Linear models in regional and interregional modeling

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Abstract: - Models, included in this paper, in contrast to the common model of inter-sector relations, focuses on products and services, but also activities with inputs in form of products and services as well. Proposed models also include regional aspect. There is described solution for the case that number of elements of products and services and number of activities are not equal, because there is a problem with finding solution.

Key-Words: - Decision-making, Models, Uncertainty, Risk, Regions

1 Introduction

Mathematical modeling methods are often used for its universality, as demonstrated by numerous scientific papers [7, 8, 16]

Reasons for process modeling are the fact that the main goal has been to offer simplified information and to improve communication among relevant stakeholders. A lot of methods exist for the modeling of regional processes. Some of these methods are used to represent the processes embedded in major enterprise resource planning software (e.g. SAP R/3, ARIS). In principle, these methods can be used to model e.g. government processes [3].

In the service or public-administration sector, a cluster oriented approach or a modified Critical Path Method have been recently used as well. [20]

Some authorities have declared that a model should make it easy to identify possible ways for improvement. It is a great advantage of clarity in the use of graphically interpretable mathematical modeling tools. To be easily understood and hence, accepted by the authorities, highly formal modeling languages like Petri Nets or the Unified Modeling Language seems to be suitable for regional process modeling or for e-Government oriented models [15]. The modeling approach should be independent of the primary purposes (e.g. customers needs, higher efficiency, cost reduction, etc.) of the re-engineering efforts.

In particular, Petri nets are a very good modeling tool for creating dynamic models where necessary to model parallel during these processes. Not always meet these tools the needs for creating models. Then it may be preferable to use purely mathematical models.

The aim of this paper is to discuss opportunities of usage of linear mathematic models in description of relations among and within regions. Attention is paid to the linear structural model, which is suitable tool for analyses, forecasting and decision-making. These characteristics make it an appropriate tool to support management decisions at the regional level.

The model can be expressed by means of tables which are used by employees of regional administration on a daily basis.

In the model we result from the fact that there could be sets of inputs, outputs and appropriate activities which a certain subject, for example region, can perform. Outputs are determined by inputs and activities which transform inputs to outputs. Each activity corresponds to a set of inputs and a set of outputs. If we choose a unit activity, then inputs are used and outputs are obtained in a known amount. In case of a change in the amount of activities, the amount of inputs and outputs is changed at a fixed rate. Also volumes of final usage inputs, external inputs and outputs beyond subject are set. So the definite problem can be modeled, of course, by most of mentioned modeling tools.

Formal balance occurs in case when total subject inputs equal total outputs. In the case of balance, subject activities secure inputs which do not participate in activities and net outputs. The aim is to reach formal balance, i.e. to find volumes of subject activities which correspond to balance.

It has to be noted that we consider formal balance. This requirement favors the use of a purely mathematical model to others.

Economic balance is a more complex term, which is comprehended variously by various authors. [13, 17] For example in the Walrasov concept in case of balance all subjects reached their goals and do not wish for any changes. In another concept the balance is reached, when anticipations of economic subjects are realized. Balance often becomes criterion of economical policy.

We stated that the aim of a model is to find the volumes of activities of subjects. In the considered model it is a solution of linear relations. General formulation of the model generally leads to a different number of rows and columns of model-describing matrixes. That leads to usage of various solution processes.

After formulation of the model, we aim to possibilities of solution processes. Besides usage of generalized inversion, we outline process of solution of large systems by means of aggregation and disaggregation. Outlined process of aggregation and disaggregation can be generalized for solving with a different number of rows and columns, but this generalization would need more extensive interpretation. The considered model can also be solved by means of optimization processes. In interpretation, the attention is drawn to possibility of using the Dantzig-Wolf decomposition. Further, one of approaches to description of uncertainty in linear structural models is outlined. Structural models should describe reality complexly and precisely.

Considering complexity, the model outlined above is amended with three problem areas, which are price problem, decision-making problem and time problem.

2 Models attribute

By economic oriented mathematic model a description of an object, event, behavior etc. simplified by means of quantities and relations is comprehend.

Quantities are then resolved into constants, parameters and variables. Constants are expressed by numerals and other symbols (+, -, decimal points, blank spaces) and express invariable values. Parameters and variables are formulated by common numbers. Parameters represent boundary line among constants (parameter can be for example interest rate, time etc.) and variables. They do not change their value within one reflection. Parameters and variables can be divided to exogenous and endogenous. Exogenous values are determined by outer facts which are not described by the model. Endogenous quantities are incident to the model and are determined by the model. They are also called unknown variables, calculated variables etc.

Relations can be divided to definitions and identities; relations describe technological aspects, legislative norms, institutionary relations, behavior etc. We consider quantities and relations

-Exact (deterministic), which do not contain random elements

-Stochastic, which contain random element.

If a model contains only deterministic quantities and relations, it is a deterministic model; a model containing random elements is called stochastic. In case of stochastic quantities we can use techniques of simulation in model application, which consist in determining random quantity by means of pseudo/random numbers and evaluation of characteristics of distribution of these quantities provided by validity of selected distribution type.

If the model covers object or event as a whole, we call it aggregate or global model. If we consider individual parts of object or partial events, we call it non-aggregate model (for example sector).

If we consider dependency of model elements and their changes on time or time period, it is a dynamic model; in opposite case it is a static model.

If we use the model select an option from a set of options according to decision-making criteria, it is decision-making or optimization model. In that case there are one or more criteria of optimality. Optimality is bound to processes (algorithms) of optimization.

3 Linear model

We will focus on some structural models and their alternatives in regionalistics, on solvability problems and on possible integration of system of structural models.

3.1 Structural models

We will describe one of possible approaches to linear modeling to be used in regionalistics and in public administration based on input-output approach. Suggested model can help to balance planning of subject development, even if model dynamization is not explicitly considered. Extent of this paper forbids covering a number of aspects, which concern the problem given.

The object of the model is a system of relations and quantities, which in certain time period concerns production and consumption in certain region. We will also have a respect to value relations. For model construction in this paper we will use following markings

e vector with coordinates representing units and with such dimension that term featuring the vector would make sense.

E represents unit matrix with such dimension that term featuring the matrix would make sense, i.e. $E = \hat{e}$.

Symbol ′ indicates transposition, i.e. interchange of rows and columns of respective matrix.

Symbol ^ indicated diagonal matrix created by vector, i.e. $\hat{x}$ in matrix with vector x coordinates on diagonal and zeros on all other positions.

Symbol X^{-1} indicates inversion matrix, i.e.
$XX^{-1} = X^{-1}X = E$.

We will state objects and subjects and their classifications, which will be considered in models.

3.1.1 Regions as subjects

System of production and consumption can be stratified according to regions. The term region is variously referred to large areas, often identified for the purposes of public administration with a territorial breakdown by NUTS. A specific role has also various subsystems in the regions, usually called micro-regions. The application of the Principles of subsidiarity in the EU aimed at public administration becomes precisely regions and micro-regions in the administration structure in the context of public, importance.

