Vibrations of cylindrical shells with circumferential cracks

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Abstract: - An approximate method for the vibration analysis of stepped shells accounting for the influence of cracks located at re-entrant corners of steps is presented. Introducing the additional compliance function it is shown that the flexibility of the shell near cracks can be prescribed by means of the compliance of the structure. The latter is coupled with the stress intensity factor, which is known from the linear elastic fracture mechanics. Theoretical analysis is developed for circular cylindrical shells, provided the shell wall has an arbitrary number of steps and circular cracks of constant depth. Various combinations of end conditions are considered.

Key words: - crack, cylinder, free vibration, elasticity, axisymmetric response, stepped shell

1. Introduction

Due to the wellknown matter that repeated and extreme loadings cause cracks and other defects deteriorating operational parameters of structures there exists an obvious need for investigation of structures accounting for the influence of cracks on the flexibility of beams, plates and shells which are widely used in technology. Special attention is to be paid to the vibration of structural members.

Considerable attention has been paid to the investigation of vibration and stability of elastic beams with cracks during last decades.

Liang, Hu, Choy [20,21], Nandwana, Maiti [25], Yang, Yi, Xie [39], Kisa, Brandon, Topcu [13], Yang, Chen [38], Ostachowicz, Krawczuk [27], Rizos, Aspragathos, Dimarogonas [33] studied the behaviour of cantilever beams with cracks located at fixed positions.

Vibration based methods for detection of damages in cantilever beams and in other simple elements have been developed by Gillich et al [11], Providakis et al [30].

The effect of cracks on the free vibration frequencies of uniform beams with arbitrary number of cracks was investigated by Lin, Chang, Wu [22] by the use of the transfer matrix method. Masoud, Jarrah, Al-Maamory [23] presented theoretical and experimental results concerning an axially loaded fixed-fixed beam with cracks.

Zheng, Fan [41] studied vibration and stability of hollow-sectional beams in the case of presence of cracks in expected cross sections. An Euler-Bernoulli beam containing multiple opening cracks and subjected to the axial force is studied by Binici [2]. Both, stability and vibration problems are considered making use of the concept of additional compliance due to a crack by Dimarogonas [7, 8], also Chondros, Dimarogonas, Yao [5], also by the authors [15- 19]. The case of inclined edge or internal cracks was studied by Nandwana and Maiti [25].

Fernandez- Saez, Rubio and Navarro [9] presented an alternative analytical method for evaluation of fundamental frequencies of cracked Euler- Bernoulli beams. This method is based on the approach of representing the crack in the beam through an elastic hinge whereas transverse deflection of the cracked beam is constructed by adding polynomial functions to that of the uncracked beam.

De Rosa [34] and Naguleswaran [24] studied axially loaded segmented beams. In the latter work beams with different axial forces in beam segments are investigated whereas De Rosa has used an exact method to derive the frequency equation for a stepped beam with follower forces at each step.

The vibration of uniform Euler-Bernoulli beams with a single edge crack was investigated by Yokoyama, Chen [40] making use of a modification of the distributed line-spring method which was suggested earlier by Rice and Levy [32] for rectangular plates with part through cracks.

The behavior of circular cylindrical shells with circumferential cracks was studied by Nikpour [26] and Petroski [29]; Lellep and Roots [18]; Lellep, Roots, Tungel [16, 17].

In the present paper the method of distributed springs is extended to free axisymmetric vibrations of elastic circular cylindrical shells. The shells under consideration have piece wise constant thickness and circular cracks of constant depth are located at cross sections with steps of the thickness.

2. Formulation of the problem and governing equations

Let us consider axisymmetric free vibrations of circular cylindrical shells of length l and radius R (Fig.1).

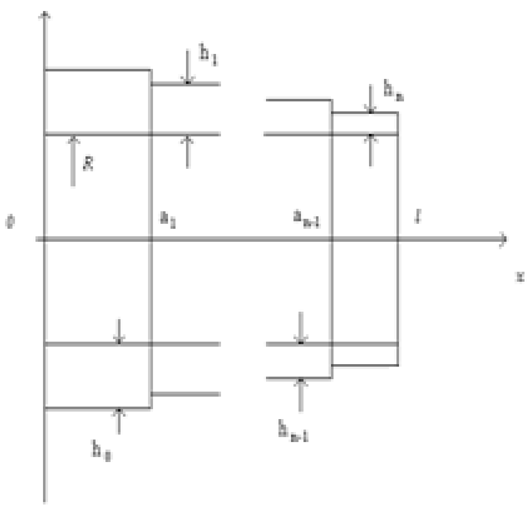

Fig.1. Stepped cylindrical shell.

