# New Exact Traveling Wave Solutions For Some Non-linear Evolution Equations By $(\frac{G'}{G})$-expansion method

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*Abstract:* In this paper, we study the application of the known generalized $(\frac{G'}{G})$-expansion method for seeking more exact traveling solutions solutions and soliton solutions of the ZK-MEW equation and the (2+1) dimensional Boiti-Leon-Pempinelli equation. As a result, we come to the conclusion that the traveling wave solutions for the two non-linear equations are obtained in three arbitrary functions including hyperbolic function solutions, trigonometric function solutions and rational solutions. The method appears to be easier and faster by means of some mathematical software.

*Key–Words:* $(\frac{G'}{G})$-expansion method, Traveling wave solutions, ZK-MEW equation, (2+1) dimensional Boiti-Leon-Pempinelli equation, exact solution, evolution equation, nonlinear equation

## 1 Introduction

In the nonlinear sciences, it is well known that many nonlinear partial differential equations are widely used to describe the complex phenomena. The powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far such as in [1-7].

Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirotas bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic function method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painleve expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

In [38], Mingliang Wang proposed a new method called $(\frac{G'}{G})$-expansion method. Recently several authors have studied some nonlinear equations by this method [39-42]. The value of the $(\frac{G'}{G})$-expansion method is that one can treat nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation. The main merits of the $(\frac{G'}{G})$-expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Our aim in this paper is to present an application of the $(\frac{G'}{G})$-expansion method to some nonlinear problems to be solved by this method. The rest of the paper is organized as follows. In Section 2, we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to the ZK-MEW equation and the (2+1) dimensional Boiti-Leon-Pempinelli equation. In the last Section, the features of the $(\frac{G'}{G})$-expansion method are briefly summarized.

## 2 Description of the $(\frac{G'}{G})$-expansion method

In this section we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables $x,\ t$, is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \qquad (2.1)$$

or in three independent variables $x,\ y$ and $t$, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, ...) = 0, \qquad (2.2)$$

where $u = u(x,t)$ or $u = u(x,y,t)$ is an unknown function, $P$ is a polynomial in $u = u(x,t)$ or $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Suppose that

$$u(x,\ t) = u(\xi), \quad \xi = \xi(x,t) \qquad (2.3)$$

or

$$u(x,\ y,\ t) = u(\xi), \quad \xi = \xi(x,y,t) \qquad (2.4)$$

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for $u = u(\xi)$

$$P(u,\ u', u'',\ ...) = 0. \qquad (2.5)$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m(\frac{G'}{G})^m +\ ... \qquad (2.6)$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \qquad (2.7)$$

$\alpha_m, ..., \lambda$ and $\mu$ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (2.6) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m-1$. The positive integer $m$ can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (2.5) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ..., \lambda$ and $\mu$.

Step 4. Assuming that the constants $\alpha_m, ..., \lambda$ and $\mu$ can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting $\alpha_m, ...$ and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

## 3 Application Of The $(\frac{G'}{G})$-Expansion Method For The ZK-MEW Equation

In the following two sections, we will apply the $(\frac{G'}{G})$-expansion method to construct the traveling wave solutions for some nonlinear partial differential equations in mathematical physics.

We begin with the ZK-MEW equation [43]:

$$u_t + a(u^3)_x + (bu_{xt} + ru_{yy})_x = 0 \qquad (3.1)$$

where $a,\ b$ and $r$ are known constants.

In order to obtain the traveling wave solutions of Eq.(3.1), we suppose that

$$u(x,\ y,\ t) = u(\xi),\ \xi = x + y - Vt \qquad (3.2)$$

$V$ is a constant that to be determined later.

By using the wave variable (3.2), (3.1) is converted into an ODE

$$-Vu' + a(u^3)' - bVu''' + ru''' = 0 \qquad (3.3)$$

Integrating (3.3) with respect to $\xi$ once, we obtain

$$C - Vu + au^3 + (r - bV)u'' = 0 \qquad (3.4)$$

where $C$ is the integration constant that can be determined later.

Suppose that the solution of (3.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i (\frac{G'}{G})^i \qquad (3.5)$$

where $a_i$ are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \qquad (3.6)$$

where $\lambda$ and $\mu$ are constants.

