A study of the impact of the initial energy in a close approach of a cloud of particles

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Abstract: In this paper we perform a study of the effects of a close approach in the three-dimensional space between a planet and a cloud of particles, with the goal of understanding the dispersion of this cloud in terms of the variations of velocity, energy, angular momentum and inclination. It is assumed that the cloud is formed at the periapsis and the particles differ only by the magnitude of the velocity at this point. In this research we use the three-dimensional circular restricted three-body problem as the basic model for this close approach. A numerical algorithm is developed and implemented to study this problem and then it is applied to a cloud of particles, based in an analytical description of the close approach maneuver in the three-dimensional space. Analytical equations based in the patched conics approximation are used to calculate the variation in velocity, angular momentum, energy and inclination of the spacecraft that performs this maneuver.

Key-Words: astrodynamics, artificial satellites, orbital dynamics, swing-by.

1 Introduction

The swing-by maneuver is a very popular technique used to decrease fuel expenditure in space missions. The most usual approach to study this problem is to divide the problem in three phases dominated by the “two-body” celestial mechanics. Other models used to study this problem are the circular restricted three-body problem (like in [1], [2] and [3]) and the elliptic restricted three-body problem ([4], [5] and [6]).

The goal of this paper is to use analytical equations to obtain the variations of velocity, energy, angular momentum and inclination of a spacecraft that passes close to a celestial body. This passage, called swing-by, is assumed to be performed around the secondary body of the system. Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, the following five variables are used: Vp, the velocity of the spacecraft at the periapsis of the orbit around the secondary body; two angles (α and β), that specify the direction of the periapsis of the trajectory of the spacecraft around M_2 in a three-dimensional space; r_p, the distance from the spacecraft to the center of M_2 in the moment of the closest approach to M_2 (periapsis distance); g, the angle between the velocity vector at periapsis and the intersection between the horizontal plane that passes by the periapsis and the plane perpendicular to the periapsis that holds $\vec{V}_p$.

It is assumed that the system has three bodies [7]: a primary (M_1) and a secondary (M_2) bodies with finite masses that are in circular orbits around their common center of mass and a third body with negligible mass that has its motion governed by the two other bodies. The result of this maneuver is a change in velocity, energy, angular momentum and inclination in the Keplerian orbit of the spacecraft around the central body.

Then, our goal is to study the change of the relative inclination of the orbits of this cloud of particles after the close approach with the planet, as well as the variations in velocity, energy and angular momentum. It is assumed that all the particles that belong to the cloud have the same orbital elements, except by the magnitude of the velocity at the periapsis that will be varied in a short interval around a nominal value to represent a group of particles that have similar orbital elements. Others studies related to the problem considered in this paper can be found in references [9] to [16].

2 Analytical Equations for fhe Swing-by in Three Dimensions

First, it is calculated the initial conditions with respect to M_2 at the periapsis [8]:

Position:

$$x_i = r_p \cos\beta \cos\alpha \tag{1}$$

$$y_i = r_p \cos\beta \sin\alpha \tag{2}$$

$$z_i = r_p \sin\beta \tag{3}$$

Velocity:

$$V_{xi} = -V_p \sin\gamma \sin\beta \cos\alpha - V_p \cos\gamma \sin\alpha \tag{4}$$

$$V_{yi} = -V_p \sin\gamma \sin\beta \sin\alpha + V_p \cos\gamma \cos\alpha \tag{5}$$

$$V_{zi} = V_p \cos\beta \sin\gamma \tag{6}$$

During the passage, it is assumed that the two-body celestial mechanics are valid and that the whole maneuver takes place in the plane defined by the vectors $\vec{r}_p$ and $\vec{V}_p$. So, the vectors $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, that are velocity vectors before and after the swing-by, respectively, with respect to M2 can be written as a linear combination of the versors associated with $\vec{r}_p$ and $\vec{V}_p$. Using $\vec{V}_\infty$ to represent both $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, since the conditions are the same for both vectors and a double solution will give the values for $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, we have:

$$\vec{V}_\infty = A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p} \tag{7}$$

Which means that:

$$\begin{aligned}\vec{V}_\infty = {} & A(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + B(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \sin\gamma\sin\beta\sin\alpha + \\ & + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{8}$$

With A, B constants that follows the relations:

$A^2 + B^2 = V_\infty^2$, where V_∞ can be obtained from $V_\infty^2 = V_p^2 - \frac{2\mu}{r_p}$, that represents the conservation of energy of the two-body dynamics. A second requirement for $\vec{V}_\infty$ is that it makes an angle δ with $\vec{V}_p$, where δ is half of the total rotation angle described by the velocity vector during the maneuver (angle between $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$). This condition can be written as:

$$\vec{V}_\infty \bullet \vec{V}_p = V_\infty V_p \cos\delta \tag{9}$$

where the dot represents the scalar product between two vectors.

