Robust Individuals Control Chart for Change Point Model

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Abstract: Control charts are used to monitor for changes in a process by distinguishing between common and special causes of variability. When a control chart signals, process engineers must initiate a search for the special cause of the process disturbance. Identifying which combination of the many process variables is responsible for a change in the process allows engineers to improve quality by preventing or avoiding changes in those variables which lead to poor quality. We examine a process-monitoring tool that not only provides speedy detection regardless of the magnitude of the process shift, but also provides useful change point statistics. Robustness against assignable causes of variation appears to be important and a likelihood ratio method is used to develop test statistics for step change shifts. The performance of the proposed methodology is demonstrated with numerical example and simulation studies.

Key-Words: Change Point Estimation; Statistical Process Control; In-Control; Out-of-Control; Outlier; Maximum-Type.

1. Introduction

The variability in process measurement comes from two basic sources: "common cause" variability, which is all sources of unavoidable random variability that can be removed only by changing the system, and "special cause" variability, which results from some potentially identifiable source that can be removed. A system is said to be in the state of statistical control when the only variability is that due to common causes. When a special cause intervenes, the process is said to be out-of-control. The most often investigated change point problem is that of the change in the mean or variance of normal variables.

In the area of mathematical statistics, the decision whether the observed series remained stationary or whether a change of a specific kind occurred is usually based upon hypotheses testing. The null hypothesis claims that the process is stationary while the alternative hypothesis claims that the process is non-stationary and the stationarity was violated in a specific way. The simplest situation is when at the beginning, a certain process (e.g. that of manufacturing process) is assumed to vary around a certain constant a_0 whereby it is assumed to be in-control. However, it can happen that as a failure of the production device, e.g., the observed characteristics suddenly start to alter around another out-of-control constant $a_1 \neq a_0$. It can happen that, on account of that sudden failure, the variance σ^2 may change as well, yet it is possible that the variance remains the same. Furthermore, sometimes the variance can be presumed to be known due to researcher's long experience with the production process.

2. Change in mean and/or variance

In some cases, it can happen that the change point may happen either in one parameter or in both (simultaneously). Then, the null hypothesis H against the alternative A can be written as follows;

$$H: Y_1, \ldots\ldots, Y_n \sim N(a, \sigma^2)$$

$$\begin{aligned} &A: \exists\, m \in \{2, \ldots, n-2\} \text{ such that} \\ &Y_1, \ldots\ldots, Y_m \sim N(a_1, \sigma_1^2) \\ &Y_{m+1}, \ldots\ldots, Y_n \sim N(a_2, \sigma_2^2) \end{aligned} \tag{1}$$

where $(a_1, \sigma_1^2) \neq (a_2, \sigma_2^2)$.

Jaromir et al. (1999) proposed the Maximum-Type test statistics to test the hypothesis in (1). This test is formulated based on the maximum likelihood approach which have the form

$$\max_{1 \le k \le n-1} \left\{ \left| \widetilde{Z}_k \right| \right\} \tag{2}$$

and

$$\max_{\lfloor \beta n \rfloor \le k \le \lfloor (1-\beta) n \rfloor} \left\{ \left| \widetilde{Z}_k \right| \right\} \tag{3}$$

where

$$\begin{aligned} \widetilde{Z}_k^2 = n \log\left(\frac{1}{n} \sum\nolimits_{i=1}^{n} \left(Y_i - \overline{Y}_n\right)^2 \right) - \\ k \log\left(\frac{1}{k} \sum\nolimits_{i=1}^{k} \left(Y_i - \overline{Y}_k\right)^2 \right) - \\ (n-k) \log\left(\frac{1}{n-k} \sum\nolimits_{i=k+1}^{n} \left(Y_i - \overline{Y}_k^0\right)^2 \right) \end{aligned} \tag{4}$$

$$\overline{Y}_k = \frac{1}{k} \sum_{i=1}^{k} Y_i \quad \text{and} \tag{5}$$

$$\overline{Y}_k^0 = \frac{1}{n-k} \sum_{i=k+1}^{n} Y_i \tag{6}$$

and β is a small positive constant less than one and $\lfloor x \rfloor$ indicates the integer part of x. The benefit of statistic (3) is that they are bounded in probability. The trimming off a 100β % portion of the sample (upper time points) implies that one assumes that the change did not occur during this time period. It is important to note that generally we take $\beta \in [0.01, 0.1]$. The decision on "How much to trim off?" depends on the subjective decision of the statistician and his/her a priori knowledge of the problem. If the statistician decides to trim off only a very small portion of the time points or no time points (observations) at all, he/she pays for it by a loss of the power of his/her test as the critical values depend rather strongly on the value of β.

