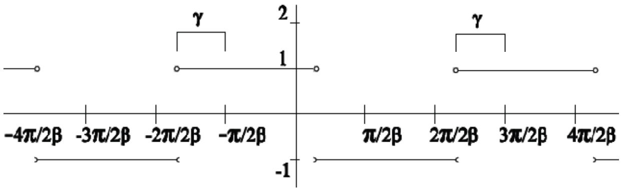


Fig. 1: The $ang_x(\beta(x+\gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_x(x) = \begin{cases} +1 \ for \ \cos(x) \ge 0 \\ -1 \ for \ \cos(x) < 0 \end{cases}$$

2.2 Angular function $ang_y(x)$

The expression of the angular function related to the (oy) axis is defined, for $K \in \mathbb{Z}$, as:

$$ang_y(\beta(x+\gamma)) = \begin{cases} +1 \ for \ 2K\pi/\beta - \gamma \le x \le (2K+1)\pi/\beta - \gamma \\ -1 \ for \ (2K+1)\pi/\beta - \gamma < x < (2K+2)\pi/\beta - \gamma \end{cases} \quad (2)$$

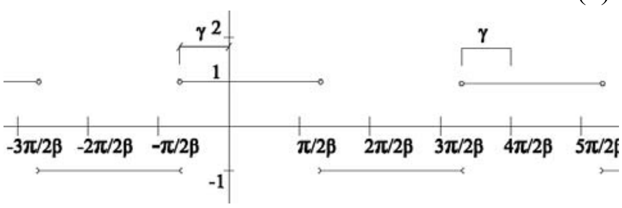


Fig. 2: The $ang_y(\beta(x+\gamma))$ waveform.

For $\beta = 1$ and $\gamma = 0$, the expression of the angular function becomes:

$$ang_y(x) = \begin{cases} +1 \ for \ \sin(x) \ge 0 \\ -1 \ for \ \sin(x) < 0 \end{cases}$$

2.3 Angular function $ang_\alpha(x)$

α (called firing angle) represents the angle width of the positive part of the function in a period, in this way, we can vary the width of the positive and the negative part by varying only α. The firing angle must be positive.

$$ang_\alpha(\beta(x+\gamma)) = \begin{cases} +1 \ for \\ (2K\pi - \alpha)/\beta - \gamma \le x \le (2K\pi + \alpha)/\beta - \gamma \\ \\ -1 \ for \\ (2K\pi + \alpha)/\beta - \gamma < x < (2(K+1)\pi - \alpha)/\beta - \gamma \end{cases} \quad (3)$$

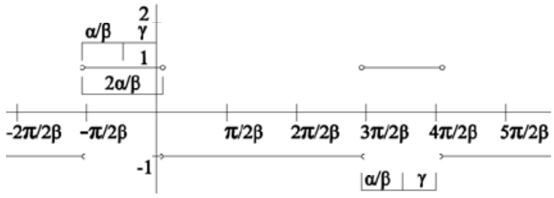


Fig. 3: The $ang_\alpha(\beta(x+\gamma))$ waveform.

3 Definition of the Rectangular Trigonometry

3.1 The Rectangular Trigonometry unit

The rectangular trigonometry unit is a rectangle centered in O (x=0; y=0) point of symmetry of the rectangle; the rectangle is symmetric about the axes (ox) and (oy) (Figure 4).
Four equations describe this rectangle:

$$Rectangle = \begin{cases} x = a \quad for \ -b \le y \le b \\ x = -a \quad for \ -b \le y \le b \\ y = b \quad for \ -a \le x \le a \\ y = -b \quad for \ -a \le x \le a \end{cases} \quad (4)$$

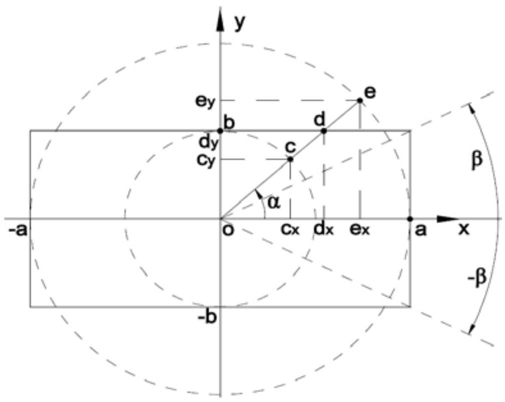


Fig. 4: The Rectangular Trigonometry unit.

It is essential to note that ‘a’ and ‘b’ must be positive. In this paper, ‘a’ is fixed to 1. One is interested to vary only a single parameter which is ‘b’.

3.2 Intersections and projections of different elements of the Rectangular trigonometry on the relative axes

From the intersections of the Rectangle with the positive parts of the axes (ox) and (oy), define respectively two circles of radii $[oa]$ and $[ob]$. These radii can be variable or constant according to the form of the rectangle.

The points of the intersection of the half-line $[od)$ (figure 4) with the internal and external circles and with the rectangle and their projections on the axis (ox) and (oy) can be described by many functions that have an extremely importance in creating plenty of signals and forms that are very difficult to be created in the traditional trigonometry.