The material of shells is an isotropic elastic material.

Let the origin of the axis Ox be at the left end of the tube. It is assumed that the thickness h of the shell is piece wise constant, e.g.

$$h(x)=h_j \qquad (1)$$

for $x \in (a_j, a_{j+1})$, where $j=0,\dots,n$

Here the quantities h_j ($j=0,\dots,n$) stand for fixed constants. Similarly, a_j ($j=0,..,n+1$) are given constants whereas it is reasonable to use notations $a_0=0$, $a_{n-1}=l$.

It is known in the linear elastic fracture mechanics (see Anderson, [1]; Broberg, [3]; Broek, [4]) that repeated loading and stress concentration at sharp corners entails cracks. Thus it is reasonable to assume that at the re-entrant corners of steps e.g. at $x = a_j$ $(j=1,\dots,n)$ cracks of depth c_j are located. For the simplicity sake we assume that these flaws are stable circular surface cracks. In the present study like in Rizos et al. [33], Chondros et al. [6], Dimarogonos [7], Kukla [14] no attention will be paid to the crack extension during operation of the structure.

Equilibrium conditions of a shell element yield (see Reddy, [31]; Soedel, [35]; Ventsel and Krauthammer, [37])

$$\begin{aligned}
&\frac{\partial N_1}{\partial x} = \rho h \frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial M}{\partial x} = Q,\\
&\frac{\partial Q}{\partial x} = \frac{N}{R} - p + \rho h \frac{\partial^2 w}{\partial t^2},
\end{aligned} \qquad (2)$$

where N_1 and N are membrane forces in the axial and circumferential directions, respectively, whereas M is the axial bending moment and Q – shear force. In Eqs. (2) u and w stand for displacements in the axial and transverse direction whereas p is the intensity of distributed transverse pressure, ρ is the material density and t stands for time. In the case of free vibrations when p=0 one can present the last two equations in the system (2) as

$$\frac{\partial^2 M}{\partial x^2} - \frac{N}{R} - \rho h_j \frac{\partial^2 w}{\partial t^2} = 0 \qquad (3)$$

for $x \in (a_j, a_{j+1})$, $j=0,..,n$.

Assume that the material of shells is an isotropic pure elastic material. A body made of a linear elastic material behaves according to the Hooke's law. In the case of axisymmetric behaviour of circular cylindrical shells Hooke's law reads as (Reddy [31], Ventsel and Krauthammer [37])

$$\begin{aligned}
N_1 &= \frac{Eh_j}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon),\\
N &= \frac{Eh_j}{1-\nu^2}(\varepsilon + \nu\varepsilon_1),\\
M &= \frac{Eh_j^3}{1-\nu^2}\chi
\end{aligned} \qquad (4)$$

for $x \in (a_j, a_{j+1})$, $j=0,...,n$.

Here E and ν stand for the Young's and Poisson's modulus, respectively.

Strain components corresponding to Eq. (3) are (Soedel [35])

$$\varepsilon_1 = \frac{\partial u}{\partial x}, \quad \varepsilon = \frac{w}{R}, \quad \chi = -\frac{\partial^2 w}{\partial x^2}. \tag{5}$$

Substituting Eq. (5) with Eq. (4) in Eq. (3) yields the equation

$$\frac{\partial^4 w}{\partial x^4} + \frac{12(1-\nu^2)}{R^2 h_j^2} w = -\frac{12\rho \cdot h_j (1-\nu^2)}{E h_j^3} \frac{\partial^2 w}{\partial t^2} \tag{6}$$

which must be satisfied for $x \in (a_j, a_{j+1})$, where $j=0,..,n$.

When deriving Eq. (6) it was taken into account that according to Eq. (4) one has

$$N = Eh\frac{w}{R} \tag{8}$$

and

$$\frac{\partial u}{\partial x} = -\nu \frac{w}{R}, \tag{9}$$

when $N_1=0$.

3. The local flexibility due to the crack

It is well known in the mechanics of solids and in the fracture mechanics that the presence of flaws or cracks in a structural member involves considerable local flexibilities. Additional local flexibility due to a crack depends on the crack geometry as well as on the geometry of the structural element and its loading. Probably the first attempt to prescribe the local flexibility of a cracked beam was undertaken by Irwin [12] who recognized the relationship between the compliance C of the beam and stress intensity factor K. Later on, Dimarogonas [7]; Chondros, Dimarogonas, Yao [5]; Rizos, Aspragathos, Dimarogonas [33]; Kukla [14] introduced so called massless rotating spring model which reveals the relationship between the stress intensity factor and local compliance of the beam. In the present paper we attempt to extend this approach for axisymmetric vibrations of circular cylindrical shells with circular cracks of constant depth.

