An Application of Fuzzy Time Series to Improve ISE Forecasting

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Abstract: - The problem of fuzzy time series forecasting plays an important role in many scientific areas such as statistics and neural networks. While forecasting fuzzy time series, most of forecasting applications use the same length of intervals. The determination of length of intervals is significant and critical in fuzzy time series forecasting. The usage of convenient performance measure may also have an important affect for forecasting studies. MSE (Mean squared error) as a performance measure is widely used in many studies. The aim of this paper is to improve fuzzy time series forecasting by using different length of intervals with neural networks according to various performance measures. For this reason, we take ISE (Istanbul stock exchange) national-100 index as a large data set for forecasting. We use various performance measures such as MSE, RMSE (Root mean squared error), MAE (Mean absolute error) and MAPE (Mean absolute percentage error) to compare forecasting performances with different length of intervals. The empirical results show that the most convenient length of intervals can be chosen as 300 by comparing overall performance of MSE, RMSE, MAE and MAPE by using neural networks.

Key-Words: - Forecasting, Fuzzy time series, Neural networks, ISE national-100 index, Performance measures, Length of intervals.

1 Introduction

The definition of fuzzy time series was firstly introduced by Song and Chissom [1, 2, 3]. They used fuzzy time series forecasting in the enrollment data of the University of Alabama and suggested a step-by-step procedure. This procedure has six steps: (1) Define and partition the universe, of discourse, (2) define fuzzy sets for the observations, (3) fuzzify the observations, (4) establish the fuzzy relationship, (5) forecast and (6) defuzzify the forecasting results [2]. Song and Chissom considered forecasting problem according to fuzzy relational equations and approximate reasoning, which takes a large amount of computation time in deriving the fuzzy relationship [2, 16].

Originally proposed by Zadeh in 1965, fuzzy theory nowadays has been applied in various areas widely [17, 18]. The fuzzy time series approaches, which are based on fuzzy sets, have been advanced to enhance the models prediction capabilities and applicability [19]. In recent years, many various applications have been used for fuzzy forecasting either to find a better forecasting result or to do faster computations [2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15]. After Song and Chissom's definitions, many fuzzy time series models have been applied to different problems such as enrollment [1, 2, 3, 4, 6, 7, 16, 20, 21, 22, 23, 24], temperature forecasting [25, 26, 27, 28], stock index forecasting [6, 7, 14, 20, 23, 26, 27, 28, 32, 33] and fuzzy systems have been applied some problems concerning with nonlinear systems [29, 30, 31]. Many forecasting studies generally use MSE for performance measure and 1000 as length of intervals [2, 3, 4, 6, 21]. But, the issue of how to affect forecasting results by choosing different performance measures and length of intervals has not been investigated properly. Using different performance measures and length of intervals may lead to different forecasting results in fuzzy time series. Hence, Assigning convenient performance measure and effective length of intervals are critical determinations.

In this study, we have used a feed forward neural network architecture, which includes one hidden layer with two hidden nodes and one output layer, to establish fuzzy relationships. To investigate the forecasting performance with different length of intervals and performance measures, we use a basic model which simply uses a neural network approach to forecast all of the observations [8]. ISE national-100 index for the years 2001-2008 is selected as the forecasting target. MSE, RMSE, MAE and MAPE are chosen as performance measures. Forecasting results are compared for different length of intervals according to these performance measures and the most convenient length of intervals is stated as 300 for all performance measures in overall. To show these things, the remaining content of this paper is organized as follows. Section 2 includes the definitions of fuzzy time series and artificial neural networks. Section 3 explains forecasting process and compares forecasting results according to performance measures and lengths of intervals. Section 4 offers some conclusions.

2 Problem Formulation

In this section, we explain the definitions of fuzzy time series and we give some information about neural network architecture.

2.1 Fuzzy Time Series

Song and Chissom first proposed the definitions of fuzzy time series [1, 2]. The some general definitions of fuzzy time series are given as follows:

Let U be the universe of discourse, where $U=\{u_1,u_2,...,u_n\}$. A fuzzy set A_i of U is defined by $A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + ... + f_{Ai}(u_n)/u_n$, where f_{Ai} is the membership function of the fuzzy set A_i, $f_{Ai}: U \to [0,1]$. u_k is an element of fuzzy set A_i and $f_{Ai}(u_k)$ is the degree of belongingness of u_k to A_i. $f_{Ai}(u_k)\in[0,1]$ and $1 \le k \le n$.