For the decision about rejection of the null hypothesis H, we need to know critical values of the suggested test statistics. It means to know their distribution under H. Since the distribution of statistics (2) and (3) are not tractable, their critical values were obtained by simulation. Each type of the test statistic (2) and (3) with different sample sizes was repeated for 10,000 times under H model and the corresponding desired percentile critical value was obtained from generated empirical distribution. One would reject H if test statistics of (2) or (3) is greater than the desired significance level α. The simulated critical values of statistic (2) and (3) are presented in Table 1-3.

For n large, Gombay and Horvath (1990) showed that the limit behavior of the studied probabilities is as follows

$$P\left(\max_{1 \le k \le n-1} \left\{ \left| \widetilde{Z}_k \right| \right\} > \frac{x + b_n}{a_n} \right) \approx 1 - \exp\left\{ -2e^{-x} \right\},$$
$$x \in R^1, \tag{7}$$

$$a_n = \sqrt{2 \log\log n} \quad \text{and}$$
$$b_n = 2 \log\log n + \log\log\log n.$$

$$P\left(\max_{\lfloor \beta n \rfloor \le k \le \lfloor (1-\beta) n \rfloor} \left\{ \left| \widetilde{Z}_k \right| \right\} > x \right)$$
$$\approx e^{-x^2/2} + e^{-x^2/2} x^2 \log \frac{1-\beta}{\beta}. \tag{8}$$

3. Maximum-Type Critical Values

n	Over-All Maximum-Type	Trimmed Maximum-Type		
		$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$
10	3.087	3.076	3.090	3.043
20	3.174	3.173	3.206	3.143
30	3.301	3.263	3.198	3.087
40	3.289	3.271	3.251	3.127
50	3.322	3.327	3.255	3.189
100	3.375	3.387	3.268	3.163

Table 1: Simulated 2.5% critical values of the Over-All Maximum-Type test statistic (2) and the corresponding Trimmed Maximum-Type statistics (3) for different trimming portions β.

n	Over-All Maximum-Type	Trimmed Maximum-Type		
		$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$
10	2.841	2.817	2.858	2.808
20	2.942	2.956	2.935	2.862
30	3.046	3.021	2.952	2.851
40	3.031	3.040	3.010	2.867
50	3.090	3.076	3.034	2.911
100	3.142	3.142	3.047	2.919

Table 2: Simulated 5% critical values of the Over-All Maximum-Type test statistic (2) and the corresponding Trimmed Maximum-Type statistics (3) for different trimming portions β.

n	Over-All Maximum-Type	Trimmed Maximum-Type		
		$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$
10	2.562	2.539	2.552	2.528
20	2.684	2.673	2.678	2.594
30	2.765	2.763	2.674	2.598
40	2.774	2.793	2.745	2.592
50	2.817	2.815	2.765	2.630
100	2.889	2.897	2.767	2.642

Table 3: Simulated 10% critical values of the Over-All Maximum-Type test statistic (2) and the corresponding Trimmed Maximum-Type statistics (3) for different trimming portions β .

4. Methodology

De Mast and Roes (2004) developed the subsequent control charting procedure that is illustrated as follow.

1. Determine the locations of possible shifts and test the significance of these shifts. Upon completion of this step, the original data is divided into intervals in which the mean of the measurements is presumed constant.

2. Estimate (using robust estimators) the means of the intervals between successive shifts and the variance of the in-control measurements.

3. On the basis of these estimates, estimate a pair of control limits for each interval. Points are identified as outliers if they fall beyond these control limits.

Let the in-control process be denoted as y_i, $i = 1,...,n$, with the following null model:

$$y_i = \mu_i + \varepsilon_i \,, \qquad \text{for} \quad i = 1,...,n \tag{9}$$

with ε_i i.i.d. $\mathrm{N}(0,\sigma^2)$.