Let us consider the crack located at the cross section $x = a_j$ and let the segments adjacent to the crack have thicknesses h_{j-1} and h_j, respectively. According to the current approach it is assumed that the slope of deflection w' is discontinuous, e.g.

$$w'(a_j + 0,t) - w'(a_j - 0,t) = \theta_j$$

where $\theta_j \neq 0$.

The quantity θ_j can be treated as an additional angle caused by the crack at $x=a_j$. If θ_j is a generalized coordinate then $M(a_j)$ can be treated as corresponding generalized force.

Let C_j be the additional compliance of the beam due to the crack located at $x=a_j$ and U_T be the extra strain energy due to the crack. These notations admit to present the relationship between the compliance C_j and the generalized force $M(a_j)$ as

$$C_j = \frac{\partial \theta_j}{\partial M(a_j)}. \tag{10}$$

It is wellknown in the fracture mechanics, that

$$C_j = \frac{\partial^2 U_T}{\partial M^2(a_j)}. \tag{11}$$

Note that equalities (10)-(11) are used in the linear elastic fracture mechanics in the case when the generalized displacement and generalized force are u_j and P_j, respectively (see Broek [4], Anderson [1], Broberg [3]).

The energy release rate G and the stress intensity factor K are related to each other as (Broek [4])

$$G = \frac{K^2}{E'}, \tag{12}$$

where $E'=E$ for the plane stress state and $E'=E/(1-\nu^2)$ for the plane deformation state. It is known that the energy release rate due to the generalized fore $M(a_j)$ is

$$G = \frac{M_j^2}{2}\frac{dC_j}{dA_j}, \tag{13}$$

where A_j stands for the crack surface area.

The stress intensity factor is defined as

$$K_j = \sigma\sqrt{\pi c_j} \cdot F(\frac{c_j}{h_j}) \tag{14}$$

(see Tada, Paris, Irwin, [36]). Here c is the crack depth and $\sigma = 6M/h^2$ whereas F stands for a function to be determined experimentally. When applying Eqs (10) – (14) for the cross section $x = a_j$ with crack depth c_j one has

$$\frac{M_j^2}{2}\frac{dC_j}{dc_j} = \frac{36M_j^2}{E'h_j^4}\pi c_j F^2(\frac{c_j}{h_j}) \tag{15}$$

provided $h_j < h_{j-1}$. Let us introduce the notation $s_j = c_j / h_j$.
Thus it follows from Eq. (15) that

$$\frac{dC_j}{ds_j} = \frac{72\pi}{E'h_j^2} s_j F^2(s_j). \tag{16}$$

After integration in (16) one obtains

$$C_j = \frac{72\pi}{E'h_j^2}\int_0^{s_j} s_j F^2(s_j) ds_j \tag{17}$$

for the plane stress state.

The function $F(s_j)$ in Eqs. (14) – (17) is called the shape function as it is different for experimental specimens of different shape. Many researchers have investigated the problem of determination of the stress intensity factor for various specimens (among others Freund and Hermann, [10]; Irwin, [12]; Tada, Paris, Irwin, [36]).

In the present study we are resorting to the data of experiments conducted by Brown and Srawley which can be approximated as (see Tada, Paris, Irwin, [36])

$$F(s_j) = 1{,}93 - 3{,}07s_j + 14{,}53s_j^2 - \\ - 25{,}11s_j^3 + 25{,}8s_j^4 \tag{18}$$

From (17), (18) after integration one obtains

$$C_j = \frac{72\pi}{E'h_j^2} f(s_j),$$

where

$$f(s_j) = 1{,}862s_j^2 - 3{,}95s_j^3 + 16{,}375s_j^4 - \\ - 37{,}226s_j^5 + 76{,}81s_j^6 - 126{,}9s_j^7 + \\ + 172{,}5s_j^8 - 143{,}97s_j^9 + 66{,}56s_j^{10} \tag{19}$$

The function (19) is employed also in papers by Dimarogonas [7], Chondros and Dimarogonas [6], Kukla [14], also in earlier papers of the authors [15-19].