Definition 1. $Y(t)$ $(t=...,0,1,2,...)$ is a subset of real numbers. Let $Y(t)$ be the universe of discourse defined by the fuzzy set $f_i(t)$. If $F(t)$ consists of $f_i(t)$ $(i=1,2,...)$, $F(t)$ is defined as a fuzzy time series on $Y(t)$ $(t=...,0,1,2,...)$ [1].

Definition 2. If there exists a fuzzy relationship $R(t-1,t)$, such that $F(t)=F(t-1)\times R(t-1,t)$, where $\times$ is an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by
$F(t-1)\to F(t)$.

Definition 3. Suppose $F(t-1)=A_i$ and $F(t)=A_j$, a fuzzy logical relationship is defined as

$$A_i \to A_j,$$

where A_i is named as left-hand side of the fuzzy logical relationship and A_j the right-hand side. Note the repeated fuzzy logical relationships are removed [2].

Definition 4. Fuzzy logical relationships can be further grouped together into fuzzy logical relationship groups according to the same left-hand sides of the fuzzy logical relationships.

For example, there are fuzzy logical relationships with the same left-hand sides (A_i):

$$A_i \to A_{j1},$$
$$A_i \to A_{j2}$$
$$\ldots$$

These fuzzy logical relationships can be grouped into a fuzzy logical relationship group as follows:

$$A_i \to A_{j1}, A_{j2}, ...$$

Definition 5. Suppose $F(t)$ is caused by $F(t-1)$ only, and $F(t)=F(t-1)\times R(t-1,t)$. For any t, if $R(t-1,t)$ is independent of t, then $F(t)$ is named a time-invariant fuzzy time series, otherwise a time-variant fuzzy time series.

Song and Chissom applied both time-invariant and time-variant models to forecast the enrollment at the University of Alabama [2,3]. The time-invariant model includes the following steps:

(1) define the universe of discourse and the intervals,
(2) partition the intervals,
(3) define the fuzzy sets,
(4) fuzzify the data,
(5) establish the fuzzy relationships,
(6) forecast,
(7) defuzzify the forecasting results.

The time-variant model includes the following steps:

(1) Define the universe of discourse and the intervals (the same as step (1) in the time-invariant model).
(2) Partition the intervals (the same as step (2) in the time-invariant model).
(3) Define the fuzzy sets (the same as step (3) in the time-invariant model).
(4) Fuzzify the data (the same as step (4) in the time-invariant model).
(5) Establish the fuzzy relationships and forecast.
(6) Defuzzify the forecasting results.

In both models, note that the establishment of fuzzy relationships, $R(t-1,t)$, and defuzzification were the critical steps for forecasting [2, 6].

We used the following steps in problem solution:

Step 1. Define and partition the universe of discourse.
Step 2. Define fuzzy sets for the observations.
Step 3. Fuzzify the observations.
Step 4. Establish the fuzzy relationship, R.
Step 5. Forecast.
Step 6. Defuzzify the forecasting results.

2.2 Artificial Neural Networks

In forecasting, artificial neural networks are mathematical models that imitate biological neural networks. Artificial neural networks consist of some elements. Determining the elements of the artificial neural networks issue that affect the forecasting performance of artificial neural networks should be considered carefully. Elements of the artificial neural networks are generally given as network architecture, learning algorithm and activation function. One critical decision is to determine the suitable architecture, that is, the number of layers, number of nodes in each layers and the number of arcs which interconnects with the nodes [34, 35]. However, there are not general rules for determining the best architecture. So, much architecture should be tried for the correct results. There are various types of artificial neural networks. One of them is called as feed forward neural networks and its architecture is shown in Fig. 1 [35].

