Towards a Mathematical Theory of Unconscious Adaptation and Emotions

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Abstract: The psychological adaptability is realized through the conscious and the unconscious mechanisms. Even though the functionality of the mechanisms involved emphasizes different approaches, the main reason for these - approaches - is the obtaining of the maximum adaptability which has to protect the person at the conscious level. The invisible unconscious mechanisms have a major role in the process of psychological adaptation. The unconscious contribution does not eliminate completely the danger of psychological collapse, and this accredits the idea of the existence of a psychological guarantee only for proper inquirer stimuli. The incapacity of the consciousness filters to mediate the transactions of unconscious offers changes the adaptability process in an approach which is not entirely controllable. The uncontrollable part validates the approach of the imposing of own unconscious offers in the environment. The emotion analyzed at the conscious level translates the uncertainty of the validation of the transactions of unconscious offers with the environment as tensional state. The mathematical approach of this paper describes the need for psychological stimulation as a facet of adaptability and the emotional mechanisms as devices of the preserving of tensional states.

Key–Words: Coordinate systems, Dynamics, Emotion, Group theory, Mechanism, Symmetry group, Unconscious adaptability

1 Introduction

The dynamics of human behaviour depends on a subtle energy which is governed by inflexible laws imposed by the functionality of the psychological system and by laws determined by the processing of stimuli.

The ratio between the homeostasis - represented by the actions of defense mechanisms [3], [4], [5], [7], [8], [9], [10], [11] - and the entropy of certain psychological measures modulates the tensional dynamics of the person. We advance the idea of the existence of a certain specific mechanism of this ratio which resets the human psychological system correctly.

Every person has certain specific instruments to solve any emotional situation. The attitude responses to psychological challenges require the selection of those behaviour strategies which minimize regret and amplify pleasure. The building of responses constitutes itself a process consisting in searching and testing of the behaviour recipes which satisfy the optimum criteria of the psychological dynamics having as purpose the avoidance of unpleasant states and the search for pleasurable states.

What is essential and what is variable in transactional "challenge-response"? The pointing out of the essential features presupposes the acting of an equivalence class of a ratio in which the engaged resources oppose a certain destructive potential. Technically speaking, in this ratio a certain tolerance of the entropy of psychological measures and a certain capacity to adjust this entropy are involved. This means that different situations could be treated in an equivalent manner. This is a striking conclusion!

If our judgment is right, it means that the human psyche approaches problematic situations in a similar way. But this fact proves the myth of psychological invincibility.

At the conscious level, a new structure is needed, which has to bring out the problematic situations. This structure should have the behaviour of a special consciousness which has to distinguish between the potentials of using its products. The interpretative potential of the conscious instance is recorded by the tensional effort - a weak structure which preserves the equivalence class - mentioned above - specific to a person. The reason for preserving it is to assure the simultaneous functioning of strong and weak psychological structures and, of course, of psychological instances. At psychological level, the entropy and the adjustment of entropy are assembled in different structures. The structures must be compatible, meaning that these are acting on the basis of certain formulae which have to assure the accurate functioning of the psychological system. The variability of the psychological energy suggests for the unconscious instance equivalence class type structures, functioning on the basis of a constant ratio ”anti-entropy / entropy” and for the conscious instance and its peripheral instance, the subconscious instance, the special structures functioning on the basis of the formula ”anti-entropy + entropy”.

The functionality of the psychological system presupposes the existence of a certain control structure of the functioning of psychological instances along predefined formulae.

The psychological dysfunction appears as an effect of the vulnerability of the mentioned formulae which result at the level of the control structure in the correct functioning of the system. Actually, this is a false image, induced probably by the presence of the possible indetermination in the formula ”anti-entropy / entropy”. The indetermination decays the recipe ”entropy-adjustment” into an infinity of possible ones, having as an effect the blocking of the approaches of unconscious nature. The decayed recipes record a null psychological effort at the conscious level. This fact could be interpreted by the appearance of a doubled unconscious instance and by the suppression of the conscious instance. The psychological conflict appears at the control structure level which receives unspecific signals from the conscious instance. In this way the conscious one will be charged with the task of the gratification of ungratifiable pulsions.

1.1 Why the Unconscious Adaptability and Emotion

The behaviour reactions are the psychological availability solutions resulted through the counteracting of certain tensional states, which are discernible or not in the specter of the conscious filters.

The attitude responses require a considerable effort of the conscious adaptation to an unconscious offer of the emotional transactions.

The systemic dynamics [18] represented by the couple ”conscious state - unconscious state” imposes the acting of some interpretative mechanisms with adaptive valences in the plan of realizing of certain buffer type plausible scenarios between the subjective reality and the un-discernible one. Surprisingly, the psychological system responds to an invincible outside reality with adaptive mechanisms which permit the including of the unconscious offers in the transactional circuit. The inclusion is realized through the preserving of certain codes specific to a person, codes which generate reactions specific to the stable structures of behaviour.

The confrontation with strong stimuli or with stimuli which require repeatedly unspecific attitudes diminishes the capacity of self-defense of the psychological system through the alteration of the codes specific to a person. These codes symbolize effective availabilities of unconscious communication with the environment.

