Thermal and Solutal Marangoni Mixed Convection Boundary Layer Flow

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Abstract:- An analysis is performed for mixed convection thermal and solute concentration Marangoni boundary layer that can be formed along the surface, which separates two immiscible fluids in surface tension flows. Similarity equations for the case in which an external pressure gradient is imposed are derived. The dimensionless governing equations of the flow field are solved numerically using the shooting method. The effect of flow parameters on the velocity, temperature and concentration are computed and presented in tables and figures.

Key-Words:- Marangoni flow, Thermosolutal, Mixed convection, Boundary layer.

1 Introduction

The study of the flow and heat transfer in an electrically conducting fluid has many practical applications in manufacturing process in industry. The thermal fluid flow problem have been extensively studied numerically, theorrtically as well as experimentally (see [1-2]). Marangoni convection induced by variation of the surface tension with temperature along a surface influences crystal growth melts and other processes with liquid-liquid or liquid-gas interfaces. Marangoni convection, due to thermally induced surface tension gradients, plays an important role in the burning and extinction of wicks fed by liquid fuels and in the ignition and spread of flames across the surfaces of liquid fuel layers [3]. The surface tension gradients for Marangoni convection can be temperature and/or concentration gradients. The numerous investigations of Marangoni convection with an imposed surface temperature gradient have been reviewed in the literature (see Arifin and Rosali [4], Arifin et al. [5], Arifin and Pop [6], Arifin and Abidin [7] and Chen and Chan [8]).

The investigations of Marangoni flow in various geometries have been reviewed by Arafune and Hirata [9] and Croll et al. [10]. Later, Arafune and Hirata [11] studied a similarity analysis for just the velocity profile for Marangoni flow that are linearly related to the surface position. Similarity solutions for surface tension that varied as a quadratic function of the temperature as would occur near a minimum has been investigated by Slavtchev and Miladinova [12]. Schwabe and Metzger [13] experimentally studied Marangoni flow on a flat surface combined with natural convectionin a unique geometry where the Marangoni and buoyancy effects could be varied independently.

The existence of the steady dissipative layers along the liquid-liquid or liquid-gas interfaces have been first studied by Napolitano [14-15] and were called Marangoni boundary layers. Napolitano and Golia [16] have shown that the fields are uncoupled when the momentum and energy resistivity ratios of the two layers and the viscosity ratio of the two fluids aremuch less than one. Furthermore, as shown by Napolitano and Russo [17], similarity solutions for Marangoni boundary layers exist when the interface temperature gradient varies as a power of the interface arc length ($\overline{x}$). The power laws for all other variables, including themean curvature, were determined.

Numerical solutions for Marangoni boundary layers have been analyzed and discussed in subsequent papers by Golia and Viviani [18,19], Pop et al. [20] and Chamkha et al. [21]. Al-Mudhaf and Chamkha [22] have studied numerically and analytically the thermosolutal Marangoni convection along a permeable surface with heat generation or absorption and a first-order chemical reaction effects. Recently, Magyari and Chamkha [23] reported exact analytical solutions for the velocity, temperature and concentration fields of steady thermosolutal MHD Marangoni convection.

The present work focuses on numerical solution for thermosolutal Marangoni mixed convection boundary layer due to imposed temperature and concentration gradients. The analysis assumes that the surface tension varies linearly with temperature and concentration. The corresponding similarity equations are then solved numerically for some values of these parameters using shooting method. The velocity, temperature and concentration profiles as well as interface velocity, heat and mass transfer at the interface are obtained and discussed.

2 Basic Equations

Consider the steady two-dimensional flow along the interface S of two Newtonian immiscible fluids where x and y are the axes of a Cartesian coordinate system as shown in Fig. 1.