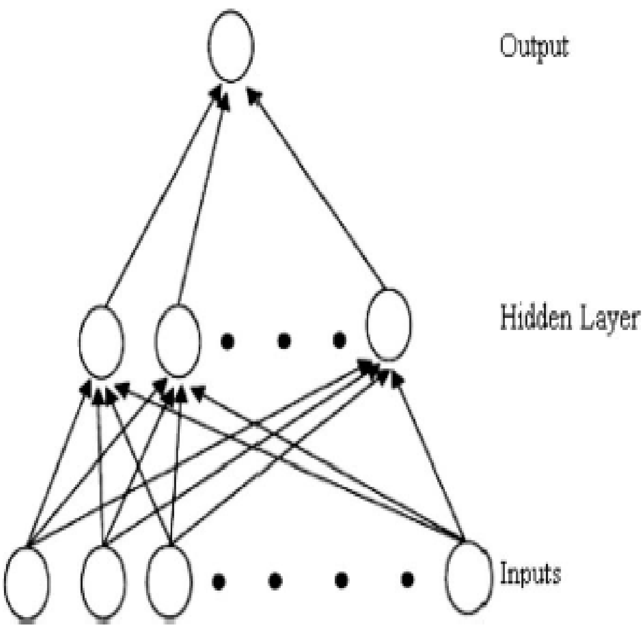

Fig. 1. Feed Forward Neural Network Architecture

In our study, we prepared a feed forward neural network architecture. We took its architecture including one input layer, one hidden layer with two hidden nodes and one output layer and we can show its architecture in Fig. 2 [8]. The function of each node for hidden and output layers can be shown with Fig. 3 [8].

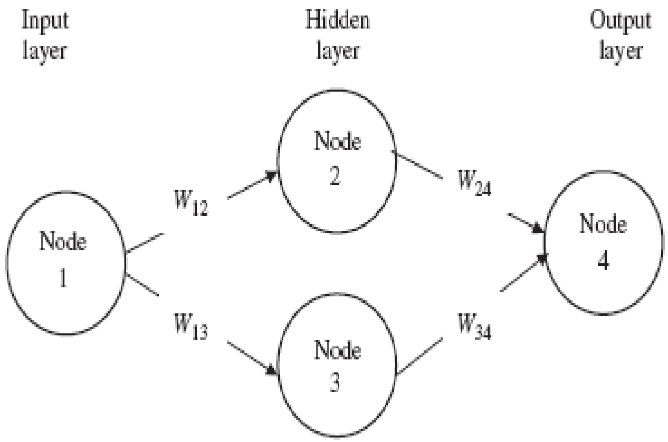

Fig. 2. Neural Network Architecture.

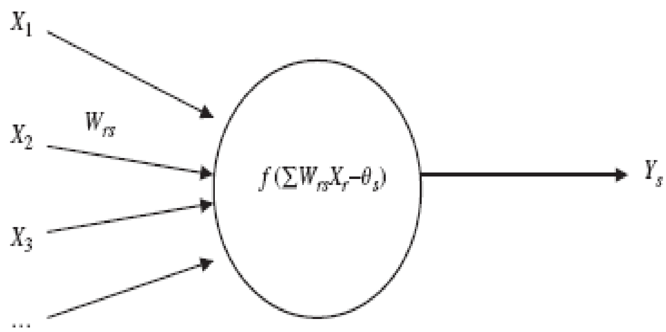

Fig. 3. A node in a neural network

3 Problem Solution

A large data set for is necessary for training a neural network and we used weekly closing prices of the ISE national-100 index for the years 2001-2008. Many studies have used a convenient ratio to separate in-samples from out-of samples ranging from 70%:30% to 90%:10% [9]. Hence, we chose the data from January to October for our estimation (in-sample) and November and December for forecasting (out-sample). So the ratio is about 83%:17%.

In this study, we used a feed forward neural network architecture. We stated its architecture as includes one input layer, one hidden layer with two hidden nodes and one output layer. We used backpropagation learning algorithm for training neural network models and sigmoid activation function for all neurons. Sigmoid activation function is $f(z)=1/1+e^{-z}$. For evaluation purposes; MSE, RMSE, MAE and MAPE were used to measure performance:

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \quad (1)$$

$$RMSE = \sqrt{MSE} \quad (2)$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |e_t| \quad (3)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} \left|\frac{e_t}{actual_t}\right| \quad (4)$$

Where $e_t = actual_t - forecast_t$ and n is forecast number.

The steps of neural network-based fuzzy time series models are given below.