The unconscious conflict generated by the acting of the codes which maintain the features of the ancestral codes, less the level of the effective availability of the unconscious communication, determines the blocking of the approaches of unconscious nature.

The adaptability to environment contains an unstable weak structure, represented by the conscious contribution and a stable strong structure which is managed by the unconscious contribution.

The psychological system records and evaluates permanently the unconscious offer of the psychological availability which is compared with the original offer. The discrepancy engages the psychological system in a perennial effort of identifying the right offer which has effects in the tensional and the emotional dynamics.

2 The Need for Psychological Stimulation as a Mechanism

How could the need for psychological stimulation be interpreted?

The existential equilibrium is dependent on the duality ”generation-consumption”. The dominant role in the dynamics of the psychological status is fulfilled by the psychological behaviour, which is the observable form of the consumption of psychological potential. The presence of the generator of psychological potential is justified by the functionality of an intrinsic consciousness of the consumption which has to identify it. The existence of the generator of psychological states seems enigmatic because it should be justified by an external component of the mentioned duality, probably in the form of a conscience of consciousness. The generation and the consumption of the psychological potential are dual elements of a system which maintain the forms of certain laws, with repeatability valences.

Surprisingly, the consumption seems to be endowed with mechanisms which allow the parallel functioning of the state corresponding to the extinction of pulsion signals and, more interestingly, of the state corresponding to the extinction of the potential state. Here, a special role is played by the mystic power of intrinsic consciousness, which changes the irrationality of the extinction of an unchallenged lack of balance into something materialized in desire.

What expresses this special kind of desire? The finality must overlap with a target image, having cathartic valences. The logic of the projection of finality type images is known by the producer (the unique consciousness) and not by the watcher (the simple consciousness). What seems to be the desire at the conscious level is the need at the unconscious level - as a dual image of desire. The autonomy and the overall efficiency of the human psyche are explained through the interpretation of the surplus of psychological energy as a need for psychological stimulation. In this context the need for psychological stimulation seems to have a special role.

The right functionality of the supervisory instance is determined by complying with certain rigid conditions of recording. Psychological lability is a consequence of the breach of the psychological system which allows the confrontation between the philogenetic and the ontogenetic. The source of deterioration of the psychological system through failed resettings is placed at the level of the factors which induce disparities between the philogenetic formulae and those resulted from the confrontation with a hostile environment. The logic and the bizarreness of this statement could accredit the idea that a balanced personality would access only those environments which maintain the overlapping between the ontogenetic and the philogenetic.

2.1 The Mathematical Function of the Need for Psychological Stimulation

A way to determine the mathematical function of the need for psychological stimulation is one which uses group theory and it consists in identifying a group of transformations which preserves a system of partial differential equations. We call this group the symmetry group [1], [2], [6], [12], [13], [14], [15].

2.1.1 The Symmetry Group of a System of Partial Differential Equations

Let Φ be a system with p independent variables $(x_1, x_2, \ldots, x_p) \equiv \mathbf{x} \in X$ and q dependent variables $\left(u^1, u^2, \ldots u^q\right) \equiv \mathbf{u} \in U$. We define the solution of the system Φ through the mapping $f : X \to U$ where $\mathbf{u} = f(\mathbf{x})$. The mapping determines a p – dimensional manifold in the space $X \times U$. We write the manifold as follows $\Gamma = \{(x, f(x))/x \in X\}$.

Let G be a local group of transformations which acts in the space $X \times U$. The system Φ is invariant from the group G if for any local solution of the system Φ, $\mathbf{u} = f(\mathbf{x})$, the transformed function $g \cdot f$ is defined for any $g \in G$, and $\tilde{\mathbf{u}} = (g \cdot f)(\widetilde{\mathbf{x}})$ is also a solution of the system.

Each component $u^l = f^l(\mathbf{x})$, where $l = 1, 2, \ldots, q$, generates partial derivatives of h degree, $u^l_J \equiv \partial_l f^l(\mathbf{x}) = \frac{\partial^h f^l}{\partial x_1^{j_1} \partial x_2^{j_2} \ldots \partial x_p^{j_p}}$. The indices $J \equiv (j_1, \ldots, j_p)$ satisfy the relation $j_1 + j_2 + \cdots + j_p = h$. For the function f we obtain qp_h different derivatives of h degree. Let us build the space $U^{(h)} = U \times U_1 \times \cdots \times U_h$ of all derivatives of degree $\leq h$ of the function f. We associate to the function f the extension of h degree $pr^{(h)} f : X \longrightarrow U^{(h)}$. Therefore $pr^{(h)} f(\mathbf{x}) \in U^h$ is a vector, containing $q(1 + p_1 + \cdots + p_h) = qp_{h+1}$ components. The group G, acting in the space $X \times U$, will induce an action in the space $X \times U^{(h)}$ called the extension of G of h degree. Suppose X is the generator of the one parameter subgroup $\left\{e^{tX} \subset G\right\}$. Then $pr^{(h)} X = \frac{d}{dt}\left(pr^{(h)} e^{tX}\right)_{t=0}$.