Balancing the order of $u''$ and $u^3$ in Eq.(3.4), we have

$$m + 2 = 3m \ \Rightarrow\ m = 1$$

So Eq.(3.5) can be rewritten as

$$u(\xi) = a_1(\frac{G'}{G}) + a_0,\ a_1 \neq 0 \qquad (3.7)$$

$a_1,\ a_0$ are constants to be determined later. Then we can obtain

$$u'(\xi) = a_1[-\lambda(\frac{G'}{G}) - \mu - (\frac{G'}{G})^2]$$

$$u''(\xi) = 2a_1(\frac{G'}{G})^3 + 3\lambda a_1(\frac{G'}{G})^2$$
$$+(\lambda^2 a_1 + 2a_1\mu)(\frac{G'}{G}) + \lambda\mu a_1$$

Substituting (3.7) into (3.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0 :\ C - Va_0 + aa_0^3 + (r - bV)\lambda\mu a_1 = 0$$

$$(\frac{G'}{G})^1 :\ 3aa_1a_0^2 + (r - bV)(\lambda^2 a_1 + 2a_1\mu)$$
$$-a_1V = 0$$

$$(\frac{G'}{G})^2 :\ 3aa_0a_1^2 + 3\lambda a_1(r - bV) = 0$$

$$(\frac{G'}{G})^3 :\ aa_1^3 + 2(r - bV)a_1 = 0$$

Solving the algebraic equations above, yields two different cases in view of the positive or negative of $\frac{r}{a[b(\lambda^2 - 4\mu) - 2]}$:

Case (I): If $\frac{r}{a[b(\lambda^2 - 4\mu) - 2]} > 0$, then

$$a_1 = \pm 2\sqrt{\frac{r}{a[b(\lambda^2 - 4\mu) - 2]}}$$

$$a_0 = \pm \lambda\sqrt{\frac{r}{a[b(\lambda^2 - 4\mu) - 2]}}$$

$$V = \frac{r(\lambda^2 - 4\mu)}{b(\lambda^2 - 4\mu) - 2}$$

$$C = 0 \qquad (3.8)$$

Substituting (3.8) into (3.7), we have

$$u(\xi) = \pm 2\sqrt{\frac{r}{a[b(\lambda^2 - 4\mu) - 2]}}\left(\frac{G'}{G}\right) \pm\lambda\sqrt{\frac{r}{a[b(\lambda^2 - 4\mu) - 2]}}$$

$$\xi = x + y - \frac{r(\lambda^2 - 4\mu)}{b(\lambda^2 - 4\mu) - 2}t \qquad (3.9)$$

Substituting the general solutions of Eq.(3.6) into (3.9), we have three types of traveling wave solutions of the ZK-MEW equation (3.1) as follows:

When $\lambda^2 - 4\mu > 0$

$$u_{1,2}(\xi) = \pm\sqrt{\frac{r(\lambda^2 - 4\mu)}{a[b(\lambda^2 - 4\mu) - 2]}} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)$$

where

$$\xi = x + y - \frac{r(\lambda^2 - 4\mu)}{b(\lambda^2 - 4\mu) - 2}t$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if

$C_1 = 1,\ C_2 = 0,\ \mu = 0,\ \lambda = 2,\ a = 2,\ b = 1,\ r = 1,$

then we can obtain the exact traveling wave solutions as follows

$$u(x, y, t) = \pm\tanh(x + y - 2t).$$

When $\lambda^2 - 4\mu < 0$

$$u_{3,4}(\xi) = \pm\sqrt{\frac{r(4\mu - \lambda^2)}{a[b(\lambda^2 - 4\mu) - 2]}} \times \left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2)}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2)}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2)}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2)}\xi}\right)$$

where

$$\xi = x + y - \frac{r(\lambda^2 - 4\mu)}{b(\lambda^2 - 4\mu) - 2}t$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if $C_1 = 1,\ C_2 = 0,\ \lambda^2 - 4\mu = -4,\ a = 2,\ b = -1,\ r = 1$, then it is obvious that

$$u(x, y, t) = \pm\tan(x + y + 2t).$$

When $\lambda^2 - 4\mu = 0$

$$u_5(\xi) = \pm\lambda\sqrt{\frac{r}{-2a}} \pm 2\sqrt{\frac{r}{-2a}}\left[\frac{2C_2 - C_1\lambda - C_2\lambda\xi}{2(C_1 + C_2\xi)}\right]$$

where

$$\xi = x + y$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if $C_1 = C_2 = 1,\ \mu = 1,\ \lambda = 2,\ a = -1,\ r = 2$, then we have

