Exact Solutions For Two Non-linear Equations

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Abstract: In this paper, we test the validity and reliability of the $(\frac{G'}{G})$-expansion method by applying it to get the exact traveling wave solutions of the (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation and the nonlinear heat conduction equation. The traveling wave solutions are obtained in three forms. Being concise and less restrictive, the method can also be applied to many other nonlinear partial differential equations.

Key–Words: $(\frac{G'}{G})$-expansion method, Traveling wave solutions, (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation, nonlinear heat conduction equation, exact solution, evolution equation, nonlinear equation

1 Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many non-linear evolution equations(NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far such as in [1-7].

In this paper, we pay attention to the analytical method for getting the exact solution of NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the homogeneous balance method [8,9], the hyperbolic tangent expansion method [10,11], the trial function method [12], the tanh-method [13-15], the non-linear transform method [16], the inverse scattering transform [17], the Backlund transform [18,19], the Hirotas bilinear method [20,21], the generalized Riccati equation [22,23], the Weierstrass elliptic function method [24], the theta function method [25-27], the sineCcosine method [28], the Jacobi elliptic function expansion [29,30], the complex hyperbolic function method [31-33], the truncated Painleve expansion [34], the F-expansion method [35], the rank analysis method [36], the exp-function expansion method [37] and so on.

In [38], Mingliang Wang proposed a new method called $(\frac{G'}{G})$-expansion method. Recently several authors have studied some nonlinear equations by this method [39-42]. The value of the $(\frac{G'}{G})$-expansion method is that one can treat nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation. The main merits of the $(\frac{G'}{G})$-expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Our aim in this paper is to present an application of the $(\frac{G'}{G})$-expansion method to some nonlinear problems to be solved by this method for the first time. The rest of the paper is organized as follows. In Section 2, we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to the (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation and the nonlinear heat conduction equation. In section 5, the features of the $(\frac{G'}{G})$-expansion method are briefly summarized.

2 Description of the $(\frac{G'}{G})$-expansion method

In this section we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables $x,\ t$, is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \tag{2.1}$$

or in three independent variables $x,\ y$ and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, ...) = 0, \tag{2.2}$$

where $u = u(x,t)$ or $u = u(x,y,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ or $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, we will give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Suppose that

$$u(x,\ t) = u(\xi), \quad \xi = \xi(x,t) \tag{2.3}$$

or

$$u(x,\ y,\ t) = u(\xi), \quad \xi = \xi(x,y,t) \tag{2.4}$$

The traveling wave variable (2.3) or (2.4) permits us reducing (2.1) or (2.2) to an ODE for $u = u(\xi)$

$$P(u,\ u', u'',\ ...) = 0. \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m (\frac{G'}{G})^m +\ ... \tag{2.6}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{2.7}$$

$\alpha_m, ..., \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (2.6) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using second order LODE (2.7), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of (2.5) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ..., \lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m, ..., \lambda$ and μ can be obtained by solving the algebraic equations in Step 3. Since the general solutions of the second order LODE (2.7) have been well known for us, then substituting $\alpha_m, ...$ and the general solutions of (2.7) into (2.6) we have traveling wave solutions of the nonlinear evolution equation (2.1) or (2.2).

3 Application Of The $(\frac{G'}{G})$-Expansion Method For The (2+1) Dimensional Boussinesq And Kadomtsev-Petviashvili Equation

In this section we will apply the $(\frac{G'}{G})$-expansion method for getting the exact traveling wave solutions of a kind of nonlinear equation. We consider the (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation:

$$u_y = q_x \tag{3.1}$$

$$v_x = q_y \tag{3.2}$$

$$q_t = q_{xxx} + q_{yyy} + 6(qu)_x + 6(qv)_y \tag{3.3}$$

In order to obtain the traveling wave solutions of (3.1), (3.2) and (3.3), similar to the section 3, we suppose that

$$u(x,\ y,\ t) = u(\xi),\ v(x,\ y,\ t) = u(\xi)$$

$$q(x,\ y,\ t) = u(\xi),\ \ \xi = ax + dy - ct \qquad (3.4)$$

a, d, c are constants that to be determined later.