Definition of letters in the Figure 4:

a: Is the intersection of the Rectangle with the positive part of the axe (ox) that gives the relative circle on (ox). It can be variable.

b: Is the intersection of the Rectangle with the positive part of the axe (oy) that gives the relative circle on (oy). It can be variable.

c: Is the intersection of the half-line $[od)$ with the circle of radius b.

d: Is the intersection of the half-line $[od)$ with the Rectangle.

e: Is the intersection of the half-line $[od)$ with the circle of radius a.

c_x: Is the projection of the point c on the ox axis.

d_x: Is the projection of the point d on the ox axis.

e_x: Is the projection of the point e on the ox axis.

c_y: Is the projection of the point c on the oy axis.

d_y: Is the projection of the point d on the oy axis.

e_y: Is the projection of the point e on the oy axis.

α: Is the angle between the (ox) axis and the half-line $[od)$.

o: Is the center (0, 0) of (ox, oy).

3.3 Definition of the Rectangular Trigonometric functions $\boldsymbol{Rfun(\alpha)}$

The traditional trigonometry contains only 6 principal functions: Cosine, Sine, Tangent, Cosec, Sec, Cotan [2]. But in the Rectangular Trigonometry, there are 32 principal functions and each function has its own characteristics. These functions give a new vision of the world and will be used in all scientific domains and make a new challenge in the reconstruction of the science especially when working on the economical side of the power electrical circuits, the electrical transmission, the signal theory and many other domains [13],[16].

The functions $Cjes(\alpha), Cmar(\alpha), Cter(\alpha)$ and $Cjes_y(\alpha)$, which are respectively equivalent to cosine, sine, tangent and cotangent. These functions are particular cases of the “*Circular Trigonometry*”. The names of the cosine, sine, tangent and cotangent are replaced respectively by *Circular Jes*, *Circular Mar, Circular Ter* and *Circular Jes-y*.

$Cjes(\alpha) \Leftrightarrow cos(\alpha)$; $Cmar(\alpha) \Leftrightarrow sin(\alpha)$

$Cter(\alpha) \Leftrightarrow tan(\alpha)$; $Cjes_y(\alpha) \Leftrightarrow cotan(\alpha)$.

The Rectangular Trigonometric functions are denoted using the following abbreviation “$Rfun(\alpha)$”:

-the first letter “R” is related to the Rectangular trigonometry.

-the word “ $fun(\alpha)$ ” represents the specific functions names that are defined hereafter: (refer to Figure 4).

• **Rectangular *Jes* functions:**

Rec. Jes: $Rjes(\alpha) = \frac{od_x}{oa} = \frac{od_x}{oe}$ (5)

Rec. Jes-x: $Rjes_x(\alpha) = \frac{od_x}{oe_x} = \frac{Rjes(\alpha)}{Cjes(\alpha)}$ (6)

Rec. Jes-y: $Rjes_y(\alpha) = \frac{od_x}{oe_y} = \frac{Rjes(\alpha)}{Cmar(\alpha)}$ (7)

• **Rectangular *Mar* functions:**

Rec. Mar: $Rmar(\alpha) = \frac{od_y}{ob} = \frac{od_y}{oc}$ (8)

Rec. Mar-x: $Rmar_x(\alpha) = \frac{od_y}{oc_x} = \frac{Rmar(\alpha)}{Cjes(\alpha)}$ (9)

Rec. Mar-y: $Rmar_y(\alpha) = \frac{od_y}{oc_y} = \frac{Rmar(\alpha)}{Cmar(\alpha)}$ (10)

• **Rectangular *Ter* functions:**

Rec. Ter: $Rter(\alpha) = \frac{Rmar(\alpha)}{Rjes(\alpha)}$ (11)

Rec. Ter-x:

$Rter_x(\alpha) = \frac{Rmar_x(\alpha)}{Rjes_y(\alpha)} = Rter(\alpha) \cdot Cter(\alpha)$ (12)

Rec. Ter-y: $Rter_y(\alpha) = \frac{Rmar_y(\alpha)}{Rjes_x(\alpha)} = \frac{Rter(\alpha)}{Cter(\alpha)}$ (13)

• **Rectangular *Rit* functions:**

Rec. Rit: $Rrit(\alpha) = \frac{od_x}{ob} = \frac{od_x}{oc} = \frac{Rmar(\alpha)}{Cter(\alpha)}$ (14)

Rec. Rit-y: $Rrit_y(\alpha) = \frac{od_x}{oc_y} = \frac{Rrit(\alpha)}{Cmar(\alpha)}$ (15)

• **Rectangular *Raf* functions:**

Rec. Raf: $Rraf(\alpha) = \frac{od_y}{oa} = Cter(\alpha).Rjes(\alpha)$ (16)

Rec. Raf-x: $Rraf_x(\alpha) = \frac{od_y}{oe_x} = \frac{Rraf(\alpha)}{Cjes(\alpha)}$ (17)

• **Rectangular *Ber* functions:**

Rec. Ber: $Rber(\alpha) = \frac{Rraf(\alpha)}{Rrit(\alpha)}$ (18)

Rec. Ber-x:

$Rber_x(\alpha) = \frac{Rraf_x(\alpha)}{Rrit_y(\alpha)} = Rber(\alpha) \cdot Cter(\alpha)$ (19)

Rec. Ber-y: $Rber_y(\alpha) = \frac{Rraf_y(\alpha)}{Rrit_x(\alpha)} = \frac{Rber(\alpha)}{Cter(\alpha)}$ (20)

3.4 The reciprocal of the Rectangular Trigonometric function

$Rfun^{-1}(\alpha)$ is defined as the inverse function of $Rfun(\alpha)$. $(Rfun^{-1}(\alpha) = 1/Rfun(\alpha))$. In this way the reduced number of functions is equal to 32 principal functions.

