Transformation of Fuzzy State Space Model of a Boiler System: A Graph Theoretic Approach

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Abstract: - Graph theory has many real-world applications and is rich in theoretical results, especially for studying interconnection among elements in natural and man-made systems. In recent studies of complex control system, directed graph was introduced to define and interpret the interconnection structure underlying the dynamics of the interacting subsystems. Similarly, Fuzzy State Space Model (FSSM) was developed for solving inverse problem in multivariable control system. Thus, this paper aims to describe the transformation of FSSM of a Boiler system using a graph theoretic approach. The main subsystems of the Boiler system are furnace, superheater, drum, riser and reheater. These subsystems are transformed into vertices whereas the interconnections between subsystems are associated with edges of the graph. Here, the Autocatalytic Set (ACS) is a subgraph, whereby each of the nodes has at least one incoming link from a node belonging to the same subgraph. The concept of ACS is integrated to reveal some properties of the new graphical representation of FSSM such that irreducible graph, adjacency matrix and the relationship of an ACS to Perron Frobenius Eigenvector. These properties indicate initial findings that will lead to further exploration in Fuzzy Graphical representation.

Key-Words: - Combined Cycle Power Plant, State Space, Multi-connected System, Adjacency Matrix

1 Introduction

Graphical representation is extremely useful to illustrate complex structure in a direct and intuitive way, and such they are widely used in many fields. Graph theory provides a mathematical modeling for studying interconnection among elements in natural and man-made systems [1,3]. In recent studies of complex control system, directed graphs were introduced to define and interpret the interconnection structure underlying the dynamics of the interacting subsystems. The subsystems were associated with vertices while interconnection with edges of the graph [2]. Theorems and algorithm of graph theory represent the behavioral properties of the system as the properties of the vertices or edges of the graph [3, 20].

The concept of autocatalysis comes from chemistry. An Autocatalytic Set (ACS) is a set of reactions that are not individually autocatalytic, but whose products catalyze one another [4]. The underlying idea of this concept is that a set of molecular species that contain, within itself, a catalyst for each of its member species. In term of graph theoretic approach, ACS is a subgraph, each of the nodes has at least one incoming link from a node belonging to the same subgraph [5]. Our interest in this study is to transform the Fuzzy State Space Model of a boiler system to Fuzzy Graph using the graph theoretic approach. The subsystems will be transformed into vertices and interconnection between subsystems will be associated with edges of the graph. Initially, the input-output variables are identified using state space approach. The state space approach is based on time domain analysis and synthesis using state variables. It is a unified method for modeling, analyzing and designing a wide range of systems [6]. This approach is well studied and it provides a good approximation in modeling engineering and biological systems [8-11]. Further discussion will focus on integrating the concept of ACS to reveal some properties of the graphical representation of FSSM.

2 A Boiler System

Boiler system consists of five main subsystems namely furnace, riser, drum, reheater and superheater [7]. Based on the typical drum-type boiler, the

feedwater is supplied to the drum where the water is evaporated. The water flows into downcomers, then furnace is used to increase the water temperature and eventually to cause evaporation. Thus the circulation of water, steam, and water and steam mixture takes place in the drum, the downcomers and the risers. Steam generated in the risers is separated in the drum where it flows through the superheater on to the high pressure turbines. It may be recycled to the boiler in the reheater where its energy content is increased.
Desuperheating spray water is introduced in the superheater for control of main steam temperature. As for the combustion process path, the risers absorb radiant heat in the furnace. The hot gases leaving the furnace transfer the heat by radiation and convection to the superheater. The heat then transferred by convection to the reheater and the economizer, before exiting the boiler via the stack. The burner tilt is used to change radiation heat distribution between the risers and the superheater. Block Diagram of a Boiler System is shown in Fig. 1[7].

3 Graphical Representation of Boiler System

Graph theory is a useful representation because the elements of the graph can be defined so as to have a one to one correspondence with the elements of many kinds of engineering systems [3]. In addition, a graph is a symbolic of network and of its connectivity which implies an abstraction of the reality that can be simplified as set of linked nodes [12]. Each link represents a relationship between pairs of elements. A directed graph $G = (V, E)$, often referred as a graph, is defined by a set V of 'nodes' and E of 'links' (or 'arcs'). The set of nodes and edges can be conveniently labeled by $V = \{v_1, v_2, v_3, ..., v_s\}$ and $E = \{e_1, e_2, e_3, ..., e_n\}$ respectively. A graph with s nodes is an s x s, denoted by $C = (c_{ij})$ called the adjacency matrix of a graph.