From the two-body celestial mechanics it is known that:

$$\sin\delta = \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}} \tag{10}$$

Using the equation for $\vec{V}_\infty$ as a function of $\vec{r}_p$ and $\vec{V}_p$, we have:

$$\vec{V}_\infty \bullet \vec{V}_p = \left(A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p}\right) \bullet \vec{V}_p = BV_p = V_\infty V_p \cos\delta \tag{11}$$

So, $B = V_\infty \cos\delta$, because $\vec{r}_p \bullet \vec{V}_p = 0$ (at the periapsis $\vec{r}_p$ and $\vec{V}_p$ are perpendicular) and $\vec{V}_p \bullet \vec{V}_p = V_p^2$.

Then, since $A^2 + B^2 = V_\infty^2 \Rightarrow A^2 = V_\infty^2 - B^2 = V_\infty^2 - V_\infty^2 \cos^2\delta = V_\infty^2(1-\cos^2\delta) = V_\infty^2 \sin^2\delta \Rightarrow A = \pm V_\infty \sin\delta$.

From those conditions, we have:

$$\begin{aligned}\vec{V}_\infty^- = {} & V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & - \sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{12}$$

$$\begin{aligned}\vec{V}_\infty^+ = {} & -V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & - \sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{13}$$

For M_2, its velocity with respect to an inertial frame ($\vec{V}_2$) is assumed to be:

$$\vec{V}_2 = (0, V_2, 0) \tag{14}$$

By using vector addition:

$$\vec{V}_i = \vec{V}_\infty^- + \vec{V}_2 = V_\infty sin\delta(\cos\beta\cos\alpha, \cos\beta\, sin\alpha, sin\beta) + \\ + V_\infty \cos\delta(-sin\gamma\, sin\beta\cos\alpha - \cos\gamma\, sin\alpha, - sin\gamma\, sin\beta\, sin\alpha + \\ + \cos\gamma\cos\alpha, \cos\beta\, sin\gamma) + (0, V_2, 0) \tag{15}$$

$$\vec{V}_0 = \vec{V}_\infty^+ + \vec{V}_2 = -V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, -\sin\gamma\sin\beta\sin\alpha + \\ + \cos\gamma\cos\alpha, \cos\beta\sin\gamma) + (0, V_2, 0) \tag{16}$$

where $\vec{V}_i$ and $\vec{V}_0$ are the velocity of the spacecraft with respect to the inertial frame before and after the swing-by, respectively.

From those equations, it is possible to obtain expressions for the variations in velocity, energy and angular momentum [8]. They are:

$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_i = -2V_\infty \sin\delta(\cos\alpha\cos\beta, \cos\beta\sin\alpha, \sin\beta) \tag{17}$$

which implies that:

$$\Delta V = \left|\Delta\vec{V}\right| = 2V_\infty \sin\delta \tag{18}$$

$$\Delta E = \frac{1}{2}\left(V_0^2 - V_i^2\right) = -2V_2 V_\infty \cos\beta\sin\alpha\sin\delta \tag{19}$$

For the angular momentum ($\vec{C}$) the results are:

$$\vec{C}_i = \vec{R}\times\vec{V}_i = d\, V_\infty (0, -\sin\beta\sin\delta + \cos\beta\cos\delta\sin\gamma, \\ \frac{V_2}{V_\infty} + \cos\alpha\cos\delta\cos\gamma + \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma) \tag{20}$$

$$\vec{C}_0 = \vec{R}\times\vec{V}_0 = d\, V_\infty (0, \sin\beta\sin\delta - \cos\beta\cos\delta\sin\gamma, \\ \frac{V_2}{V_\infty} + \cos\alpha\cos\delta\cos\gamma - \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma) \tag{21}$$

Where $\vec{R} = (d, 0, 0)$ is the position vector of M_2. Then:

$$\Delta\vec{C} = \vec{C}_0 - \vec{C}_i = 2d\, V_\infty \sin\delta(0, \sin\beta, -\cos\beta\sin\alpha) \tag{22}$$

and $$\left|\Delta\vec{C}\right| = 2dV_\infty \sin\delta\left(\cos^2\beta\sin^2\alpha + \sin^2\beta\right)^{1/2} \tag{23}$$

Using the definition of angular velocity $\omega = \frac{V_2}{d}$ it is possible to get:

$$\omega\Delta C_Z = -2V_2 V_\infty \cos\beta\sin\alpha\sin\delta = \Delta E \tag{24}$$