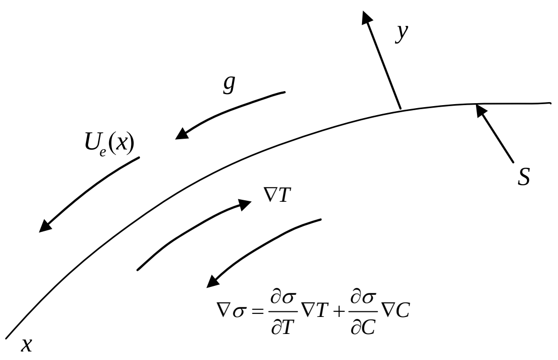

Fig 1 : Physical Model

We assume that the temperature and concentration at the interface are $T_s(x)$ and $C_s(x)$, respectively. Under the usual boundary layer approximations, the basic governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \Gamma g\beta(T - T_m) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad (4)$$

where u and v are the velocity components along x and y axes, $u_e(x)$ is the external velocity, T is the fluid temperature, C is the solutal concentration, α is the thermal diffusivity, D is the mass diffusivity and ν is the kinematic viscosity. For $\Gamma = -1$ refers to buoyancy forces which are favourable to the Marangoni flow and the buoyancy forces are opposing to the Marangoni flow if $\Gamma = +1$. The surface tension gradients that are responsible for the Marangoni mixed convection flow can be due to the gradients of temperature and/or solutal concentration. The boundary conditions of equations (1) – (4) are

$$\begin{gathered} v = 0, \quad T = T_s(x), \quad C = C_s(x), \\ \mu\frac{\partial u}{\partial y} = \sigma_T\frac{\partial T}{\partial x} + \sigma_c\frac{\partial C}{\partial x} \quad \text{on} \quad y = 0, \end{gathered} \qquad (5)$$

$$u \to u_e(x), \quad T \to T_m, \quad C \to C_m \quad \text{as} \quad y \to \infty, \qquad (6)$$

where μ is the dynamic viscosity, σ_T and σ_c are rates of change of surface tension with temperature and solute concentration, respectively. The fourth condition of (5) represents the Marangoni coupling condition at the interface, having considered for the surface tension given by

$$\begin{gathered} \sigma = \sigma_m - \sigma_T\left(T - T_m\right) - \sigma_C\left(C - C_m\right), \\ \sigma_T = -\frac{\partial\sigma}{\partial T}, \quad \sigma_C = -\frac{\partial\sigma}{\partial C}. \end{gathered} \qquad (7)$$

The subscript m in equation (7) denotes values pertaining to the hydrostatic state, assumed uniform and chosen as the reference state, which coincides, in this case, with the external conditions. The directions of the driving actions depend on the orientation of the temperature and solutal concentration gradients in liquid ∇T and ∇C , and on the signs of the thermodynamics coefficients σ_T and σ_c . We now define the following non-dimensional variables:

$$x = L_0 + XL, y = \delta LY, u = U_C U, v = \delta U_C V,$$

$$T = T_m + \theta\Delta T, C = C_m + \phi\Delta C, u_e(x) = U_C U_e(x), \tag{8}$$

where L_0 locates the origin of the curvilinear abscissa x, L is the extension of the relevant interface S, ΔT and ΔC are positive increments of temperature and solute concentration linked to the temperature and solute concentration gradients imposed on the interface. Further, δ is a scale factor in the direction normal to the interface and U_C is the reference velocity which are defined as δ = Re$^{-1/3}$ and $U_C = \nu/(L\delta^2)$ with $\mathrm{Re} = \sigma_T \Delta TL/\nu\mu$ being the Reynolds number. Substituting (8) into equations (1)-(4), we obtain the following non-dimensional equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{9}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = U_e\frac{dU_e}{dX} + \frac{\partial^2 U}{\partial Y^2} - \Gamma\lambda\theta, \tag{10}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial Y^2}, \tag{11}$$

$$U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial Y^2}, \tag{12}$$

and the boundary conditions (5) and (6) reduce to

$$\begin{gathered} V = 0, \theta = \theta_s(X), \phi = \phi_s(X), \\ \frac{\partial U}{\partial Y} = \frac{\partial \theta}{\partial X} + \varepsilon\frac{\partial \phi}{\partial X} \text{ on } Y = 0 \end{gathered} \tag{13}$$

$$U \to U_e(X), \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad Y \to \infty \tag{14}$$