Assuming that a single shift has occurred. Then model (9) is written as follows:

$$\begin{cases} y_i = \mu_1 + \varepsilon_i & \text{for} \quad i = 1,...,\tau \\ y_i = \mu_2 + \varepsilon_i & \text{for} \quad i = \tau+1,...,n \end{cases} \tag{10}$$

with ε_i i.i.d. $\mathrm{N}(0,\sigma^2)$;

The main aim is to estimate the location shift, τ. There are two change point detection methods to determine the location of shifts τ and test the significance of these shifts, namely proposed by De Mast and Roes (2004). Let refer this method as DMRCP method henceforth. In this paper, we consider the Maximum-Type change point detection method [4]. The desired critical values for testing the significance of location shifts are displayed in Tables 1-3. De Mast and Roes (2004) proposed a robust procedure to estimate μ_1 and μ_2 to accommodate the presence of possible outliers. According to De Mast and Roes (2004) robust procedure, $\hat{\mu}_1[\tau]$ and $\hat{\mu}_2[\tau]$ be the solution of

$$\sum_{i=1}^{\tau} \psi\left(\frac{(y_i - \mu_1)}{cs_0[\tau]} \right) = 0 \tag{11}$$

where

$$s_0[\tau] = median\{|y_i - m|\}_{i=1,...,n}$$

with $m = m_1[\tau]$ if $1 \le i \le \tau$ and $m = m_2[\tau]$ if $\tau + 1 \le i \le n$ for which $m_1[\tau]$ and $m_2[\tau]$ are the medians of $y_1,..., y_\tau$ and $y_{\tau+1},..., y_n$, respectively. ψ is an odd function and c is a tuning constant. It can be observed that $\hat{\mu}_1[\tau]$ and $\hat{\mu}_2[\tau]$ are the M-estimates for the location based on the scale initial estimate denoted as $s_0[\tau]$. The the final scale estimate is given by

$$\hat{\sigma}^2[\tau] = \frac{\sqrt{n}cs_0[\tau]\begin{pmatrix} \sum_{i=1}^{\tau}\psi^2((y_i - \hat{\mu}_1[\tau])/cs_0[\tau]) \\ + \sum_{i=\tau+1}^{n}\psi^2((y_i - \hat{\mu}_1[\tau])/cs_0[\tau]) \end{pmatrix}^{1/2}}{\begin{vmatrix} \sum_{i=1}^{\tau}\psi'((y_i - \hat{\mu}_1[\tau])/cs_0[\tau]) \\ + \sum_{i=\tau+1}^{n}\psi'((y_i - \hat{\mu}_1[\tau])/cs_0[\tau]) \end{vmatrix}} \tag{12}$$

with ψ' indicating the derivative of ψ . The scale estimate obtained by employing the asymptotic variance of the M-estimators for location in order to estimate the standard deviation of the error. This kind of scale estimator in Eq. (12) is called an A-estimators by Lax (1985). There are many psi function to be chosen from, and in this study, De Mast and Roes (2004) proposed to use bisquare function which is given by

$$\psi(u) = \begin{cases} u\left(1-u^2\right)^2, & |u| \le 1, \\ 0, & |u| > 1. \end{cases} \qquad (13)$$

Hence, in other words, observations further away from the mean will be more and more downweighted, and observations beyond $cs_0[\tau]$ are totally rejected. De Mast and Roes (2004) pointed out that the value $c = 9$ for the tuning constant seem to perform well.

It is important to note that once the shift has been detected, the data are split into two groups $y_1,..., y_\tau$ and $y_{\tau+1},..., y_n$. Repeating the same process to both groups, justifying whether more shifts can be detected, De Mast and Roes (2004) recommended that process will be continued until the size of the groups becomes smaller than 4 or any other selected minimum value.

Let $\hat{\mu}_1,..., \hat{\mu}_k$ be the estimated means of the groups in between the detected shifts $\hat{\tau}_2,...\hat{\tau}_k$, the standard deviation of the error is estimated by:

$$\hat{\sigma} = \frac{n(cs_0)\begin{pmatrix} \sum_{i=1}^{\hat{\tau}_1} \psi^2((y_i - \hat{\mu}_1)/cs_0) \\ + ...+ \sum_{i=\hat{\tau}_{k-1}+1}^{n} \psi^2((y_i - \hat{\mu}_k)/cs_0) \end{pmatrix}^{1/2}}{\sqrt{n-k}\left| \begin{matrix} \sum_{i=1}^{\hat{\tau}_1} \psi'((y_i - \hat{\mu}_1)/cs_0) \\ + ...+ \sum_{i=\hat{\tau}_{k-1}+1}^{n} \psi'((y_i - \hat{\mu}_k)/cs_0) \end{matrix} \right|} \qquad (14)$$

The Median Absolute Deviation (MAD) is used for the initial scale, s_0 which is defined as $s_0 = median\{y_i - m\}$ where m denoting the median of the corresponding subgroup. The factor $\sqrt{n}$ in the numerator of Eq. (14) is replaced by $\sqrt{n^2(n-k)}$ for the sake of accounting the loss of degrees of freedom in the estimation of $\mu_1,..., \mu_k$.