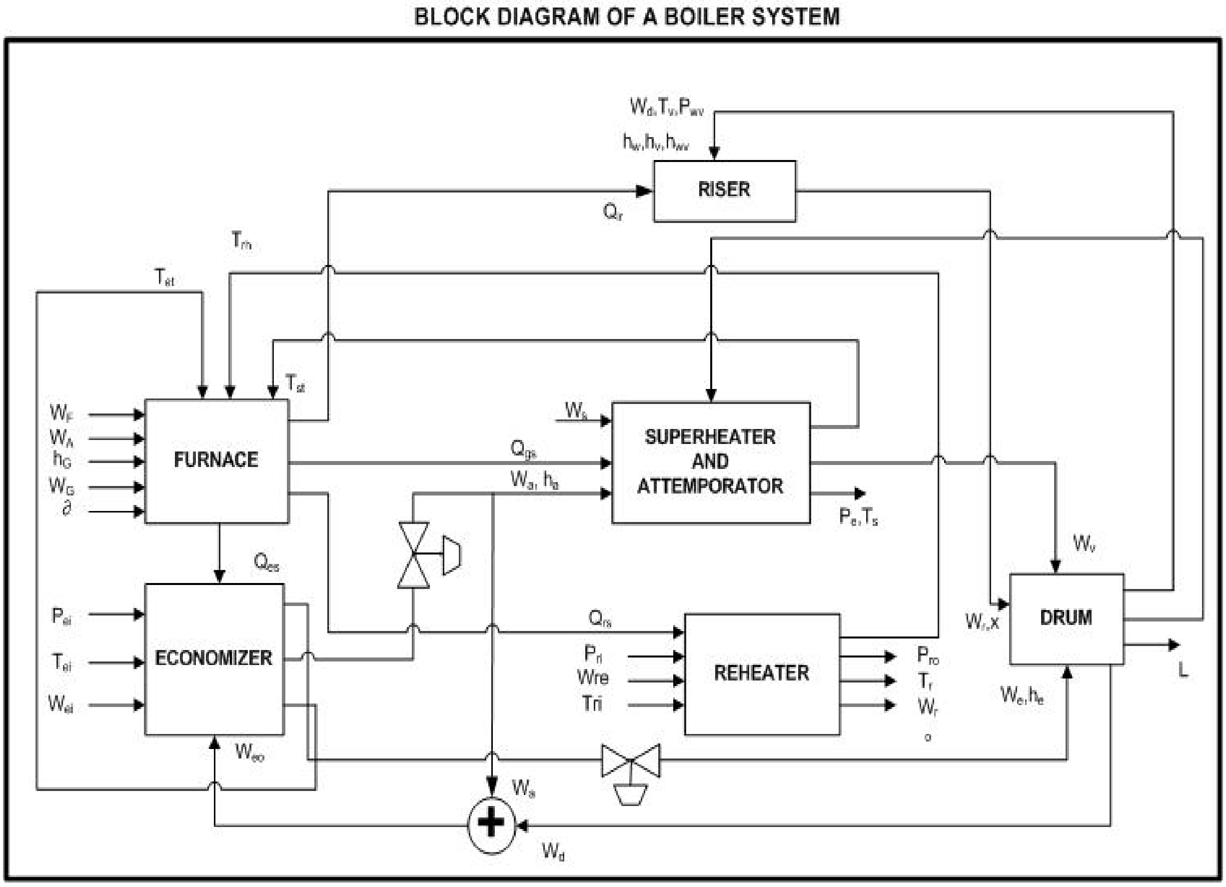


Fig. 1. Block Diagram of a Boiler System

The boiler system can be visualized in the form of a graph as shown in Fig. 2. Initially, the state space model is used to determine the input and output parameters for each subsystem as shown in Table 1 [10, 13]. All the subsystems except economiser and the input-output variables involve in the process are grouped into two distinct sets namely the set of vertices, V and the set of edges, E. An 'environment' is used to represent the systems outside the boiler. Each subsystem in the Boiler and the environment will be represented by a vertex while parameter sources will be the edge. These two sets are listed below:

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

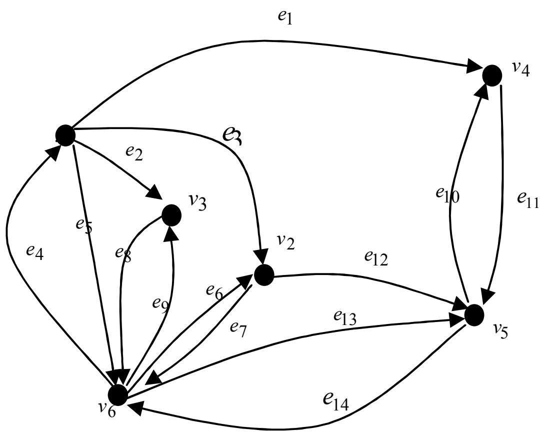
and,

$$E = \{e_1, e_2, e_3, e_4 ....., e_{14}\}$$

The construction of the graphical representation of boiler system can be regarded as the proof by construction of the following proposition.

Proposition 3.1
The boiler system of a combined cycle power plant in [7] can be presented by a graph $G_{cc} = \{V, E\}$: where a set $V(G)$ of vertices corresponds to the subsystems related to the boiler and a set $E(G)$ of edges corresponds to the input-output variables associated with the components.

However, the above graphical representation of a boiler system is static. Therefore, a more comprehensive graphical representation of the system is needed and outlined in the following section.



Vertex	Subsystem	Edge	Source for Input and output Parameter:
v_1	Furnace (S_{gF1})	e_1	Q_{ir}
v_2	Superheater (s_{gF2})	e_2	Q_{rs}
v_3	Reheater (s_{gF3})	e_3	Q_{gs}
v_4	Riser (s_{gF4})	e_4	$w_{F,} w_A, w_G$
v_5	Drum (s_{sF5})	e_5	P_G, Q_{is}, Q_{es}
v_6	Environment (s^*_{gF6})	e_6	w_s
		e_7	P_s, T_s
		e_8	T_r, P_{ro}
		e_9	h_{ir}
		e_{10}	h_w, w_d
		e_{11}	x, w_r
		e_{12}	w_v
		e_{13}	w_e, w_r
		e_{14}	w_d