Each region can be comprehended as territorial object, which is subject to human interest and it is administered by a subject or subjects. The term “region” has to contain delimitation of territory and delimitation of human interest and efforts with consideration to time dimension. Administration is performed by subjects with the aim to organize and manage fulfillment of interest and effort.

We will use designation subject for a region. That will accent decision-making, administration and production-consumption management system aspects. Distribution of production-consumption system to partial elements will generate necessity to include relations among partial units into research.

Subject can be region, nation or a set of regions or nations. In wider concept it can be even a corporation etc.

In model we will consider subjects or their sets. Classification of subjects will be designated K_S. Certain subject will be designated $s \in K_S$. All subjects will be marked “*” and all subjects except subject considered will be marked “+“.

3.1.2 Activities

Activities (processes, activities) considered in the model can be realized in certain time period[1] with certain intensity and they have their inputs and outputs. Condition for implementation of activities is presence of primary factors such as work, capital, and land.

Volume of activity intensity influences volume of inputs and outputs. In linear models we presume direct proportionality between volume of individual input or output and volume of activity intensity. Activity intensity volume can be defined variously. For example we define volume of outputs of selected item or total output volume.

Intensity of activity determined by the volume of output of item selected is used in cases when model determines input distribution. Activity intensity determined by total value of output is used in cases when we model both distribution and value relations. In that case it is necessary to consider default prices, which allows determining output value.

If activity intensity if defined, we can consider unit activity intensity.

Activities (processes) are performed by a subject or a group of subjects.

Activity classification is designated K_P. If it is appropriate, we will add designation of subject considered in classification, for example ${}^{s}K_P$.

3.1.3 Inputs and outputs

For specification of the modeled process are essential inputs and outputs. While performing activities, each activity has various inputs and outputs which correspond to various products, services and other items (for example money).

Inputs and outputs can be expressed in physical units or in value units. Physical units are used to model distribution relations. If we model both distribution and value relations, it is appropriate to use value units. In that case it is necessary to consider default prices which allow determining value unit as a product of respective physical unit and default price.

Classification of inputs and outputs is designated K_V. Inputs and outputs are called item.

3.1.4 Primary factors

Performing activities requires necessary amount of labor, capital, land, but also products and services, which are inputs, but are not generated in current

[1] In statistic models time or time periods will not be explicitly considered.

production-consumption system. These items are called primary factors. Classification of items, which are not generated, but are used in the system will be designated K_F. Classification of types of capital and land will be designated K_K and classification of types of labor will be designated K_L. Usage of matrixes in modeling requires usage of necessary classifications of rows and columns of matrix. Subject of both classifications can be items of inputs and outputs, activities used by subjects and the subjects themselves. Let us presume that classification used for rows and columns correspond to items of inputs and outputs K_V and that classification used for columns corresponds to activities (processes), i.e. K_P. In this case we consider model representing relation of items of inputs and outputs and processes. This model would be the base of our analysis. Economic-mathematical structural models can also be used for analysis of other relations in production-consumption system. We will state following approaches:

- model expressing relation among items, not considering activities, i.e. classification of rows and columns are the same and equal K_V; this case is used for example in models of inter-branch relations, models of inter-product relations etc.
- model expressing relation among subjects manifesting in value of mutual supplies of items of inputs and outputs, classification of rows and columns are the same and equal K_S; this case is used for example in model of inter-regional relations containing values of total volume of transfers between regions.

Model expressing relation among items of inputs and outputs and subjects, classification of rows is K_V and corresponds to items and classification of columns is K_S and corresponds to subjects.

If the research should cover more models, we have to assure consistency of all models considered. This consistence can be reached by detailed classification of default data and by using methods of aggregation and disaggregation.

If subjects can work with their own prices or use various measures to formulate prices, we can introduce price and exchange rate aspects. These issues arise in process of introducing individual currencies within various groupings. Formally it is possible for individual subjects to introduce price vectors and to design costs and revenues of subjects or costs and revenues related to activities. Price levels can be solved analogically to processes, which were used to formulate price types[2]. We will also presume that quantities related to inputs and outputs are expressed in value form. That would allow countability of data and direct aggregation within system of models. On the background of this presumption there is an existence of default prices of items and factors. Prices of factors can be in certain cases derived from sacrificed prices.

3.1.5 Data of model

Let us consider solid subject $s \in K_S$.

Let us consider such model expressed by matrixes and vectors that classification of rows is the same for all subjects and corresponds to a set of input and output elements included to classification K_V and column classification corresponds to activities $j \in {}^sK_P$, which are performed by subject considered.

Let us designate

${}^sx$ vector [3] with coordinates ${}^sx_j$ expressing intensity of activities $j \in {}^sK_P$ of subject considered,

${}^sp$ vector with coordinates ${}^sp_i$ expressing value, which enables subject to express item related to commodity $i \in K_V$ by means of selected general equivalent; for instance price, price index etc.,

${}^sy$ vector with coordinates ${}^sy_i$ expressing final usage (final consumption) $i \in K_V$ of subject considered,

${}^{s+}v$ vector with coordinates ${}^sv_i$ expressing external output of element $i \in K_V$ used outside subject considered,

${}^su$ vector with coordinates ${}^su_j$ expressing value added by processing by activity $j \in {}^sK_P$ assigned with intensity ${}^sx_j$,

${}^sX$ matrix of items ${}^sx_{ij}$, where $i \in K_V$ and $j \in {}^sK_P$, which represent volume of input $i \in K_V$ to activity $j \in {}^sK_P$ of subject considered with intensity of activity ${}^sx_j$,

${}^sZ$ matrix of items ${}^sz_{ij}$, where $i \in K_V$ and $j \in {}^sK_P$, which represent volume of output $i \in K_V$ from activity $j \in {}^sK_P$ of subject considered with intensity of activity ${}^sx_j$,

${}^{+s}d$ vector with coordinates ${}^sd_i$ expressing external input of item $i \in K_V$, which originates in subjects outside subject considered.

We divide inputs of subject considered to inputs used for generating output of subject considered (intermediate inputs) and inputs which do not participate in activities of subject (final inputs).

Input of elements generated in surroundings of subject considered, which enters to process of

[2] Sekerka B., Kýn O., Hejl L.: Price Systems Computable from Input/Output Coefficients (4) /in/ Proceedings of the 4th International Conference on Input/Output Analysis, Geneva 1968 „Contributions to Input/Output Analysis“ (editor A. P. Carter) 1969, North Holland Publishing Company, Amsterdam, London pp. 183-203

[3] Vectors will be comprehended as columnar vectors

consumption of subject considered, will be called external input and Outputs of elements generated by activity of subject considered and enters to its surroundings, will be called external output. External inputs and external outputs are vectors. In model we presume that external input cannot be used as external output.

3.1.6 Model formulation

Let us consider classifications $i \in K_V$ and $j \in {}^sK_P$. We shall consider activities of subject s, which have their own inputs and outputs and relation with other subjects.