E.g.: $Rjes^{-1}(\alpha) = \frac{1}{Rjes(\alpha)}$

3.5 Definition of the Absolute Rectangular Trigonometric functions $\bar{R}fun(\alpha)$

The *Absolute Rectangular Trigonometry* is introduced to create the absolute value of a function by varying only one parameter without using the absolute value "| |". The advantage is that we can change and control the sign of a Rectangular Trigonometric function without using the absolute value in an expression. We will show some functions to get an idea about the importance of this new definition. To obtain the absolute Rectangular trigonometry for a specified function (e.g.: $Rjes(\alpha)$) we must multiply it by the corresponding Angular Function (e.g.: $(ang_x(\alpha))^i$) in a way to obtain the original function if i is even, and to obtain the absolute value of the function if i is odd (e.g.: $|Rjes(\alpha)|$).

If the function doesn't have a negative part (not alternative), we multiply it by $(ang_x(\beta(\alpha-\gamma)))^i$ to obtain an alternating signal which the form depends on the value of the frequency "β" and the translation value "γ". By varying the last parameters, we can get a multi form signal.

• $\bar{R}jes_i(\alpha) = (ang_x(\alpha))^i \cdot Rjes(\alpha)$ (21)

$$= \begin{cases} (ang_x(\alpha))^1 \cdot Rjes(\alpha) = |Rjes(\alpha)| & if\ i = 1 \\ (ang_x(\alpha))^2 \cdot Rjes(\alpha) = Rjes(\alpha) & if\ i = 2 \end{cases}$$

• $\bar{R}jes_{i,x}(\alpha) = (ang_x(\alpha-\gamma))^i \cdot Rjes_x(\alpha)$ (22)

$$= \begin{cases} ang_x(\alpha-\gamma) \cdot Rjes_x(\alpha) & if\ i = 1 \\ Rjes_x(\alpha) & if\ i = 2 \end{cases}$$

• $\bar{R}jes_{i,y}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Rjes_y(\alpha)$ (23)

$$= \begin{cases} ang_y(2\alpha) \cdot Rjes_y(\alpha) = |Rjes_y(\alpha)| & if\ i = 1 \\ Rjes_y(\alpha) & if\ i = 2 \end{cases}$$

• $\bar{R}mar_i(\alpha) = \left(ang_y(\alpha)\right)^i \cdot Rmar(\alpha)$ (24)

• $\bar{R}mar_{i,x}(\alpha) = \left(ang_y(2\alpha)\right)^i \cdot Rmar_x(\alpha)$ (25)

• $\bar{R}mar_{i,y}(\alpha) = (ang_x(\alpha-\gamma))^i \cdot Rmar_y(\alpha)$ (26)

• $\bar{R}rit_i(\alpha) = (ang_x(\alpha))^i \cdot Rrit(\alpha)$ (27)

And so on…

3.6 Original formulae of the rectangular trigonometry

In this section, a brief review on some remarkable formulae formed using the rectangular trigonometric functions.

• $\frac{Rjes_{x_b}(x)}{Rjes_{y_b}(x)} = Cter(x) = \tan(x)$

In fact: $\frac{Rjes_{x_b}(x)}{Rjes_{y_b}(x)} = \frac{Rjes_b(x)/Cjes(x)}{Rjes_b(x)/Cmar(x)} = \frac{Cmar(x)}{Cjes(x)} = Cter(x) = \tan(x)$

• $(Rjes_b(x))^2 \cdot \left(\frac{a}{b}Cter(x)\right)^2 - (Rmar_b(x))^2 = 0$

• $\left(Rjes_b{}^{-1}(x)\right)^2 = \left(Rjes_{x_b}{}^{-1}(x)\right)^2 + \left(Rjes_{y_b}^{-1}(x)\right)^2$

• $Rjes_{x_b}{}^{-1}(x) \cdot Rmar_{x_b}{}^{-1}(x) + Rjes_{y_b}{}^{-1}(x) \cdot Rmar_{y_b}{}^{-1}(x)$
$= Rjes_b{}^{-1}(x) \cdot Rmar_b{}^{-1}(x) = \left(\frac{a}{b}Cter(x)\right)^{\text{ang}_\beta(x)\cdot\text{ang}_\beta(x-\pi)}$

4 A survey on the Rectangular Trigonometric functions

As previous sections, a brief study on the Rectangular trigonometry is given, and some examples are treated to show multi form signals made using the characteristic of this trigonometry. Four functions are given with examples.