According to the concept of massless rotating spring

$$\theta_j = -\frac{Eh_j^3}{12K_j(1-\nu^2)} \cdot w''(a_j+, t) \tag{20}$$

one can equalize $K_j = 1/C_j$ and obtaining thus

$$K_j = \frac{E'h_j^2}{72\pi \cdot f(s_j)}, \tag{21}$$

where

$$f(s_j) = \int_o^{s_j} \xi F^2(\xi) d\xi \tag{22}$$

4. Solution of equations of motion

Evidently, it is reasonable to look for the general solution of the equation (6) in the form

$$w(x,t) = X_j(x)T(t) \tag{23}$$

for $x \in (a_j, a_{j+1})$

where $X_j(x)$ and $T(t)$ are functions of the single variable. Differentiating Eq.(23) with respect to x and t and substituting in Eq.(6) leads to the equation

$$X_j^{IY}(x)T(t) + \frac{12(1-\nu^2)}{R^2h_j^2} X_j(x)T(t) = \\ = -\rho \cdot \frac{12(1-\nu^2)}{Eh_j^2} X_j(x)\ddot{T}(t) \tag{24}$$

where the notation

$$X_j^{IY}(x) = \frac{d^4 X_j}{dx^4},$$
$$\ddot{T}(t) = \frac{d^2 T}{dt^2} \qquad (25)$$

is used. Note that Eq.(24) must be satisfied for $x \in (a_j, a_{j+1})$ for $j = 0,....,n.$

Separating variables in Eq.(24) one easily obtains

$$\ddot{T} + \omega^2 T = 0 \qquad (26)$$

Eq. (25) gives the solution

$$T = \alpha \sin(\omega\omega + \delta) , \qquad (27)$$

α and δ being arbitrary constants. Assume that at the initieal time instant

$$w(x,0) = 0,$$

and therefore,

$$T(0) = 0.$$

Substituting $T(0)=0$ in (27) yields $\delta=0$. Thus it follows from Eq.(27) that

$$T = \alpha sin\omega t. \qquad (28)$$

Finally, it follows from (24) that

$$X_j^{IY} - r_j^4 X_j = 0 \qquad (29)$$

for $x \in (a_j, a_{j+1}), j = 0,....,n.$ Here

$$r_j^4 = \omega^2 \cdot \frac{12\rho(1-v^2)}{Eh_j^2} - \frac{12(1-v^2)}{R^2 h_j^2}, \qquad (30)$$

whereas ω stands for the frequency of free vibrations of the shell.

The general solution of the linear fourth order equation (30) can be presented as

$$X_j(x) = A_j \sin(r_j x) + B_j \cos(r_j x) + + C_j \sinh(r_j x) + D_j \cosh(r_j x) . \qquad (31)$$

Note that Eq. (31) holds good for $x \in (a_j, a_{j+1}),$ $j = 0,....,n.$

In this paper we consider cylindrical shells whose ends are fixed in arbitrary manner. If, for instance, the left-hand end of the tube is simply supported, then $w(0,t)=0$, $M(0,t)=0$. Consequently, one has

$$X_0(0) = 0,$$
$$X_0''(0) = 0. \qquad (32)$$

In the case of the absolutely free left hand end $M(0,t)=Q(0,t)=0$. Since according to (2) $M'=Q$ one has

$$X_0''(0) = 0,$$
$$X_0'''(0) = 0. \qquad (33)$$

If the left hand end of the shell is clamped then evidently

$$X_0(0) = 0,$$
$$X_0'(0) = 0. \qquad (34)$$

Similarily, for the supported right hand one has

$$X_n(l) = 0,$$
$$X_n''(l) = 0. \qquad (35)$$

Whereas in the case of a free right hand end

$$X_n''(l) = 0,$$
$$X_n'''(l) = 0. \qquad (36)$$

5. Intermediate conditions

In order to prescribe the requirements imposed on functions $X_j(x)$. Let us denote one has to distinguish the left and right hand limits,

$$Z(a_j \pm 0) = \lim_{x \to a_j \pm 0} Z(x), \qquad (37)$$

where $Z(x)$ is a function depending on x.

Resorting to physical considerations one can state that the displacement $w(x,t)$, bending moment M and shear force $Q(x,t)$ must be continuous at cross

sections $x = a_j,\ j = 0,....,n$ where steps of the thickness and cracks are located.
According to the Hooke's law the bending moment M reads

$$M(x,t) = -\frac{Eh_j^3}{12(1-\nu^2)} w''(x,t), \qquad (38)$$

provided $x \in (a_j, a_{j+1})$. The last equality shows that $M(x,t)$ is continuous, if $h^3 w''(x,t)$ is continuous. Similarly we can conclude that the shear force is continuous if the quantity $h^3 w'''(x,t)$ is continuous when passing the cross sections $x = a_j$. However, the quantity $w'(x,t)$ has finite jumps at $x=a_j$, as showm above. Finally one obtains the system of intermediate conditions at $x = a_j,\ j = 0,....,n$

$$\begin{aligned} &X_j(a_j+0) - X_{j-1}(a_j-0) = 0,\\ &X'_j(a_j+0) - X'_{j-1}(a_j-0) + p_j X''_j(a_j-0) = 0,\\ &h_j^3 X''_j(a_j+0) - h_{j-1}^3 X''_{j-1}(a_j-0) = 0,\\ &h_j^3 X'''_j(a_j+0) - h_{j-1}^3 X'''_{j-1}(a_j-0) = 0. \end{aligned} \qquad (39)$$

In the system (39) the notation

$$p_j = -\frac{Eh_j^3}{12(1-\nu^2)K_j} \qquad (40)$$

is used.