Balance is set in case when total subject inputs equal to total subject outputs. In case of balance, subject activity ensures inputs, which do not participate in activities and external outputs. It is apparent from notation above that we can state

$$ {}^sX\, e + {}^sy + {}^{s+}v + \Delta^s z = {}^sZ\, e + {}^{+s}d $$

$$ e'\, {}^sX + {}^su' = e'\, {}^sZ\, . $$

The first equation corresponds to division relations. Vector of sources of considered subject represents output vector ${}^sZ\, e$ and external inputs vector of subject is ${}^{+s}d$. Vector ${}^sZ\, e + {}^{+s}d$ has to, is case of balance, correspond to total demand vector

$$ {}^sX\, e + {}^sy + {}^{s+}v. $$

Total subject output demand vector consists of demand

${}^sX\, e$ which corresponds to inter-inputs of items $i \in K_V$ for output generation,

${}^sy$ which corresponds to final usage of considered subject of items $i \in K_V$,

^{s}v which corresponds to external output of considered subject of items $i \in K_V$.

Vector $\Delta^s z$ is a residual vector. It is desirable that this vector equals to the zero vector. It will not be further considered.

We can adjust this equation to following form

$$ {}^sX\, e + {}^sy + {}^{s+}v - {}^{+s}d = {}^sZ\, e $$

$$ {}^sX\, e + {}^sf = {}^sZ\, e $$

$$ {}^sY\, e = {}^sf, $$

where

$$ {}^sf = {}^sy + {}^{s+}v - {}^{+s}d $$

$$ {}^sY = {}^sZ - {}^sX\, . $$

Equation

$$ e'\, {}^sX + {}^su' = e'\, {}^sZ $$

corresponds to value relations. Value of output $e'\, {}^sZ$ se equals value of inputs $e'\, {}^sX$ and gross added value expressed by vector u.

It is apparent from equations quoted that

$$ e'\, {}^sX\, e + e'\, {}^sf = e'\, {}^sZ\, e $$

$$ e'\, {}^sX\, e + {}^su'e = e'\, {}^sZ\, e. $$

Last equations results in condition

$$ {}^su'e = e'\, {}^sf\, . $$

The equations quoted are always valid for default data.

If vector ${}^sf$ or ${}^su$ is changed, it is possible that imbalance will be set. Balance can be achieved again by changing intensities of activities of considered subject, or by changing valuation of items. It is also possible to use combination of methods mentioned.

Change of intensities of subject's activities will be expressed by change of vector e to new vector ${}^sx$. Change in valuation will be expressed by change of vector e to new vector ${}^sp$. This vector can be interpreted as change of prices expressed for individual items by price index.

We define

$$ {}^sA = {}^sX\, {}^s\hat{x}^{-1} $$

$$ {}^sB = {}^sZ\, {}^s\hat{x}^{-1}. $$

Let us presume that we can apply following equations for arbitrary intensities of activities ${}^sx$

$$ {}^sA\, {}^s\hat{x} = {}^sX $$

$$ {}^sB\, {}^s\hat{x} = {}^sZ $$

and that matrixes ${}^sA$ and ${}^sB$ are constant.

Elements ${}^sa_{ij}$ of matrix ${}^sA$ represent input i created with unit intensity of activity j of considered subject.

Elements ${}^sb_{ij}$ of matrix ${}^sB$ represent output i created with unit intensity of activity j of considered subject.

We can therefore state following equations for arbitrary vector ${}^sf$

$$ {}^sA\, {}^sx + {}^sf = {}^sB\, {}^sx $$

$$ {}^sC\, {}^sx = {}^sf\, , $$

where

$$ {}^sC = {}^sB - {}^sA\, . $$

Solution of obtained relation is vector ${}^sx$, which, with given values ${}^sA$, ${}^sB$ and ${}^sf$, is the most suitable for given scheme. Solution is acquired according to scheme character and to rank of scheme matrix and extended scheme matrix.

If the scheme does not have a solution (rank of matrix differs from rank of extended matrix), then we search for new vector x, which can properly approximate solution. We will get back to this problem further in the text. If the scheme has more solutions, we can use optimalization methods, eventually special methods resulting from character of solved problem.

Let us presume change of vector u'. In this case vector e' in relation

$$ e'\, {}^sX + {}^su' = e'\, {}^sZ $$

changes, i.e. we will search for vectors such ${}^sp$, for which following equation is valid

$$ {}^sp'\, {}^sX + {}^su' = {}^sp'\, {}^sZ\, . $$

If we multiply stated equation from right side by vector ${}^{s}\hat{x}^{-1}$, we will get

${}^{s}p'\,{}^{s}X\;{}^{s}\hat{x}^{-1} + {}^{s}u'\,{}^{s}\hat{x}^{-1} = {}^{s}p'\,{}^{s}Z\;{}^{s}\hat{x}^{-1}$

${}^{s}p'\,{}^{s}A + {}^{s}u'\,{}^{s}\hat{x}^{-1} = {}^{s}p'\,{}^{s}B$.

By this way we obtain basic equation for change of value by means of indexes ${}^{s}p$. In the same application we can consider ${}^{s}p$ vector of price indexes.

Solution of this relation will be named vector ${}^{s}p$, which is, with given values ${}^{s}A$, ${}^{s}B$, ${}^{s}x$ and ${}^{s}u$, the most suitable solution of the given scheme. Solution is acquired according to scheme character and to rank of scheme matrix and extended scheme matrix.

If the relation among items and activities is such that activity output j is exactly one item j, we can presume equality of rows' and columns' classifications. In this case matrix B equals unit matrix E. If $(E\text{-}A)$ is regular, therefore the problem has exactly one solution.

3.1.7 Primary sources

The model stated would not be complex if it did not consider quantities which condition performing activities. Activities require factors such as services of work, capital etc. In modeling we will call them primary sources. Primary sources are dependent on model. Primary sources are bound to activities.

It is necessary to choose classification similar to items' classification. In matrix characterizing necessity of primary sources individual items of sources are considered in rows. In columns there are activities of subject considered.

${}^{s}G$ matrix of items ${}^{s}g_{ij}$, where $i\in{}^{s}K_F$, and $j\in{}^{s}K_P$, which represent volume of primary source $i\in K_F$ required by subject to activity $j\in{}^{s}K_P$ by intensity of activity ${}^{s}x_j$;

${}^{s}L$ matrix of items ${}^{s}l_{ij}$, where $i\in{}^{s}K_L$ and $j\in{}^{s}K_P$, which represent volume of work of type $i\in{}^{s}K_L$ required by subject to activity $j\in{}^{s}K_P$ by intensity of activity ${}^{s}x_j$;

${}^{s}K$ matrix of items ${}^{s}k_{ij}$, where $i\in{}^{s}K_K$ a $j\in{}^{s}K_P$, which represent volume of capital of type $i\in{}^{s}K_K$ required by subject to activity $j\in{}^{s}K_P$ by intensity of activity ${}^{s}x_j$;

We will set

$${}^{s}\overline{G} = {}^{s}G\;{}^{s}\hat{x}^{-1} \quad (1)$$

$${}^{s}\overline{L} = {}^{s}L\;{}^{s}\hat{x}^{-1} \quad (2)$$

$${}^{s}\overline{K} = {}^{s}K\;{}^{s}\hat{x}^{-1}\,. \quad (3)$$

We consider matrixes ${}^{s}\overline{G}$, ${}^{s}\overline{L}$, ${}^{s}\overline{K}$ known and constant.