Fig. 2. Graphical Representation of a Boiler System

Table 1. State Space Representation of a Boiler System

Sub-system	Matrices A, B and C in state space equations	State Vector	Input-Output Vector
Superheater	$A=\begin{pmatrix} -\frac{k_s}{M_sC_{st}}\left(\frac{1}{f_s}(p_v-p_s)\rho_v\right)^{\frac{1}{2}} & \frac{k_s}{M_sC_{st}}\left(\frac{1}{f_s}(p_v-p_s)\rho_v\right)^{\frac{1}{2}}\left(\frac{R_s}{p_s(c_{ps})^2}(h_s-h_{ref}+c_{ps}T_{ref})^2\right) \\ \frac{1}{v_s}\left(\frac{\rho_v k_s p_v}{f_sQ_s}-\frac{\rho_v k_s p_s}{f_sQ_s}\right) & -\frac{1}{v}\left(\frac{\rho_v k_s p_v}{f_sQ_s}-\frac{\rho_v k_s p_s}{f_sQ_s}\right)\left(\frac{R_s}{p_s(c_{ps})^2}(h_s-h_{ref}+c_{ps}T_{ref})^2\right) \end{pmatrix}$ $B=\begin{pmatrix} \frac{1}{M_sC_{st}} & 0 \\ 0 & -\frac{1}{v_s} \end{pmatrix}$ $C=\begin{pmatrix} 0 & \left(\frac{R_sh_s}{c_{ps}}-\frac{h_{ref}}{h_s-h_{ref}+c_{ps}T_{ref}}-\frac{T_{ref}c_{ps}}{h_s-h_{ref}+c_{ps}T_{ref}}\right) \\ 0 & \frac{R_s}{p_s(c_{ps})^2}(h_s-h_{ref}+c_{ps}T_{ref})^2 \\ \left(\frac{\rho_v k_s p_v}{f_sQ_s}-\frac{\rho_v k_s p_s}{f_sQ_s}\right) & \left(\frac{\rho_v k_s p_v}{f_sQ_s}-\frac{\rho_v k_s p_s}{f_sQ_s}\right)\left(\frac{R_s}{p_s(c_{ps})^2}\right)(h_s-h_{ref}+c_{ps}T_{ref})^2 \end{pmatrix}$	$\vec{x}(t)=\begin{pmatrix} T_{st} \\ \rho_s \end{pmatrix}$	$\vec{u}(t)=\begin{pmatrix} Q_{gs} \\ w_s \end{pmatrix}$ $\vec{y}(t)=\begin{pmatrix} p_s \\ T_s \\ w_v \end{pmatrix}$
Riser	$A=\begin{pmatrix} -\frac{k_rJ}{M_rc_{rt}} & 0 \\ \frac{k_rJ}{\rho_rV_r} & -\frac{w_r}{\rho_rV_r} \end{pmatrix}$, $B=\begin{pmatrix} \frac{1}{M_rc_{rt}} & 0 \\ 0 & \frac{h_w}{\rho_rV_r} \end{pmatrix}$ $C=\begin{pmatrix} 0 & \frac{1}{h_v-h_{wr}} \end{pmatrix}$	$\vec{x}(t)=\begin{pmatrix} T_{rt} \\ h_r \end{pmatrix}$	$\vec{u}(t)=\begin{pmatrix} Q_r \\ w_d \end{pmatrix}$ $\vec{y}(t)=(x)$
Reheater	$A=\begin{pmatrix} -\frac{k_{rh}w_{ri}}{M_{rc}C_{rh}} & 0 \\ \frac{k_{rh}w_{ri}}{V_{rh}} & -\frac{w_{ro}}{\rho_{rh}} \end{pmatrix}$, $B=\begin{pmatrix} \frac{1}{M_{rc}C_{rh}} & 0 \\ 0 & \frac{w_{ri}}{V_{rh}} \end{pmatrix}$ $C=\begin{pmatrix} 0 & R_r\left[\frac{h_{ro}-h_{ref}+c_{pr}T_{ref}}{c_{pr}}\right] \\ 1 & 0 \end{pmatrix}$	$\vec{x}(t)=\begin{pmatrix} T_{rh} \\ x_{rh} \end{pmatrix}$	$\vec{u}(t)=\begin{pmatrix} Q_{rs} \\ h_{ri} \end{pmatrix}$ $\vec{y}(t)=\begin{pmatrix} P_{ro} \\ T_r \end{pmatrix}$
Drum	$A=\begin{pmatrix} -\frac{v_{dow}}{V_L} & 0 \\ 0 & -\frac{v_{dow}}{V_L} \end{pmatrix}$, $B=\begin{pmatrix} 1 & 1-x \\ h_e & (1-x)h_{wr} \end{pmatrix}$ $C=\begin{pmatrix} 0 & \frac{1}{V_L\rho_w} \\ \frac{v_{dow}}{V_L} & 0 \end{pmatrix}$	$\vec{x}(t)=\begin{pmatrix} m_{dL} \\ x_{Dl} \end{pmatrix}$	$\vec{u}(t)=\begin{pmatrix} w_e \\ w_r \end{pmatrix}$ $\vec{y}(t)=\begin{pmatrix} h_w \\ w_d \end{pmatrix}$
Furnace	$A=\left(-\frac{k_FR_{EG}T_g}{V_F}\right)$, $B=\begin{pmatrix} \frac{1}{V_F} & \frac{1}{V_F} & \frac{1}{V_F} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$, $C=\begin{pmatrix} \frac{\theta T_gR_{EG}k_f}{(1-\theta)w_{EG}}(Q_{gs}+w_{EG}c_{gs}T_{gr}) \\ \frac{T_gR_{EG}k_f}{w_{EG}}(Q_{gs}+w_{EG}c_{gs}T_{gr}) \\ k_fR_{EG}T_gc_{gs}(T_{gr}-T_{ge}) \\ k_fR_{EG}T_gc_{gs}(T_{ge}-T_{gl}) \\ R_{EG}T_g \end{pmatrix}$	$\vec{x}=\rho_{EG}$	$\vec{u}(t)=\begin{pmatrix} w_F \\ w_A \\ w_G \end{pmatrix}$ $\vec{y}(t)=\begin{pmatrix} Q_{ir} \\ Q_{is} \\ Q_{rs} \\ Q_{es} \\ p_G \end{pmatrix}$