It follows from Eq. (30) that

$$r_j^4 h_j^2 = \omega^2 \frac{12\rho\cdot(1-\nu^2)}{E} - \frac{12(1-\nu^2)}{R^2}.$$

The product $h_j r_j^2$ takes common value for each interval $x \in (a_j, a_{j+1})$, $j = 0,....,n.$ Therefore, it is reasonable to introduce a real number k so that

$$r_j = \frac{k}{\sqrt{h_j}} \qquad (41)$$

for each $j = 0,....,n.$

In order to determine the deflected shape of a shell generator one has to specify functions $X_j(x)$ for each $j = 0,....,n.$ So far, functions $X_j(x)$ are given by Eq. (31) which involves unknown constants $A_j,\ B_j,\ C_j,\ D_j$, e.g. totally 4n+4 unknowns. These constants can be determined from boundary and intermediate conditions, e.g. Eqs. (32) – (36). Since the number of equations in Eq. (40) is 4n the total number of available conditions is also 4n+4, as might be expected. In order to demonstrate the solution procedure in a greater detail let us consider the shell clamped at the left hand end and simply supported at right hand end. The end conditions entail the boundary conditions (32), (35) for functions $X_0(x)$ and $X_n(x)$. Making use of (31), (32), (35) one obtains

$$B_0 = D_0 = 0 \qquad (42)$$

and

$$\begin{aligned} &A_n \sin r_n l + B_n \cos r_n l +\\ &+ C_n \sinh r_n l + D_n \cosh r_n l = 0,\\ &- A_n \sin r_n l - B_n \cos r_n l +\\ &+ C_n \sinh r_n l + D_n \cosh r_n l = 0. \end{aligned} \qquad (43)$$

Evidently, Eq. (43) can be converted into the system

$$\begin{aligned} &A_n \sin r_n l + B_n \cos r_n l = 0,\\ &C_n \sinh r_n l + D_n \cosh r_n l = 0. \end{aligned} \qquad (44)$$

Making use of Eq. (31) one can present the system (39) as

$$\begin{aligned} &A_{j-1} \sin r_{j-1} a_j + B_{j-1} \cos r_{j-1} a_j +\\ &+ C_{j-1} \sinh r_{j-1} a_j + D_{j-1} \cosh r_{j-1} a_j -\\ &-(A_j \sin r_j a_j + B_j \cos r_j a_j +\\ &+ C_j \sinh r_j a_j + D_j \cosh r_j a_j) = 0,\\ &r_{j-1}(A_{j-1} \cos r_{j-1} a_j - B_{j-1} \sin r_{j-1} a_j +\\ &+ C_{j-1} \cosh r_{j-1} a_j + D_{j-1} \sinh r_{j-1} a_j) -\\ &r_j(A_j(\cos r_j a_j - p_j r_j \sin r_j a_j) +\\ &+ B_j(-\sin r_j a_j - p_j r_j \cos r_j a_j) +\\ &+ C_j(\cos hr_j a_j + p_j r_j \sinh r_j a_j) +\\ &+ D_j(\sinh r_j a_j + p_j r_j \cosh r_j a_j) = 0, \end{aligned} \qquad (45)$$

$$
\begin{aligned}
&h_{j-1}^3 r_{j-1}^2(-A_{j-1}\sin r_{j-1}a_j - B_{j-1}\cos r_{j-1}a_j +\\
&+C_{j-1}\sinh r_{j-1}a_j + D_{j-1}\cosh r_{j-1}a_j)-\\
&-h_j^3 r_j^2(-A_j \sin r_j a_j - B_j \cos r_j a_j +\\
&+C_j \sinh r_j a_j + D_j \cosh r_j a_j) = 0,\\
&h_{j-1}^3 r_{j-1}^3(-A_{j-1}\cos r_{j-1}a_j + B_{j-1}\sin r_{j-1}a_j +\\
&+C_{j-1}\cosh r_{j-1}a_j + D_{j-1}\sinh r_{j-1}a_j)-\\
&-h_j^3 r_j^3(-A_j \cos r_j a_j + B_j \sin r_j a_j +\\
&+C_j \cosh r_j a_j + D_j \sinh r_j a_j) = 0
\end{aligned}
$$

for $j = 0,....,n.$

It is worthwhile to emphasize that equations (42)-(45) serve for determination of unknowns A_j, B_j, C_j, D_j ($j = 0,....,n$). However, this is an algebraic linear homogeneous system of equations. It is well known that a non-trivial solution of this system exists if the determinant of this system vanishes.