Following equations hold true

$${}^{s}\overline{G}\;{}^{s}x = {}^{s}g \quad (4)$$

$${}^{s}\overline{L}\;{}^{s}x = {}^{s}l \quad (5)$$

$${}^{s}\overline{K}\;{}^{s}x = {}^{s}k\,. \quad (6)$$

Stated vectors represent total number of items, work and capital available at disposal for subject considered to its activities.

3.1.8 Model version

We consider model expressed by relation

${}^{s}A\,{}^{s}x + {}^{s}f = {}^{s}B\,{}^{s}x.$

In the model, matrixes ${}^{s}A$ and ${}^{s}B$ are relatively stable, because they correspond to technologies, which can be considered stable in sufficiently short period of time.

It is appropriate in more detailed analyses and analyses of values to distinguish inputs into activities and final usage according to whether they do or do not have their own or external input. Outputs should be distinguished between those that are meant for own consumption and for consumption elsewhere.

For this reason we will state conventions set before:

Symbol * means count of all values of respective index[4] ,

Symbol $^+$ means count of all values of respective index except index of subject considered.

Therefore

${}^{*s}X$ means matrix of items ${}^{*s}x_{ij}$, where $i\in K_V$ and $j\in K_P$, which represent total volume of input i to activity j of subject s with intensity of activity characteristic ${}^{s}x_j$,

${}^{s*}Z$ means matrix of items ${}^{s*}z_{ij}$, where $i\in K_V$, and $j\in K_P$, which represent total volume of input i to activity j of subject s with intensity of activity characteristic ${}^{s}x_j$.

Equation stated before

${}^{s}X\,e + {}^{s}y + {}^{s}v - {}^{s}d = {}^{s}Z\,e$

can be transcribed to form

${}^{*s}X\,e + {}^{*s}y + {}^{s+}v - {}^{+s}d = {}^{s*}Z\,e.$

Total input to other subjects ${}^{+s}d$ can be used by subject s as intermediate or for final usage. We can therefore state

${}^{+s}d = {}^{+s}X\,e + {}^{+s}y,$

where

${}^{+s}d$ is vector, whose coordinates ${}^{+s}q_i$ represent input of item $i\in K_V$ generated by external subjects, which subject s consumes

[4] We should note that sometimes in terms we use point instead of countable index, for example $\sum_j a_{ij}\,x_j$; by means of this marking we can easily write terms in following form $a_{i\cdot}\,x_{\cdot}$.

${}^{+s}y$ is vector, whose coordinates ${}^{+s}y_i$ represent final consumption of item i generated by external subjects and consumed by subject s

${}^{+s}X$ is matrix of items ${}^{+s}x_{ij}$, where $i \in K_V$, and $j \in {}^sK_P$, which represent volume of input $i \in K_V$ generated by external subjects to activity $j \in {}^sK_P$ of subject s.

Total output of subject provided to other subjects is represented by vector ${}^{s+}v$. This vector with coordinates ${}^{s+}v_i$ represent output of item $i \in K_V$ generated by subject s, which is consumed by external subjects.

We can easily observe that following holds true

${}^{s+}v = {}^{s+}Z\, e$,

where

${}^{s+}Z$ is matrix of units ${}^{s+}z_{ij}$, where $i \in K_V$, and $j \in {}^sK_P$, which represent volume of output $i \in K_V$ generated by subject s by activity $j \in {}^sK_P$, which is not consumed by subject s.

Let us define

${}^{+s}A$ as matrix, whose items ${}^{+s}a_{ij}$ represent usage of $i \in K_V$ generated by external subjects and usable by subjects s by its unit activity $j \in {}^sK_P$.

${}^{s+}B$ as matrix, whose items ${}^{s+}b_{ij}$ represent output $i \in K_V$ generated by subject s by its unit intensity of activity $j \in {}^sK_P$ and which is usable by external subjects[5]

If a subject does not generate or consume a commodity, the respective values will be zero.

Let us stem from relation

${}^{*s}X\, e + {}^{*s}y + {}^{s+}v - {}^{+s}d = {}^{s*}Z\, e.$

This relation is relatively stable. Flows (inputs and outputs) with consideration of subject of origin and subject of usage are influenced by trading conditions and business policy and are therefore variable. Relations within one subject are influenced by conditions related to the subject and are relatively stable.

If we consider intensities of activities given by vector ${}^sx$, we can state

$$ {}^{*s}X\ {}^s\hat{x}^{-1}\ {}^s\hat{x}\, e + {}^{*s}y + {}^{s+}v - {}^{+s}d = {}^{*s}Z\ {}^s\hat{x}^{-1}\ {}^s\hat{x}\, e \quad (7) $$

$$ {}^{*s}A\, {}^sx + {}^{*s}y + {}^{s+}v = {}^{s*}B\, x^s + {}^{+s}d $$

$$ {}^{ss}A\, {}^sx + {}^{ss}y + {}^{+s}A\, x^s + {}^{+s}y + {}^{s+}v = {}^{ss}B\, {}^sx + {}^{s+}B\, {}^sx + {}^{+s}d. $$

We can comprehend that last relation can be divided to relations

${}^{ss}A\, {}^sx + {}^{ss}y = {}^{ss}B\, {}^sx$

${}^{+s}A\, x^s + {}^{+s}y = {}^{+s}d$

${}^{s+}v = {}^{s+}B\, {}^sx.$

[5] From technological viewpoint we can presume stability of coefficients of matrixes ${}^{*s}A$, ${}^{s*}B$. Matrixes ${}^{+s}A$, ${}^{s+}B$ are constituted based on relations between subject and its surroundings.

Relations stated represent balance condition related to subject s towards other subjects. Statement ${}^{s+}B\, {}^sx$ represents the supply of subject s to other subjects. Statement ${}^{+s}A\, x^s + {}^{+s}y$ represents the demand of subject s after items produced by other subjects.

It results from statements above that

-External input of subject considered can be used for performing activities of considered subject and for its final usage. Meanwhile input for ensuring activities depends on intensities of performed activities within subject and within respective matrix ${}^{+s}A$.

- output of considered subject for usage by external subjects is determined by intensity of activities of considered subject and by matrix ${}^{s+}B$. Output ${}^{s+}B\, {}^sx$ represent supply of considered subject to other subjects.

Model results in important finding related to inputs and outputs. It is apparent from following equations

${}^{ss}A\, {}^sx + {}^{ss}y = {}^{ss}B\, {}^sx$

${}^{+s}A\, x^s + {}^{+s}y = {}^{+s}d$

${}^{s+}v = {}^{s+}B\, {}^sx.$

that we can determine vectors ${}^{+s}d$ and ${}^{s+}v$, under conditions of constant structures ${}^{ss}A$, ${}^{ss}B$, ${}^{+s}A$, ${}^{s+}B$ and given values ${}^{ss}y$, ${}^{+s}y$, by finding intensities of activities ${}^sx$.