4 Fuzzy State Space Model

The fuzzy systems, developed by based on specific items of fuzzy logic such as rule bases, fuzzy values and others are nonlinear transfer elements, with multivariable inputs and single variable output [15,18]. Fuzzy State Space Model (FSSM) of a furnace was introduced and followed by Fuzzy State Space algorithm with the purpose of solving the inverse problem where the input parameters that achieve the desired outcome can be deduced [10]. In this algorithm, the uncertainties in the parameters are presented by fuzzy numbers [14] which are integrated in the state space model of the system. The algorithm is flexible as the input parameter can be described as approximately as desired at the initial stages of the control process. The development of the algorithm is based on the three phases of the fuzzy system, namely, fuzzification, fuzzy environment and defuzzification. The fuzzy algorithm has been shown to give good parameter estimation for a superheater system [13,17]. Based on [14], the definition of FSSM of a multivariable dynamic system is given as follows:

Definition 4.1

A Fuzzy State Space Model of a multivariable dynamic system is defined as

$$S_{gF}: \quad \dot{x}(t) = A\,x(t) + B\,\tilde{u}(t)$$
$$\tilde{y}(t) = C\,x(t)$$

where $\tilde{u}$ denotes the fuzzified input vector $[u_1, u_2, \ldots. u_n]^T$ and $\tilde{y}$ denotes the fuzzified output vector $[y_1, y_2, \ldots. y_m]^T$ with initial conditions as $t_0 = 0$ and $x_0 = x(t_0) = 0$. The elements of state matrix $A_{p\times p}$, input matrix $B_{p\times n}$, and output matrix $C_{m\times p}$ are known to specified accuracy. The multi-connected systems of FSSM can be viewed as a system of FSSM, S_{gF} which is a collection of subsystems S_{gF1}, S_{gF2}, …, S_{gFj} $(1,2,\ldots., j)$. The system is based on the source of input parameters; either the input parameters are directly from one system or from multiple systems.

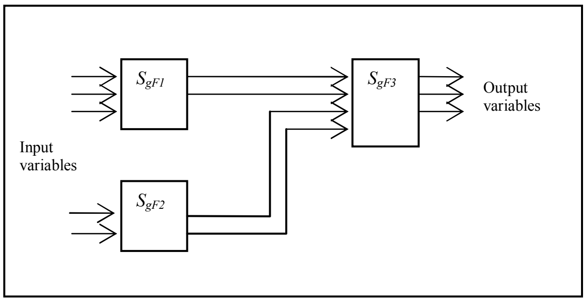


Fig. 3. A multiconnected System

For each of S_{gF} in multi-connected system, it can be transformed into a point in the Euclidean *n*-space where the elements of *A, B* and *C* matrices can be written as coordinates of a point in a finite dimensional space. Thus, Fuzzy state space model can be embedded in Euclidean space by using the following transformation:

Theorem

Given a fuzzy state space systems

$$S_{gFj} = (A, B, C) \in A_{p\times p} \times B_{p\times n} \times C_{m\times p}$$

Such that

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pp} \end{bmatrix},$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{p1} & \cdots & b_{pn} \end{bmatrix} \; C = \begin{bmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mp} \end{bmatrix}$$

.

The transformation

$$\theta : A_{p\times p} \times B_{p\times n} \times C_{m\times p} \rightarrow E^{p^2 + pn + mp}$$

$$\ni \theta(S_{gFj}) = (a_{11}, \ldots, a_{pp}, b_{11}, \ldots, b_{pn}, c_{11}, \ldots, c_{mp})$$

is a bijective map.

Proof

(i) Let $S_{gF} = S_{gF}^*$

$$\Rightarrow (A, B, C) = (A', B', C')$$
$$\Rightarrow A = A', B = B', C = C'$$
$$\Rightarrow (a_{11}, \ldots.., a_{pp}) = (a'_{11}, \ldots.., a'_{pp}),$$
$$(b_{11}, \ldots.., b_{pn}) = (b'_{11}, \ldots.., b'_{pn}),$$
$$(c_{11}, \ldots.., c_{mp}) = (c'_{11}, \ldots.., c'_{mp})$$
$$\Rightarrow a_{11} = a'_{11}, \ldots\ldots., a_{pp} = a'_{pp}, b_{11} = b'_{11}, \ldots\ldots.., b_{pn} = b'_{pn},$$
$$c_{11} = c'_{11}, \ldots\ldots.., c_{mp} = c'_{mp}$$
$$\therefore (a_{11}, \ldots.., a_{pp}, b_{11}, \ldots., b_{pn}, \ldots.., c_{11}, \ldots.., c_{mp})$$
$$= (a'_{11}, \ldots.., a'_{pp}, b'_{11}, \ldots., b'_{pn}, \ldots.., c'_{11}, \ldots.., c'_{mp})$$