For this study the following conditions are taken:

- $a = 1$
- $b > 0$ the height of the rectangle from the center.
- $Cter(\beta) = \tan(\beta) = \frac{b}{a} \Rightarrow \beta = \text{arcCter}\left(\frac{b}{a}\right) \pm k\pi$
- $\beta = \arctan\left(\frac{b}{a}\right) = \text{arcCter}\left(\frac{b}{a}\right)$ and $\beta \in]0; \frac{\pi}{2}[$ only the positive part is considered because $b > 0$.

4.1 Determination of the Rectangular *Jes* function

The rectangular form in the figure 4 is written with four equations (4). Thus, given (5), the rectangular *Jes* function can be determined using these functions. In fact:

- $Rjes(x) = 1$ For $\alpha \in [-\beta; \beta]$
- $Rjes(x) = \frac{b}{a.\text{Cter}\,(\alpha)}$ For $\alpha \in]\beta; \pi - \beta]$
- $Rjes(x) = -1$ For $\alpha \in]\pi - \beta; \pi + \beta]$
- $Rjes(x) = \frac{-b}{a.\text{Cter}\,(\alpha)}$ For $\alpha \in]\pi + \beta; 2\pi - \beta[$

Thus the expression of the rectangular *Jes* can be unified by using the angular function expressions (2) and (3), therefore the expression become:

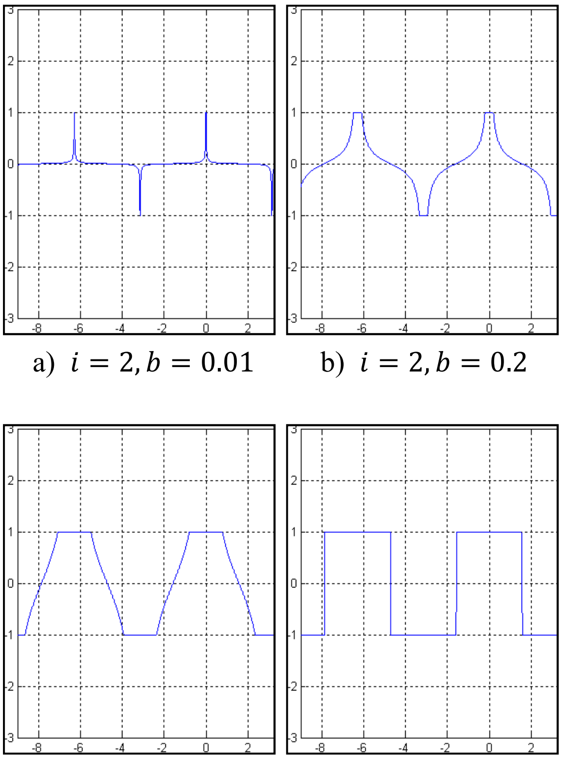
$$Rjes_b(x) = ang_y(x+\beta)\cdot\left(\frac{a}{b}\text{Cter}(x)\right)^{\frac{\text{ang}_\beta(\text{x})+\text{ang}_\beta(\text{x}-\pi)}{2}} \quad (28)$$

• Expression of the Absolute Rectangular *Jes*:

$$\bar{R}jes_{i,b}(x) = Rjes_b(x)\cdot\left(ang_x(x)\right)^i \quad (29)$$

• Multi form signals made by $\bar{R}jes_{i,b}(x)$:

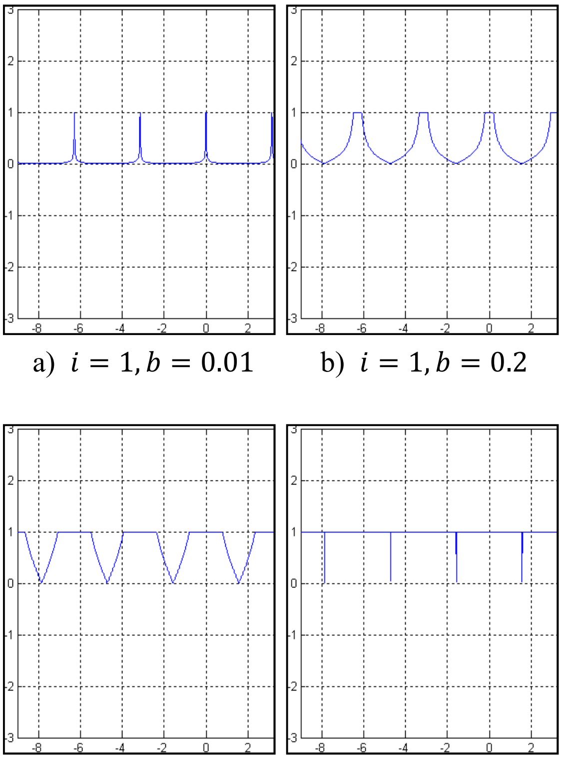
Figures 5 and 6 represent multi form signals obtained by varying two parameters (i and b). For the figures 5.a to 5.d the value of $i = 2$, for the figures 6.a to 6.d the value of $i = 1$.



a) $i = 2, b = 0.01$ b) $i = 2, b = 0.2$

c) $i = 2, b = 1$ d) $i = 2, b = 100$

Fig. 5: multi form signals of the function $\bar{R}jes_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



a) $i = 1, b = 0.01$ b) $i = 1, b = 0.2$

c) $i = 1, b = 1$ d) $i = 1, b = 100$

Fig. 6: multi form signals of the function $\bar{R}jes_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

Important signals obtained using this function: Alternate impulse train (positive and negative part), Impulse train (positive part only), square, rectangle, saturated saw signal, saturated triangle signal, continuous signal …

These types of signals are widely used in power electronics, electrical generator and in transmission of analog signals [16].