Where P_r and Sc are usually Prandtl and Schmidt numbers, $\lambda \geq 0$ is the Marangoni mixed convection parameter, which is defined as $\lambda = g\beta\Delta TL/U_c$ and the Marangoni parameter, $\varepsilon = Ma_c / Ma_T$. The equations (9) - (12) can be transformed into the corresponding ordinary differential equations by the following transformations (see Pop et al. [20])

$$U = u_e X^p, \quad \theta_s = -t_0 X^\beta, \quad \phi_s = -c_0 X^\gamma,$$

$$U = u_0 X^q f'(\eta), \quad \theta = -t_0 X^\beta g(\eta),$$
$$\phi = -c_0 X^\gamma h(\eta), \quad \eta = \frac{Y}{l_0 X^r},$$

where $f'(\eta), g(\eta)$ and $h(\eta)$ represent the velocity, temperature and solutal concentration profiles in the similarity plane and η being the similarity variable, c_0, l_0, t_0 and u_0 are constant scale factors to be further determined and prime denotes differentiation with respect to η. On using equations (9)-(12), the boundary conditions (13) and (14) and the similarity transformation above, we obtain, after some algebra,

$$p = q = \frac{2\beta - 1}{3}, \quad r = \frac{2-\beta}{3}, \quad \gamma = \beta.$$

Particularly interesting cases are: $\beta = 1/2$, interface and external velocities are constant, $\beta = 1$, the temperature and the solute concentration gradients at the interface are constant and $\beta = 2$, the thickness of the dissipative (velocity) boundary layer is constant. (see Pop et al. [20]). The constant scale factors c_0, l_0, t_0 and u_0 must satisfy the following conditions:

$$u_0 l_0^2 = \frac{3}{\beta+1}, \quad \frac{l_0 t_0}{u_0} = \frac{1}{\beta}, \quad \frac{c_0}{t_0} = 1.$$

These relations show that the nature of the boundary layers influence only the scale factors c_0, l_0, t_0 and u_0. If we take $t_0 = 1$ or $c_0 = 1$ then l_0 and u_0 are uniquely determined as

$$l_0 = \left(\frac{3}{1+\beta}\right)^{1/3} \beta^{-1/3}, \quad u_0 = \left(\frac{3}{1+\beta}\right)^{1/3} \beta^{2/3}$$

The transformed ordinary differential equations are

$$f''' + ff'' + \frac{2\beta - 1}{\beta + 1}\left(1 - f'^2\right) + \Gamma\lambda g = 0 \tag{15}$$

$$\frac{1}{Pr}g'' + fg' - \frac{3\beta}{1+\beta}f'g = 0 \tag{16}$$

$$\frac{1}{Sc}h'' + fh' - \frac{3\beta}{1+\beta}f'h = 0 \tag{17}$$

along with the boundary conditions

$$f(0)=0,\quad f''(0)=-1-\varepsilon,\qquad g(0)=1,\quad h(0)=1 \tag{18}$$

$$f(\infty)=1,\quad g(\infty)=0,\quad h(\infty)=0. \tag{19}$$

3 Results and Discussion

The system of ordinary differential equations (15)-(17) subject to the boundary conditions (18) and (19) has been solved numerically for various values of β when $\lambda=0$ (Marangoni forced convection) and $\lambda=1$ using the shooting method. We considered both favourable (aiding Marangoni effect, $\Gamma=-1$) and contrary (opposing Marangoni effect, $\Gamma=+1$) flow cases. Numerical solutions of the problem described by equations (1) – (4) for Marangoni forced convection ($\lambda=0$) have been obtained by Pop et al. [20] using a special adapted version of the Keller-box method. Thus, for $\lambda=0$, $P_r=0.7$, $Sc=0.22$ and $Sc=0.6$, we have obtained the numerical values for the reduced interface velocity, $f'(0)$, heat transfer at the interface, $-g'(0)$ and mass transfer, $-h'(0)$, from the interface as shown in Tables 1 and 2 for various values of β in the case of $\varepsilon=0$ and $\varepsilon=1$, respectively. Results obtained by Pop et al. [20] are also included in these tables. It is seen that the present results are in good agreement with those of Pop et al. [20].