$$u(x, y, t) = \pm 2\frac{1}{1 + x + y}.$$

Case (II): If $\frac{r}{a[b(\lambda^2 - 4\mu) - 2]} < 0$, then

$$a_1 = \pm 2i\sqrt{\frac{-r}{a[b(\lambda^2 - 4\mu) - 2]}}$$

$$a_0 = \pm\lambda i\sqrt{\frac{-r}{a[b(\lambda^2 - 4\mu) - 2]}}$$

$$V = \frac{r(\lambda^2 - 4\mu)}{b(\lambda^2 - 4\mu) - 2}$$

$$C = 0 \qquad (3.10)$$

Substituting (3.10) into (3.7), we have

When $\lambda^2 - 4\mu > 0$

$$u_{1,2}(\xi) = \pm i\sqrt{\frac{-r(\lambda^2-4\mu)}{a[b(\lambda^2-4\mu)-2]}}\times$$
$$\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)$$

where

$$\xi = x + y - \frac{r(\lambda^2-4\mu)}{b(\lambda^2-4\mu)-2}t$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if

$C_1 = 1,\ C_2 = 0,\ \mu = 0,\ \lambda = 2,\ a = 2,\ b = 1,\ r = -1,$

then we can obtain the exact traveling wave solutions as follows

$$u(x,y,t) = \pm i\tanh(x+y+2t).$$

When $\lambda^2 - 4\mu < 0$

$$u_{3,4}(\xi) = \pm i\sqrt{\frac{r(\lambda^2-4\mu)}{a[b(\lambda^2-4\mu)-2]}}\times$$
$$\left(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2)}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2)}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2)}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2)}\xi}\right)$$

where

$$\xi = x + y - \frac{r(\lambda^2-4\mu)}{b(\lambda^2-4\mu)-2}t$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if $C_1 = 1,\ C_2 = 0,\ \lambda^2 - 4\mu = -4,\ a = 2,\ b = -1,\ r = -1$, then it is obvious that

$$u(x,y,t) = \pm i\tan(x+y-2t).$$

When $\lambda^2 - 4\mu = 0$

$$u_5(\xi) = \pm\lambda i\sqrt{\frac{r}{2a}} \pm 2i\sqrt{\frac{r}{2a}}.$$
$$\left[\frac{2C_2 - C_1\lambda - C_2\lambda\xi}{2(C_1+C_2\xi)}\right]$$

where

$$\xi = x + y$$

$C_1$ and $C_2$ are two arbitrary constants. In particular, if $C_1 = C_2 = 1,\ \mu = 1,\ \lambda = 2,\ a = 1,\ r = 2$, then we have

$$u(x,y,t) = \pm 2i\frac{1}{1+x+y}.$$

# 4 Application Of The $(\frac{G'}{G})$-Expansion Method For The (2+1) Dimensional Boiti-Leon-Pempinelli Equation

We consider the (2+1) dimensional Boiti-Leon-Pempinelli equation:

$$u_{ty} = (u^2 - u_x)_{xy} + 2v_{xxx} \qquad (4.1)$$

$$v_t = v_{xx} + 2uv_x \qquad (4.2)$$

Supposing that

$$\xi = k(x + y - ct) \qquad (4.3)$$

By (4.3), (4.1) and (4.2) are converted into ODEs

$$-cu'' = (u^2)'' - ku''' + 2kv''' \qquad (4.4)$$

$$-cv' = kv'' + 2uv' \qquad (4.5)$$

Suppose that the solution of (4.4) and (4.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i(\frac{G'}{G})^i \qquad (4.6)$$

$$v(\xi)=\sum_{i=0}^{n}b_i(\frac{G'}{G})^i \qquad (4.7)$$

where $a_i, b_i$ are constants, $G=G(\xi)$ satisfies the second order LODE in the form:

$$G''+\lambda G'+\mu G=0 \qquad (4.8)$$

where $\lambda$ and $\mu$ are constants.

Balancing the order of $u'''$ and $v'''$ in Eq.(4.6), the order of $v''$ and $uv'$ in Eq.(4.7), then we can obtain $m+3=n+3,\ n+2=m+n+1 \Rightarrow\ m=n=1$, so Eq.(4.6) and (4.7) can be rewritten as

$$u(\xi)=a_1(\frac{G'}{G})+a_0,\ a_1\neq 0 \qquad (4.9)$$

$$v(\xi)=b_1(\frac{G'}{G})+b_0,\ b_1\neq 0 \qquad (4.10)$$

$a_1,\ a_0,\ b_1,\ b_0$ are constants to be determined later.