For each identified group, $j = 1,..., k$, De Mast and Roes (2004) proposed the following control limits:

$$UCL_j = \hat{\mu}_j + h\sqrt{\frac{\hat{\tau}_{j+1} - \hat{\tau}_j - 1}{\hat{\tau}_{j+1} - \hat{\tau}_j}}\,\hat{\sigma}$$
$$LCL_j = \hat{\mu}_j - h\sqrt{\frac{\hat{\tau}_{j+1} - \hat{\tau}_j - 1}{\hat{\tau}_{j+1} - \hat{\tau}_j}}\,\hat{\sigma} \qquad (15)$$

We define $\hat{\tau}_1 = 0$ and $\hat{\tau}_{k+1} = n$. The conventional value of $h = 3$ is employed and the factor $\sqrt{\frac{\hat{\tau}_{j+1} - \hat{\tau}_j - 1}{\hat{\tau}_{j+1} - \hat{\tau}_j}}$ is introduced to explain the connection between the y_i and the control limits.

5. Numerical Example: Hybrid Microcircuits Data

In this section, we will apply the proposed procedure to analyze a data taken from Camil and Ron (1998) for verification purposes. Raw materials used in manufacturing of hybrid microcircuits consist of components, dyes, pastes and ceramic substrates. The substrates plate undergo a process of printing and firing during which layers of conductors, dielectric, resistors and platinum or gold are added to the plates. Subsequent production steps consist of laser trimming, mounting and reflow soldering or chip bonding. The last manufacturing stage is the packaging and sealing of complete modules.

Five dimensions of substrate plates are considered here, with labels (a, b, c), (W, L). The first three are determined by the laser inscribing process. The last two are outer physical dimensions. These outer dimensions are measured on a different instrument than the first three, using a different sample substrates.

In this study, we will consider only the first dimension 'a' which measured the substrate plates. The Average Moving Range (AMR) chart and the proposed robust control charts for these individual length measurement of 'a' are presented in Figure 1 and Figure 2 respectively. The numerical results are performed using S-PLUS language.

The plot of Figure 1 indicates that there are two signals, which are observations number 19 and 31. The two signals would suggest that the length measurements is not completely stable in the sample, which is in itself a valuable suggestion of assignable cause. It can be seen that the AMR chart detects the larger outliers. It does not, however, provide indications about the presence of the shifts. The standard deviation of the in-control process is estimated to be 2.125, which seems too large. The large estimate can be explained by the fact that it includes the additional variation that caused by the shifts and by the fact that the AMR chart is not robust. The AMR chart does not provide clear information about their number or the time points on which they occur. Furthermore, the detected shifts are not incorporated into the analysis, and, as a consequence, the chart is less sensitive in detecting the remaining assignable causes.

On the other hand, the robust control chart reveals the presence of several assignable causes, three isolated disturbances occurred which are indicated by observation number 19, 20 and 31 as well as one shifts in which it happen in between observation number 18 and 19. Observation number 22 which is located very close to border line of second pair of lower control limit would suggest that the point might be a possible outlier. On the contrary, this point seems still within the control limit of the AMR chart. Removing the outliers and correcting the remaining measurements for the outliers, the process engineer is left with measurement that can be described considerably well by a normal distribution with standard deviation 1.962. This value is smaller than the the standard deviation of the AMR chart due to the fact that the effects of outlying observations have been downweighted.

Once the special cause has been identified, the necessary action can then be taken to rectify or improve the process. Figure 2 gives important indications of their characteristic and the time points on which they occurred. Based on the chart, it also implies that the measurement length has shifted between observation 18 and observation 19.

In addition to these shifts, the length measurement appeared to be in statistical control, indicating that no assignable causes should be sought on the basis of these measurements.

It seems that the robust control chart is preferred as it provides the most revealing description of this dataset.