4.2 Determination of the Rectangular *Mar* function

The rectangular form in the figure 4 is written with four equations (4). Thus, given (8), the rectangular *Mar* function can be determined using these functions. In fact:

- $Rmar(x) = \frac{a.\text{Cter}\,(\alpha)}{b}$ For $\alpha \in [-\beta; \beta]$
- $Rmar(x) = 1$ For $\alpha \in]\beta; \pi - \beta]$
- $Rmar(x) = \frac{-a.\text{Cter}\,(\alpha)}{b}$ For $\alpha \in]\pi - \beta; \pi + \beta]$
- $Rmar(x) = -1$ For $\alpha \in]\pi + \beta; 2\pi - \beta[$

Thus the expression of the rectangular *Mar* can be unified by using the angular function expressions (2) and (3), therefore the expression become:

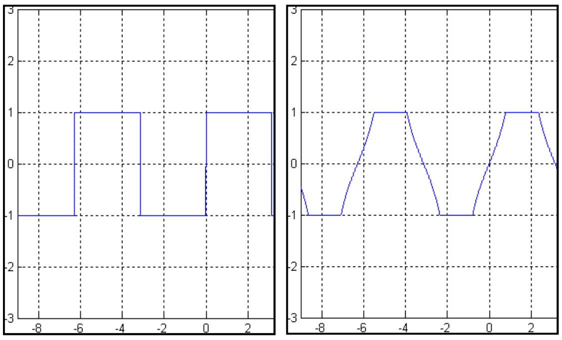
$$Rmar_b(x) = ang_y(x+\beta)\cdot\left(\frac{a}{b}\text{Cter}(x)\right)^{\frac{1-\text{ang}_\beta(x)\cdot\text{ang}_\beta(x-\pi)}{2}} \quad (30)$$

• Expression of the Absolute Rectangular *Mar*:

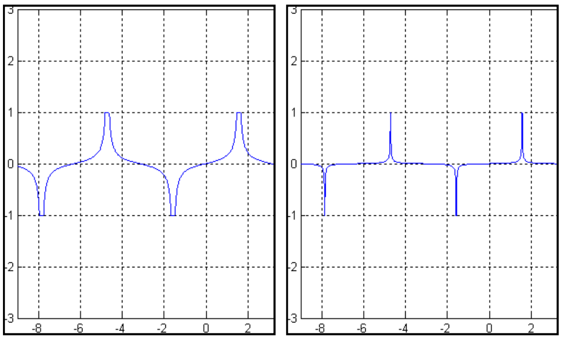
$$\bar{R}mar_{i,b}(x) = Rmar_b(x)\cdot\left(ang_y(x)\right)^i \quad (31)$$

• Multi form signals made by $\bar{R}mar_{i,b}(x)$:

Figures 7 and 8 represent multi form signals obtained by varying two parameters (i and b). For the figures 7.a to 7.d the value of $i = 2$, for the figures 8.a to 8.d the value of $i = 1$.

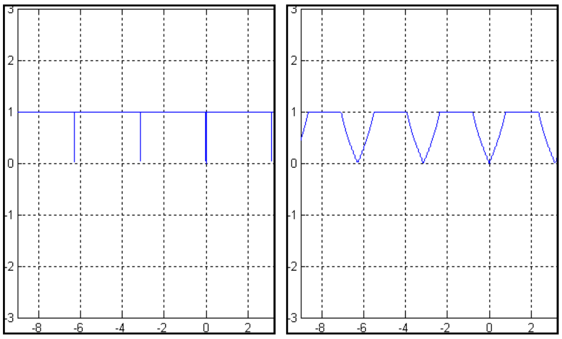


a) $i = 2, b = 0.01$ b) $i = 2, b = 1$

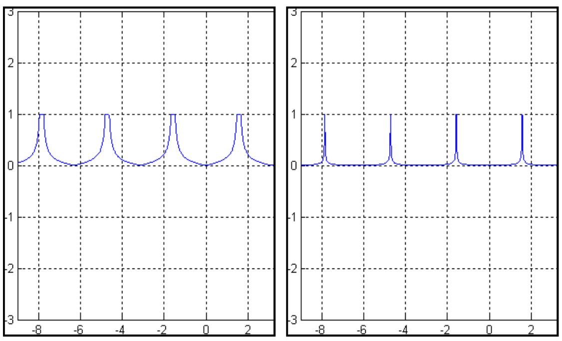


c) $i = 2, b = 10$ d) $i = 2, b = 100$

Fig. 7: multi form signals of the function $\bar{R}mar_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



a) $i = 1, b = 0.01$ b) $i = 1, b = 1$



c) $i = 1, b = 10$ d) $i = 1, b = 100$

Fig. 8: multi form signals of the function $\bar{R}mar_{i,b}(x)$ for $i = 1$ and for different values of $b > 0$.