For the inclination, the results are the following:

$$\mathrm{Cos}(i_i) = \frac{C_{iZ}}{\left|\vec{C}_i\right|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta\sin\delta + \cos\beta\cos\delta\sin\gamma}{\dfrac{V_2}{V_\infty} + \cos\alpha\cos\delta\cos\gamma + \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma}\right)^2}} \tag{25}$$

$$\mathrm{Cos}(i_o) = \frac{C_{oZ}}{\left|\vec{C}_o\right|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta\sin\delta - \cos\beta\cos\delta\sin\gamma}{\dfrac{V_2}{V_\infty} + \cos\alpha\cos\delta\cos\gamma - \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma}\right)^2}} \tag{26}$$

where $\vec{C}_i$ and $\vec{C}_o$ are the initial and final angular momentum, respectvely, i_i and i_o are the initial and final inclinations, respectevely, and the subscript Z stands for the z-component of the angular momentum.

4 Results

With those equations available, the given initial conditions (values of r_p, v_p, α, β, γ) are varied in any desired range and the effects of the close approach in the orbit of the spacecraft are studied.

Figures 1 to 16 show the results. It was assumed that a satellite explodes when passing by the periapsis in a given position. In those examples, this position is given by α = 30°, β = 45°. Then, a reference value was used for the direction of the velocity: γ = 60°. Two different values were used for the velocity at periapsis (v_p = 4.0 and v_p = 4.5) and two different values were used for the periapsis distance (r_p = 1.5 r_J, r_p = = 5.0 r_J), all of them expressed in canonical units. The vertical axis shows the difference between the value (inclination, velocity, energy and angular momentum) of every single particle and a reference value, that is the value that would exist if no explosion occurred, assumed to be the value of the particle that remains with the nominal values of γ. The horizontal axis shows the value of γ, in radians.

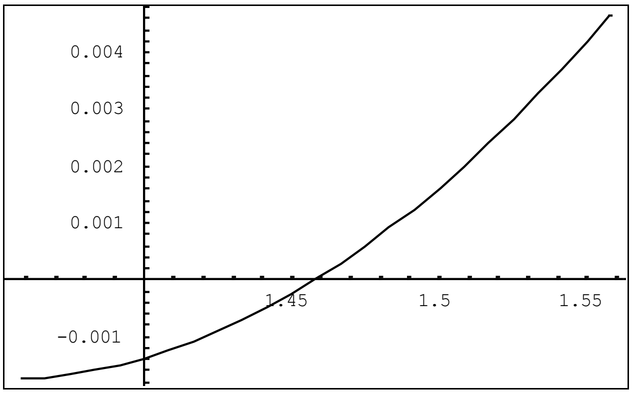

Fig. 1 – Variation in Inclination (rad) for r_p = 1.5 r_J and v_p = 4.0.

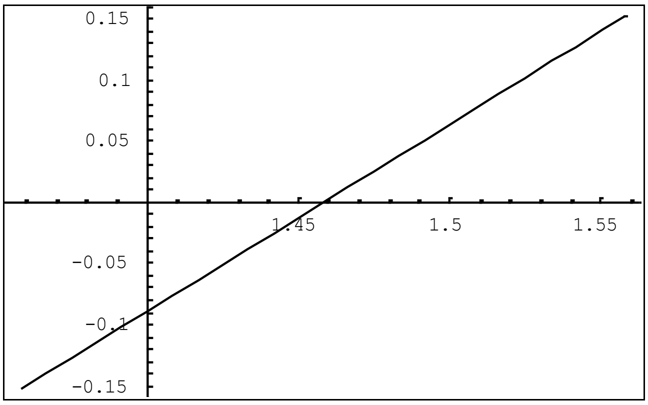

Fig. 2 – Variation in Velocity for r_p = 1.5 r_J and v_p = 4.0.

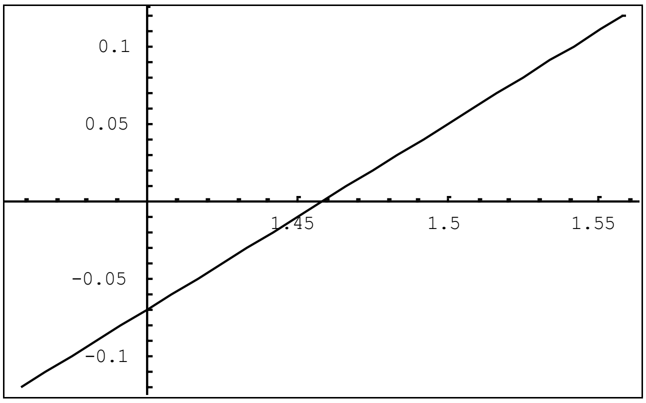

Fig. 3 – Variation in Angular momentum for r_p = 1.5 r_J and v_p = 4.0.