Intensities of activities can be obtained by solving equation

${}^{ss}A\, {}^sx + {}^{ss}y = {}^{ss}B\, {}^sx$

${}^{ss}C\, {}^sx = {}^{ss}y$,

where

${}^{ss}C = {}^{ss}B - {}^{ss}A.$

Other possible process is to find solution for ${}^sx$ in relation

$$ {}^{*s}A\, {}^sx + {}^{*s}y + {}^{s+}v = {}^{s*}B\, x^s + {}^{+s}d $$

with constant structures ${}^{*s}A$, ${}^{s*}B$ and given values of ${}^{*s}y$, ${}^{+s}d$, ${}^{s+}v$.

Problem of solving relations stated above is that we can state that classification of inputs and outputs and classification of activities does not necessarily have to have the same number of items. That is why we will further outline possibilities of finding intensities of activities.

3.1.9 Solution of linear relation sets

Structural models generally require solution of linear equations. Existence and process of solving in practical problems depends on character of problem solved. From the practical viewpoint a case, where there is number of relations higher than number of unknown variables, will require different approach to a case with number of relations lower than number of unknown variables. In the process of searching for solution, great role will be played by the range of solved tasks.

There are various methods of solving linear models, for instance direct and iterative methods. There are also methods of solving linear systems which use simulation of pseudo-random number and Markov processes. There methods would deserve more attention than currently paid to them in literature.

That is why one of decisive elements in modeling is choice of resolution. In the following paragraph we will focus on some solution approaches. [1]

3.1.10 Solvability of model by iteration aggregation and disaggregation process

In the beginning of this paper we stated various classifications used in model. But we did not occupy ourselves with general features.

We consider a set of some classified elements. Let us state general characteristics of classification:

-Classification of elements has to be complete, i.e. there are no elements unassigned to considered set of elements;

-Each of classified elements has to be exactly in one item of items classification.

In some cases it is allowed to group items into item groups. This grouping can be easily described by aggregation matrix *S*. Rows of the matrix *S* correspond to groups of items and columns correspond to items of classifications. Item s_{ij} of matrix *S* equals one, if item of classification *j* is included in group *i*; in opposite case it equals zero. Matrix *S* has the attribute of having counts of columns always equal one. It is obvious that groups of items can be items of new classification.

Opposite of grouping is classification of groups of items to individual items. Quantification raises the problem of weights, which we exemplify on vector *x*, which has countable coordinates. Let us choose matrix of aggregation *S*. For aggregated vector x^+ the following statement holds true

$S\,x = x^+$.

Let us define matrix of weights

$$G(x) = (\hat{x}^+)^{-1} S\ \hat{x}\,. \qquad (8)$$

By means of matrix of weights we can state

$$G'(x) = \hat{x}\ S'(\hat{x}^+)^{-1} \qquad (9)$$

$$G'(x)\ x^+ = \hat{x}\ S'\ e^+ \qquad (10)$$

$$G'(x)\ x^+ = x\,, \qquad (11)$$

Where e^+ is vector of units, whose number corresponds to number of groups.

Hypothetically we can consider very detailed model

$$A\,x + y = B\,x\,, \qquad (12)$$

Where matrix *B* is unit matrix and *y* is vector.

In the considered model, the number of columns and rows is equal and very large (large scale model). There is, under certain circumstances, solution of this model, which can be obtained by iterative aggregation and disaggregation process in form

$$A\,G'(x_n)\ x^+ + y = x_{n+1}\,, \qquad (13)$$

where

$$x^+ = (E - SAG'(x_n))^{-1} Sy \qquad (14)$$

And default vector x_0 is given. Conditions of convergence of this process were proven.

Generally we can consider two different aggregation matrixes; one for aggregation of rows and second for aggregation of columns. That way we can obtain aggregated model, where earlier considered classification K_V corresponds to rows and K_P corresponds to columns.

3.1.11 Solution by generalized inversion

Considering general approach to linear model, number of rows and columns characterizing model does not have to be equal. That is the reason why we state general process of solving models with rectangular matrixes.

In analysis stated we will stem from electronic scripts of Department of Mathematics at the Faculty of Constructions of ČVUT[6] [4]

Let *A* be matrix of type (m,n).

In practical applications we often consider system of linear relations

$A\,x = b$.

It would be appropriate if each of these systems had a solution. That cannot hold true for classic solution considering condition on rank of scheme matrix and rank of extended scheme matrix. Therefore we generalize term of solving of scheme.

For purpose of generalization we define a scalar product of vectors *u* and *v* with coordinates u_i a v_i i=1, 2, ..., n by relation

$$(u,v) = \sum_{i=1}^{n} u_i\ v_i\ . \qquad (15)$$

We will introduce quadratic form

$F(x) = (Ax-b, Ax-b)$.

It can be shown that there exists $\underline{x}$, for which the following equation holds true

$F(\underline{x}) = \min_x F(x) = d$.

[6] See for instance Ivo Marek, Petr Mayer a Bohuslav Sekerka: Úvod do numerické matematiky. Presentation for informatics auditors. Winter or summer half 2/2, electronic text of department of mathematics of faculty of construction, ČVÚT, http://mat.fsv.cvut.cz /skripta/nummet/ams.pdf

Let us consider $M = \{x; F(x) = d\}$. This set is non-empty, isolated and convex. There is exactly one item $x^* \in M$ for which holds true

$\|x^*\| = \min_{x \in M} \|x\|$.

Vector x^* is called normal solution of scheme $Ax = b$. Sometimes we use title in sense of least squares method.

Generalized inversion operators

We inspect set or relations[7]

$AXA = A, \quad XAX = X, \quad (AX)^* = AX, \quad (XA)^* = XA.$

This set corresponds to exactly one matrix X of type (n,m), which is called (Moore - Penrose) pseudo-inverse matrix to matrix A and is destined A^+.

If matrix A is regular, it becomes apparent that $A^+ = A^{-1}$.

Solution of scheme $A\,x = b$

Let us stem from scheme

$A\,x = b$,

Where A is matrix of type (m,n), x is vector with n coordinates and b is vector with m coordinates. Let us define such vectors b^1 and b^2 that there exists vector c^1, for which holds true[8]

$b^1 = A\,c^1$

$A^*\,b^2 = 0.$

This holds true

$A^*A\,x = A^*\,b = A^*b^1 + A^*b^2 = A^*A\,c^1.$

Scheme

$A^*A\,x = A^*\,b$

is called normal scheme. This scheme has always a classic solution.

$x = (A^*A)^{-1}\,A^*\,b$.

Let us state

$x^+ = A^+\,b.$

It can be proven that

$\underline{x} = x^+ + y,$

where

$A\,y = 0.$

The proof is beyond the range of this paper.

3.2 Multilevel models

Extent of tasks has led to thoughts of decomposition. Decomposition means division of given task to solutions of tasks with smaller amount of conditions. Smaller amount of conditions leads to higher amount of variables. Advantage of decomposition procedures is the fact that we do not have to consider all variables.

[7] We should note that A^* means matrix Hermitov adjoin matrix to matrix A.

[8] $b^1 \in \mathrm{range}(A)$, $b^2 \in \ker(A^*)$

Multi-level models are related to decomposition problems. Division of nation to self-government units or division of territory administration to administration of regions can serve as an example. In this context we will distinguish top and lower level.