Suppose the system Φ is represented by m differential equations $\Xi^i\left(\mathbf{x}, \mathbf{u}^{(h)}\right) = 0, \;\; i = 1, 2, \ldots, m$. The system of differential equations is associated with the manifold $V_\Xi \subset X \times U^{(h)}$ which is determined through the cancellation of the function $\Xi : X \times U^h \to R^m$, respectively $V_\Xi = \left\{\left(\mathbf{x}, \mathbf{u}^{(h)}\right) / \Xi\left(\mathbf{x}, \mathbf{u}^{(h)}\right) = 0\right\}$. We say that G is a symmetry group for the system Φ if the manifold V_Ξ is invariant for the group $pr^{(h)} G$.

Using the following hypothesis:

a. There is a system of partial differential equation of h degree with p independent variables and q dependent variables where the jacobian of the system has maximum rank everywhere;

b. for any point $\left(x_0, u_0^{(h)}\right)$ there is a solution $\mathbf{u} = f(\mathbf{x})$ defined in the neighborhood of $\mathbf{x}_0$, such that $\mathbf{u}_0^{(h)} = pr^{(h)} f(\mathbf{x}_0)$;

c. G is a local group of transformations, which acts in the space $X \times U$,

the condition of the symmetry group for the system Φ results: $pr^h X \left[\Xi\left(\mathbf{x}, \mathbf{u}^{(h)}\right)\right] = \mathbf{0}$.

Let $\Xi : X \times U^h \to R^m$ be a differentiable function. The total derivative with respect to x_i is given by the function $D_i \Xi : X \times U^{h+1} \to R^m$, where the operator D_i results from the expression $D_i = \frac{\partial}{\partial x_i} + \sum_{l=1}^q \sum_{J_i} u^l_{J_i} \frac{\partial}{\partial u^l_J}$ with $1 \leq i \leq p$, $J_i = (j_1, \ldots, j_p)$. Let us note the expression $D_1^{j_1} D_2^{j_2} \ldots D_p^{j_p}$ through the symbol D^J. The extension of h degree of the generator X defined through the relation $pr^{(h)} X = \frac{d}{dt}\left(pr^{(h)} e^{tX}\right)_{t=0}$ is calculated using the expression $pr^{(h)} X = X + \sum_{l=1}^q \sum_J \varphi_l^J \left(\mathbf{x}, \mathbf{u}^{(h)}\right) \frac{\partial}{\partial u^l_J}$, where X is supposed to be a smooth vector field on $X \times U$, having the shape $X = \sum_{i=1}^{p} \xi_i(\mathbf{x},\mathbf{u}) \frac{\partial}{\partial x_i} + \sum_{l=1}^{q} \varphi_l(\mathbf{x},\mathbf{u}) \frac{\partial}{\partial u^l}$. From the relation $\varphi_l^J = D^J\left(\varphi_l - \sum_{i=1}^{p} u_i^l \xi_i\right) + \sum_{i=1}^{p} u_{Ji}^l \xi_i$, where $u_i^l = \frac{\partial u^l}{\partial x_i}$, $l = 1, 2, \ldots, q$, the coefficients φ_l^J result. The coefficients ξ_i and φ_l constitute the symmetry group of the system of partial differential equations.

2.1.2 The Function of the Need for Psychological Stimulation

The equation (see [16], [17])

$$\frac{\partial^2 u}{\partial \bar{t}^2} = \frac{\left(o\bar{t}+p\right)^2}{4}\left(\frac{\partial^2 u}{\partial m^2} + \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial b^2}\right) \quad (1)$$

where u - tensional state, $\bar{t}$ - physical time, m - the ratio between the current neuropsychological stimulation and the minimal one needed for the activation of attention, v - apperception, b - the amplitude of the tensional state, o and p are constant describes the dynamics of tensional states in which the whole (tensional) potential is used for specific processings determined by the psychological functionality. This equation is obtained from the metrics formula:

$$ds^2 = \left(id\bar{t}\right)^2 + \frac{4}{\left(o\bar{t}+p\right)^2}\left[(dm)^2 + (dv)^2 + (db)^2\right]$$

We generalize the equation (1) by including the term C which indicates an unused tensional potential, called the need for psychological stimulation.

$$\frac{\partial^2 u}{\partial \bar{t}^2} = \frac{\left(o\bar{t}+p\right)^2}{4}\triangle_3 u + Cu \quad (2)$$

We will apply the theory presented above for the equation (2). The generator for the symmetry group of this equation writes: $L = \xi\left(\bar{t},m,v,b,u\right)\frac{\partial}{\partial \bar{t}} + \eta\left(\bar{t},m,v,b,u\right)\frac{\partial}{\partial m} + \zeta\left(\bar{t},m,v,b,u\right)\frac{\partial}{\partial v} + \lambda\left(\bar{t},m,v,b,u\right)\frac{\partial}{\partial b} + \varphi\left(\bar{t},m,v,b,u\right)\frac{\partial}{\partial u}$

The extension of the second degree for this generator has the mathematical expression:

$$\begin{aligned} pr^{(2)}L = L + \varphi^{\bar{t}}\frac{\partial}{\partial u_{\bar{t}}} + \varphi^{m}\frac{\partial}{\partial u_m} + \varphi^{v}\frac{\partial}{\partial u_v} + \\ \varphi^{b}\frac{\partial}{\partial u_b} + \varphi^{\bar{t}\bar{t}}\frac{\partial^2}{\partial u_{\bar{t}\bar{t}}} + \varphi^{mm}\frac{\partial^2}{\partial u_{mm}} + \\ \varphi^{vv}\frac{\partial^2}{\partial u_{vv}} + \varphi^{bb}\frac{\partial^2}{\partial u_{bb}} + .. \end{aligned} \quad (3)$$