The problem is solved up to the end numerically.

6. Numerical results

Calculations have been carried out for shells with one and two steps. Special attention is paid to shells with unsymmetric end conditions.

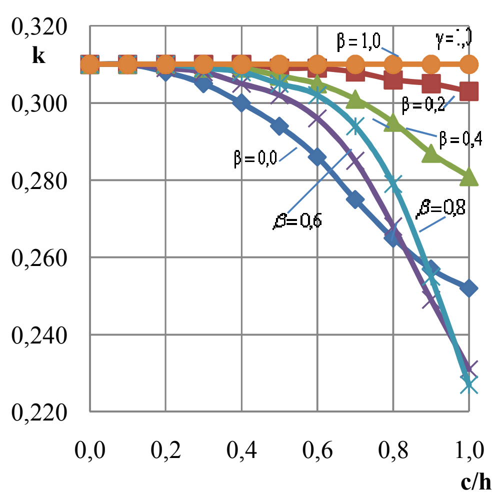

Fig.2: Cylindrical shell clamped at the left end with simply supported right hand end, constant thickness (γ=1,0).

The results calculations presented in Figs. 2, 4, 6, 9 are obtained for shells clamped at the left hand and simply supported at the right hand end whereas Figs. 3, 5, 7, 9 are associated with cantilever shells.

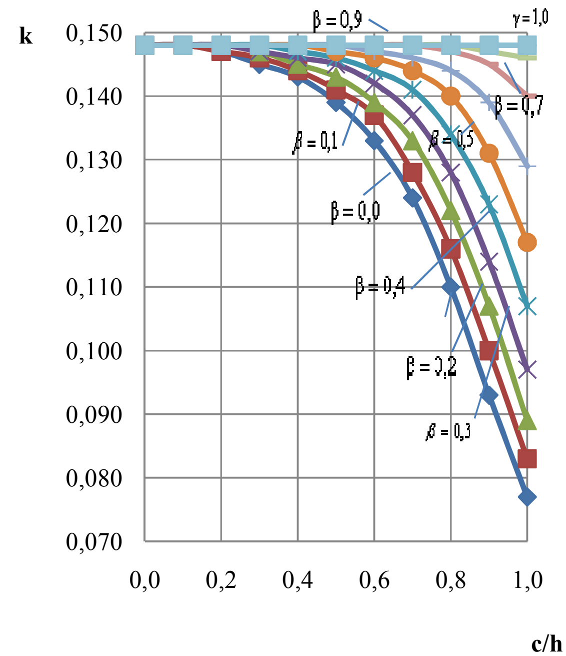

Fig.3: Cylindrical shell clamped at the left end with free right hand end, constant thickness (γ=1,0).

In the latter case the right hand end of the shell is absolutely free.

Figs. 2-7 correspond to shells with a unique step, e.g. to shells with thicknesses h_0 and h_1. The cases of shells with two cracks and two stepped shells are considered in Figs. 8-9.

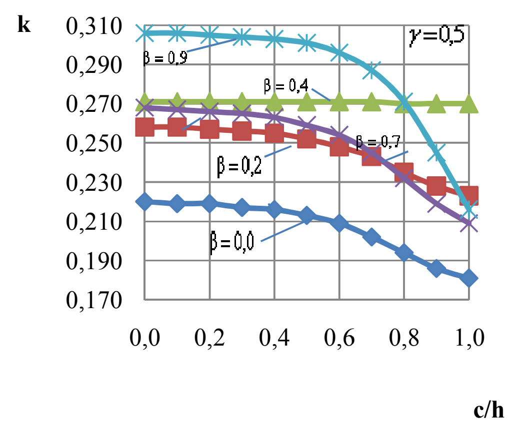

Fig.4: Cylindrical shell clamped at the left end with simply supported right hand end, the case *γ*=0.5.

It is assumed herein that the material of the shell is a homogeneous elastic material.

In calculations cylindrical shells of length *l*=1,2*m* clamped at the left hand end are considered.

The largest thickness has been taken h_0=0,009*m*. Whereas the notation $\gamma=h_1/h_0$, $\beta=a_1/l$ is used.