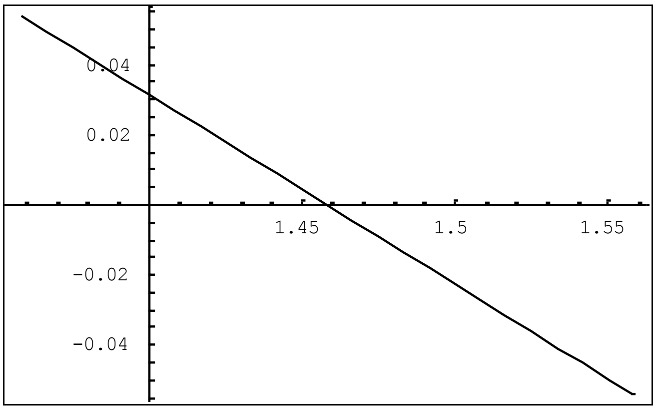

Fig. 4 – Variation in Energy for r_p = 1.5 r_J and v_p = 4.0.

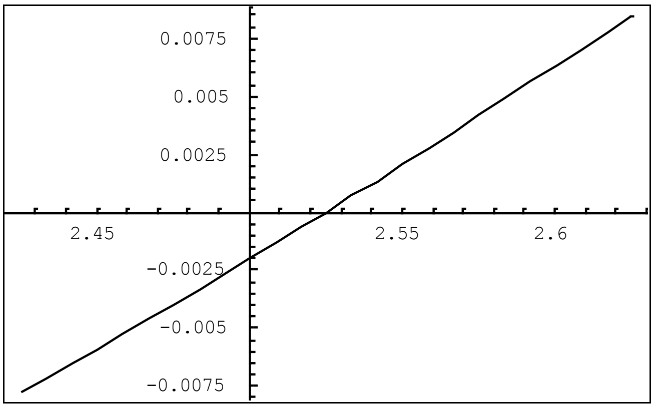

Fig. 5 – Variation in Inclination (rad) for $r_p = 1.5\ r_J$ and $v_p = 4.5$.

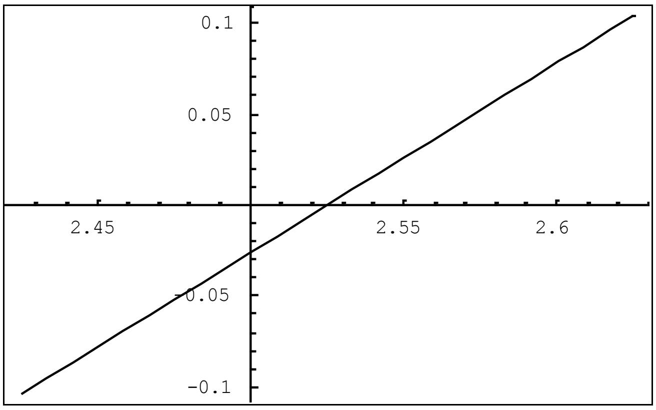

Fig. 6 – Variation in Velocity for $r_p = 1.5\ r_J$ and $v_p = 4.5$.

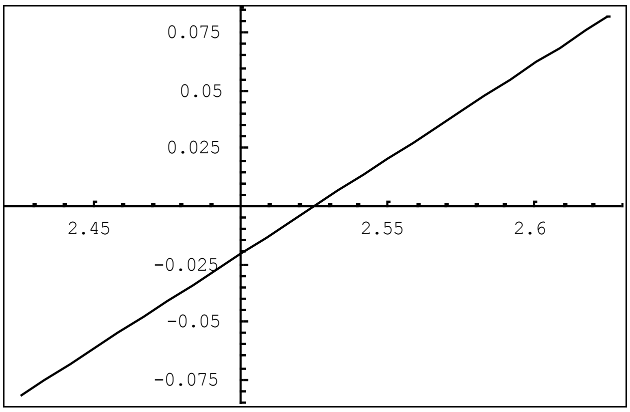

Fig. 7 – Variation in Angular momentum for $r_p = 1.5\ r_J$ and $v_p = 4.5$.