Top level generally has central resources at disposal, which, in process of distribution, are passed on to subjects of lower level. Meanwhile we presume possibility of decision-making of subjects on lower levels.

Decomposition by authors Dantzig G. B., Wolfe P., enables us to solve problem stated. In this task we consider delimitating conditions and purpose function for top level. Besides that we also consider conditions specific for lower levels. The task is to optimize purpose function for top level under all conditions, i.e. conditions for both top and lower level conditions.

Formally we can express the problem like this:

We consider n lower levels j=1, …, n. Each lower level corresponds to a set of relations

${}^jA\ {}^jx = {}^jb$.

Top level corresponds to relation

$$\sum_{j=1}^{n} \overline{A}_j\ {}^jx = b\,. \tag{16}$$

Top purpose function is

$$f = \sum_{j=1}^{n} {}^jc'\ {}^jx\ . \tag{17}$$

Goal is to optimize purpose function f (to minimize or maximize) under set conditions.

In process of solving by means of the simplex algorithm we set purpose functions for each lower level in following form

$$({}^jc' - u'\,\overline{A}_j)\ {}^jx\,. \tag{18}$$

Vector u results from relation

$$c'_B\ B^{-1} = (u',\ {}_\lambda u')\,, \tag{19}$$

where B is matrix with corresponding base used for solution by means of simplex algorithm. Vector u corresponds to conditions for top level and ${}_\lambda u$ corresponds to conditions for lower level.

If we include a vector into base, then we have to, due to rules of simplex algorithm, exclude one column form base. We will determine vectors to new base u and ${}_\lambda u$. We repeat the process until we find optimal solution.

Default solution can be found by introducing artificial variables as in simplex method; these variables are then in purpose function assigned with so high value (in case of minimization) that it will ensure absence of artificial variable in the solution. [18,19]. Other possibility could be a econometric orineted models, which is beyond the scope of this paper

4 Price effects

4.1 Price determination

We presume constant prices in model. If subjects are allowed to work with their own prices, or to use different measures for formulating price, we can introduce price aspects and exchange rate aspects into model. These issues come forth in course of introducing individual currencies within various clusters.

Balance has been formulated for both commodity flows and for value flows, including currency impacts. If we consider, for instance, subject *s*, then price index ${}^{s}p$ is determined by value added in process. We can have certain requirements in this value.

$${}^{s}p'\,{}^{s}A + {}^{s}u'\,{}^{s}\hat{x}^{-1} = {}^{s}p'\,{}^{s}B\,.$$

Let us presume that each subject s has its own price level expressed by vector ${}^{s}p$. This price level is determined by relations between inputs and outputs which are determined by matrixes ${}^{s}A$, ${}^{s}B$, intensities of production activities ${}^{s}x$ and by vector ${}^{s}u$, where all price generating elements of production are grouped. Among these elements there are also costs of production factors, i.e. wages, capital, rent, expected (presumed, desired) revenue and other elements.

Price level of given subject can be derived from quantities own to the subject, not considering the fact whether subjects use the same or different currency. Price levels can be solved analogically to processes used for formulation of price types[9]. [9]

4.2 Price exchange coefficients

Even in case when subjects belong to one currency territory and therefore it is not possible for subject to perform its own currency economic policy, differentiation of price levels could remain, because price generating elements can vary among subjects. Adjustment of subject economy to changes has to be performed by other adaptation processes than monetary policy also in case of currency union.

Adaptation processes are generally influenced by unreservedness of economy, product diversification of subjects, labor market (geographical mobility, adjustment of wages and prices), capital market, ground market, financial market; fiscal transfers, production price and production factor flexibility etc.

Fiscal policy can be usable to certain extent. Usability rate will depend on decision-making mechanisms in currency union.

Vital element of price generating elements of production is wages. It was an interesting task to find out attributes of relation between price level and level of production in real and stable prices according to selected territorial units and to focus to research of delay of changes between quantities considered. This topic was the first step of research focused on influence of price generating elements and adjusting mechanisms to changes of these elements.

It results from stated model approach that it is possible even in case of currency union to define exchange coefficients of price levels of individual subjects and to exchange rates by these exchange coefficients.

We consider a transfer of items from subject *t* to subject *s* formulated by vector ${}^{ts}d$. Value of this transfer for subject *t* is determined by price level ${}^{t}p$ and by expression ${}^{t}p'\,{}^{ts}d$ and for subject *s* by price level ${}^{s}p$ and by expression ${}^{s}p'\,{}^{ts}d$.

If we formulate transfers in price level of the subject, where they originated and we want to reformulate them in price level of subject that uses it, we can introduce price level exchange coefficients. In order to exchange these transfers we can introduce exchange coefficients such $\hat{p}_{t/s}$ that following formula holds true

$${}^{t}p = \hat{p}_{t/s}\,{}^{s}p.$$

4.3 Relation between price-formation elements and autonomous input

If we consider subject s, then price index ${}^{s}p$ is determined by value added by processing. We can have certain requirements to this value.

$${}^{s}p'\,{}^{s}A + {}^{s}u'\,{}^{s}\hat{x}^{-1} = {}^{s}p'\,{}^{s}B\,.$$

If we multiply this relation by vector ${}^{s}x$, we will get

$${}^{s}p'\,{}^{s}A\,{}^{s}x + {}^{s}u'\,{}^{s}\hat{x}^{-1}\,{}^{s}x = {}^{s}p'\,{}^{s}B\,{}^{s}x$$

$${}^{s}p'\,{}^{s}A\,{}^{s}x + {}^{s}u'\,e = {}^{s}p'\,{}^{s}B\,{}^{s}x$$

Analogically we will stem from relation

$${}^{s}A\,{}^{s}x + {}^{s}y + {}^{s}v + \Delta{}^{s}z = {}^{s}B\,{}^{s}x + {}^{s}d,$$

which we multiply by vector ${}^{s}p'$ in order to get

[9] Sekerka B., Kýn O., Hejl L.: Price Systems Computable from Input/Output Coefficients (4) /in/ Proceedings of the 4th International Conference on Input/Output Analysis, Geneva 1968 "Contributions to Input/Output Analysis" (editor A. P. Carter) 1969, North Holland Publishing Company, Amsterdam, London pp. 183-203

${}^{s}p'\,{}^{s}A\,{}^{s}x + {}^{s}p'\,{}^{s}y + {}^{s}p'\,{}^{s}v + {}^{s}p'\,\Delta^{s}z = {}^{s}p'\,{}^{s}B\,{}^{s}x + {}^{s}p'\,{}^{s}d.$

By subtracting obtained relations we will get

${}^{s}p'\,{}^{s}y + {}^{s}p'\,{}^{s}v + {}^{s}p'\,\Delta^{s}z - {}^{s}u'\,e = {}^{s}p'\,{}^{s}d,$

which results in

${}^{s}u'\,e = {}^{s}p'\,{}^{s}y + {}^{s}p'\,{}^{s}v + {}^{s}p'\,\Delta^{s}z - {}^{s}p'\,{}^{s}d.$

Relation stated is simplified, if we consider residual element equal zero. Then we can state

${}^{s}u'\,e = {}^{s}p'\,{}^{s}y + {}^{s}p'\,{}^{s}v - {}^{s}p'\,{}^{s}d.$

Appreciation of vector ${}^{s}d$ requires attention. This vector represents inputs to system from other subjects and is expressed in prices ${}^{+}p$. That is why we have to use exchange coefficient so that we can write ${}^{s}p'\,{}^{s}d$.

