













$$\bar{S}_{1,2} = \left( \alpha\mu + \gamma\mu - 2 \pm \sqrt{(\alpha - \gamma)^2 \mu^2 - 4} \right) N / (2\alpha\mu).$$

The period two cycle exists when the square root is real, i.e., when  $(\alpha - \gamma)\mu > 2$ .

The stability is determined by  $|a|_{S=\bar{S}_1}|a|_{S=\bar{S}_2}| < 1$ .

From this condition it can be shown that the period 2 cycle  $\bar{S}_{1,2}$  is locally asymptotically stable if  $2 < (\alpha - \gamma)\mu < \sqrt{6}$ , Otherwise the cycle  $\bar{S}_{1,2}$  is unstable.

To find the period  $2^n$  cycle, let  $f^{2^n}(S(t)) = S(t)$  and solve for equilibrium points ( $\hat{S}$ ). The stability is considered by  $|a|_{S=\hat{S}_1}|a|_{S=\hat{S}_2} \dots |a|_{S=\hat{S}_n}| < 1$ . It is extremely complicated, if not impossible, to find these higher-order limit cycles by analytical methods, and therefore numerical methods are useful.

### 5 Numerical Result

This section shows the numerical solutions for many different type of time scales. We begin by looking at how the behavior of the solutions changes for a combination of continuous and discrete time scales.

#### 5.1 Combination of continuous and discrete time scales.

For the same values of parameters  $\alpha = 3.6$ ,  $\gamma = 0.9$  and  $N = 100$ , there are various behaviors of the solution depending on the values of the parameters  $l$  and  $h$ . Fig. 3 shows that the positive equilibrium is asymptotically stable on the continuous time scale  $\mathbb{P}_{1,0}$  i.e., for discrete jump  $h = 0$ . Fig. 4 shows that the positive equilibrium on time scale  $\mathbb{P}_{1,1}$  is also asymptotically stable, i.e., for continuous region and jump both of length 1. The gaps in the solution are due to the discrete time jumps  $\mu$  of length  $h = 1$ . In the real world application, the mosquito population increases drastically with the onset of heavy rainfall. Therefore, the dengue fever has a high in transmission in rainy season [24]. Fig. 5 shows that the positive equilibrium on time scale  $\mathbb{P}_{0,00001,1}$  is unpredictable. Fig. 3-Fig. 5 show that as the length of the continuous interval increases the positive stable equilibrium point is reached more quickly. The first jumps in the values of  $S$  in Fig. 4 and Fig. 5 are the same and are from  $S(t) = 23.7578$  to  $S(t+1) = 27.1672$ . They are the same because the

size of the jump in  $S$  is fixed by the  $S$  value before the jump is 23.7578 and the length of the time jump  $h = 1$ . However, later jumps in  $S$  in the two figures are different because the size of  $S$  at the end of a continuous region depends on the length of the continuous region.

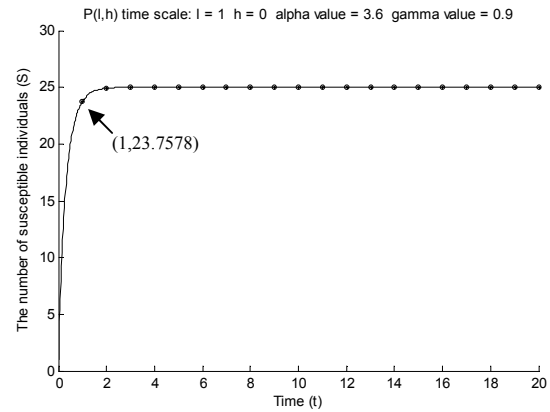


Fig. 3 The time series solution of (12) on time scale  $\mathbb{P}_{1,0}$  therefore  $\mu = 0$  with  $\alpha = 3.6$ ,  $\gamma = 0.9$  and  $N = 100$ . The result appears as a non-oscillatory solution. For  $h = 0$ , the time scale is a continuous time scale.

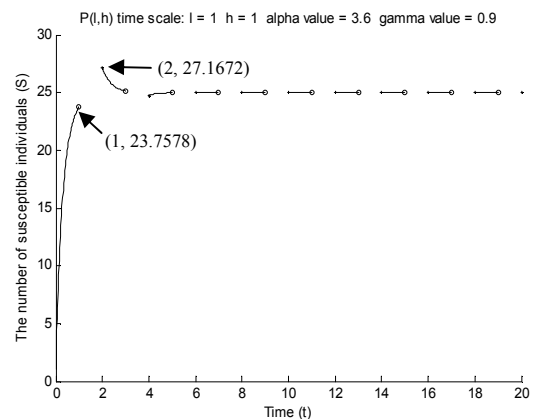


Fig. 4 The time series solution of (12) on time scale  $\mathbb{P}_{1,1}$  therefore  $\mu = 1$  with  $\alpha = 3.6$ ,  $\gamma = 0.9$  and  $N = 100$ . The result appears as a non-oscillatory solution which is similar to the result in Fig. 3 for a continuous time scale. The result disappears in some intervals because of the discrete time jump.