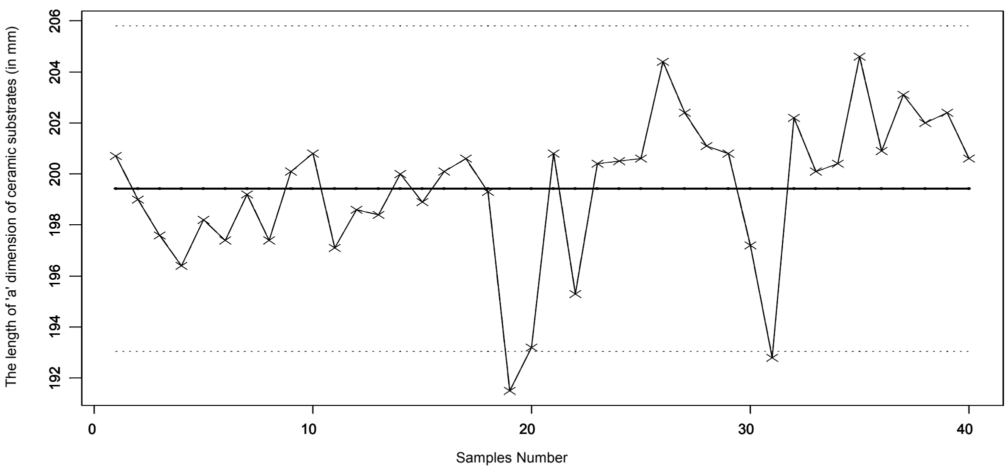

Figure 1: AMR Control Chart for Hybrid Microcircuits.

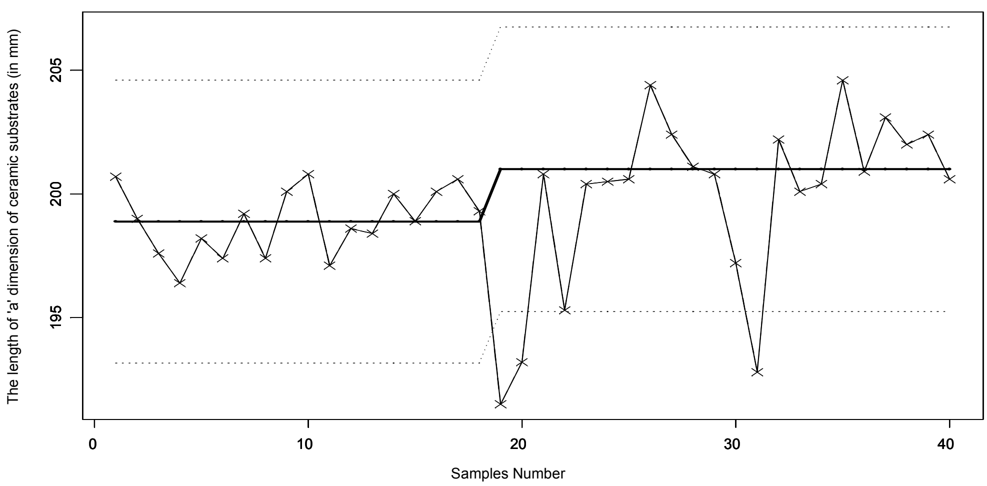

Figure 2: Proposed Robust Control Chart for Hybrid Microcircuits.

Magnitude of shift, δ	Sample size, n					
	$n=10$	$n=20$	$n=30$	$n=40$	$n=50$	$n=100$
0.5	0.1180	0.0608	0.0560	0.0474	0.0492	0.0556
1	0.2018	0.1761	0.1804	0.2005	0.2157	0.2542
1.5	0.3265	0.3512	0.3898	0.4163	0.4350	0.4605
2	0.4734	0.5442	0.5868	0.6005	0.6221	0.6307
2.5	0.6021	0.6973	0.7366	0.7521	0.7531	0.7634
3	0.7191	0.8136	0.8386	0.8395	0.8474	0.8517

Table 4: Proportion of detected shifts depending on magnitude of the shift and sample size for change in mean for trimmed portion of $\beta = 0.05$.