4.3 Determination of the Rectangular *Jes*-x function

The rectangular form in the figure 4 is written with four equations (4). Thus, given (6), the rectangular *Jes*-x function can be determined using these functions. In fact:

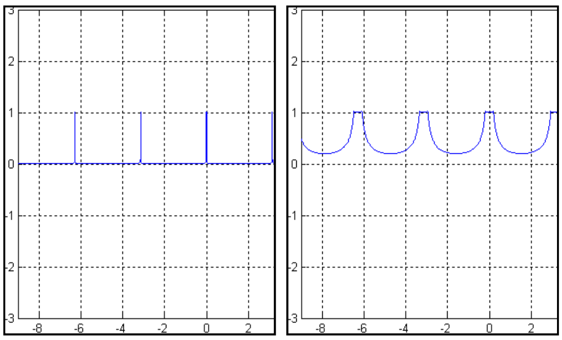
$$Rjes_{x_b}(x) = \frac{Rjes(x)}{Cjes(x)}$$
$$= \frac{ang_\gamma(x+\beta)}{Cjes(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(\mathrm{x})+\mathrm{ang}_\beta(\mathrm{x}-\pi)}{2}} \qquad (32)$$

• Expression of the Absolute Rectangular *Jes*-*x*:

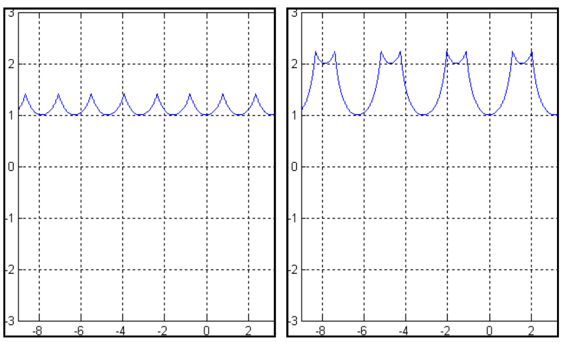
$$\bar{R}jes_{x^i_{b;\gamma}}(x) = Rjes_{x_b}(x) \cdot \left(ang_x(x-\gamma)\right)^i \qquad (33)$$

• Multi form signals made by $\bar{R}jes_{x^i_{b;\gamma}}(x)$:

Figures 9 and 10 represent multi form signals obtained by varying two parameters (i and b). For the figures 9.a to 9.d the value of $i = 2$, for the figures 10.a to 10.e the value of $i = 1$.

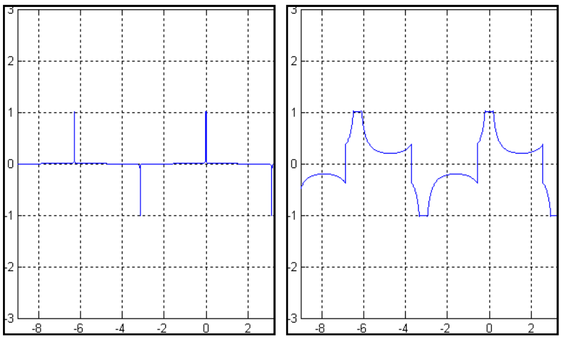


a) $i = 2, b = 0.01$ b) $i = 2, b = 0.2$

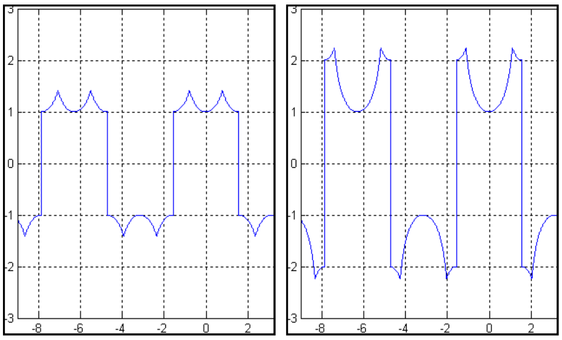


c) $i = 2, b = 1$ d) $i = 2, b = 2$

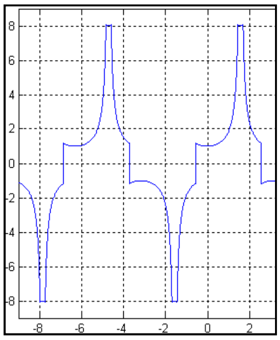
Fig. 9: multi form signals of the function $\bar{R}jes_{x^i_b}(x)$ for $i = 2$ and for different values of b.



a) $b = 0.01,\ \gamma = 0$ b) $b = 0.2, \gamma = 1$



c) $b = 1, \gamma = 0$ d) $b = 2, \gamma = 0$



e) $b = 8, \gamma = 1$

Fig. 10: multi form signals of the function $\bar{R}jes_{x^i_{b;\gamma}}$ for $i = 1$ and for different values of b and γ.

In electronic application and in modeling of signals sources, this function is very important especially in the controlled part of an electrical unit as in motor drives, robotics, and many other industrial electronic applications. The main advantage is that by varying the 3 parameters, one can control easily the input and the output power of a controlled circuit especially when a low or high impulse signal is required. The other advantage is that the $\left(\int_t^{t+period} Rjes_{x_b}(x)dx\right)$ can takes values from 0 to ∞ by varying only one parameter "b".