Substituting (4.9) and (4.10) into (4.4) and (4.5) and collecting all the terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq.(4.4):

$$(\frac{G'}{G})^0:\ 2kb_1\lambda^2\mu-2ka_1\mu^2-ca_1\lambda\mu$$
$$-ka_1\lambda^2\mu-2a_1a_0\lambda\mu+4kb_1\mu^2-2a_1^2\mu^2=0$$

$$(\frac{G'}{G})^1:\ -2ca_1\mu-ca_1\lambda^2-6a_1^2\lambda\mu$$
$$+2kb_1\lambda^3+16kb_1\lambda\mu-4a_1a_0\mu$$
$$-ka_1\lambda^3-2a_1a_0\lambda^2-8ka_1\lambda\mu=0$$

$$(\frac{G'}{G})^2:\ 14kb_1\lambda^2-6a_1a_0\lambda-8a_1^2\mu$$
$$-7ka_1\lambda^2-8ka_1\mu-4a_1^2\lambda^2$$
$$-3ca_1\lambda+16kb_1\mu=0$$

$$(\frac{G'}{G})^3:\ -12ka_1\lambda-2ca_1-4a_1a_0$$
$$-10a_1^2\lambda+24kb_1\lambda=0$$

$$(\frac{G'}{G})^4:\ -6ka_1+12kb_1-6a_1^2=0$$

For Eq.(4.5):

$$(\frac{G'}{G})^0:\ -kb_1\lambda\mu+cb_1\mu+2b_1a_0\mu=0$$

$$(\frac{G'}{G})^1:\ -2kb_1\mu-kb_1\lambda^2+cb_1\lambda$$
$$+2b_1a_0\lambda+2b_1a_1\mu=0$$

$$(\frac{G'}{G})^2:\ cb_1+2b_1a_1\lambda-3kb_1\lambda+2b_1a_0=0$$

$$(\frac{G'}{G})^3:\ -2kb_1+2b_1a_1=0$$

Solving the algebraic equations above yields:

$$a_1=k,\ a_0=a_0$$

$$b_1=k,\ b_0=b_0$$

$$k=k,\ c=k\lambda-2a_0 \qquad (4.11)$$

where $a_0, b_0, k$ are arbitrary constant, $k \neq 0$.

Substituting (4.11) into (4.9) and (4.10), yields:

$$u(\xi) = k(\frac{G'}{G}) + a_0 \qquad (4.12)$$

$$v(\xi) = k(\frac{G'}{G}) + b_0 \qquad (4.13)$$

where

$$\xi = k[x + y - (k\lambda - 2a_0)t]$$

Substituting the general solutions of (4.8) into (4.12) and (4.13), we have:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{\lambda^2 - 4\mu}}{2}$$

$$\cdot \left( \frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right)$$

$$+a_0$$

$$v_1(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{\lambda^2 - 4\mu}}{2}$$

$$\cdot \left( \frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right)$$

$$+b_0$$

Where

$$\xi = k[x + y - (k\lambda - 2a_0)t],$$

$a_0, b_0, k$ are arbitrary constant, $k \neq 0$.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{4\mu - \lambda^2}}{2}$$

$$\cdot \left( \frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right)$$

$$+a_0$$

$$v_2(\xi) = -\frac{k\lambda}{2} + \frac{k\sqrt{4\mu - \lambda^2}}{2}$$

$$\left( \frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right)$$

$$+b_0$$

Where

$$\xi = k[x + y - (k\lambda - 2a_0)t],$$

$a_0, b_0, k$ are arbitrary constant, $k \neq 0$.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{2k(C_1 + C_2\xi)} + a_0$$

$$v_3(\xi) = \frac{k(2C_2 - C_1\lambda - C_2\lambda\xi)}{2(C_1 + C_2\xi)} + b_0$$

Where

$$\xi = k[x + y - (k\lambda - 2a_0)t],$$

$a_0, b_0, k$ are arbitrary constant, $k \neq 0$.

## 5 Conclusions

In this paper we have seen that the traveling wave solutions of the ZK-MEW equation and the (2+1) dimensional Boiti-Leon-Pempinelli equation are successfully found by using the $(\frac{G'}{G})$-expansion method. Now we briefly summarize the method in the following.

The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an $m$-th degree polynomial in $(\frac{G'}{G})$, where $G = G(\xi)$ is the general solutions of a second order LODE. The positive integer $m$ is determined by the homogeneous balance between the highest order derivatives and nonlinear terms appearing in the reduced ODE, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the method.

Compared to the methods used before, one can see that this method is direct, concise and effective. As we can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equations resulted, so we can also avoids tedious calculations. This method can also be used to many other nonlinear equations.

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