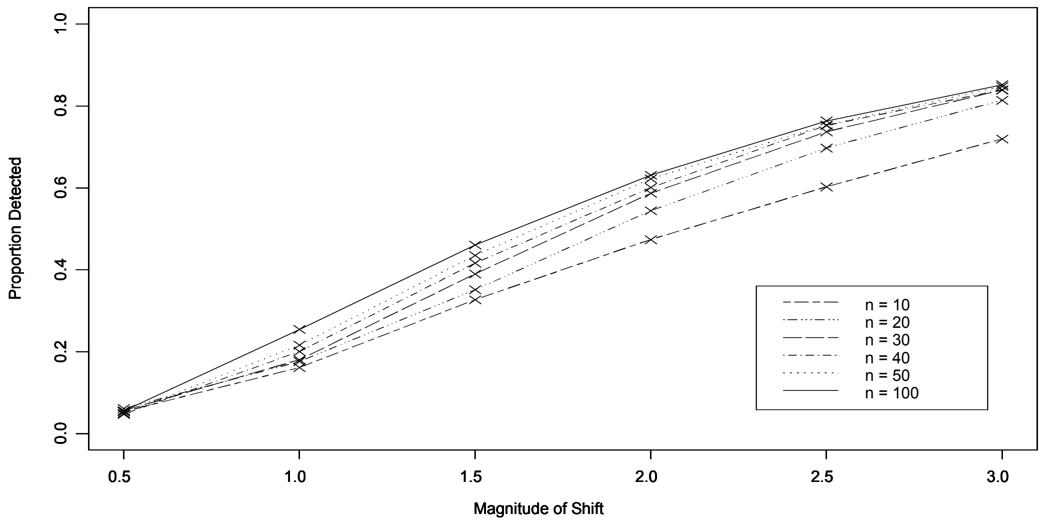

Figure 3: Proportion of detected shifts depending on magnitude of the shift and sample size for change in mean for trimmed portion of $\beta = 0.05$.

Sample Size		Magnitude of Shift, δ					
		δ =0.5	δ =1.0	δ =1.5	δ =2.0	δ =2.5	δ =3.0
n =30	$\bar{\hat{\tau}}$	14.84	15.08	15.01	14.99	14.99	15.00
	Standard Error ($\bar{\hat{\tau}}$)	0.29046	0.22691	0.14180	0.08976	0.04394	0.02450
n =50	$\bar{\hat{\tau}}$	24.94	24.99	25.02	25.00	24.98	25.01
	Standard Error ($\bar{\hat{\tau}}$)	0.55348	0.33362	0.14920	0.06274	0.03063	0.01763

Table 5: Simulation Result: Average Change Point Estimates and Associated Standard Error for Change Point for various sample sizes.

	Magnitude of Shift, δ					
	δ =0.5	δ =1.0	δ =1.5	δ =2.0	δ =2.5	δ =3.0
$\hat{P}(\hat{\tau}=\tau)$	0.0534	0.1833	0.3916	0.5910	0.7331	0.8320
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 1)$	0.1202	0.3322	0.6017	0.7991	0.9075	0.9589
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 2)$	0.1739	0.4296	0.7082	0.8782	0.9591	0.9844
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 3)$	0.2230	0.4996	0.7681	0.9172	0.9769	0.9927
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 4)$	0.2647	0.5525	0.8071	0.9388	0.9840	0.9957
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 5)$	0.3089	0.5940	0.8316	0.9517	0.9874	0.9972
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 6)$	0.3471	0.6287	0.8509	0.9591	0.9895	0.9980
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 7)$	0.3914	0.6601	0.8690	0.9652	0.9908	0.9982
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 8)$	0.4352	0.6914	0.8835	0.9700	0.9918	0.9985
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 9)$	0.4815	0.7203	0.8964	0.9734	0.9930	0.9986
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 10)$	0.6069	0.7500	0.9089	0.9797	0.9932	0.9988

Table 6: Simulation Results for Different Magnitudes of Process Change Based on 10,000 Trials, n =30.

	Magnitude of Shift, δ					
	δ =0.5	δ =1.0	δ =1.5	δ =2.0	δ =2.5	δ =3.0
$\hat{P}(\hat{\tau}=\tau)$	0.0462	0.2141	0.4336	0.6131	0.7580	0.8550
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 1)$	0.1118	0.3954	0.6743	0.8307	0.9244	0.9710
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 2)$	0.1627	0.5011	0.7859	0.9144	0.9731	0.9976
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 3)$	0.2018	0.5721	0.8493	0.9490	0.9873	0.9995
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 4)$	0.2364	0.6235	0.8855	0.9704	0.9945	0.9997
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 5)$	0.2708	0.6654	0.9097	0.9798	0.9972	0.9999
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 6)$	0.2991	0.6966	0.9275	0.9853	0.9983	1.0000
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 7)$	0.3264	0.7217	0.9371	0.9899	0.9988	
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 8)$	0.3488	0.7430	0.9454	0.9923	0.9994	
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 9)$	0.3698	0.7617	0.9519	0.9940	0.9996	
$\hat{P}(\lvert\hat{\tau}-\tau\rvert \le 10)$	0.3925	0.7764	0.9569	0.9950	1.0000	

Table 7: Simulation Results for Different Magnitudes of Process Change Based on 10,000 Trials, n =50.