4.4 Determination of the Rectangular *Rit* function

The rectangular form in the figure 4 is written with four equations (4). Thus, given (14), the rectangular *Rit* function can be determined using these functions. In fact:

$$Rrit_b(x) = \frac{Rmar(x)}{Cter(x)}$$

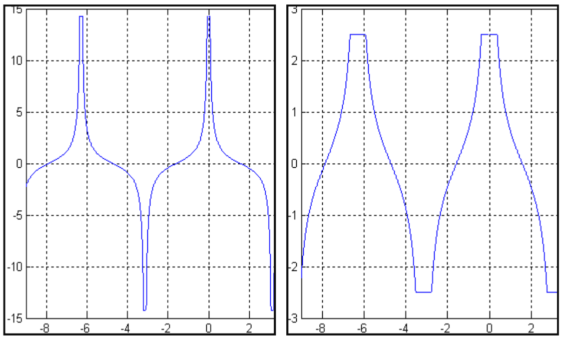
$$= \frac{ang_y(x+\beta)}{Cter(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{1-\mathrm{ang}_\beta(x)\cdot\mathrm{ang}_\beta(x-\pi)}{2}} \qquad (34)$$

• Expression of the Absolute Rectangular *Rit*:

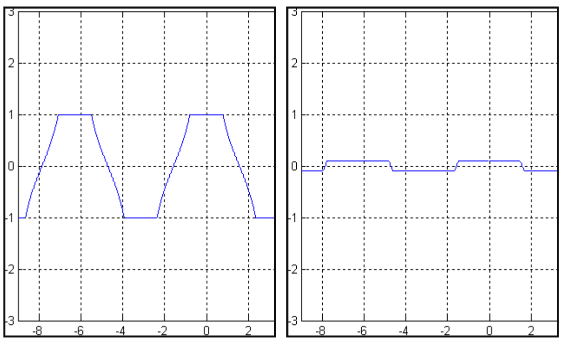
$$\bar{R}rit_{i,b}(x) = Rrit_b(x) \cdot \left(ang_x(x)\right)^i \qquad (35)$$

• Multi form signals made by $\bar{R}rit_{i,b}(x)$:

Figures 11 and 12 represent multi form signals obtained by varying two parameters (i and b). For the figures 11.a to 11.d the value of $i = 2$, for the figures 12.a to 12.e the value of $i = 1$.

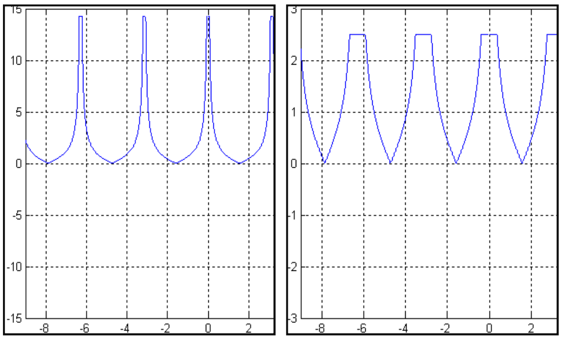


a) $i = 2, b = 0.07$ b) $i = 2, b = 0.4$

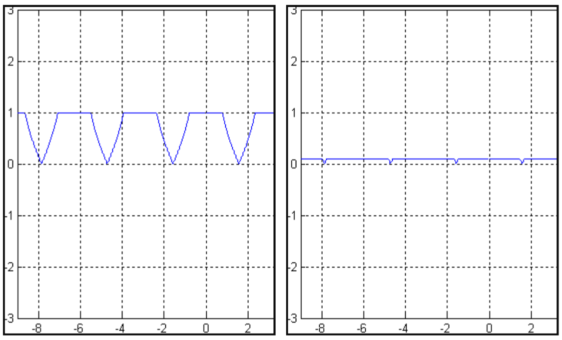


c) $i = 2, b = 1$ d) $i = 2, b = 10$

Fig. 11: multi form signals of the function $\bar{R}rit_{i,b}(x)$ for $i = 2$ and for different values of $b > 0$.



a) $i = 1, b = 0.07$ b) $i = 1, b = 0.4$



c) $i = 1, b = 1$ d) $i = 1, b = 10$

Fig. 12: multi form signals of the function $\bar{R}rit_{i,b}(x)$ for $i = 1$ and for different values for $b > 0$.

5 Example using the rectangular trigonometry

In this section, an example using the rectangular trigonometry is treated. For this, consider the following function:

$$y^2 = \left(k.rect_T(x - x_0).Rjes_{x_b}(\varphi.x)\right)^2 \qquad (36)$$

With k is the amplitude of the signal,
b is the controlled parameter,
T is the period of the rectangular function,
x_0 is the center of the rectangular function,
φ is the frequency of the function $Rjes_{x_b}$.

In fact, the rectangular function $rect_T(x - x_0)$, which is illustrated in figure 13 is defined as:

$$rect_T(x - x_0) = \begin{cases} 1 & for\ x_0 - \frac{T}{2} \le x \le x_0 + \frac{T}{2} \\ 0 & otherwise \end{cases} \qquad (37)$$

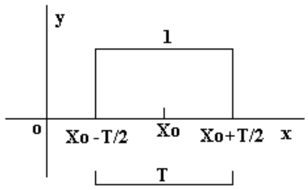


Fig. 13: $rect_T(x - x_0)$ function form

Then: $y = \pm k.rect_T(x - x_0).Rjes_{x_b}(\varphi.x)$ (38)
For a particular case, let $k = 1$, $T = \pi$, $x_0 = 0$ and $\varphi = 1$.