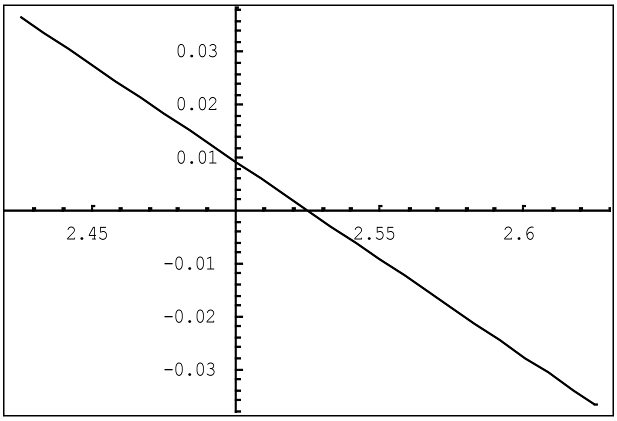

Fig. 8 – Variation in Energy for $r_p = 1.5\ r_J$ and $v_p = 4.5$.

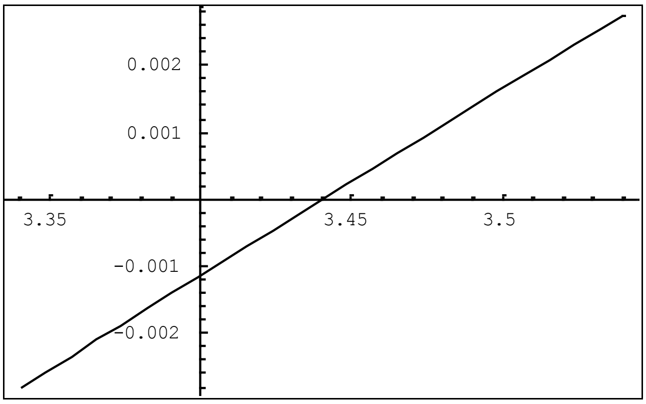

Fig. 9 – Variation in Inclination (rad) for $r_p = 5.0\ r_J$ and $v_p = 4.0$.

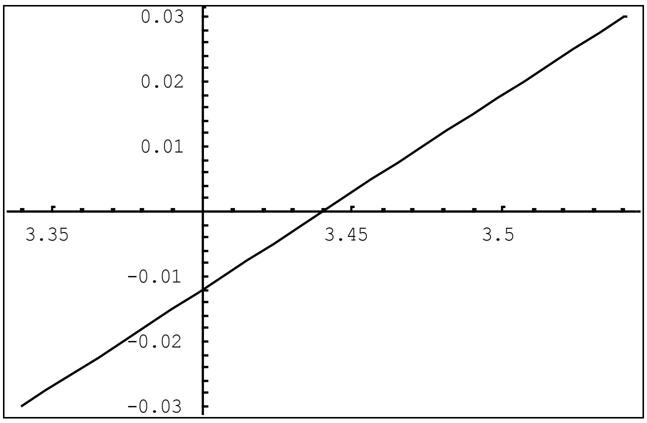

Fig. 10 – Variation in Velocity for $r_p = 5.0\ r_J$ and $v_p = 4.0$.

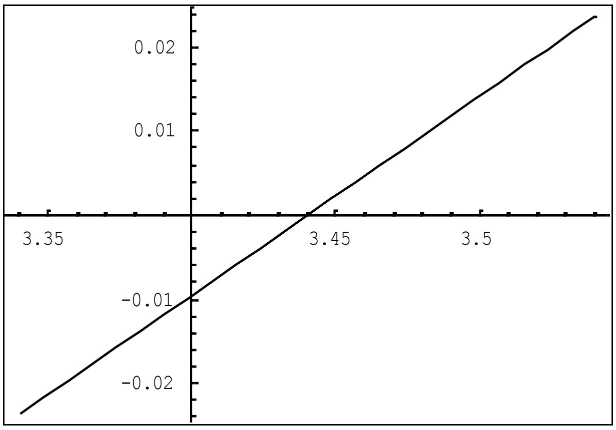

Fig. 11 – Variation in Angular momentum for $r_p = 5.0\ r_J$ and $v_p = 4.0$.

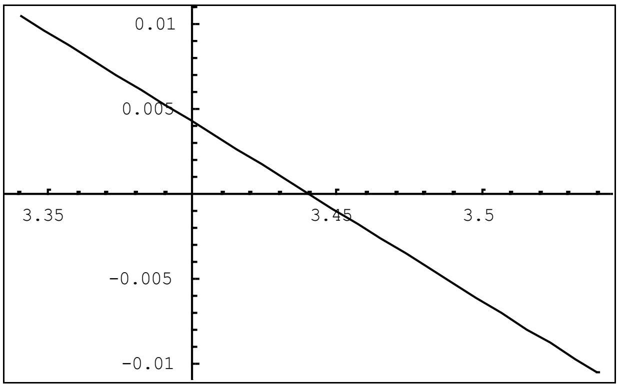

Fig. 12 – Variation in Energy for $r_p = 5.0\ r_J$ and $v_p = 4.0$.