The condition $pr^{(2)}X\left[\Xi\left(\mathbf{x},\mathbf{u}^{(h)}\right)\right] = \mathbf{0}$ becomes:

$$\varphi^{\bar{t}\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{4}\left(\varphi^{mm} + \varphi^{vv} + \varphi^{bb}\right) - C\varphi = 0$$

Using the relations:

$$\begin{aligned} \varphi^{\bar{t}\bar{t}} = D_{\bar{t}}^2\left(\varphi - u_{\bar{t}}\xi - u_m\eta - u_v\zeta - u_b\lambda\right) + \\ u_{\bar{t}\bar{t}\bar{t}}\xi + u_{\bar{t}\bar{t}m}\eta + u_{\bar{t}\bar{t}v}\zeta + u_{\bar{t}\bar{t}b}\lambda \end{aligned} \quad (4)$$

$$\begin{aligned} \varphi^{mm} = D_{m}^2\left(\varphi - u_{\bar{t}}\xi - u_m\eta - u_v\zeta - u_b\lambda\right) + \\ u_{mm\bar{t}}\xi + u_{mmm}\eta + u_{mmv}\zeta + u_{mmb}\lambda \end{aligned} \quad (5)$$

$$\begin{aligned} \varphi^{vv} = D_{v}^2\left(\varphi - u_{\bar{t}}\xi - u_m\eta - u_v\zeta - u_b\lambda\right) + \\ u_{vv\bar{t}}\xi + u_{vvm}\eta + u_{vvv}\zeta + u_{vvb}\lambda \end{aligned} \quad (6)$$

$$\begin{aligned} \varphi^{bb} = D_{b}^2\left(\varphi - u_{\bar{t}}\xi - u_m\eta - u_v\zeta - u_b\lambda\right) + \\ u_{bb\bar{t}}\xi + u_{bbm}\eta + u_{bbv}\zeta + u_{bbb}\lambda \end{aligned} \quad (7)$$

where $D_{\bar{t}} = \left(\frac{\partial}{\partial \bar{t}} + u_{\bar{t}}\frac{\partial}{\partial u}\right)$, $D_m = \left(\frac{\partial}{\partial m} + u_m\frac{\partial}{\partial u}\right)$, $D_v = \left(\frac{\partial}{\partial v} + u_v\frac{\partial}{\partial u}\right)$, $D_b = \left(\frac{\partial}{\partial b} + u_b\frac{\partial}{\partial u}\right)$, the following system, which corresponds with the psychological reality, results:

$$\varphi_{\bar{t}\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{4}\left(\varphi_{mm} + \varphi_{vv} + \varphi_{bb}\right) - C\varphi = 0 \quad (8)$$

$$\eta_{\bar{t}\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{4}\left(\eta_{mm} + \eta_{vv} + \eta_{bb}\right) = 0 \quad (9)$$

$$\zeta_{\bar{t}\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{4}\left(\zeta_{mm} + \zeta_{vv} + \zeta_{bb}\right) = 0 \quad (10)$$

$$\lambda_{\bar{t}\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{4}\left(\lambda_{mm} + \lambda_{vv} + \lambda_{bb}\right) = 0 \quad (11)$$

$$\eta_m = \zeta_v = \lambda_b = \frac{1}{2}\xi_{\bar{t}} \quad (12)$$

$$\eta_{\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{2}\xi_m = 0 \quad (13)$$

$$\zeta_{\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{2}\xi_v = 0 \quad (14)$$

$$\lambda_{\bar{t}} - \frac{\left(o\bar{t}+p\right)^2}{2}\xi_b = 0 \quad (15)$$

$$\zeta_m + \eta_v = 0 \quad (16)$$

$$\lambda_m + \eta_b = 0 \quad (17)$$

$$\lambda_v + \zeta_b = 0 \quad (18)$$

$$\xi_u = \eta_u = \zeta_u = \lambda_u = \varphi_u = \varphi_{uu} = 0 \quad (19)$$

The solution of this system is:

$$\xi = 2a_1\bar{t} + a_2 \quad (20)$$

$$\eta = a_1 m - a_3 v + a_4 b + a_5 \quad (21)$$

$$\zeta = a_1 v + a_3 m - a_6 b + a_7 \quad (22)$$

$$\lambda = a_1 b - a_4 m + a_6 v + a_8 \quad (23)$$

$$\varphi = f\left(r, \bar{t}\right) \quad (24)$$

where a_i are parameters of the symmetry group and $r \equiv (m,\ v,\ b)$.