If we divide final input to its components (citizens, investments, administration and its representatives) and if we divide components of value added by processing to components (citizens' revenues are one of them), then we can formulate relation ${}^{s}p'\,{}^{s}c = w$, where ${}^{s}c$ is vector of citizens' consumption and quantity w represents their revenues. It is obvious that the process stated can help to increase model precision.

5 Dynamization of models

5.1 Material and time local character of models

In course of model formulation we resulted from data, which were determined statistically. These data allowed us to pass over to relative indicators, which were represented for instance by matrixes ${}^{s}A$ and ${}^{s}B$. It is clear that solution of considered linear system of relations does exist, because it is determined by default data. If we perform changes in quantities, for instance changing vectors ${}^{s}y$ and ${}^{s}u$,, we could encounter problems with existence of solution and solution itself, in case that structure illustrating matrixes remain unchanged. That shows evidence to local characteristic of models considered.

5.2 Role of expectation and decision-making in modeling

In previous reading we considered activities, which – with subject – constituted basic elements of model. We searched for such activity intensities, which would ensure balance in individual subjects.

Let us focus on decision of subjects. When we take a peek to future, we should think of activities performed by subject in future. In current activities the subject should focus on their development, maintenance of stable intensities or their inhibition.

Let us focus on new activities. Introduction of a new activity requires precise conception of future inputs and outputs, of requirements on external inputs and of requirements on primary sources. We will presume that this conception is expresses in project, which corresponds to one future activity. In this case, project has to be described formally by quantities as if it was an activity. That way we can define within subject considered economic project j as follows:

Economic project j of subject s is determined by quantities

${}^{s}a_{ij}$ represents input i with unit activity intensity in project j,

${}^{s}b_{ij}$ represents output i with unit activity intensity in project j,

${}^{s}x_{j}$ represents projected activity intensity in project j.

From values stated it is possible to derive contribution of project to final usage. Besides values corresponding to repetition of performing activities within project during its lifetime, it is necessary to take into consideration primary sources related to project activities. The values are

${}^{s}g_{ij}$ represents volume of primary source of type i, which would be needed for activity j with intensity ${}^{s}x_{j}$ during project realization,

${}^{s}l_{ij}$ represents volume of labor of type i, which would be needed for activity j with intensity ${}^{s}x_{j}$ during project realization,

${}^{s}k_{ij}$ represents volume of capital of type i, which would be needed for activity j with intensity ${}^{s}x_{j}$ during project realization,

Values stated correspond to expectation and decision-making of subject considered.

Implementation of project is conditioned by investments of subject considered, which would ensure required primary sources and manifests itself in vector ${}^{s}y$ and eventually in vector ${}^{s}d$.

Each project is characterized by time of its implementation into system. If we presume that production-consumption system was founded based on inclusion of individual economical projects, then we can derive dynamics of system from these projects and from period of inclusion of these projects into system in accordance with decision-making or respective subjects.

In process of including or excluding projects it is important to realize that each of projects passes through various living period during its lifetime. Generally we consider period of construction, development, stagnation and period of ending. These periods correspond to various activity intensities, which can be estimated with consideration of time, which passed from including the project to the system.

5.3 Technique of scenarios

Calculations of trends and estimations of probability distributions of random quantities result from statistic data. Based on statistic data we estimate future. Changes estimated in the long term period are often not realized, because often outer factors are not considered in necessary extent.

Common prognostic and planning methods are generally based on extrapolation of current trends. These methods generally do not consider turning points of development in certain areas.

For better coping with problems indicated we elaborated technique of scenarios. Technique of scenarios is special prognostic technique. Qualitative and quantitative information from various aspects are elaborated. Previous values serve only for comprehension of mutual relations.

Technique of scenarios is, for its complex conception, adapted to usage of additional considerations and searching for new development trends, so that future could be better estimated. A scenario is a description of image of future situation. This conception is supplemented by trajectory of development which would lead to that future situation.

In technique of scenarios we work with various alternatives of future development. Generally we stem from pessimistic and optimistic viewpoints. Then we can derive respective strategic plans. By means of continuous comparison of real achieved development with data from scenarios we can perform respective corrections according to needs. By means of technique of scenarios, more conceivable development alternatives are reflected to future, while considering all possible expected requirements and changes of all areas influencing development from viewpoint of current knowledge. We should try to mutually interconnect them. Results then allow for respective conclusions to be defined.

Scenarios show development alternatives. Development is formulated in such way, that it starts with description of current status and ends by possible situation in the future. Technique of scenarios starts with analysis of current state respective observed sphere. All the quantitative and qualitative information available are taken into consideration. We set premises of development for main factors, which influence it. Possible disruptive events are pointed out. Then we can design alternatives of development for future; it should also take notice of transparency of all processes and work steps.

Scenario is an important orientation support for strategic planning. In strategic reflections we can work out alternative plans for individual scenarios. When certain events occur, management of organization can react more easily and quicker to situation incurred. Results of scenarios should lead to improvement and simplification of decision.

5.4 Dynamization approaches

In process of analysis of dynamization we search for adaptation processes in order to achieve new solution with change of data of linear static model or adaptation processes become part of model.

In process of analysis we focus on discrete time order. Time values will take whole number values from set..., -2, -1, 0, 1, 2, ...,. Time 0 can be defined as present time, negative values as past and positive numbers as future. Besides time values we can consider time periods. Time period k can be comprehended as time interval <k-1, k).

We consider solid subject *s*.

Increase of intensities of activities during time is ensured in structural models by means of investments, which are one of final consumption elements. Subject in this process decides about volume of investments.

Investments influence volume of capital by ensuring its restoration and increase. Investments ensuring restoration of capital are called restoration investments and investments ensuring increase of capital are called development investments.

Increase of capital creates prerequisites for increase of activities intensities.

Let us outline the described process.

Let us presume that activity $j \in K_P$ performed by subject $s \in K_S$ in range ${}^s x_j$ uses capital of type $i \in K_K$ in volume of ${}^s k_{ij}$. We will designate vector ${}^s \mathrm{k}_j$ representing capital used for activity j in range ${}^s x_j$. Coordinates i of vector ${}^s k_j$ correspond to classification K_K.

Let us presume that capital attrition in its individual units i∈KK in unit time period will be characterized by such vector of parameters sα, that ${}^s\hat{\alpha}\ {}^s k_j$ represent attrition of capital, which needs to be replaced.