In the opposing flow case, $\Gamma=+1$, the Prandtl number is taken as $P_r=0.7$, while in the aiding flow case, $\Gamma=-1$, the Prandtl number is $P_r=5$. It should also be noticed that for $\Gamma=+1$, numerical computation converges for any value of P_r , whereas for $\Gamma=-1$, the numerical computation does not converge for $P_r<2.8$ (see also Chamkha et al. [21]). The values for the reduced interface velocity, $f'(0)$, heat transfer at the interface, $-g'(0)$ and mass transfer, $-h'(0)$ from the interface with $\lambda=1, Sc=0.22$ and $Sc=0.6$ and $Sc=0.75$ are tabulated in Tables 3 and 4 for various values of β in the case of $\Gamma=+1$ and $\Gamma=-1$, respectively. It is seen that the mass transfer increases with the increase of the Schmidt number for both $\Gamma=+1$ and $\Gamma=-1$.

Figures 1- 12 present the numerical results for the reduced velocity, $f'(\eta)$, temperature, $g(\eta)$ and concentration, $h(\eta)$ profiles for various values of parameters β and ε with $\lambda=1$, $Sc=0.22$ for both opposing ($\Gamma=+1$) and aiding ($\Gamma=-1$) cases. It is observed that the effect of increasing the similarity parameter, β at any given Marangoni parameter, ε , results in the reduction of the velocity, temperature and solutal concentration levels and thinning the corresponding boundary layers for both $\Gamma=+1$ and $\Gamma=-1$ cases.

Table 1: Values of $f'(0), -g'(0)$ and $-h(0)$ for ε = 0, λ = 0 (forced convection), Pr=0.7

Sc					0.22		0.60	
	f'(0)		-*g*'(0)		-*h*'(0)			
β	Present	Pop et al. [20]	Present	Pop et al. [20]	Present	Pop et al. [20]	Present	Pop et al. [20]
0.5	1.98428	1.98426	1.35415	1.35416	0.72506	0.72496	1.24584	1.24583
1.0	1.69688	1.69687	1.44235	1.44233	0.77991	0.77894	1.32869	1.32869
1.5	1.60908	1.60907	1.50675	1.50675	0.81726	0.81720	1.38867	1.38867
2.0	1.56595	1.56595	1.55062	1.55062	0.84237	0.84232	1.42944	1.42944
3.0	1.52301	1.52306	1.60536	1.60536	0.87350	0.87349	1.48028	1.48028
4.0	1.50151	1.50150	1.63791	1.63791	0.89199	0.89195	1.51049	1.51049
5.0	1.48857	1.48856	1,65943	1,65944	0.90419	0.90415	1.53047	1.53047
∞	1.43623	1.43623	1.76455	1.76456	0.96359	0.96357	1.62796	1.62796

Table 2: Values of $f'(0), -g'(0)$ and $-h(0)$ for ε = 1, λ =0 (forced convection), Pr=0.7

Sc					0.22		0.60	
	f'(0)		-*g*'(0)		-*h*'(0)			
β	Present	Pop et al. [20]	Present	Pop et al. [20]	Present	Pop et al. [20]	Present	Pop et al. [20]
0.5	2.73363	2.73362	1.54508	1.54507	0.80624	0.80616	1.41769	1.41769
1.0	2.27461	2.27460	1.61711	1.61711	0.85442	0.85436	1.48585	1.48585
1.5	2.12622	2.12621	1.67527	1.67527	0.88905	0.88900	1.54012	1.54012
2.0	2.06197	2.06196	1.71597	1.71597	0.91275	0.91272	1.57799	1.57799
3.0	1.97716	1.97715	1.76756	1.76756	0.94250	0.94247	1.62595	1.62595
4.0	1.93938	1.93937	1.79856	1.79857	0.96026	0.96023	1.65474	1.65474
5.0	1.91657	1.91654	1.81917	1.81917	0.97203	0.97201	1.67387	1.67387
∞	1.82343	1.82343	1.92073	1.92074	1.02979	1.02977	1.76810	1.76810

Table 3: Values of $f'(0), -g'(0)$ and $-h(0)$ for **ε = 0, λ = 1, Γ = +1 (opposing case), P_r =0.7**