By using the wave variable (3.4), Eq.(3.1)-(3.3) can be converted into ODEs

$$du' - aq' = 0 \qquad (3.5)$$

$$av' - dq' = 0 \qquad (3.6)$$

$$(a^3 + d^3)q''' - cq' - 6auq'$$
$$-6aqu' - 6dvq' - 6dqv' = 0 \qquad (3.7)$$

Integrating the ODEs above, we obtain

$$du - aq = g_1 \qquad (3.8)$$

$$av - dq = g_2 \qquad (3.9)$$

$$(a^3 + d^3)q'' - cq - 6auq - 6dvq = g_3 \qquad (3.10)$$

Supposing that the solution of the ODEs above can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{l} a_i (\frac{G'}{G})^i \qquad (3.11)$$

$$v(\xi) = \sum_{i=0}^{m} b_i (\frac{G'}{G})^i \qquad (3.12)$$

$$q(\xi) = \sum_{i=0}^{n} c_i (\frac{G'}{G})^i \qquad (3.13)$$

where a_i, b_i, c_i are constants and $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \qquad (3.14)$$

where λ and μ are constants.

Balancing the order of u' and q' in Eq.(3.8), the order of v' and q' in Eq.(3.9) and the order of q''' and vq' in Eq.(3.10), we have $l + 1 = n + 1,\ m + 1 = n + 1,\ n + 3 = m + n + 1\ \Rightarrow\ l = m = n = 2$. So Eq.(3.11)-(3.13) can be rewritten as

$$u(\xi) = a_2(\frac{G'}{G})^2 + a_1(\frac{G'}{G}) + a_0,\ a_2 \neq 0 \qquad (3.15)$$

$$v(\xi) = b_2(\frac{G'}{G})^2 + b_1(\frac{G'}{G}) + b_0,\ b_2 \neq 0 \qquad (3.16)$$

$$u(\xi) = c_2(\frac{G'}{G})^2 + c_1(\frac{G'}{G}) + c_0,\ c_2 \neq 0 \qquad (3.17)$$

$a_i,\ b_i,\ c_i$ are constants to be determined later.

Substituting Eq.(3.15)-(3.17) into the ODEs (3.8)-(3.10), collecting all terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

For Eq.(3.8)

$$(\frac{G'}{G})^0 :\ a_0 d - ac_0 - g_1 = 0$$

$$(\frac{G'}{G})^1 :\ a_1 d - ac_1 = 0$$

$$(\frac{G'}{G})^2 :\ a_2 d - ac_2 = 0$$

For Eq.(3.9)

$$(\frac{G'}{G})^0 :\ ab_0 - g_2 - dc_0 = 0$$

$$(\frac{G'}{G})^1 :\ ab_1 - dc_1 = 0$$

$$(\frac{G'}{G})^2:\ dc_2 - ab_2 = 0$$

For Eq.(3.10)

$$(\frac{G'}{G})^0:\ -g_3 + d^3c_1\lambda\mu - cc_0$$
$$+2a^3c_2\mu^2 - 6db_0c_0 + 2d^3c_2\mu^2$$
$$-6aa_0c_0 + a^3c_1\lambda\mu = 0$$

$$(\frac{G'}{G})^1:\ 6a^3c_1\lambda\mu + a^3c_1\lambda^2 + 6d^3c_2\lambda\mu$$
$$-6aa_0c_1 + 2a^3c_1\mu - 6db_0c_1$$
$$-cc_1 + d^3c_1\lambda^2 + 2d^3c_1\mu$$
$$-6db_1c_0 - 6aa_1c_0 = 0$$

$$(\frac{G'}{G})^2:\ -6aa_2c_0 + 3a^3c_1\lambda + 4d^3c_2\lambda^2$$
$$-6aa_0c_2 + 3d^3c_1\lambda - 6db_2c_0$$
$$-6aa_1c_1 + 8d^3c_2\mu - 6db_1c_1$$
$$-6db_0c_2 + 4a^3c_2\lambda^2 - cc_2 + 8a^3c_2\mu = 0$$

$$(\frac{G'}{G})^3:\ 10d^3c_2\lambda - 6aa_2c_1 - 6aa_1c_2$$
$$+2d^3c_1 - 6db_2c_1 + 2a^3c_1$$
$$-6db_1c_2 + 10a^3c_2\lambda = 0$$

$$(\frac{G'}{G})^4:\ -6aa_2c_2 + 6d^3c_2 - 6db_2c_2 = 0$$

Solving the algebraic equations above yields:

$$a_2 = a^2,\ a_1 = a^2\lambda,\ a_0 = a_0$$

$$b_2 = d^2,\ b_1 = d^2\lambda,\ b_0 = b_0$$

$$c_2 = ad,\ c_1 = ad\lambda,\ c_0 = c_0$$

$$g_1 = da_0 - ac_0, g_2 = ab_0 - dc_0$$

$$g_3 = \frac{1}{ad}(d^5\lambda^2a^2\mu - c_0a^4\lambda^2d - 8c_0a^4d\mu$$
$$+6a^3c_0^2 - c_0d^4\lambda^2a - 8c_0d^4\mu a$$
$$+6d^3c_0^2 + 2a^5d^2\mu^2 + 2d^5a^2\mu^2$$
$$+a^5\lambda^2d^2\mu)$$

$$c = -\frac{1}{ad}(-a^4\lambda^2d - 8a^4\mu d + 6a^3c_0$$
$$+6a^2a_0d - a\lambda^2d^4 - 8d^4\mu a$$
$$+6d^2b_0a + 6d^3c_0)$$

where $a_0, b_0, c_0, \lambda, \mu$ are arbitrary constants.

Substituting the results above into (3.15)-(3.17), we have

$$u(\xi) = a^2(\frac{G'}{G})^2 + a^2\lambda(\frac{G'}{G}) + a_0$$

$$v(\xi) = d^2(\frac{G'}{G})^2 + d^2\lambda(\frac{G'}{G}) + b_0$$

$$q(\xi) = ad(\frac{G'}{G})^2 + ad\lambda(\frac{G'}{G}) + c_0$$

$$\xi = x + y + \frac{1}{ad}(-a^4\lambda^2d - 8a^4\mu d$$
$$+6a^3c_0 + 6a^2a_0d - a\lambda^2d^4$$
$$-8d^4\mu a + 6d^2b_0a + 6d^3c_0)t \qquad (3.18)$$

Substituting the general solutions of Eq.(3.6) into (3.18), we have three types of traveling wave solutions of the (2+1) dimensional BKP equation (3.1)-(3.3) as follows:

When $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = -\frac{a^2\lambda^2}{4} + \frac{a^2}{4}(\lambda^2 - 4\mu).$$
$$\left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2$$
$$+a_0$$

$$v_1(\xi) = -\frac{d^2\lambda^2}{4} + \frac{d^2}{4}(\lambda^2 - 4\mu).$$
$$\left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2$$
$$+b_0$$

$$q_1(\xi) = -\frac{ad\lambda^2}{4} + \frac{ad}{4}(\lambda^2 - 4\mu).$$
$$\left(\frac{C_1 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh \frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2$$
$$+c_0$$

where

$$\xi = x + y + \frac{1}{ad}(-a^4\lambda^2 d - 8a^4\mu d$$
$$+6a^3c_0 + 6a^2a_0d - a\lambda^2d^4$$
$$-8d^4\mu a + 6d^2b_0a + 6d^3c_0)t$$

a_0, b_0, c_0, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = -\frac{a^2\lambda^2}{4} + \frac{a^2}{4}(4\mu - \lambda^2).$$
$$\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2$$
$$+a_0$$

$$v_2(\xi) = -\frac{d^2\lambda^2}{4} + \frac{d^2}{4}(4\mu - \lambda^2).$$
$$\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2$$
$$+b_0$$

$$q_2(\xi) = -\frac{ad\lambda^2}{4} + \frac{ad}{4}(4\mu - \lambda^2).$$
$$\left(\frac{-C_1 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin \frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2$$
$$+c_0$$

where

$$\xi = x + y + \frac{1}{ad}(-a^4\lambda^2 d - 8a^4\mu d$$
$$+6a^3c_0 + 6a^2a_0d - a\lambda^2d^4$$
$$-8d^4\mu a + 6d^2b_0a + 6d^3c_0)t$$

. a_0, b_0, c_0, C_1, C_2 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{a_2^2C_2^2}{(C_1 + C_2\xi)^2} - \frac{a^2\lambda^2}{4} + a_0$$

$$v_3(\xi) = \frac{d_2^2C_2^2}{(C_1 + C_2\xi)^2} - \frac{d^2\lambda^2}{4} + b_0$$

$$q_3(\xi) = \frac{adC_2^2}{(C_1 + C_2\xi)^2} - \frac{ad\lambda^2}{4} + c_0$$

where

$$\xi = x + y + \frac{1}{ad}(-a^4\lambda^2 d - 8a^4\mu d$$
$$+6a^3c_0 + 6a^2a_0d - a\lambda^2d^4$$
$$-8d^4\mu a + 6d^2b_0a + 6d^3c_0)t$$

a_0, b_0, c_0, C_1, C_2 are arbitrary constants.