The generators of the symmetry group, are written:

$$a_1 = 1:\ X_1 = 2\bar{t}\frac{\partial}{\partial \bar{t}} + m\frac{\partial}{\partial m} + v\frac{\partial}{\partial v} + b\frac{\partial}{\partial b} \quad (25)$$

$$a_2 = 1:\ X_2 = \frac{\partial}{\partial \bar{t}} \quad (26)$$

$$a_4 = 1:\ X_4 = b\frac{\partial}{\partial m} - m\frac{\partial}{\partial b} \quad (27)$$

$$a_3 = 1:\ X_3 = m\frac{\partial}{\partial v} - v\frac{\partial}{\partial m} \quad (28)$$

$$a_6 = 1:\ X_6 = v\frac{\partial}{\partial b} - b\frac{\partial}{\partial v} \quad (29)$$

$$a_5 = 1:\ X_5 = \frac{\partial}{\partial m} \quad (30)$$

$$a_7 = 1:\ X_7 = \frac{\partial}{\partial v} \quad (31)$$

$$a_8 = 1:\ X_8 = \frac{\partial}{\partial b} \quad (32)$$

$$f \neq 0:\ X_{10} = f\left(m, v, b, \bar{t}\right)\frac{\partial}{\partial u} \quad (33)$$

Integrating Lie's equations

$$\frac{dp}{da_i} = X_i p,\ p \equiv \left(\bar{t}, m, v, b\right) \quad (34)$$

$$\frac{du}{da_i} = X_i u,\ p(0) = p,\ u(0) = u \quad (35)$$

the finite transformations which correspond to each one-dimensional group results:

$$a_1 : \left(r, \bar{t}, u\right) \to \left(rd, \bar{t}d^2, u\right) \quad (36)$$

$$a_2 : \left(r, \bar{t}, u\right) \to \left(r, \bar{t} + a_2, u\right) \quad (37)$$

$$\rho : \left(r, \bar{t}, u\right) \to \left(Rr, \bar{t}, u\right) \quad (38)$$

$$\omega : \left(r, \bar{t}, u\right) \to \left(r + \omega, \bar{t}, u\right) \quad (39)$$

$$\mu : \left(r, \bar{t}, u\right) \to \left(r, \bar{t}, u + \mu f\right) \quad (40)$$

where

$$d = \exp(a_1),\ \rho \equiv (a_3, a_4, a_6),$$

$$R \in SO(3),\ \omega \equiv (a_5, a_7, a_8).$$

Combining the above transformations in the space of the variables $\left(r, \bar{t}\right)$, the group results:

$$\left(r, \bar{t}\right) \to g\left(r, \bar{t}\right) \equiv \left[(Rr + \omega)\, d, \left(\bar{t} + a_2\right) d^2\right] \quad (41)$$

The elements of this group could be characterized through the symbol:

$$g \equiv (d, a_2, \omega, R) = (d)(a_2)(\omega, R) \quad (42)$$

Using the transformations (36)-(39), we obtain

$$f_{(d)} = \overline{C},\ f_{(a_2)} = \overline{C},\ f_{(\omega,R)} = \overline{C} \quad (43)$$

where $\overline{C}$ is a constant.

Taking into account the relation (42) and combining the above functions, we have the following result

$$f_g\left(r, \bar{t}\right) = \overline{C} \quad (44)$$

The relations (2) and (44) show that the need for psychological stimulation is governed by the following rule

$$C\left[g\left(r, \bar{t}\right)\right] - C\left(r, \bar{t}\right) = 0 \quad (45)$$

Namely, the need for psychological stimulation is a constant specific to a person.

$$C = constant \quad (46)$$

3 The Emotion as a Support of the Psychological Defense Mechanism

The dynamics of the psychological experience records deviations from an optimal register which has to favour the transformation of the personality towards the saturation formulae.

At the psychological level, the internalisation of the references develops the entropy capacities with destructive and restoration valences.

The transformation of the inner representation into behaviour acts dissociates the reference type reality in the subjective reality as an image of a self – created scenario and in the mediated reality which has to maintain the self – created scenarios. The alteration of the subjective reality inhibits the dynamics of the personality but it activates the psychological resources, directing them towards the maintaining of the affected subjective reality.

The formulae of the psychological structuration pressupose the existence of certain equilibriums between the elements of the mediated reality which necessitate the existence of a restoration and self - generation capacity. The deletion of the self - generation capacity as an effect of self – satisfaction determines the activation of certain germination processes of the elements of subjective and mediated reality.

At the level of the subjective reality, the appearance of the unspecific contents involves an entire affective emotional regression caused by the priority of their interpretation. Moreover, the subjective reality changes into a potential reality determined by the approach of identifying of certain specifics which do not exist!

The lack of identification of the authentic subjective reality engages the psychological system in the rebuild approach but without the elements mentioned above. If the approach becomes effective, the subjective reality will be changed into an altered mediated reality which does not maintain the self – created scenarios. The chance of restoration consists in the act of maximal dissociation of the altered subjective reality, followed by the correct interpretation of the specific elements and their structuration in the class of subjective reality. The persistence of the subjective realities amplifies the inner communication up to saturation.

The subjective reality and the mediated reality define the interpretative elements of the pure desire (personal) and of the negotiated desires (resulted and interpreted from the communication with the environment).

The psychological system has certain specific codification resources which assign a psychological experience to each element of the subjective and mediated reality. The dynamics of the psychological experience is determined by the changes of the elements of the two types of realities. The using of the realities, subjective and mediated, is determined by the commutators supervised by the sensors of detection and transformation of the psychological tonus not used in needs.