Step 1: Define and partition the universe of discourse The universe of discourse for observations, $U = [starting, ending]$, is defined. After the length of intervals, l, is determined, the U can be partitioned into equal –length intervals $u_1, u_2, ..., u_b$, $b = 1,...$ and their corresponding midpoints $m_1, m_2, ..., m_b$ respectively.

$$u_b = \left[starting + (b-1)\times l, starting + b \times l\right],$$
$$m_b = \frac{\left[starting + (b-1)\times l, starting + b \times l\right]}{2}.$$

We can show data set with date and ISE national-100 index values in Table 1. Table 2, Table 3, Table 4, Table 5, Table 6, Table 7, Table 8 and Table 9 show different length of intervals with starting and ending points according to ISE national-100 index for the year from 2001 to 2008 respectively..

Table 1. ISE national-100 index with dates.

Date	**ISE**
05-01-2001	10007,30000
12-01-2001	10964,89000
19-01-2001	11400,70000
26-01-2001	10742,79000
02-02-2001	10546,66000
09-02-2001	9545,86000
16-02-2001	10169,50000
23-02-2001	8344,94000
02-03-2001	9513,77000
09-03-2001	9513,77000
16-03-2001	8763,42000
23-03-2001	8365,64000
30-03-2001	8022,72000
06-04-2001	8236,80000
13-04-2001	8940,25000
20-04-2001	9658,35000
27-04-2001	12363,01000
04-05-2001	12529,69000
11-05-2001	11779,31000
18-05-2001	12738,55000
25-05-2001	11967,96000
01-06-2001	11270,98000
08-06-2001	12138,26000
15-06-2001	11839,75000
22-06-2001	10879,57000
29-06-2001	11204,24000
06-07-2001	10168,80000
13-07-2001	8940,84000
20-07-2001	9684,55000
27-07-2001	9765,10000
03-08-2001	10101,58000
10-08-2001	9400,63000
17-08-2001	9736,95000
24-08-2001	10049,66000
31-08-2001	9878,88000
07-09-2001	9628,06000
14-09-2001	7937,87000
21-09-2001	7306,38000
28-09-2001	7625,87000
05-10-2001	7598,91000
12-10-2001	8431,49000
19-10-2001	8822,63000
26-10-2001	9915,52000
02-11-2001	10069,72000
09-11-2001	10080,92000
16-11-2001	11370,89000
23-11-2001	11719,40000
30-11-2001	11633,93000
07-12-2001	12662,84000
14-12-2001	12761,86000
21-12-2001	12721,20000
28-12-2001	13782,76000
…	
…	
…	
04-01-2008	52529,88000
11-01-2008	51920,59000
18-01-2008	48657,46000
25-01-2008	45497,24000
01-02-2008	44452,08000
08-02-2008	41957,86000
15-02-2008	44305,27000
22-02-2008	45592,48000
29-02-2008	44776,88000

07-03-2008	41537,22000
14-03-2008	42585,91000
21-03-2008	39719,08000
28-03-2008	39501,17000
04-04-2008	41362,71000
11-04-2008	41980,11000
18-04-2008	42641,06000
25-04-2008	43593,80000
02-05-2008	43425,85000
09-05-2008	42034,70000
16-05-2008	42498,82000
23-05-2008	39960,99000
30-05-2008	39969,63000
06-06-2008	39645,54000
13-06-2008	38295,72000
20-06-2008	37912,85000
27-06-2008	35829,40000
04-07-2008	34300,37000
11-07-2008	35006,46000
18-07-2008	37946,57000
25-07-2008	37556,95000
01-08-2008	42984,66000
08-08-2008	40949,91000
15-08-2008	42194,42000
22-08-2008	40894,27000
29-08-2008	39844,48000
05-09-2008	39115,63000
12-09-2008	37033,87000
19-09-2008	36370,16000
26-09-2008	36556,61000
03-10-2008	34553,00000
10-10-2008	28495,93000
17-10-2008	25870,17000
24-10-2008	24176,68000
31-10-2008	27832,93000
07-11-2008	26648,17000
14-11-2008	25425,26000
21-11-2008	21965,96000
28-11-2008	25714,98000
05-12-2008	24034,70000
12-12-2008	24936,93000
19-12-2008	26205,40000
26-12-2008	26498,96000

Table 2. Different length of intervals for the year 2001.