In other words,

$\theta(S_{gF})=\theta(S_{gF}^{*})$ where θ is a function

(ii) Let $\theta(S_{gF})=\theta(S_{gF}^{*})$,
therefore,

$$(a_{11},.....,a_{pp},b_{11},.....b_{np},c_{11},.....,c_{mp}) = (a_{11}^{'},.......,a_{pp}^{'},b_{11}^{'}......b_{np}^{'},c_{11}^{'},......,c_{mp}^{'})$$, and

$a_{11}=a_{11}^{'},..........,a_{pp}=a_{pp}^{'}$, $b_{11}=b_{11}^{'},..........,b_{np}=b_{np}^{'}$, $c_{11}=c_{11}^{'},..........,c_{mp}=c_{mp}^{'}$

$$\Rightarrow \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pp} \end{bmatrix} = \begin{bmatrix} a_{11}^{'} & \cdots & a_{1p}^{'} \\ \vdots & \ddots & \vdots \\ a_{p1}^{'} & \cdots & a_{pp}^{'} \end{bmatrix},$$

$$\begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{np} \end{bmatrix} = \begin{bmatrix} b_{11}^{'} & \cdots & b_{1p}^{'} \\ \vdots & \ddots & \vdots \\ b_{n1}^{'} & \cdots & b_{np}^{'} \end{bmatrix},$$

$$\begin{bmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{np} \end{bmatrix} = \begin{bmatrix} c_{11}^{'} & \cdots & c_{1p}^{'} \\ \vdots & \ddots & \vdots \\ c_{m1}^{'} & \cdots & c_{mp}^{'} \end{bmatrix}$$

$A=A^{'}, B=B^{'}, C=C^{'} \Rightarrow (A,B,C)=(A^{'},B^{'},C^{'})$

(iii) Pick any point, $r\in E^{pp+np+mp}$
Then , $r=(a_{11},.....,a_{pp},b_{11},.....b_{np},c_{11},.....,c_{mp})$
where $a_{ij},b_{ij},c_{ij}\in R$.
Consequently, $\exists\, S_{gF}\in A_{p\times p}\times B_{n\times p}\times M_{m\times p}$
such that

$$\theta(S_{gF})=(a_{11},.....,a_{pp},b_{11},.....b_{np},c_{11},.....,c_{mp})$$

with

$$A=\begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pp} \end{bmatrix},\ B=\begin{bmatrix} b_{11} & \cdots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{mp} \end{bmatrix}$$

$$C=\begin{bmatrix} c_{11} & \cdots & c_{1p} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mp} \end{bmatrix}$$

Since r is arbitrary thus θ is onto.
Therefore, $S_{gF}=S_{gF}^{*}\ \therefore \theta$ is one to one

Table 2. FSSM to Euclidean *n*-space

Sub-system	Order of matrices	k	Vertex (S_i^*) in Euclidean *n*-space
Super-heater	$A_{2\times2}, B_{2\times2}, C_{3\times2}$	14	$S_3^*=\left[a_{11},a_{12},a_{21},a_{22},0,0,0,0,0,0,\frac{1}{M_sC_{st}},0,-\frac{1}{v_s},0,0,0\right]$
Riser	$A_{2\times2}, B_{2\times2}, C_{1\times2}$	10	$S_2^*=\left[a_{11},a_{12},a_{21},a_{22},0,0,0,0,0,\frac{1}{M_sC_{rt}},0,-\frac{h_w}{\rho_rV_r},0,0,0\right]$
Reheater	$A_{2\times2}, B_{2\times2}, C_{2\times2}$	12	$S_4^*=\left[a_{11},a_{12},a_{21},a_{22},0,0,0,0,0,\frac{1}{M_sC_{rh}},0,-\frac{w_{ri}}{V_{rh}},0,0,0\right]$
Drum	$A_{2\times2}, B_{2\times2}, C_{2\times2}$	12	$S_5^*=\left[-\frac{v_{dow}}{V_L},0,0,-\frac{v_{dow}}{V_L},0,0,0,0,0,0,1,(1-x),h_e,(1-x)h_{wr},0,0\right]$
Furnace	$A_{1\times1}, B_{1\times9}, C_{5\times1}$	15	$S_1^*=\left[-\frac{k_FR_{EG}T_G}{V_F},0,0,0,0,0,0,0,0,0,\frac{1}{V_F},\frac{1}{V_F},\frac{1}{V_F},0,0,0\right]$