The reason for the existence of the psychological defense mechanism is given by the presence of a dynamics of psychological experience which has to be adjusted in the integrity area of the psychological system. The adjustment is done by choosing certain languages which change the elements of the realities, subjective and mediated, into modulators of psychological experience more accessible for the psychological system.

But, the psychological defense mechanism is not in any way the magic element which generates the psychological dynamics. Simultaneously with the dynamics of the subjective and mediated realities, there exists a dissonant reality between the final act and the initial one, which functions virtually and generates new commutators or uses the commutators of the elements of reality. The commutators, as interpretative devices, mediate the relation between the cognitive dissonance and the organic behaviour retort. These are in fact the interpretative equalizing languages of these two acts. The effective overlapping is realized by the emotional act which constitutes the support for the psychological defense mechanism.

The invincibility of the psychological system is explained through its endowment with invariant elements which maintain the antagonistic constructions, having for their main task the assurance of the functionality of autonomy. The dynamics of the situations shows forms of the psychological defense mechanism meant for the adjustment of the psychological experience. However, all the psychological mechanisms constitute devices of psychological integrity. The emotional act constitutes a mechanism of the psychological availability which has the capacity to eliminate the discrepancy between the image of the satisfied desire and the present psychological image. This one is in fact a transformation act of a present psychological state into a future psychological state.

The dissociation of the experiential dowry into essential facts and detail type facts is noticed by the psychological system in the conscious and the unconscious plane. The filters of consciousness are incapable of identifying the logic of the essential, which is additionally constituted by the elements of mediated reality that hide the discernible logic of the emotional poles specific to subjective reality.

The asymmetric orientation of the emotional act towards the future shows its optimistic facet, which is discernible to the unconscious state. Moreover, the emotional act is specialized, at the level of the unconscious state, in identifying the essential facts.

The crystallization of the personality is realized through the acting of the essential references which endow the psychological system. The activation is produced through the experiential act. The experiences generate tensional states and these are determined by desires. The emotion as a support of desire maintains the tensional states. Therefore, the emotion manifests itself as a tensional psychological state and as a mechanism in which the transformations of the parameters of the psychological dynamics have to preserve the tensional state. The main consequence of this statement results in the existence of two laws: one for the tensional psychological dynamics (see the equation (1)) and the other for emotions. Both equations must be preserved by the transformations of the parameters of the psychological dynamics which define in fact the emotional mechanism.

In the following chapters we will determine and analyze the laws of the emotional mechanism and the law of emotions.

3.1 A Mathematical Theory of Emotional Mechanism

The equation (2), where C is a constant, describes the tensional state dynamics.

The solution of this equation can be obtained through the separation method of variables. The separated solutions can be described in different coordinate systems. The coordinate ones are obtained using the theory of the symmetry groups and also, these have to preserve the equation (2).

The connection between the symmetry group of a partial differential equation and the separation method of variables consists in the fact that the separated solutions are common eigenfunctions for the symmetry operators of the equation.

All the coordinate systems mentioned above are described through the system of n equations with eigenvalues:

$$\begin{array}{l} E_j u = \lambda_j u \quad if\ j = 1, 2, ...n-1 \\ \Omega u = 0 \end{array} \tag{47}$$

where E_j are the symmetry operators of the second degree for the equation (2) and Ω is a differential operator of the second degree which is applied to the variable u.

The equation (2) suggests symmetries only for the variables (m,v,b). The coordinate systems which result from the system (47) preserve the Helmholtz type equations. The vectorial expression of these equations defines the law of emotions.

3.2 The Mathematical Laws of Emotional Mechanism

The generator L (see the equations (20)-(24)) becomes:

$$\begin{array}{r} L = \left(2a_1\bar{t} + a_2\right)\partial_{\bar{t}} + \left(a_1 m - a_3 v + a_4 b + a_5\right)\partial_m + \\ + \left(a_1 v + a_3 m - a_6 b + a_7\right)\partial_v + \\ + \left(a_1 b - a_4 m + a_6 v + a_8\right)\partial_b + f\left(r, \bar{t}\right)\partial_u \end{array} \tag{48}$$

where $\partial_x = \frac{\partial}{\partial x}$, $x \equiv \left(\bar{t},\ m,\ v,\ b\right)$, $r \equiv (m,\ v,\ b)$ or

$$\begin{array}{r} L = f\left(r, \bar{t}\right)\partial_u + a_1\left(2\bar{t}\partial_{\bar{t}} + m\partial_m + v\partial_v + b\partial_b\right) + \\ + a_2\partial_{\bar{t}} + a_3\left(m\partial_v - v\partial_m\right) + a_4\left(b\partial_m - m\partial_b\right) + \\ + a_6\left(v\partial_b - b\partial_v\right) + a_5\partial_m + a_7\partial_v + a_8\partial_b \end{array} \tag{49}$$