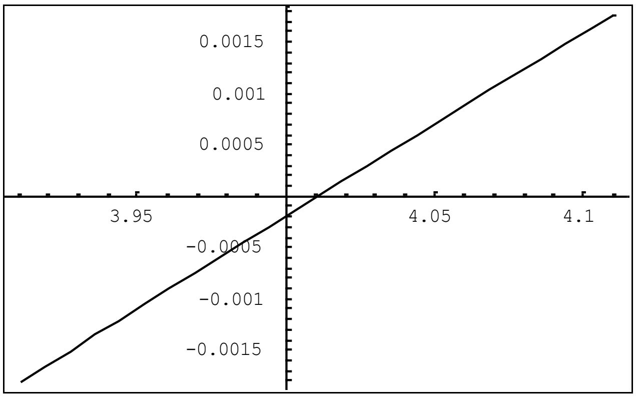

Fig. 13 – Variation in Inclination (rad) for $r_p = 5.0\ r_J$ and $v_p = 4.5$.

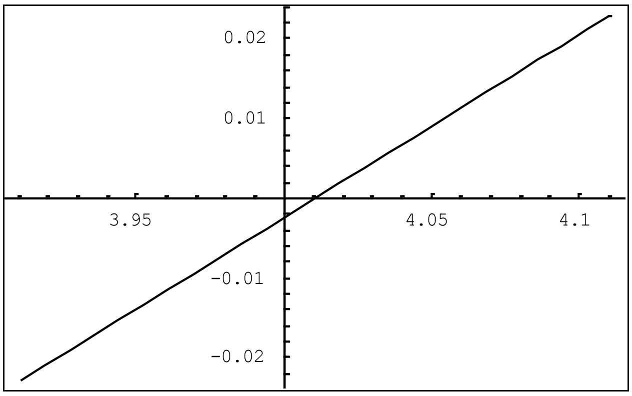

Fig. 14 – Variation in Velocity for $r_p = 5.0\ r_J$ and $v_p = 4.5$.

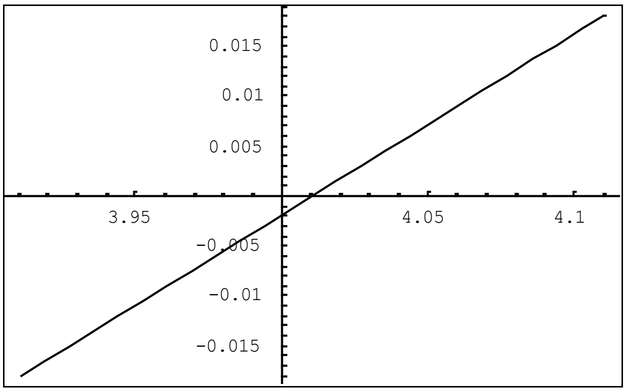

Fig. 15 – Variation in Angular momentum for $r_p = 5.0\ r_J$ and $v_p = 4.5$.

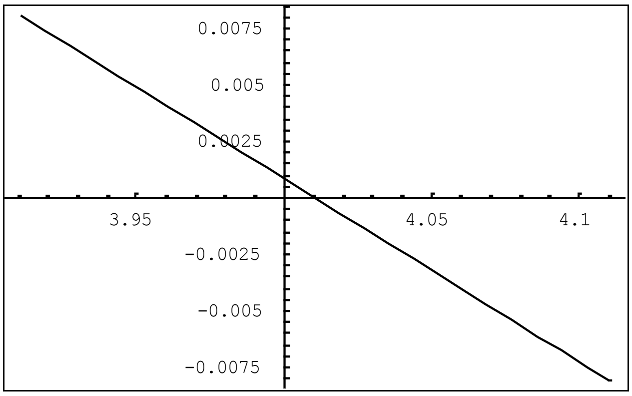

Fig. 16 – Variation in Energy for $r_p = 5.0\ r_J$ and $v_p = 4.5$.

5 The Three-dimensional Circular Restricted Problem

For the numerical simulations, the equations of motion for the spacecraft are assumed to be the ones given by the three-dimensional restricted circular three-body problem. The standard dimensionless canonical system of units is used, which implies that: the unit of distance is the distance between M_1 and M_2; the mean angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; the mass of the smaller primary (M_2) is given by μ = $m_2/(m_1+m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is (1-μ); the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is one.

There are several systems of reference that can be used to describe the three-dimensional restricted three-body problem [8]. In this paper the rotating system is used.

In the rotating system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis (x) is the line that connects the two primaries at any time. It rotates with a variable angular velocity in a such way that the two massive primaries are always on this axis. The vertical axis (y) is perpendicular to the (x) axis. In this system, the positions of the primaries are: $x_1=-\mu$, $x_2=1-\mu$, $y_1=y_2=0$.