The period of $Rjes_{x_b}(x)$ is equal to π. In fact:
$Rjes_{x_b}(x + \pi) =$

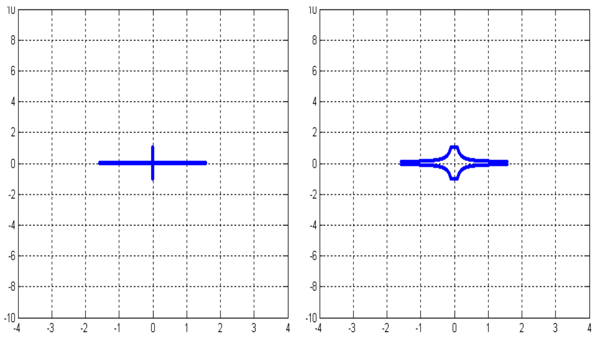
$$\frac{ang_y(x+\pi+\beta)}{Cjes(x+\pi)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x+\pi)\right)^{\frac{\mathrm{ang}_\beta(x+\pi)+\mathrm{ang}_\beta(x+\pi-\pi)}{2}}$$

$$= \frac{-ang_y(x+\beta)}{-Cjes(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x-\pi)+\mathrm{ang}_\beta(x)}{2}} =$$

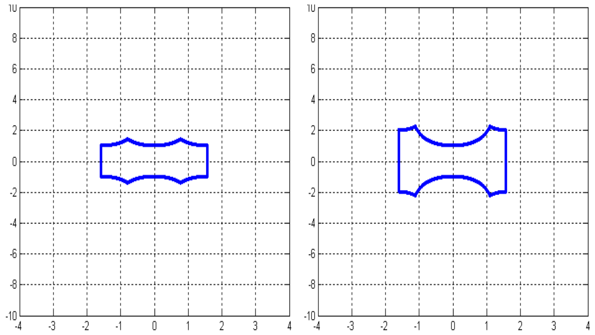
$$\frac{ang_y(x+\beta)}{Cjes(x)} \cdot \left(\frac{a}{b}\mathrm{Cter}(x)\right)^{\frac{\mathrm{ang}_\beta(x-\pi)+\mathrm{ang}_\beta(x)}{2}} = Rjes_{x_b}(x)$$

Thus, the equation (38) becomes:
$y = \pm rect_\pi(x).Rjes_{x_b}(x)$ (39)

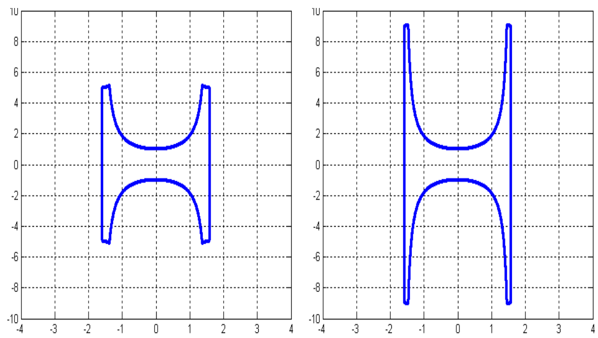
When the parameter b is varied, this new function (39) can be transformed to many shapes as illustrated in figures 14.a to 14.f



a) $b = 0.001$ b) $b = 0.1$



c) $b = 1$ d) $b = 2$



e) $b = 5$ f) $b = 9$

Fig. 14: Different shapes of the y function for different values of b.

6 A survey on the application of the rectangular trigonometry in engineering domain

As we saw in the previous study, the main goal of the rectangular trigonometry is to produce a huge number of multi form signals using a single function by varying some parameters of this function, for this reason we can imagine the importance of this trigonometry in all domains especially in telecommunication, signal theory, electrical and electronic engineering.
Particularly in electrical engineering: motor drives, robotics, or other electronic applications need many controlled circuits that produce different type of signals. In a goal to increase the efficiency, by using this trigonometry, one can develop a circuit that produces multiple forms of signals that respond to requirements.
The functions of this trigonometry are easily programmed and simulated with softwares as Matlab and Labview, and many circuits can be formed to describe these functions.

7 Conclusion

In this paper, an original study in trigonometry is introduced. The rectangular unit and its trigonometry functions are presented and analyzed. In fact the proposed rectangular trigonometry is a new form of trigonometry that permits to produce multiple forms of signals by varying some parameters; it can be used in numerous scientific domains and particularly in mathematics and in engineering. For the case treated in this paper, 32 rectangular trigonometric functions are defined, and for each rectangular function, many periodic signals are produced by varying some parameters.

The rectangular trigonometry functions will be widely used in electronic domain especially in power electronics. Thus, several studies will be improved and developed after introducing the new functions of this trigonometry. Some mathematical expressions and electronic circuits will be replaced by simplified expressions and reduced circuits.

In order to illustrate the importance of this trigonometry, some functions are defined briefly and an example is treated.

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