Therefore the Lie algebra of the symmetry group of the equation (2) is formed by the real subalgebra

$$\begin{array}{l} \Lambda = a_3 P_2 + a_4 P_3 + a_6 P_4 + a_5 P_5 + \\ + a_7 P_6 + a_8 P_7 + a_2 P_8 + a_1 P_9, / a_i \in R \end{array} \tag{50}$$

where:

$$P_2 = m\frac{\partial}{\partial v} - v\frac{\partial}{\partial m} \tag{51}$$

$$P_3 = b\frac{\partial}{\partial m} - m\frac{\partial}{\partial b} \tag{52}$$

$$P_4 = v\frac{\partial}{\partial b} - b\frac{\partial}{\partial v} \tag{53}$$

$$P_5 = \frac{\partial}{\partial m} \tag{54}$$

$$P_6 = \frac{\partial}{\partial v} \tag{55}$$

$$P_7 = \frac{\partial}{\partial b} \tag{56}$$

$$P_8 = \frac{\partial}{\partial \bar{t}} \tag{57}$$

$$P_9 = 2\bar{t}\frac{\partial}{\partial \bar{t}} + m\frac{\partial}{\partial m} + v\frac{\partial}{\partial v} + b\frac{\partial}{\partial b} \tag{58}$$

and by the infinite dimensional subalgebra of the operators $g\left(r, \bar{t}\right)\partial_u$.

The operators P_i satisfy the relations of commutation

$$\begin{array}{c} [P_2, P_3] = P_4, [P_2, P_4] = -P_3, [P_2, P_5] = -P_6, \\ [P_2, P_6] = P_5, [P_2, P_7] = 0, [P_2, P_8] = 0, \\ [P_2, P_9] = 0, [P_3, P_4] = P_2, [P_3, P_5] = P_7, \\ [P_3, P_6] = 0, [P_3, P_7] = -P_5, [P_3, P_8] = 0, \\ [P_3, P_9] = 0, [P_4, P_5] = 0, [P_4, P_6] = -P_7, \\ [P_4, P_7] = P_6, [P_4, P_8] = 0, [P_4, P_9] = 0, \\ [P_5, P_9] = P_5, [P_6, P_9] = P_6, [P_7, P_9] = P_7, \\ [P_5, P_6] = [P_5, P_7] = [P_5, P_8] = \\ = [P_6, P_7] = [P_6, P_8] = [P_7, P_8] = 0, \\ [P_8, P_9] = 2P_8 \end{array}$$

The equation (2), expressed through the differential operators P_i, becomes:

$$\Omega \mathrm{u} \equiv \left[\mathrm{P}_8^2 - \frac{\left(\mathrm{o}\bar{\mathrm{t}} + \mathrm{p}\right)^2}{4}\left(\mathrm{P}_5^2 + \mathrm{P}_6^2 + \mathrm{P}_7^2\right) - \mathrm{C}\right]\mathrm{u} = 0 \tag{59}$$

The symmetry operators of one degree

$$P_5;\ P_6;\ P_7;\ P_2;\ P_3;\ P_4$$

satisfy the commutation relations:

$$\begin{array}{c} [\Omega, P_2] = [\Omega, P_3] = [\Omega, P_4] = \\ = [\Omega, P_5] = [\Omega, P_6] = [\Omega, P_7] = 0 \end{array}$$

The algebra of the symmetry operators, $P_2,\ P_3,\ P_4,\ P_5,\ P_6,\ P_7$, is isomorphous with the following $E(2)$ groups:

$$\left(m',v'\right) = g_1\left(m,v\right) =$$

$$= \left(-v\sin\theta + m\cos\theta + f_1,\ v\cos\theta + m\sin\theta + c_1\right),$$

$$\left(v',b'\right) = g_2\left(v,b\right) =$$

$$= \left(-b\sin\theta + v\cos\theta + f_2,\ b\cos\theta + v\sin\theta + c_2\right),$$

$$\left(b',m'\right) = g_3\left(b,m\right) =$$

$$= \left(-m\sin\theta + b\cos\theta + f_3,\ m\cos\theta + b\sin\theta + c_3\right)$$

$$g_i \in E\left(2\right);\ f_i,\ c_i \in R;\ \theta \in [0,2\pi)$$

We say that an operator $R \in \Lambda$ is placed on the same orbit with the operator T if there exists $g \in E\left(2\right)$ and $c \in R$, $c \neq 0$ such that $R = cT^g$.

The operators:

$$P_2^2, P_3^2, P_4^2, P_5^2, P_6^2, P_7^2, P_2P_3, P_2P_4, P_2P_5, P_2P_6,$$

$$P_2P_7, P_3P_4, P_3P_5, P_3P_6, P_3P_7, P_4P_5, P_4P_6, P_4P_7,$$

$$P_5P_6, P_5P_7, P_6P_7, [P_2,P_3]_+, [P_2,P_4]_+, [P_2,P_5]_+,$$

$$[P_2,P_6]_+, [P_2,P_7]_+, [P_3,P_4]_+, [P_3,P_5]_+,$$

$$[P_3,P_6]_+, [P_3,P_7]_+, [P_4,P_5]_+, [P_4,P_6]_+,$$

$$[P_4,P_7]_+, [P_5,P_6]_+, [P_5,P_7]_+, [P_6,P_7]_+$$

where $[P_i,P_j]_+ = P_iP_j + P_jP_i$ are the symmetry operators of the second degree for the equation (2). The algebra of these operators contains the following orbits:

$$P_5^2, P_2^2, [P_2,\ P_5]_+, P_2^2 - l_1^2 P_5^2,$$

$$P_6^2, P_2^2, [P_2,\ P_6]_+, P_2^2 - l_2^2 P_6^2,$$

$$P_5^2, P_3^2, [P_3,\ P_5]_+, P_3^2 - l_3^2 P_5^2,$$

$$P_7^2, P_3^2, [P_3,\ P_7]_+, P_3^2 - l_4^2 P_7^2,$$

$$P_6^2, P_4^2, [P_4,\ P_6]_+, P_4^2 - l_5^2 P_6^2,$$

$$P_7^2, P_4^2, [P_4,\ P_7]_+, P_4^2 - l_6^2 P_7^2,$$

where l_i^j are constant.