2001		
Intervals	Starting	Ending
100	7300	13800
200	7300	13900
300	7300	13900
400	7200	14000
500	7300	13800
600	7300	13900
700	7000	14000
800	6900	14100
900	6900	14100
1000	7000	14000

Table 3. Different length of intervals for the year 2002.

2002		
Intervals	Starting	Ending
100	8800	14300
200	8800	14400
300	8700	14400
400	8800	14400
500	8800	14300
600	8600	14600
700	8800	14400
800	8800	14400
900	8500	14800
1000	8600	14600

Table 4. Different length of intervals for the year 2003.

2003		
Intervals	Starting	Ending
100	9400	18300
200	9400	18400
300	9400	18400
400	9300	18500
500	9400	18400
600	9400	18400
700	9300	18400
800	9100	18700
900	9400	18400
1000	9400	18400

Table 5. Different length of intervals for the year 2004.

2004		
Intervals	Starting	Ending
100	16500	25000
200	16500	25100
300	16400	25100
400	16500	25300
500	16500	25000
600	16300	25300
700	16200	25300
800	16400	25200
900	16300	25300
1000	16300	25300

Table 6. Different length of intervals for the year 2005.

2005		
Intervals	Starting	Ending
100	23500	39800
200	23500	39900
300	23400	39900

400	23500	39900
500	23400	39900
600	23300	40100
700	23300	40100
800	23300	40100
900	23100	40200
1000	23200	40200

Table 7. Different length of intervals for the year 2006.

2006		
Intervals	Starting	Ending
100	33100	46900
200	33100	46900
300	33100	46900
400	33000	47000
500	33000	47000
600	33100	46900
700	33000	47000
800	32800	47200
900	32800	47200
1000	33000	47000

Table 8. Different length of intervals for the year 2007.

2007		
Intervals	Starting	Ending
100	38100	58100
200	38100	58100
300	38100	58200
400	38100	58100
500	38100	58100
600	37900	58300
700	38000	58300
800	38100	58100
900	37800	58500
1000	38100	58100

Table 9. Different length of intervals for the year 2008.

2008		
Intervals	Starting	Ending
100	21900	52600
200	21900	52700
300	21800	52700
400	21900	52700
500	21800	52800
600	21700	52900
700	21900	52700
800	21700	52900
900	21500	53000
1000	21800	52800

Step 2: Define fuzzy sets for the observations

Each linguistic observation, A_i, can be defined by the intervals $u_1, u_2, ..., u_b$.

$$A_i = f_{Ai}\left(u_1\right)/u_1 + f_{Ai}\left(u_2\right)/u_2 + ... + f_{Ai}\left(u_b\right)/u_b.$$

For the year 2001 for 100 as length of intervals, each linguistic observation, A_i, was defined as follows:

$$\begin{aligned}
A_1 &= 1.0/u_1 + 0.5/u_2 + 0/u_3 + ... + 0/u_{63} + 0/u_{64} + 0/u_{65} \\
A_2 &= 0.5/u_1 + 1.0/u_2 + 0.5/u_3 + ... + 0/u_{63} + 0/u_{64} + 0/u_{65} \\
&\vdots \\
A_{64} &= 0/u_1 + 0/u_2 + 0/u_3 + ... + 0.5/u_{63} + 1.0/u_{64} + 0.5/u_{65} \\
A_{65} &= 0/u_1 + 0/u_2 + 0/u_3 + ... + 0/u_{63} + 0.5/u_{64} + 1.0/u_{65}
\end{aligned}$$

Step 3: Fuzzify the observations

For example, an observation is fuzzified to A_i if the maximal degree of membership of that observation is in A_i. Some observations are shown for 100 length of intervals in Table 10.

Table 10. Fuzzy ISE national-100 index for 2001

Date	ISE	Fuzzy ISE
14-09-2001	7937.87	7
21-09-2001	7306.38	1
28-09-2001	7625.87	4
05-10-2001	7598.91	3
12-10-2001	8431.49	12
19-10-2001	8822.63	16
26-10-2001	9915.52	27
02-11-2001	10069.72	28
09-11-2001	10080.92	28
16-11-2001	11370.89	41
23-11-2001	11719.40	45
30-11-2001	11633.93	44
07-12-2001	12662.84	54
14-12-2001	12761.86	55
21-12-2001	12721.20	55
28-12-2001	13782.76	65

Step 4: Establish the fuzzy relationship with feed forward neural network

The index numbers of A_i of F_{t-1} series are taken as input values and the index numbers of A_i of F_t series are taken as target values for each $A_i \rightarrow A_j$ for the neural network model. These fuzzy logical relationships became the input and target values for neural network and some of them are listed in Table 11.