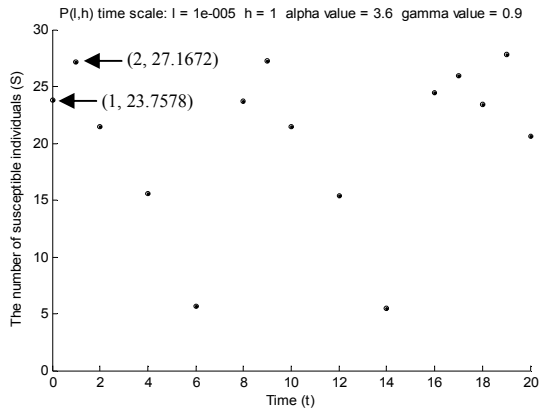


Fig. 5 The time series solution of (12) on time scale  $\mathbb{P}_{0.00001,1}$  therefore  $\mu=1$  with  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result appears unpredictable solution. The length of continuous time interval is very small and time scale approximates a discrete time scale.

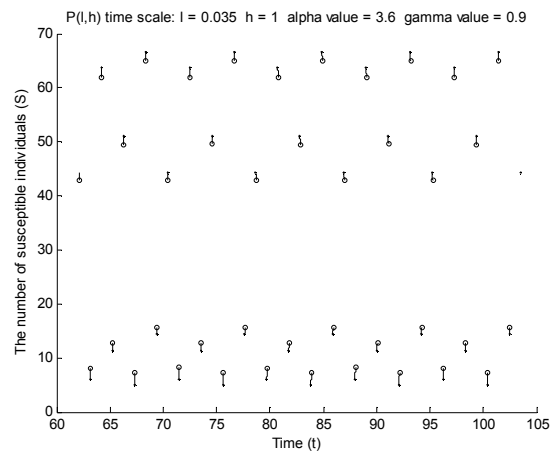


Fig. 8 The time series solution of (12) on time scale  $\mathbb{P}_{0.035,1}$  with  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result appears as a period eight cycle with a very small continuous interval.

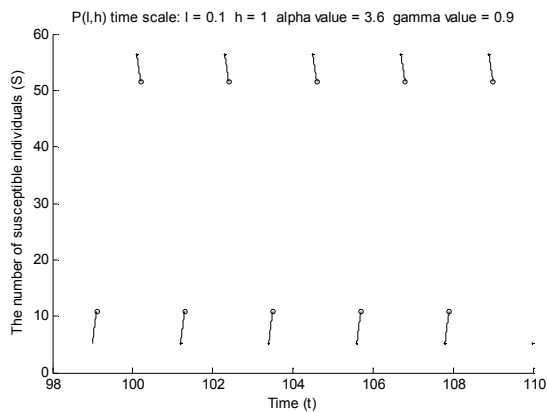


Fig. 6 The time series solution of (12) on time scale  $\mathbb{P}_{0.1,1}$  with  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result appears as a period two cycle with some continuous interval.

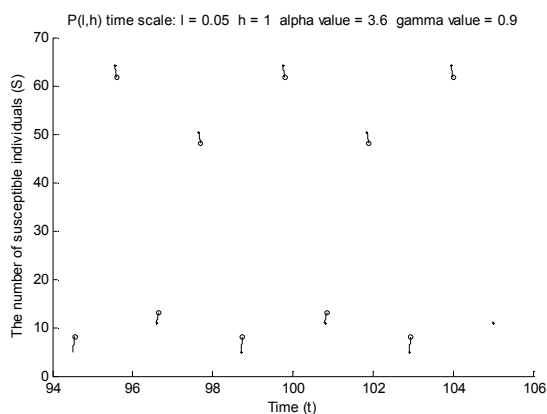


Fig. 7 The time series solution of (12) on time scale  $\mathbb{P}_{0.05,1}$  with  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result appears as a period four cycle with some small continuous interval.