By using this transformation, the Fuzzy State Space Model of the boiler system can be embedded in Euclidean space. The detailed is shown in Table 2. The Euclidean k-space for each vertex is determined by the total size of A, B and C matrices in each subsystem of FSSM. These vertices will be embedded into the same Euclidean n-space. 15 is the maximum value of k which will represent the size of the n-space. The mapping of FSSM to Euclidean space will be the initial stage for further exploration view of FSSM. A new concept of Autocatalytic set will be adopted to define some properties of the new graphical representation of FSSM.

5 Autocatalytic Set (ACS)

The concept of autocatalysis comes from chemistry. A catalyst is a substance that facilitate a chemical reaction. In autocatalysis, a product of reaction serves as a catalyst for the same reaction. The concept of Autocatalytic Set (ACS) was introduced particularly in understanding the construction of molecules in the chemical compound in a substance [4,5,19]. The underlying idea of this concept is that a set of molecular species that contain, within itself, a catalyst of its member species. Such a set of molecular species can collectively self-replicate under certain circumstances even if none of its component molecular species can individually self replicate. In general, it is a collection of entities (molecules, people, nations, institutions) that produces as outputs the same elements which are necessary inputs for expanding the collection. Stuart Kauffman theory adopted this property in understanding the origin of life [23]. Jain & Krishna initiated the ACS in term of graph theoretic approach, They imagined that a node in a directed graph is to represent a molecular species and a link from j to i as signifying that j is a catalyst for i. Based on this idea, ACS for any directed graph is define as below [5]:

Definition 5.1
An autocatalytic set (ACS) is a subgraph, each of whose nodes has at least one incoming link from a node to the same subgraph.

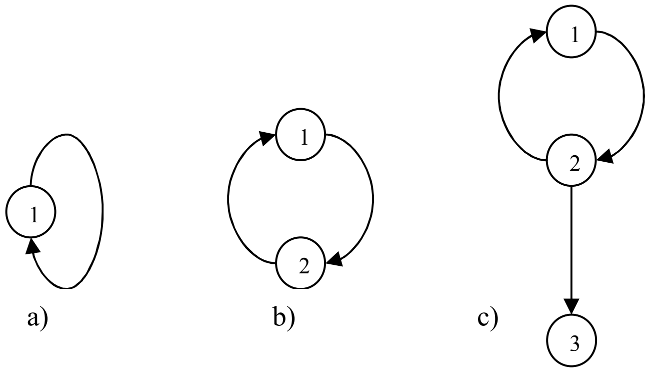


Fig. 4 (a) A 1-cycle, the simplest ACS (b) A 2-cycle ACS (c) An ACS, but not cycle

Fig. 4 shows examples of ACS [12]. By using the above definition and proposition 3.1 we can state the following:

Proposition 5.1
The graph G_{cc} is an autocatalytic set

This proposition has inspired to explore further features of the graph. This led to the following discussion of related properties of the graph in the next section.

6 Related Properties of the Graph

6.1 Irreducible Graph

The first property to look at is to determine whether the graph is reducible or irreducible. Irreducible graphs are, by definition, strongly connected graph. [23]. A graph is said to be irreducible if each node in the graph has access to every other node [1]. In addition, an irreducible graph is always an ACS [5]. Therefore, the graphical representation of Boiler System, G_{cc} shown in fig. 2 is an ACS hence, it is irreducible graph. If a graph is irreducible then its adjacency matrix is also irreducible[23] thus, we can conclude that in any dynamical system such as boiler, the matrix C is irreducible means that the system cannot be reduced. In other words, the system must be treated as a whole in studying its behavior.

6.2 The Adjacency Matrix

The network of interactions between the parameters and the subsystems in the boiler system shown in Fig. 2, can be specified using the 6 x 6 matrix C called adjacency matrix. The rows and the columns of the matrices represent the nodes in the graph and the entries C_{ij} represent the link between the *ith* node and *jth* node [1]. A directed weighted link from node *i* to node *j* is denoted by the following values.

$$C_{ij} = \begin{cases} 1 & \text{if there exist a link i to j} \\ 0 & \text{if there is no link from i to j} \end{cases}$$

This gives the following adjacency matrix *C* of the graph, G_{cc}.

$$C_{G_{cc}} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The special feature denoted by the adjacency matrix shows that the autocatalytic set of the graph is that the every row must contain at least one non-zero other element of the *j*th column. This relate to the ACS of the graph whereby any node can have at least one incoming link from any other nodes in the graph. All the elements of the principle diagonal are zeros showing that there is no self-replicating of any of the elements or there is no self-cycle link within a subsystem.