In this system, the equations of motion for the massless particle are [8]:

$$\ddot{x}-2\dot{y}=x-(1-\mu)\frac{x+\mu}{r_1^3}-\mu\frac{x-1+\mu}{r_2^3} \tag{27}$$

$$\ddot{y}+2\dot{x}=y-(1-\mu)\frac{y}{r_1^3}-\mu\frac{y}{r_2^3} \tag{28}$$

$$\ddot{z}=-(1-\mu)\frac{z}{r_1^3}-\mu\frac{z}{r_2^3} \tag{29}$$

where r_1 and r_2 are the distances from M_1 and M_2.

6 Algorithm to Solve the Problem

A numerical algorithm to solve the problem has the following steps:

1. Arbitrary values for the three parameters r_p, V_p, α, β and γ are given;

2. With these values the initial conditions in the rotating system are computed. The initial position is the point (X_i, Y_i, Z_i) and the initial velocity is (V_{Xi}, V_{Yi}, V_{Zi}), given by equations (1) to (6).

2. With these initial conditions, the equations of motion are integrated forward in time until the distance between M_2 and the spacecraft is larger than a specified limit d. At this point the numerical integration is stopped and the energy (E_+) and the angular momentum (C_+) after the encounter are calculated;

4. Then, the particle goes back to its initial conditions at the point P, and the equations of motion are integrated backward in time, until the distance d is reached again. Then the energy (E_-) and the angular momentum (C_-) before the encounter are calculated. The criteria to stop numerical integration is the distance between the spacecraft and M_2. When this distance reaches the value d = 0.5 (half of the semimajor axis of the two primaries) the numerical integration is stopped.

With this algorithm available, the given initial conditions (values of r_p, v_p, α, β, γ) are varied in any desired range and the effects of the close approach in the orbit of the spacecraft are studied.

Figures 17 to 22 show the results. It was assumed that a satellite explodes when passing by the periapsis in a given position. In those examples, this position is given by α = 20°, β = 30°. Then, a reference value was used for the direction of the velocity: γ = 45°. Two different values were used for the velocity at periapsis (v_p = 2.5 and v_p = 3.5) and three different values were used for the periapsis distance (r_p = 0.005, r_p = 0.007, r_p = 0.009), all of them expressed in canonical units. Then, a numerical integration is used to obtain the inclination of every particle. The vertical axis shows the difference between the inclination of every single particle and the inclination of the satellite before the explosion in radians, assumed to be the inclination of the particle that remains with the nominal values of γ. The horizontal axis shows the value of γ, also in radians.

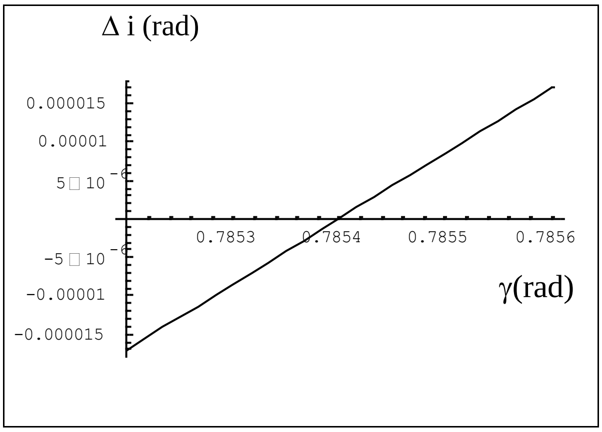

Fig. 17: Variation in inclination for r_p = 0.005, v_p = 2.5, α = 20°, β = 30°, γ = 45°.

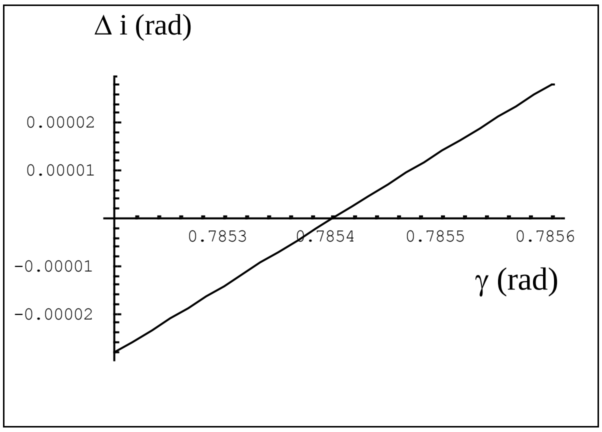

Fig. 18: Variation in inclination for r_p = 0.007, v_p = 2.5, α = 20°, β = 30°, γ = 45°.