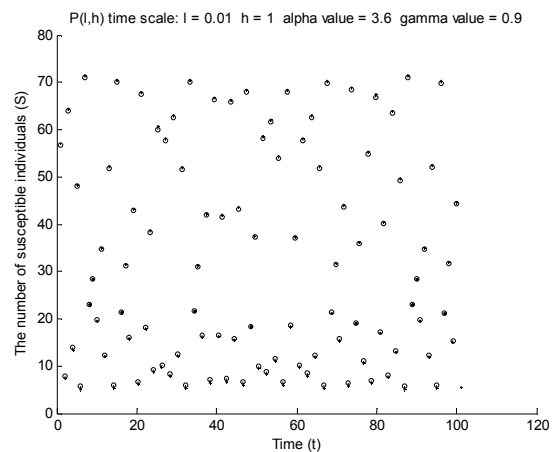


Fig. 9 The time series solution of (12) on time scale  $\mathbb{P}_{0.01,1}$  with  $\mu=1$ ,  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result appears unpredictable solution. The length of continuous time interval is small. The behavior on this time scale is similar to the behavior on a discrete time scale.

Fig. 10 shows how the behavior of the solution changes as the length of the continuous time region is reduced keeping the discrete time jump unchanged. The parameter values are  $\alpha=3.6$ ,  $\gamma=0.9$  and  $N=100$ . The result is the bifurcation path to chaos shown in Fig. 10. Two nontrivial equilibria occur until  $l=0.1965$ .



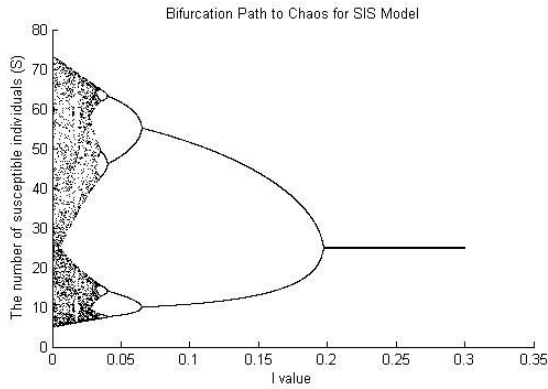


Fig. 10 The bifurcation diagram of  $l$ . The parameter values in (12) are:  $\alpha = 3.6, \gamma = 0.9$ .

We will now look at the discrete time scale case in more detail.

**5.2 Discrete Time Scales**

In the discrete time scale, the period four cycle can be obtained by numerical computation of the equilibrium points of  $f^4(S(t))=S(t)$ . For parameter values  $\alpha = 3.6, \gamma = 0.9$ , and  $N = 100$ , the system of dynamic equations has a stable period two cycle when  $\mu \in (0.740741, 0.907218)$ , and a stable period four cycle when  $\mu \in (0.907218, 0.942256)$ .

From [3], [25], the SIS epidemic model can be transformed to the discrete logistic model

$$x(\sigma(t)) = f(x(t)) = rx(t)(1 - x(t)),$$

by the substitutions  $x(t) = \frac{\alpha\mu I(t)}{N(1 - \gamma\mu + \alpha\mu)}$  and

$$r = 1 - \gamma\mu + \alpha\mu.$$

$r$  is a bifurcation parameter in the logistic model while  $\mu$  is a bifurcation parameter in the SIS epidemic model. However, as stated above  $r$  and  $\mu$  are related by  $r = 1 - \gamma\mu + \alpha\mu$ .

$0 < (\alpha - \gamma)\mu < 2$  gives the inequality  $1 < 1 + (\alpha - \gamma)\mu < 3$ , which corresponds to the condition  $1 < r < 3$ , which is the condition for asymptotic stability of a non-zero equilibrium point for the logistic model.

From [26], the ratio  $(\mu_n - \mu_{n-1})/(\mu_{n+1} - \mu_n)$  is equivalent to  $(r_n - r_{n-1})/(r_{n+1} - r_n)$  which is called the Myrberg or Feigenbaum number  $\delta$ .

From analysis (see, e.g., [6]) this ratio approaches a constant,

$$\delta = \lim_{n \rightarrow \infty} (\mu_n - \mu_{n-1})/(\mu_{n+1} - \mu_n) \approx 4.669202$$

Since

$$\frac{(r_n - r_{n-1})}{(r_{n+1} - r_n)} = \frac{((1 + (\alpha - \gamma)\mu_n) - (1 + (\alpha - \gamma)\mu_{n-1}))}{((1 + (\alpha - \gamma)\mu_{n+1}) - (1 + (\alpha - \gamma)\mu_n))} = \frac{(\mu_n - \mu_{n-1})}{(\mu_{n+1} - \mu_n)}$$

Some numerical estimates of the Feigenbaum number are given in Table II. These estimates are, however, subject to appreciable numerical errors as the limit for  $\delta$  approaches 0/0.