Sc			0.22	0.60	0.75
β	*f*'(0)	-*g*'(0)	-*h*'(0)		
0.5	2.28754	1.45749	0.77262	1.33992	1.51321
1.0	1.89754	1.52833	0.82032	1.40709	1.58580
1.5	1.77492	1.58502	0.85429	1.46005	1.64425
2.0	1.71451	1.62466	0.87751	1.49698	1.68517
3.0	1.65443	1.67492	0.90664	1.54373	1.73709
4.0	1.62441	1.70512	0.92404	1.57181	1.76831
5.0	1.60639	1.72521	0.93557	1.59047	1.78907
∞	1.53390	1.82430	0.99219	1.68247	1.86962

Table 4: Values of $f'(0), -g'(0)$ and $-h(0)$ **for ε = 0, λ = 1, Γ = -1 (aiding case), P_r =5.0**

Sc			0.22	0.60	0.75
β	*f*'(0)	-*g*'(0)	-*h*'(0)		
0.5	1.82846	3.66459	0.69756	1.20026	1.35283
1.0	1.58929	3.90666	0.75766	1.29050	1.45204
1.5	1.51855	4.08449	0.79740	1.35375	1.52231
2.0	1.48061	4.20524	0.82380	1.39631	1.56967
3.0	1.44981	4.35492	0.85634	1.44903	1.62844
4.0	1.43226	4.44428	0.87550	1.48022	1.66324
5.0	1.42216	4.50328	0.88813	1.50082	1.68619
∞	1.38053	4.79059	0.94925	1.60077	1.79779

4 Conclusion

A numerical computation was carried out for the steady thermosolutal Marangoni mixed convection boundary layer flow. The conditions for the existence of similarity solutions were found and the full boundary layer equations were reduced to similarity or ordinary differential equations. The velocity, temperature and concentration profiles as well as the velocity, heat and mass transfer at the interface were determined and discussed in detail.

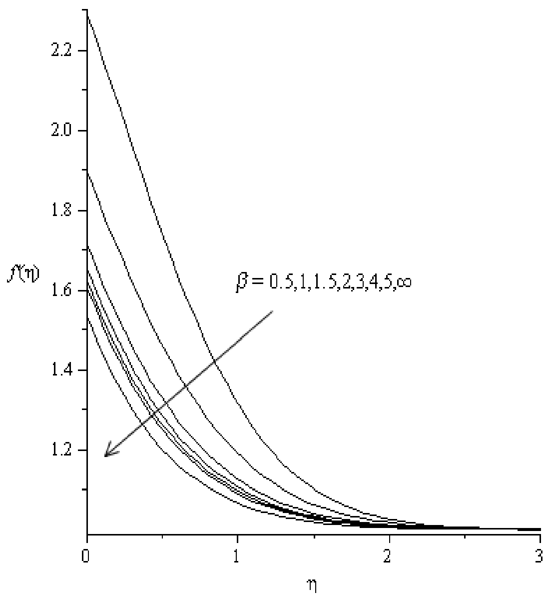

Fig. 1: Velocity profiles $f'(\eta)$ for $\varepsilon = 0,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (opposing)

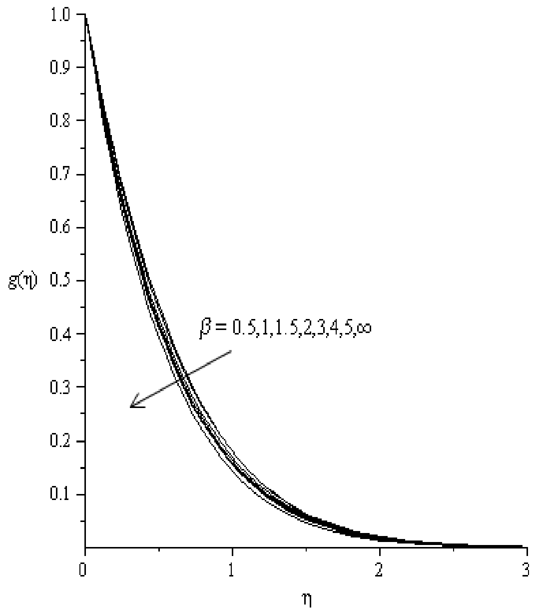

Fig. 2: Temperature profiles $g(\eta)$ for $\varepsilon = 0,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (opposing)