Vector of total investment with coordinates corresponding to classification KV will be designated ${}^s_I y$. Vector of investments, which influence capital participating in activity j∈KP will be designated ${}^{s}_{I}y_j$. It is obvious that following holds true

$$\sum_{j\in K_P} {}^{s}_{I}y_j = {}^{s}_{I}y \quad (20)$$

Let us presume that for all j∈KP there exists constant transformation such table T with rows corresponding to classification KK and columns corresponding to classification KV, that it transforms investment vector to vector, which corresponds to increment of capital and attrition. We can therefore state

$$T\ {}^{s}_{I}y_j = \Delta^s k_j + {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (21)$$

Matrix T allows selection of such items of classification KV, that results of this selection are items of classification KK. Arbitrary row i∈KK consists of zeros except position of selected item, which equals one.

T T′=E holds true.

T′T is matrix with number of rows and columns corresponding to classification KV. Its items, except diagonal ones, equals zero. Diagonal items equal either zero or one. One is in such places, where item i∈KV corresponds to item of classification KK . We can therefore state

$${}^{s}_{I}y_j = T'(\Delta^s k_j + {}^{s}\hat{\alpha}\ {}^{s}k_j) \quad (22)$$

Of which results following

$$\Delta\ {}^{s}k_j = T\ {}^{s}_{I}y_j - {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (23)$$

Now we complete this consideration with period in which values of respective quantities are implemented.

New capital ${}^{s}k_{j,t+1}$ offers executing investments ${}^{s}_{I}y_t$ equals

$${}^{s}k_{j,t+1} = {}^{s}k_{j,t} + \Delta^s k_{j,t} \quad (24)$$

In previous reading we established constant coefficients $\bar{k}_j$ by means of relation

$${}^{s}\bar{k}_j = {}^{s}k_j\ {}^{s}x_j^{-1} \quad (25)$$

$${}^{s}k_j = {}^{s}x_j\ {}^{s}\bar{k}_j, \quad (26)$$

Which can be transcribed to this form

$${}^{s}x_j\ e = {}^{s}\hat{\bar{k}}_j^{-1}\ {}^{s}k_j \quad (27)$$

It is necessary to realize that intensity of activity is scalar and used capital is expressed by vector consisting of coordinates i∈KK .

Change of capital

$$\Delta^s k_j = T\ {}^{s}_{I}y_j - {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (28)$$

Can be expressed approximately in form (term $\Delta^s x_j\ \Delta^s\bar{k}_j$ is considered to be very small vector)

$$\Delta^s k_j = \Delta({}^{s}\bar{k}_j\ {}^{s}x_j) = {}^{s}x_j\ \Delta^s\bar{k}_j + {}^{s}\bar{k}_j\ \Delta^s x_j \quad (29)$$

If structure of capital is not changed by investments, i.e. $\Delta^s\bar{k}_j$ is zero vector, then we obtain relation between capital addition and intensity of activity

$$\Delta^s k_j = {}^{s}\bar{k}_j\ \Delta^s x_j \quad (30)$$

From previous reading we know that following holds true

$$T\ {}^{s}_{I}y_j = \Delta^s k_j + {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (31)$$

$$T\ {}^{s}_{I}y_j = {}^{s}\bar{k}_j\ \Delta^s x_j + {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (32)$$

$${}^{s}_{I}y_j = \Delta^s x_j\ T'\ {}^{s}\bar{k}_j + T'\ {}^{s}\hat{\alpha}\ {}^{s}k_j \quad (33)$$

If we count the last relation for j∈KP, we will obtain

$${}^{s}_{I}y = \sum_{j\in K_P} {}^{s}_{I}y_j = \sum_{j\in K_P} \Delta^s x_j\ T'\ {}^{s}\bar{k}_j + \sum_{j\in K_P} T'\ {}^{s}\hat{\alpha}_j\ {}^{s}k_j \quad (34)$$

We will perform operations:

We will pose $${}^{s}_{\alpha}k = \sum_{j\in K_P} T'\ {}^{s}\hat{\alpha}_j\ {}^{s}k_j$$ which marks vector of total capital consumption in subject s. (35)

We will group column vectors $T'\ {}^{s}\bar{k}_j$ for j∈KP into matrix ${}^{s}\bar{K}$, where rows correspond to classification KV and columns correspond to classification for KP.

We will create columnar vector sx, whose coordinates for j∈KP equal sxj .

We consider relation

${}^{s}A\ {}^{s}x + {}^{s}f = {}^{s}B\ {}^{s}x$.

This relation can be transcribed to form

$${}^{s}A\ {}^{s}x + {}^{s}f - {}^{s}_{I}y + {}^{s}\bar{K}\ \Delta^s x + {}^{s}_{\alpha}k = {}^{s}B\ {}^{s}x \quad (36)$$

If we take time into consideration, we will obtain

$${}^{s}A\ {}^{s}x_t + {}^{s}f_t - {}^{s}_{I}y_t + {}^{s}\bar{K}\ \Delta^s x_t + {}^{s}_{\alpha}k = {}^{s}B\ {}^{s}x_t \quad (37)$$

$${}^{s}A\ {}^{s}x_t + {}^{s}f_t - {}^{s}_{I}y_t + {}^{s}\bar{K}\ {}^{s}x_{t+1} - {}^{s}\bar{K}\ {}^{s}x_t + {}^{s}_{\alpha}k = {}^{s}B\ {}^{s}x_t \quad (38)$$

Derived relation covers dynamics of production-consumption system expressed by dynamics of intensities of individual activities.

That was one of possible approaches to description of dynamics within linear model. One of the other approaches stems from presumption that intensities' of activities additions induce investments according to the acceleration principle.

6 Future research direction

The presented model allows us to analyze economic structure as a whole and related to its components. Besides planning on lower levels of decision-making, the model can be used for monitoring external effects on partial elements of economy such as finance, investment and other elements.

Usage of linear models has local characteristics in sense that we stem from a state of modeled unit in certain time or during a certain time period. The data obtained contain performed activities. From this viewpoint we are balanced, since we know, for example, performed purchases, which equal performed sales. At this point we can consider future changes. If we consider these changes small, we can use linear relations; in other case it would be necessary to perform deeper analyses of dependencies. Even there, possibilities of further research are apparent.

Besides validation of processes described in economic practice, it is advisable to recognize own model and methods of its solution with consideration of other aspects, which were not considered in the paper. It is for example demographical aspects, product substitutability, generalization of transformation of inputs to outputs, elaboration of decision approaches, usage of resources and concurrence to national economical policy and its elements etc.

It would be desirable in further research to analyze concurrence and connection of work on models with accounting standards and with statistic data. This aspect is required mainly for filling models with data.

7 Conclusion

In the paper we suggested possible approach to modeling regional and inter-regional relations based on linear structural models. Structural linear models can be used in various areas of economic life. Economic numbers serve as a base. These numbers keep changing according to economic movement. Structural models allow uncovering these changes transparently.

In the introduction part I stated that each structural model should be complex and consistent. By complexicity we meant the fact that all significant aspects of modeled subject were taken into consideration. With such a low range it would not be possible to describe all aspects of the problem considered. The aspiration was to provide the reader with a general view on the problem and to draw attention to some non-traditional approaches to modeling and to model solving.

Practical usage requires both knowledge of respective techniques, but also knowledge of modeled object. Structural linear modeling becomes a tool for analyses and consequent syntheses, a tool for forecasting and planning and also one of the tools for economic analyses of a country and regions.

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