Step 5: Forecast

We took a basic model which uses a neural network approach to forecast all of the observations. Suppose $F(t-1)=A_{i'}$. To facilitate calculation, we set i' as the input for forecasting. Suppose the output from the neural network is j'. We say that fuzzy forecast is $A_{j'}$. In other words, $F(t)=A_{j'}$ [8].

Table 11. Fuzzy logical relationships

Input	Target
7	1
1	4
4	3
3	12
12	16
16	27
27	28
28	28
28	41
41	45
45	44
44	54
54	55
55	55
55	65

Step 6: Defuzzify the forecasting results

The defuzzified forecasts are equal to midpoints of the intervals which correspond to fuzzy forecasts obtained by neural networks.

We used these steps for each year of ISE national-100 index weekly closing prices from the year 2001-2008. We took length of intervals from 100 to 1000 to investigate forecast performance. We calculated MSE, RMSE, MAE and MAPE performance measures for all years and all length of intervals. After doing same operations for every year and finding all performance measures, we compared lengths of intervals for these performance measures. Lengths of intervals with performance measures for the year from 2001 to 2008 are shown in Table 12, Table 13, Table 14, Table 15, Table 16, Table 17, Table 18 and Table 19. Lengths of intervals with performance measures are shown in Table 20.

Table 12. Performance measures with lengths of intervals for 2001

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	394,69	3,13	272958,46	522,45
200	244,84	1,97	83599,79	289,14
300	387,15	3,14	273389,79	522,87
400	566,92	4,52	471405,57	686,59
500	412,42	3,21	324632,23	569,77
600	170,08	1,40	34623,79	186,07
700	390,05	3,04	342539,79	585,27
800	241,44	2,13	73036,68	270,25
900	494,10	3,90	395557,34	628,93
1000	695,61	5,62	724978,01	851,46

Table 13. Performance measures with lengths of intervals for 2002

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	326,58	2,51	136702,09	369,73
200	424,82	3,28	224663,87	473,99
300	415,38	3,31	188637,87	434,32
400	513,62	3,90	376563,43	613,65
500	405,05	3,14	211093,43	459,45
600	458,06	3,53	284379,87	533,27
700	501,25	3,86	339595,43	582,75
800	214,19	1,78	60967,87	246,92
900	630,28	4,93	461855,87	679,60
1000	236,50	1,91	75827,43	275,37

Table 14. Performance measures with lengths of intervals for 2003

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	727,07	4,22	895680,11	946,40
200	952,07	5,52	1426028,36	1194,16
300	275,86	1,75	96610,86	310,82
400	133,53	0,82	20042,36	141,57
500	179,41	1,03	62354,36	249,71
600	189,45	1,18	61626,36	248,25
700	307,62	1,78	168776,61	410,82
800	231,90	1,45	59388,36	243,70
900	541,09	3,28	421001,61	648,85
1000	723,86	4,49	803379,36	896,31

Table 15. Performance measures with lengths of intervals for 2004

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	272,51	1,13	115250,71	339,49
200	198,67	0,84	42791,83	206,86
300	226,27	0,94	88489,16	297,47
400	333,23	1,38	191792,27	437,94
500	516,95	2,16	367222,49	605,99
600	511,87	2,20	326668,71	571,55
700	905,84	3,79	1100446,05	1049,02
800	305,08	1,27	173620,27	416,68
900	305,45	1,26	186177,38	431,48
1000	312,80	1,33	111770,94	334,32

Table 16. Performance measures with lengths of intervals for 2005

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	592,33	1,59	499639,35	706,85
200	665,95	1,87	582225,35	763,04
300	415,95	1,13	192927,79	439,24
400	534,40	1,39	464394,24	681,46
500	366,55	1,02	167758,01	409,58
600	617,91	1,65	532251,12	729,56
700	711,00	1,97	586562,24	765,87
800	664,55	1,80	631501,79	794,67
900	217,66	0,60	64093,12	253,17
1000	215,34	0,61	69533,35	263,69