Magnitude of shift, δ	Sample size, n			
	$n=30$		$n=50$	
	DMRCP	Max-Type	DMRCP	Max-Type
1	0.2031	0.2073	0.2150	0.2232
2	0.5638	0.5910	0.5738	0.6131
3	0.7986	0.8320	0.8204	0.8550

Table 8: Proportion of detected shifts depending on magnitude of the shift and sample size for change in mean using two different methods.

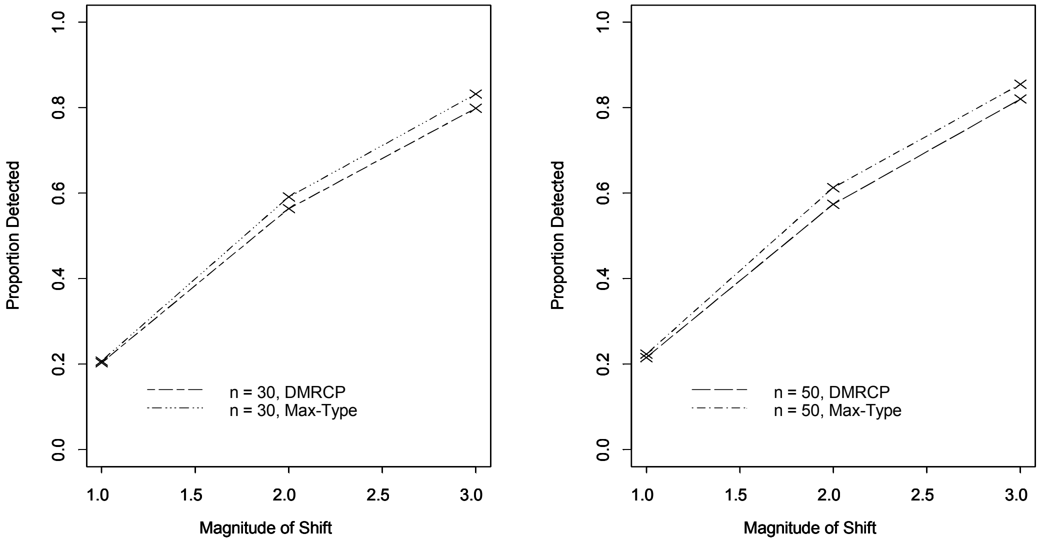

Figure 4: Proportion of detected shifts depending on magnitude of the shift and sample size for change in mean using two different methods.

6. Simulation Study

The first simulation study was conducted to assess the performances of the De Mast and Roes (2004) chart at various magnitude of shifts and sample of size 10, 20, 30, 40, 50, 100. Namely called as DMR chart hereinafter. The DMR chart is evaluated based on the proportion of correctly detected the location of shifts in the case that where there is a change in mean. The individual observations x are generated from a normal distribution with mean $\mu_0 = 0$ and $\sigma = 1$, where δ represents the size of the shift in the mean. A special case of trimmed portion $\beta = 0.05$ is chosen in this study. The size of the shift is measured in terms of process standard deviation. Hence, the size of the shift is defined as $\delta = (\mu_1 - \mu_0)/\sigma$. Without loss of generality, $\mu_0 = 0$ is the initial in-control process mean and μ_1 is the process mean after the shift, and let δ = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0. The position of the shift τ is set at middle point of the measurements. For instance, if sample is 10, then the position of the change will be set as $\tau = 5$. From Table 4, it is evident that as the magnitude of shift increases, the proportion of correctly detected shift increases accordingly. The performance of the DMR chart also improved when the sample size increases ranging from 10 to 100.