In particular, when $\lambda > 0, \mu = 0, C_1 \neq 0, C_2 = 0$, we can deduce the soliton solutions of the (2+1) dimensional Boussinesq and Kadomtsev-Petviashvili equation as:

$$u(\xi) = \frac{a^2\lambda^2}{4}[sech^2(\tfrac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi) - 2] + a_0$$
$$v(\xi) = \frac{b^2\lambda^2}{4}[sech^2(\tfrac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi) - 2] + b_0$$
$$q(\xi) = \frac{ad\lambda^2}{4}[sech^2(\tfrac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi) - 2] + c_0$$

4 Application Of The $(\frac{G'}{G})$-Expansion Method For The Nonlinear Heat Conduction Equation

We consider the nonlinear heat conduction equation first:

$$u_t - (u^2)_{xx} = pu - qu^2 \qquad (4.1)$$

where $p,\ q$ are known constants.

In order to obtain the traveling wave solutions of Eq.(4.1), we suppose that

$$u(x,\ y) = u(\xi),\ \xi = kx + \omega t \qquad (4.2)$$

k, ω are constants that to be determined later.

By using the wave variable (4.2), (4.1) is converted into an ODE

$$\omega u' - k^2(u^2)'' = pu - qu^2 \qquad (4.3)$$

Suppose that the solution of (4.3) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i(\frac{G'}{G})^i \qquad (4.4)$$

where a_i are constants.

Balancing the order of u' and $(u^2)''$ in Eq.(4.3), we have $m + 1 = 2m + 2 \ \Rightarrow\ m = -1$. So we make a variable $u = v^{-1}$, then (4.3) is converted into

$$-\omega v^2 v' - 6k^2(v')^2 + 2k^2 vv'' = pv^3 - qv^2 \quad (4.5)$$

Suppose that the solution of (4.5) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$v(\xi) = \sum_{i=0}^{n} b_i(\frac{G'}{G})^i \qquad (4.6)$$

where b_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \qquad (4.7)$$

where λ and μ are constants.

Balancing the order of v^2v' and vv'' in Eq.(4.5), we have $2n + n + 1 = n + n + 2 \ \Rightarrow\ n = 1$. So Eq.(4.6) can be rewritten as

$$v(\xi) = b_1(\frac{G'}{G}) + b_0,\ b_1 \neq 0 \qquad (4.8)$$

$b_1,\ b_0$ are constants to be determined later.

Substituting (4.6) into (4.5) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0:\ qb_0^2 - 6k^2b_1^2\mu^2 + \omega b_1 b_0^2\mu$$
$$-pb_0^3 + 2k^2b_0b_1\lambda\mu = 0$$

$$(\frac{G'}{G})^1:\ 2\omega b_1^2b_0\mu + \omega b_1b_0^2\lambda - 10k^2b_1^2\lambda\mu$$
$$+2k^2b_0b_1\lambda^2 + 4k^2b_0b_1\mu - 3pb_1b_0^2$$
$$+2qb_1b_0 = 0$$

$$(\frac{G'}{G})^2:\ 2\omega b_1^2b_0\lambda + \omega b_1b_0^2 + qb_1^2$$
$$-4k^2b_1^2\lambda^2 + 6k^2b_0b_1\lambda - 8k^2b_1^2\mu$$
$$-3pb_1^2b_0 + \omega b_1^3\mu = 0$$

$$(\frac{G'}{G})^3:\ -6k^2b_1^2\lambda + 2\omega b_1^2b_0 + 4k^2b_0b_1$$
$$-pb_1^3 + \omega b_1^3\lambda = 0$$

$$(\frac{G'}{G})^4: \ \omega b_1^3 - 2k^2 b_1^2 = 0$$

Solving the algebraic equations above, yields:

Case 1: when $\lambda^2 - 4\mu > 0$

$$b_1 = \pm\frac{q}{p}\sqrt{\frac{1}{\lambda^2-4\mu}}$$

$$b_0 = \frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{\lambda^2-4\mu}}\lambda\right]$$

$$k = \pm\sqrt{\frac{q}{4\lambda^2-16\mu}},$$

$$\omega = \pm\frac{p\sqrt{\lambda^2-4\mu}}{2(-\lambda^2+4\mu)}$$

So (4.8) is rewritten as:

$$v(\xi) = \pm\frac{q}{p}\sqrt{\frac{1}{\lambda^2-4\mu}}(\frac{G'}{G}) + \frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{\lambda^2-4\mu}}\lambda\right]$$

$$\xi = \pm\sqrt{\frac{q}{4\lambda^2-16\mu}}x \pm \frac{p\sqrt{\lambda^2-4\mu}}{2(-\lambda^2+4\mu)}t \qquad (4.9)$$

Substituting the general solutions of Eq.(4.7) into (4.9), we have

$$v_1(\xi) = \mp\frac{q\lambda}{2p}\sqrt{\frac{1}{\lambda^2-4\mu}} \pm \frac{q}{2p}$$

$$\cdot\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)$$

$$+\frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{\lambda^2-4\mu}}\lambda\right]$$

Then

$$u_1(\xi) = (v_1(\xi))^{-1}.$$

Where

$$\xi = \xi = \pm\sqrt{\frac{q}{4\lambda^2-16\mu}}x \pm \frac{p\sqrt{\lambda^2-4\mu}}{2(-\lambda^2+4\mu)}t,$$

C_1 and C_2 are two arbitrary constants.

Case 2: when $\lambda^2 - 4\mu < 0$

$$b_1 = \pm\frac{q}{p}\sqrt{\frac{1}{4\mu-\lambda^2}}i$$

$$b_0 = \frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{4\mu-\lambda^2}}\lambda i\right]$$

$$k = \pm\sqrt{\frac{q}{-4\lambda^2+16\mu}}i,$$

$$\omega = \pm\frac{p\sqrt{4\mu-\lambda^2}}{2(-\lambda^2+4\mu)}i$$

So (4.8) is rewritten as:

$$v(\xi) = \pm\frac{q}{p}\sqrt{\frac{1}{4\mu-\lambda^2}}i(\frac{G'}{G}) + \frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{4\mu-\lambda^2}}\lambda i\right]$$

$$\xi = \pm\sqrt{\frac{q}{-4\lambda^2+16\mu}}ix \pm \frac{p\sqrt{4\mu-\lambda^2}}{2(-\lambda^2+4\mu)}it \qquad (4.10)$$

Substituting the general solutions of Eq.(4.7) into (4.10), we have

$$v_2(\xi) = \mp\frac{q\lambda}{2p}\sqrt{\frac{1}{4\mu-\lambda^2}}i \pm \frac{qi}{2p}$$

$$\cdot\left(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)$$

$$+\frac{q}{2p}\left[1 \pm \sqrt{\frac{1}{4\mu-\lambda^2}}\lambda i\right]$$

Then

$$u_1(\xi) = (v_1(\xi))^{-1}.$$

Where

$$\xi = \xi = \pm\sqrt{\frac{q}{4\lambda^2-16\mu}}ix \pm \frac{p\sqrt{\lambda^2-4\mu}}{2(-\lambda^2+4\mu)}it,$$

C_1 and C_2 are two arbitrary constants.

5 Conclusions

In this paper we have seen that the traveling wave solutions of the generalized Burgers equation and the (2+1) dimensional dispersive equation are successfully found by using the $(\frac{G'}{G})$-expansion method. Now we briefly summarize the method in the following.

The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in $(\frac{G'}{G})$, where $G = G(\xi)$ is the general solutions of a second order LODE. The positive integer m is determined by the homogeneous balance between the highest order derivatives and nonlinear terms appearing in the reduced ODE, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the method. The main merits of the $(\frac{G'}{G})$-expansion method over the other methods are that it gives more general solutions with some free parameters and it handles NLEEs in a direct manner with no requirement for initial/boundary condition or initial trial function at the outset.

Compared to the methods used before, one can see that this method is direct, concise and effective. As we can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equations resulted, so we can also avoids tedious calculations. This method can also be used to many other nonlinear equations.

6 Acknowledgements

I would like to thank the anonymous referees for their useful and valuable suggestions.

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