The coordinate systems $(x_1,\ x_2, \ldots\ldots, x_n)$ for which we can apply the separation method of variables result from the system of n equations with eigenvalues (47).

The orbits mentioned above correspond to the following coordinate systems: cartesian system $\{(m,v)\,,\ (v,b)\,,\ (m,b)\}$, polar system (r,θ), parabolic system $\left(x = \frac{\xi^2-\eta^2}{2}; y = \xi\eta\right)$ where $(x,y) = \{(m,v)\,,\ (v,b)\,,\ (m,b)\}$ and elliptic system $(x = k_i\cosh\alpha\cos\beta,\ y = k_i\sinh\alpha\sin\beta)$ where $(x,y) = \{(m,v)\,,\ (v,b)\,,\ (m,b)\}$ and k_i are constant. These systems constitute, taking into account the chapter 3, the emotional mechanism. Moreover, they preserve the following Helmholtz type equations:

$$\left(\frac{\partial^2}{\partial m^2} + \frac{\partial^2}{\partial v^2} + k_1^2\right) w = 0 \qquad (60)$$

$$\left(\frac{\partial^2}{\partial v^2} + \frac{\partial^2}{\partial b^2} + k_2^2\right) w = 0 \qquad (61)$$

$$\left(\frac{\partial^2}{\partial m^2} + \frac{\partial^2}{\partial b^2} + k_3^2\right) w = 0 \qquad (62)$$

where w – represents the variable "emotion".

Let $\overrightarrow{\triangle}^*$ be an operator defined as:

$$\overrightarrow{\triangle}^* = (r_1,\ r_2,\ r_3)$$

where $r_1 = \frac{\partial^2}{\partial m^2} + \frac{\partial^2}{\partial v^2} + k_1^2$, $r_2 = \frac{\partial^2}{\partial v^2} + \frac{\partial^2}{\partial b^2} + k_2^2$, $r_3 = \frac{\partial^2}{\partial m^2} + \frac{\partial^2}{\partial b^2} + k_3^2$ and U a vector field having the components w,w,w.

Then

$$\overrightarrow{\triangle}^* \cdot \overrightarrow{U} = \mathbf{0} \qquad (63)$$

This equation defines the mathematical law of emotions.

4 Conclusion

1. The functionality of psychological instances is governed by dynamic laws, all of them containing the same terms. The laws are different as to form and the existence of the common kernel of terms is explained by the necessity of the simultaneous functioning of psychological instances and by the maintaining of the invariant law of the unconscious instance. The invincibility of the unconscious instance is determined by invariance law of the equivalence class type. The conscious instance, endowed with interpretative potential, presupposes laws which preserve the functionality of the unconscious instance. On the other hand the endowing of the conscious instance with rigid limits of potentialities, within which it can function effectively, turns this instance into an extremely vulnerable component. The maximum vulnerability of the conscious instance takes the form of deletion. The deletion is determined by the need of the psychological system to function with its natural unconscious instance in cooperation with a cloned instance;
2. The dynamics of the anti-entropy is determined by an inert component, manifested through the constant need for psychological stimulation; it is further determined by an active component which has to satisfy

adaptation to the inquiring environment. The conclusion is surprising because it shows that in the process of adaptation, the psychological system does not use its whole available capacity. Moreover, the purpose of the inert component might be the stimulation or the inhibition of the inquiring environment. In this way adaptation becomes efficient;

3. Taking into account the conclusions presented above, we can conclude that an essential component of the psychological equilibrium is given by the preserving of the ancestral constant that is the need for psychological stimulation. The deletion of the possibility of the specific expression of a person and the impossibility of avoidance of the unpleasant state are determined by the alteration of the ancestral constant mentioned above;

4. The emotion manifests itself as a tensional psychological state and as a psychological mechanism meant to preserve the tensional state;

5. The vector form of Helmholtz type equations describes the emotions as tensional states. The dynamics of emotions is dependent only on apperception, on the ratio between the current neuropsychological activation and the minimal one needed for the activation of attention and on the amplitude of the tensional states;

6. The coordinate systems which preserve the equation of the dynamics of tensional states define the psychological mechanism of emotions. The reason for the existence of this mechanism is given by its capacity to maintain the tensional psychological state;

7. The presence of the psychological defense mechanism is determined by the existence of emotions. This mechanism has to adjust permanently the tensional state in the integrity area of the psychological system. Therefore the tensional psychological state is dependent on time. The emotions as tensional states are determined by tensional psychological parameters and not by time. For this reason the equation of emotions does not contain time as an explicit parameter, even though these are in fact tensional psychological states.

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