Table 17. Performance measures with lengths of intervals for 2006

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	84,53	0,22	9431,97	97,12
200	559,04	1,43	327398,52	572,19
300	91,48	0,24	9489,08	97,41
400	171,20	0,44	46986,08	216,76
500	475,71	1,22	247307,08	497,30
600	582,33	1,50	383846,08	619,55
700	291,73	0,74	127716,85	357,37
800	270,93	0,70	85290,08	292,04
900	370,64	0,96	210533,08	458,84
1000	216,67	0,55	58305,41	241,47

Table 18. Performance measures with lengths of intervals for 2007

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	476,86	0,86	291926,27	540,30
200	339,80	0,62	127517,38	357,10
300	392,13	0,70	248514,71	498,51
400	671,31	1,21	567134,71	753,08
500	435,53	0,79	340393,38	583,43
600	564,36	1,03	347563,60	589,55
700	332,34	0,59	207613,60	455,65
800	349,00	0,63	166396,04	407,92
900	407,69	0,74	251866,04	501,86
1000	538,07	0,97	449975,60	670,80

Table 19. Performance measures with lengths of intervals for 2008

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	621,21	2,47	388175,46	623,04
200	271,21	1,12	98444,71	313,76
300	445,49	1,89	421351,71	649,12
400	921,21	3,81	1160761,71	1077,39
500	494,40	1,99	345012,71	587,38
600	151,04	0,60	36849,21	191,96
700	833,71	3,35	727319,21	852,83
800	515,65	2,16	498739,71	706,22
900	657,99	2,74	703636,46	838,83
1000	268,05	1,08	83930,21	289,71

Table 20. Performance measures with lengths of intervals

	Performance Measures			
	MAE	**MAPE**	**MSE**	**RMSE**
100	436,97	2,02	326220,55	518,17
200	457,05	2,08	364083,73	521,28
300	**331,21**	**1,64**	**189926,37**	**406,22**
400	480,68	2,18	412385,05	576,06
500	410,75	1,82	258221,71	495,33
600	405,64	1,76	250976,09	458,72
700	534,19	2,39	450071,22	632,45
800	349,09	1,74	218617,60	422,30
900	453,11	2,30	336840,11	555,20
1000	400,86	2,07	297212,54	477,89

According to results in Table 20, the most convenient length of intervals for ISE national-100 index is 300. We can decide this result for all performance measures. Table 20 is prepared with averages of performance measures for all years. Performance measures are listed in Table 21 from 2001 to 2008 for 300 as length of intervals.

Table 21. Performance measures for 300

	Performance Measures			
	MAE	MAPE	MSE	RMSE
2001	387,15	3,14	273389,79	522,87
2002	415,38	3,31	188637,87	434,32
2003	275,86	1,75	96610,86	310,82
2004	226,27	0,94	88489,16	297,47
2005	415,95	1,13	192927,79	439,24
2006	91,48	0,24	9489,08	97,41
2007	392,13	0,70	248514,71	498,51
2008	445,49	1,89	421351,71	649,12

4 Conclusion

Deciding length of intervals and choosing performance measures have important issues to forecast fuzzy time series by using neural networks. This study applies a backpropagation neural network to forecast fuzzy time series with MSE, RMSE, MAE and MAPE performance measures and length of intervals from 100 to 1000. Three important operations were investigated with forecasting fuzzy time series. First, by using ISE national-100 index for the years 2001-2008 which is a large data set for training a neural network is used for forecasting target and separated into in-sample (estimation) and out-of-sample (forecasting). The ratio was 83%:17%. Second, we used a basic model for forecasting and MSE, RMSE, MAE and MAPE for performance measures are compared with together. All performance measures gave the same results for the final model. Third, the experimental results show that 300 as length of intervals outperforms other lengths of intervals in overall performance of MSE, RMSE, MAE and MAPE for forecasting ISE national-100 index. As a result, determination of length of intervals and choosing performance measures have affect the forecasting results in fuzzy time series and it is necessary to investigate other lengths of intervals and performance measures.

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