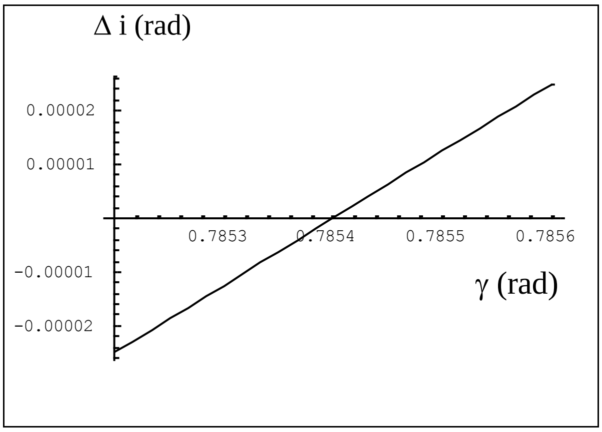

Fig. 19: Variation in inclination for r_p = 0.009, v_p = 2.5, α = 20°, β = 30°, γ = 45°.

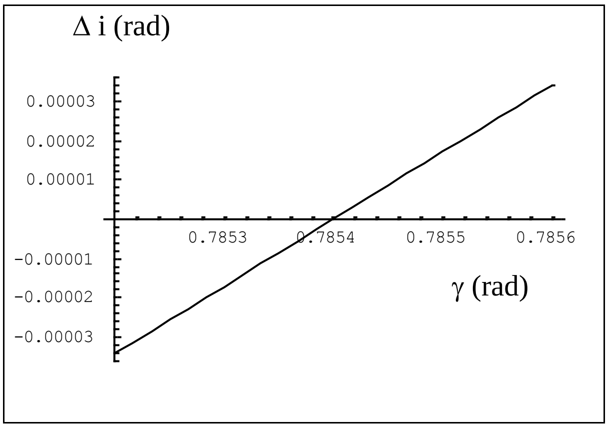

Fig. 20: Variation in inclination for r_p = 0.005, v_p = 3.5, α = 20°, β = 30°, γ = 45°.

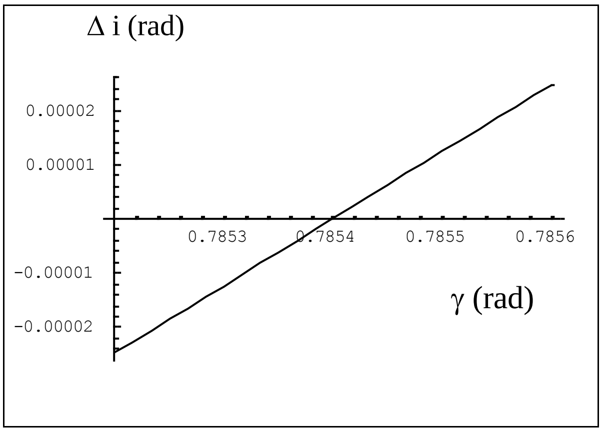

Fig. 21: Variation in inclination for r_p = 0.007, v_p = 3.5, α = 20°, β = 30°, γ = 45°.

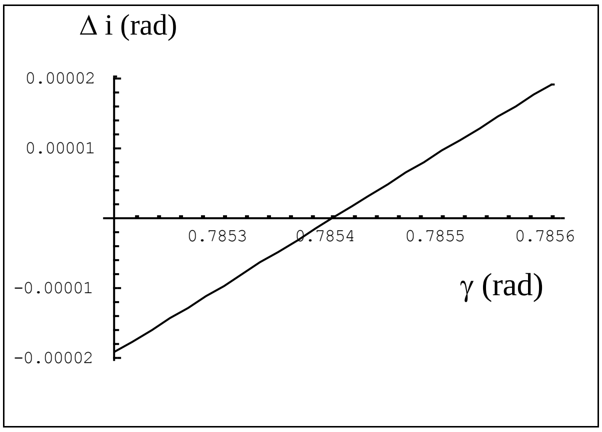

Fig. 22: Variation in inclination for r_p = 0.009, v_p = 3.5, α = 20°, β = 30°, γ = 45°.

From those plots it is clear that the relation is near linear in all the situation studied. Those results can quantify the dispersion of the inclinations of the individual particles due to the close approach.

5 Conclusion

In this paper, analytical equations based in the patched conics approximation were used to calculate the variation in velocity, angular momentum, energy and inclination of a cloud of particles that performs a swing-by maneuver. The results show the distribution of those quantities for each particle of the cloud. After that a numerical algorithm is developed to calculate the same variations. As an example, the variation in inclination is calculated. The results are very similar, so different initial conditions are used to get new results. Those results can be used to estimate the position of each individual particle in the future.

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