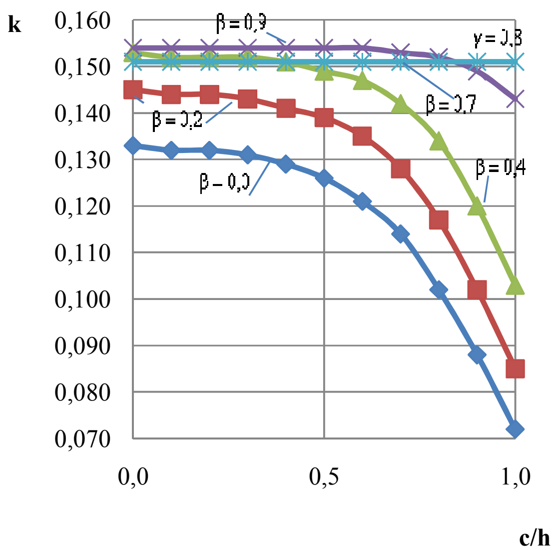

Fig.5: Cylindrical shell clamped at the left end with free right hand end, the case γ=0,8.

The influence of the non-dimensional crack depth *c/h* on the characteristic number *k* is depicted in Fig.2. Here and henceforth we take $h=min(h_0, h_1)$.

In Fig.2 the sensitivity of the number *k* (and thus the eigen frequency *ω*) with respect to crack length *c* and with respect to the location of the crack $a_1=\beta l$ is shown. Here $h_0= h_1$, e.g. the shell is of constant thickness. It can be seen from Fig.2 that the number *k* depends on the crack length in a complicated manner. For instance, the lowest eigen frequency (the smallest number *k*) corresponds to the case $\beta=0$ (the crack is located at the root section) until $c< 0,8 h_1$. If $c >0,8h_1$ in Fig.2 then minimum of *k* is achieved when β>0.

In Fig.3 the same situation as in Fig.2 is presented in the case of a cylindrical shell clamped at the left hand with free right hand end. It can be seen from Fig.3 that the larger the number *β* the higher the value *k*.

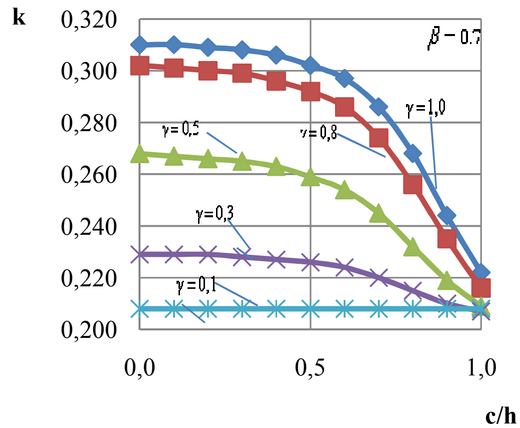

Fig.6. Cylindrical shell clamped at the left end with simply supported right hand end, the case *β*=0.5

In Fig.4 the number *k* is shown for stepped shells with h_1=0,5h_0 for shell clamped at the left hand with simply supported right hand end. Similar results for cantilever shells with h_1=0,8h_0 are presented in Fig.5. Here different curves correspond to different locations of the step, e.g. to different values of $\beta=a_1/l$. It can be seen from Fig.4 that the number *k* decreases when the crack length *c* increases in the case of a fixed value of *β*. It is interesting to note that in the range of small cracks the number *k* is almost unsensitive with respect to small changes of the crack length. However, in the range of larger cracks when c>0,6 h_1 this sensitivity is more obvious. Similar matters can be observed in Fig.5. Here the number *k* is weakly sensitive with respect to the crack growth, if β>0,7.

In Figs. 6-7 the dependence between the characteristic number *k* and the crack length *c* is shown for stepped shells with step coordinates a_1=0,7*l* and simply supported right hand end and with step coordinates a_1=0,2*l* and free right hand end, respectively. Here different curves correspond to different values of $\gamma=h_1/h_0$.

In Figs. 8-9 shells with two steps and two cracks are considered. In Figs. 8 and 9 the ratios of thicknesses are denoted $\gamma_j=h_j/h_0$; *j*=1,2. Curves 1, 2, 3, 4 in Figs.8-9 are associated with the crack locations at a_2=0,3*l*; a_2=0,4*l*; a_2=0,7*l*; a_2=0,9*l*, respectively, whereas a_1=0,1*l*.

In Fig.8 the results corresponding to cantilever shells with two cracks are presented (the case γ_1=0,2; γ_2=0,5).