We will now analyze the performance of the change point estimator through the second simulation study. The idea is very similar, assuming that the process is initially in control, with observations coming from a normal distribution with a known mean of μ_0 and a known standard deviation of σ_0. However, after an unknown point in time τ (known as the process change point), the process location changes from μ_0 to $\mu_1 = \mu_0 \pm \delta\sigma_0$, where δ is the unknown magnitude of the change. We also assume that once this step change in the process location occurs, the process remains at the new level of μ_1, until the special cause has been identified and removed. We investigated two different sample sizes, n = 30 and n = 50. For instance, when the sample size is 50, observations were randomly generated from standard normal distribution for observations 1, 2, 3,…, 25. Then, starting with subgroup 26, observations were randomly generated from a normal distribution with mean δ and standard deviation 1. This procedure was repeated a total of 10,000 times for each of the values of δ that were studied, namely δ = 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0. For each simulation run, the change point estimator was computed. The average of the estimates from the 10,000 simulation runs was computed along with its standard error.

In Table 5, $\bar{\hat{\tau}}$ are tabulated in which the average change point estimate from the simulation runs for various sizes of change in the process mean together with its corresponding standard error estimates. As the actual change point for the simulations was at time 25, the average estimated time of the process change, $\bar{\hat{\tau}}$, should be close to 25. It is noted that when the process step change of standardized magnitude δ = 1, the average estimated time of the process change was 24.94, which is fairly close to the actual change point of 25. While for a standardized process location change of size δ = 2, the average estimated time of the change is 25.00. Meanwhile, when δ = 3, the average estimated time of the change is 25.01. Hence, on average, the Maximum-Type change point estimator of the time of the process change is considerably close to the actual time of the change, regardless of the magnitude of the change.

The observed frequency in which the estimates of the time of the step was within m observations of the actual time of the change, for m = 0, 1, 2, . . . , 10 , is shown in Table 6. This provides an indication of the precision of the estimator. The proportion of the 10,000 runs where the estimated time of the change was within $\pm m$ of the actual change should be high and should increase as m increases. Referring to table 6, we observe that the precision increases with increases in sample size n for each δ.

In the case of n = 30, when the process step change of magnitude δ = 2, the average of the estimated 10,000 change point estimates was 14.99. The change point estimator identified correctly the change point in 59.10 % of the trials. Our proposed estimate was within one observation of the actual change point in 79.91% of the trials, and within two observations of the actual change point in 87.82% of the trials.

However, when the process step changes of magnitude is δ = 3, the estimator demonstrated a good performance in identifying the time of change. The average of the estimated 10,000 change point estimates was 15.00. For step changes of this magnitude, the estimator exactly identified the time of the change in 83.00% of the trials and was within one (two) observations of the time of the actual process change in 95.89% (99.44%) of the trials.

Turning to the case of n = 50, we notice that of the 10,000 simulation trials conducted for δ = 2, 61.31% of those simulation trials identified the change point precisely. It was in 83.07% of the trials, the change point was estimated to be within ± 1 of the actual time of the process change. Also, in 91.44% of the trials, the estimate was within ± 2 observations; in 94.90% of the trials, it was within ± 3 observations; and in 99.50% of the trials, it was within ± 10 observations of the actual time of the process change.

For a process step change of magnitude δ = 3, the change point estimator identified exactly the change point in 85.50% of the trials. The estimate was within one observations of the true change point in 97.10% of the trials, and within two observations of the actual change point in 99.76% of the trials.

Lastly, in table 8, we compare the proportion of detected shifts depending on magnitude of the shift and sample size for change in mean using two different methods. This comparison is done for different standardized magnitudes of the step change in the process mean. Based on Figure 4, it appears to be that when the process readings follow a normal distribution and statistically independent. Maximum-Type method compares favorably with DMRCP method, (see De Mast and Roes (2004)) and gives a better performance if the underlying distribution of change cause is normal.

7. Conclusions

If process engineers could determine when the process changed, their search could be narrowed down to finding which aspect of the process changed at that time. This could allow them to identify the special cause more quickly, and to use that information to improve the quality of the process or product sooner. Thus, it can be seen that regardless of the magnitude of the change in the process mean, our proposed estimator produces a useful estimates of the time of the process change.

The conventional Shewhart control charts for individual measurement are widely used in the monitoring processes. However, these charts require the assumption that the process variables are normally distributed and thus, are very sensitive to the presence of occasional outliers. Hence, the philosophy of the robust individual control chart is more in keeping with the desire to provide robust limits in the face of non-normal distributions or errors in data collection. The robust control chart offers some significant advantages over existing control chart. This paper has presented a simple alternative robust univariate variable control chart by using Maximum-Type change point detection formulation for monitoring the process change in mean and variance. The robust control charting method efficiently monitor contaminated data processes and process shift. Classical charts are not a preference for process monitoring where contamination may exist.

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