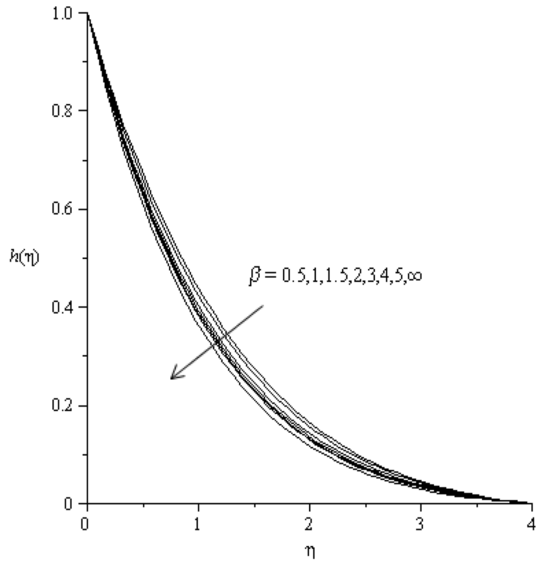

Fig. 3: Concentration profiles $h(\eta)$ for $\varepsilon = 0,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (Opposing)

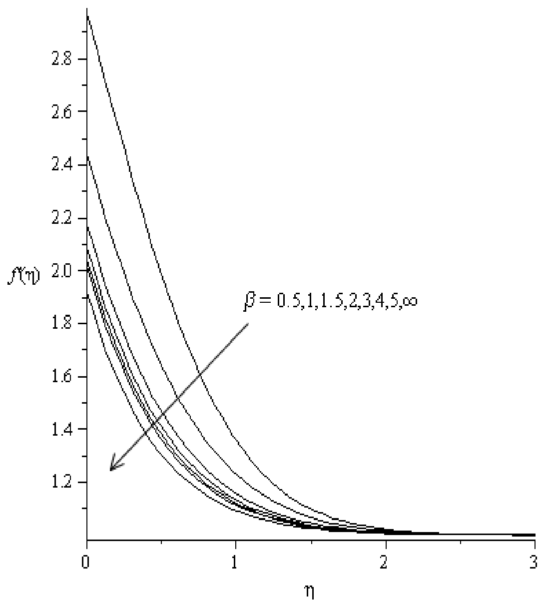

Fig. 4: Velocity profiles $f'(\eta)$ for $\varepsilon = 1,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (opposing)

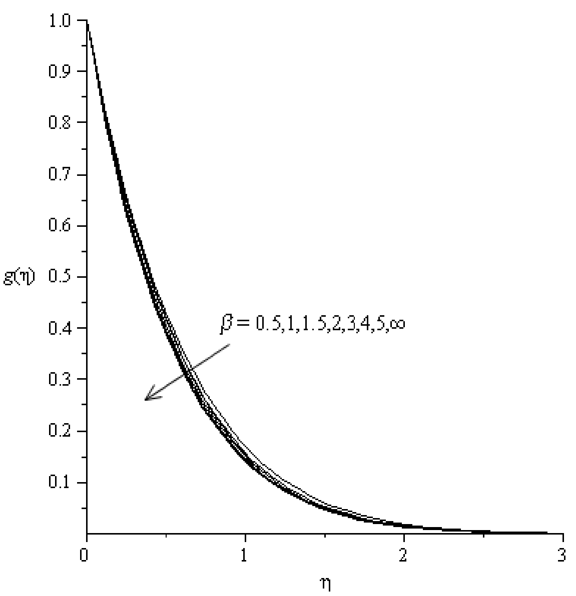

Fig. 5: Temperature profiles $g(\eta)$ for $\varepsilon = 1,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (opposing)