6.3 Relationship of an ACS to Perron Frobenius Eigenvector (PFE)

The ACS is a useful graph-theoretic construct in its connection with the PFE. The Perron–Frobenius eigenvalue of a graph is an indicator of the existing ACS in a graph [22, 23]. Let *x* be a PFE of a graph where the set of all nodes *i* for which x_i is non zero. The subgraph induced by these nodes is known as the subgraph of the PFE *x*. Thus, if all the component of the PFE are none-zero then the subgraph of the PFE is the entire graph . Finally, if $\lambda_1 \geq 1$, then the subgraph of any PFE of C is an ACS [5,24] . By using the same adjacency matrix, $C_{G_{cc}}$, further investigation to determine the relationship of ACS and PFE was carried out.

Here the eigenvalues λ and their respective eigenvectors *x* were calculated using mathlab version 7. The following are the result,

$$\lambda_1 = 2.6412,\ x_1 = \begin{pmatrix} -0.5316 \\ -0.5152 \\ -0.2177 \\ -0.0962 \\ -0.2541 \\ -0.5750 \end{pmatrix},$$

$$\lambda_2 = -1.7757,\ x_2 = \begin{pmatrix} -0.4009 \\ 0.1660 \\ -0.3411 \\ 0.2813 \\ -0.4995 \\ 0.6057 \end{pmatrix},$$

$$\lambda_3 = -0.5892,\ x_3 = \begin{pmatrix} -0.6488 \\ 0.6759 \\ -0.2235 \\ -0.2018 \\ 0.1189 \\ 0.1317 \end{pmatrix},$$

$$\lambda_4 = 0.0000,\ x_4 = \begin{pmatrix} 0.0000 \\ -0.7071 \\ 0.7071 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{pmatrix},$$

$$\lambda_5 = 0.7237,\ x_5 = \begin{pmatrix} 0.2087 \\ 0.0624 \\ 0.4346 \\ 0.6605 \\ -0.4780 \\ 0.3145 \end{pmatrix},$$

$$\lambda_6 = -1.0000,\ x_6 = \begin{pmatrix} 0.7071 \\ -0.7071 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \end{pmatrix}$$

For any non-negative irreducible matrix, the PF theorem states that there exists an eigenvalue, which is real and larger than or equal to all other eigenvalues in magnitude [22]. The largest value eigenvalue is called PF eigenvalue of the matrix, which is denoted by $\lambda(C)$. The PF eigenvalue of matrix $C_{G_{cc}}$ is,

$$\lambda_1\left(C_{G_{cc}}\right) = 2.6412$$

Further, the theorem also states that there exist an eigenvector of C corresponding to the value of $\lambda_1\left(C_{G_{cc}}\right)$ referred to as PF eigenvector (PFE), whose all the components are real and non-negative. The PFE of $\lambda_1\left(C_{G_{cc}}\right)$ is given as :

$$\text{PFE} = \begin{pmatrix} 0.5316 \\ 0.5152 \\ 0.2177 \\ 0.0962 \\ 0.2541 \\ 0.5750 \end{pmatrix}$$

The PFE above, shows that all the components of PFE are none-zero. Therefore, the subgraph of PFE is the entire graph G_{cc}. Based on the result above, and by the proposition by [5, 24], it is confirmed that the graph G_{cc} is an ACS.

The discussion on the algebraic properties (the structure of the eigenvectors of its adjacency matrix) highlighted in this section provide a good understanding of the graph representing the system.

7 Conclusion

The static graphical representation of the boiler system has been successfully modeled using the basic graph theoretical concept. We finally outlined general procedure in construction of graphical presentation for fuzzy state space model of a system. A sample construction of a boiler system and related properties is presented. This new approach will simplify the schematic diagram of interconnection of subsystems in a boiler. Thus, the graphical representation will lead to the development of fuzzy graphical representation of a boiler where the method to determine the membership values will be discussed in future papers.

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