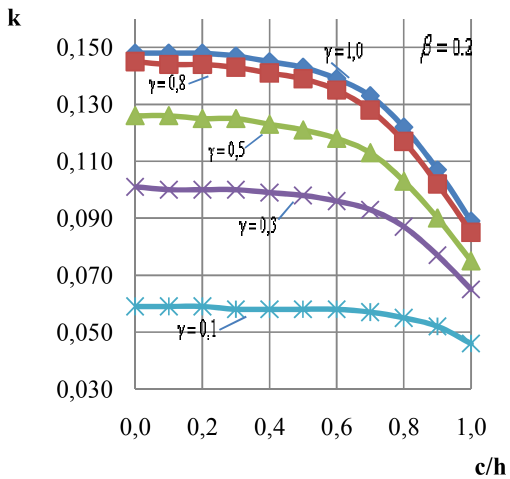

Fig.7: Cylindrical shell clamped at the left end with free right hand end, the case β=0,2.

In Fig.9 the results corresponding to shells with two cracks are presented (the case $\gamma_1 = \gamma_2 = 1{,}0$). In this case the shell is clamped at the left end and simply supported at the right hand end.

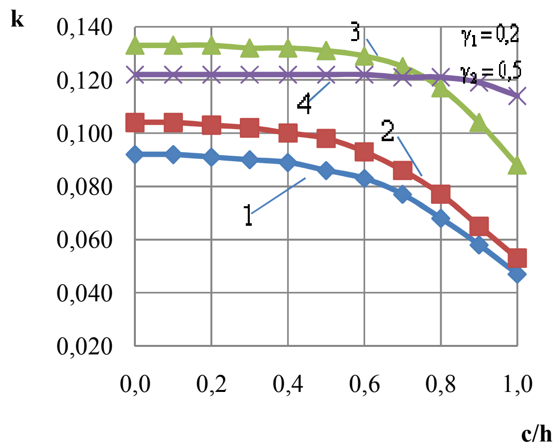

Fig.8: Cylindrical shell clamped at the left end with free right hand end, two stepped.

Thus the numerical results show that for fixed values of geometrical parameters the number k decreases when crack length c increases.

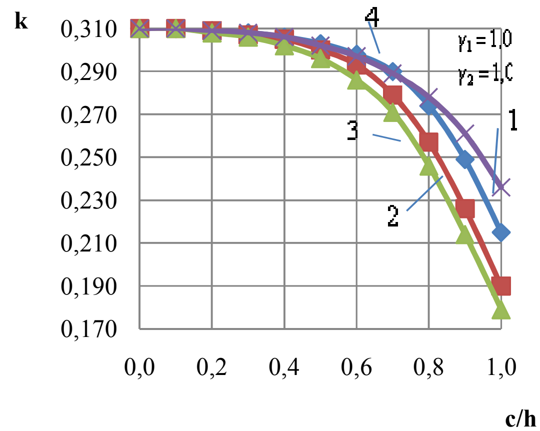

Fig.9: Cylindrical shell clamped at the left end with simply supported right hand end,two cracks.

The results of calculations for simply supported shells are compared with those obtained by Pellicano [28] in Table 1. Here the shell of constant thickness without any crack is studied whereas the first five natural frequencies of axisymmetric modes are presented in Table 1. The parameters of the shell are following: E=2,1·10^{11}N/m^2; ν=0,3; ρ=7800kg/m^3; l=0,2m; R=0,2m and h=R/20.

Table 1.Natural frequencies: present theory vs. exact and finite-elements results.

Mode	Natural frequencies (Hz)		
n	Present method	Exact by F.Pellicano	F.Pellicano [28]
1	4,162·10^3	4140,74	4140,77
2	4,794·10^3	4788,34	4788,66
3	6,895·10^3	6890,30	6891,43
4	1,066·10^4	10655,3	10657,7
5	1,591·10^4	15900,6	15904,7

It is seen from Table 1 that the natural frequencies corresponding to the current analytical method are in good agreement with the natural frequencies obtained by F.Pellicano.

7. Concluding remarks

Free vibrations of circular cylindrical shells of piece wise constant thickness are treated under the condition that at re-entrant corners of steps circular cracks of constant length are located. Cracks are considered as stationary surface cracks, problems related to re-distribution of stresses and strains due to extension of crack are not treated herein.

However, the attention is paid to the determination of vibration characteristics of cylindrical shells in the cases of various combinations of end conditions.

Calculations showed that the support conditions as well as crack parameters have essential influence on vibration characteristics. It was established that if the crack location is fixed then the maximal value of the characteristic number k is achieved if the crack length is equal to zero. On the other hand, for fixed crack length and variable ratio of thicknesses maximum of the number k is achieved if the shell has constant thickness. For the fixed ratio of thicknesses and fixed other parameters the number k decreases when the crack length increases.

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