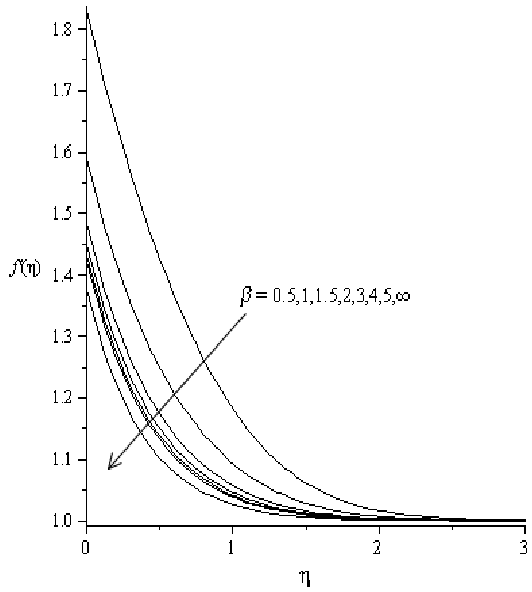

Fig. 7: Velocity profiles $f'(\eta)$ for $\varepsilon = 0,\ \lambda = 1,\ P_r = 5.0,\ Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

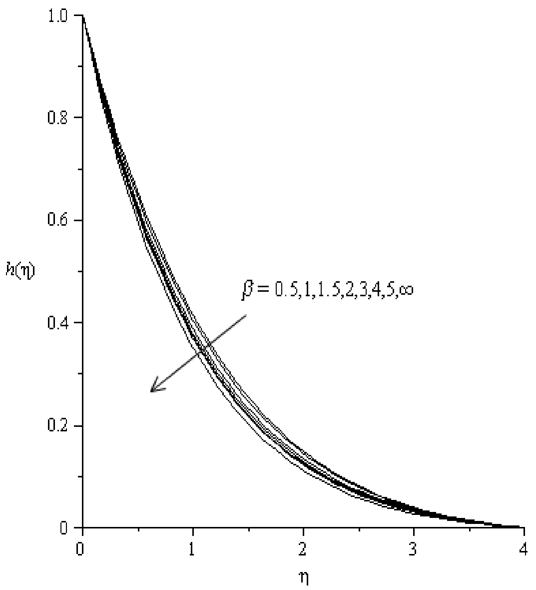

Fig. 6: Concentration profiles $h(\eta)$ for $\varepsilon = 1,\ \lambda = 1,\ P_r = 0.7,\ Sc = 0.22$ in the case of $\Gamma = +1$ (opposing)

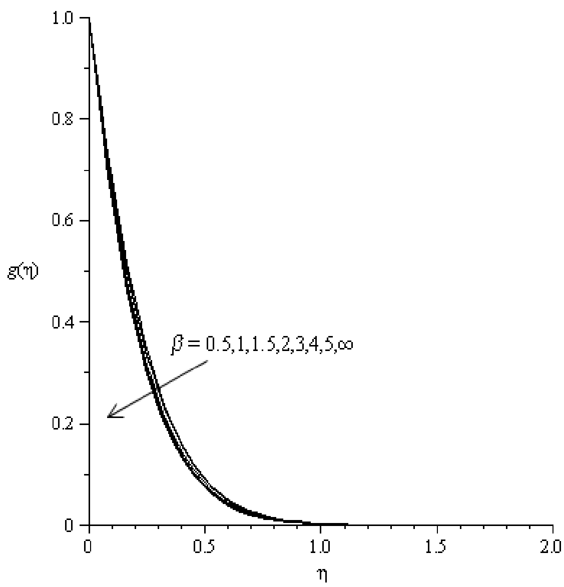

Fig. 8: Temperature profiles $g(\eta)$ for $\varepsilon = 0,\ \lambda = 1,\ P_r = 5.0,\ Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

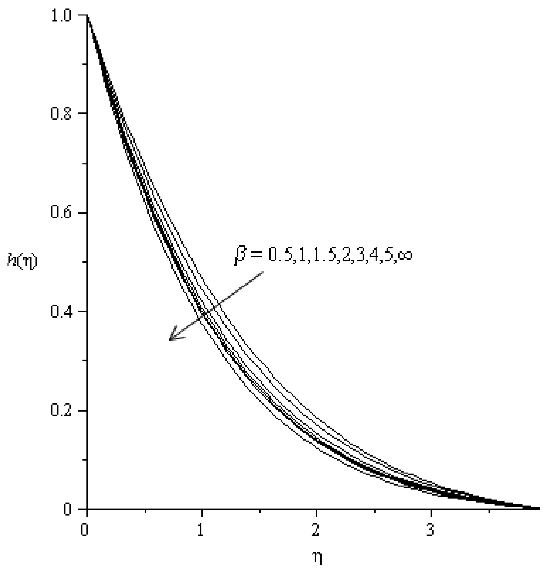

Figure 9: Concentration profiles $h(\eta)$ for $\varepsilon = 0$, $\lambda = 1$, $P_r = 5.0$, $Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