TABLE 2  
The Feigenbaum Constant

n	$\mu_n$	$\mu_n - \mu_{n-1}$	$(\mu_n - \mu_{n-1})/(\mu_{n+1} - \mu_n)$
1	0.740741		
2	0.907218	0.166477	4.751327
3	0.942256	0.035038	4.656831
4	0.94978	0.007524	4.66685
5	0.951392	0.001612	4.665595
6	0.951738	0.000346	4.688442
7	0.951811	7.37E-05	

Fig. 11 shows how the solution behavior of (13) changes for  $\alpha = 3.6, \gamma = 0.9$ . For  $\mu \in (0, 0.740741)$  the discrete equation has a stable equilibrium point, which corresponds with the stable equilibrium point of the continuous SIS model. For  $\mu = 1$ , the solution is chaotic.

As shown in [6] (see also [27]), if there exists a period 3 cycle, then there exists chaotic behavior. For  $\alpha = 3.6, \gamma = 0.9$ , the bifurcation diagrams show a period three cycle for  $\mu \in (1.0476, 1.0524)$  and also show chaos.

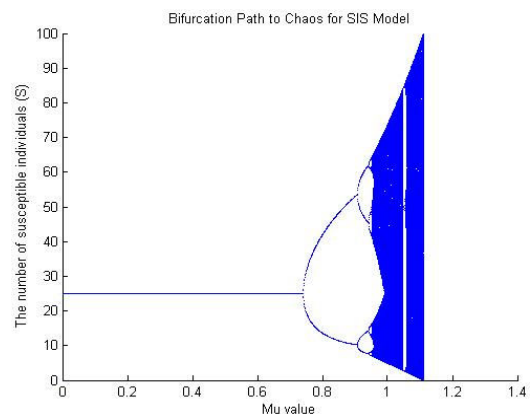


Fig. 11 The bifurcation diagram of  $\mu$ . The parameter values in (13) are:  $\alpha = 3.6, \gamma = 0.9$ .

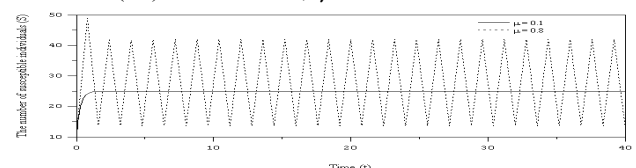


Fig. 12 The time series solution of (13) with  $\alpha = 3.6, \gamma = 0.9$  and  $N = 100$ . The non-oscillatory solution occurs when  $\mu = 0.1$  while oscillating period-2 solution occurs when  $\mu = 0.8$ .

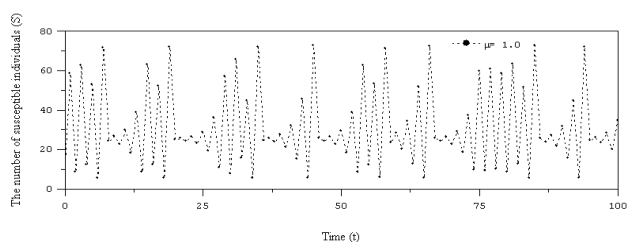


Fig. 13 The time series solution of (13) with  $\alpha = 3.6$ ,  $\gamma = 0.9$ , and  $N = 100$ . The chaos occurs when  $\mu = 1$ .

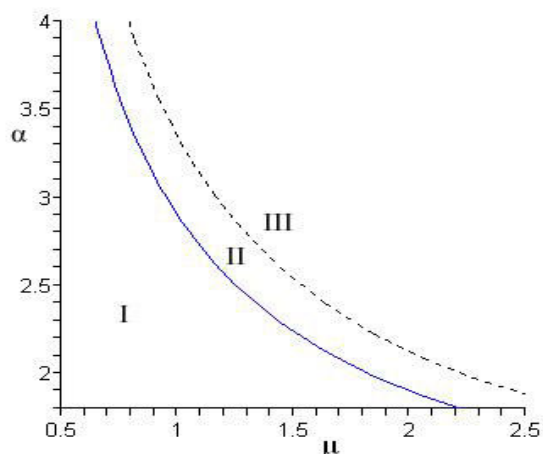


Fig. 14 The x-axis is  $\mu$  and y-axis is  $\alpha$ . Area I is region of stable equilibrium point, area II is region of stable period two cycle, and area III is region of stable higher period cycles and chaos. Parameter value  $\gamma = 0.9$ .

## 6 Conclusion

In this paper, the time space of the model is important because the behavior is changed in each model. For the continuous interval, if it is big enough, then the behavior of the model is similar to continuous model. If the continuous interval is small, then the behavior of the model is close to discrete model. Since the collecting of data could not be continuous, therefore the discrete model is important. The results of the analysis in this paper show that for an SIS model, the predictions of the model depend critically on the time scales used.

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