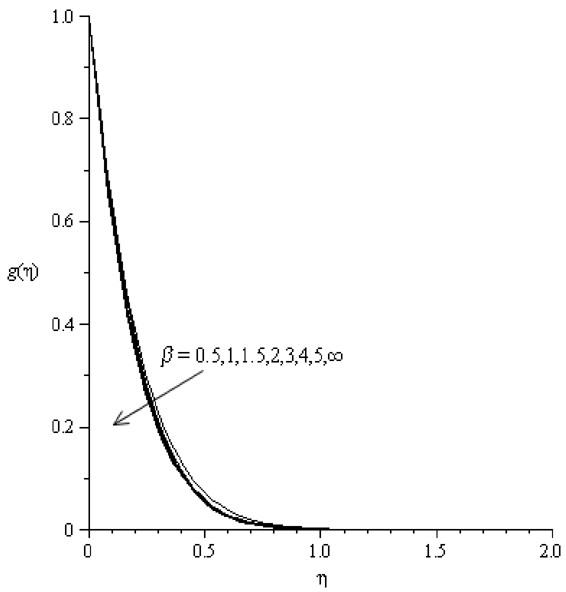

Fig. 11: Temperature profiles $g(\eta)$ for $\varepsilon = 1$, $\lambda = 1$, $P_r = 5.0$, $Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

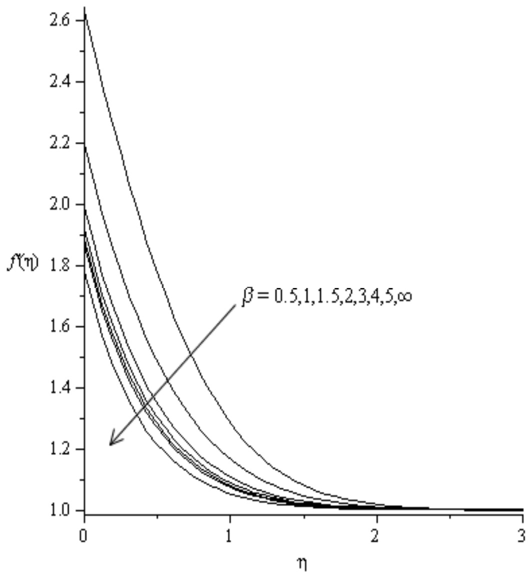

Fig. 10: Velocity profiles $f'(\eta)$ for $\varepsilon = 1$, $\lambda = 1$, $P_r = 5.0$, $Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

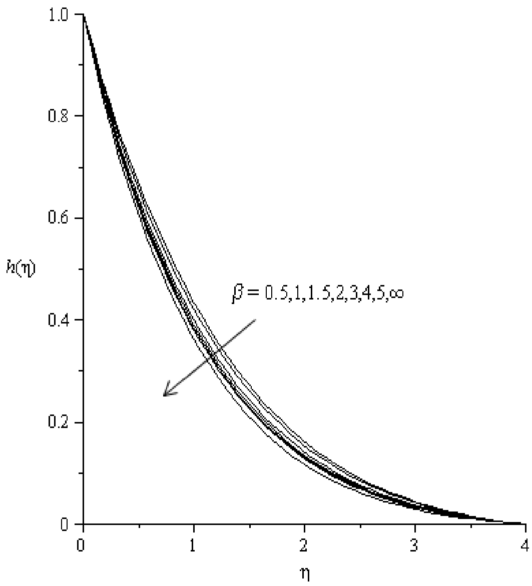

Fig. 12: Concentration profiles $h(\eta)$ for $\varepsilon = 1$, $\lambda = 1$, $P_r = 5.0$, $Sc = 0.22$ in the case of $\Gamma = -1$ (aiding)

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