Evaluation of Steady State Vector of Fuzzy Autocatalytic Set of Fuzzy Graph Type-3 of an Incineration Process

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Abstract: - Fuzzy Autocatalytic Set (FACS) of Fuzzy Graph of Type-3 incorporates the concept of fuzzy, graph and autocatalytic set. It was initially defined and used in the modeling of a clinical waste incineration process which has produced more accurate results than using crisp graph. As it is a newly developed theory, FACS seems to have great potentials in generating new mathematical theory. This paper employs Markov chain to the evaluation of steady state of an incineration process. Novel definition of transition probability matrix of FACS is presented. Steady state vector of Markov chain for incineration process is determined and graph of convergence of its norm difference is presented. This study led to some relation of Markov process and Perron-Frobenius Theorem.

Key-Words: - Fuzzy Autocatalytic Set, Fuzzy Graph, Incineration Process, Perron-Frobenius Theorem, Transition Probability Matrix, Markov process.

1 Introduction

The emergence of fuzzy graph to autocatalytic sets has instigated a new concept named Fuzzy Autocatalytic Set (FACS) [1,2]. A clinical waste incineration process in Malacca (schematic diagram given in Fig. 2) is modeled formally using crisp graph as below.

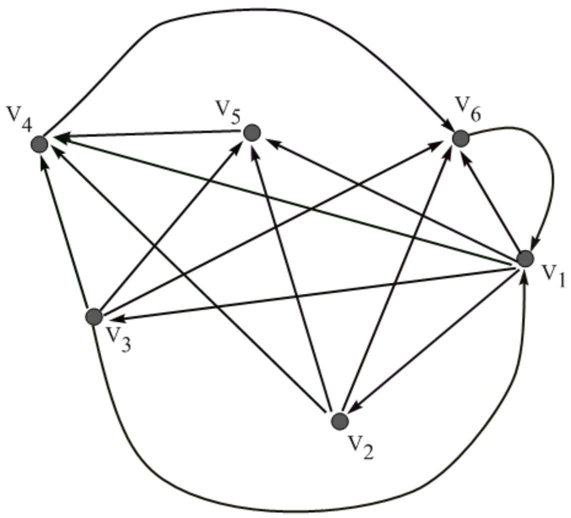


Fig. 1. Crisp Graph for clinical waste incineration process

V_1: Waste (particularly clinical waste)
V_2: Fuel
V_3: Oxygen
V_4:Carbon Dioxide
V_5:Carbon Monoxide
V_6: Other gases including water

However the interpretation of the graph at the end of the process did not signify the product of the process [1]. This led us to use the new concept, FACS [1, 2], and in particular using fuzzy graph of type-3 [3]. Several new results of FACS which linked to Perron Frobenius Theorem have been discussed in previous studies [2, 4, 5, 6, 7]. In this paper, we focus on the study of FACS of an incineration process from a new perspective by using Markov chain and evaluate its steady state vector.

2 A Fuzzy Graph Type 3

Rosenfeld [8] has defined fuzzy graph in which he has considered fuzzy graph to consist both fuzzy set for vertices as well as for the edges. Yeh and Bang [9] also coined a special case of graph fuzziness where only the edges are fuzzy and the vertices remain as a crisp set. After the pioneering work of Rosenfeld and Yeh and Bang in 1975, where some cases of graph fuzziness have been defined and basic theoretic concepts have been formulated, Blue et. al [3] further generalized the catalog of various fuzziness possible in graph where five types of fuzzy graphs are introduced. However, Sabariah [1] and Tahir et. al [2] have formalized the five types of fuzzy graphs described by Blue et. al in the following.

Definition 2.1

Fuzzy graph is a graph G_F satisfying one of the fuzziness (G_F^i of the i^{th} type) or any of its combination:

1) $G_F^1 = \{G_{1_F}, G_{2_F}, G_{3_F}, ..., G_{n_F}\}$ where fuzziness is on G_F^i for i = 1,2,3,…,n.
2) ii. $G_{2_F} = \{V, E_F\}$ where the edge set is fuzzy.
3) iii. $G_F^3 = \{V, E(t_F, h_F)\}$ where both the vertex and edge sets are crisp, but the edges have fuzzy heads and tails.
4) iv. $G_F^4 = \{V_F, E\}$ where the vertex set is fuzzy.
5) v. $G_F^5 = \{V, E(w_F)\}$ where both the vertex and edge sets are crisp but the edges have fuzzy weights.

3 Autocatalytic Set

The concept of autocatalytic set (ACS) was first introduced in the context of catalytically interacting molecules, Kauffman [10]; Rossler [11]. However, Jain and Krishna [12] have been formalized the autocatalytic set in terms of graph theoretical concept (see Fig.3) as follows:

Definition 3.1

An autocatalytic set is a subgraph, each of whose nodes has at least one incoming link from a node belonging to the same subgraph.

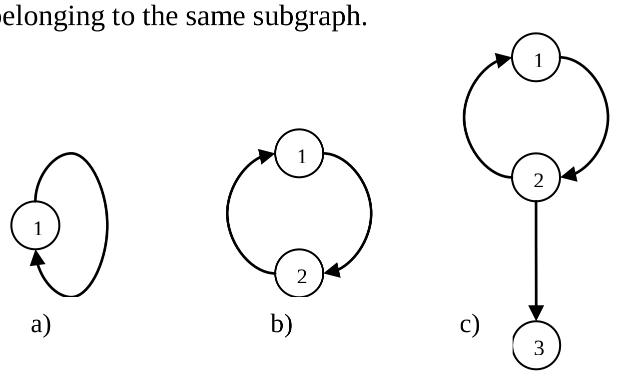


Fig. 3. (a) A 1-cycle, the simplest ACS (b) A 2-cycle ACS (c) An ACS, but not cycle

A graph with s nodes is completely specified by an $s \times s$ matrix, $C = (c_{ij})$ called the adjacency matrix of the graph.

Definition 3.2.

The adjacency matrix of a graph $G = G(V, E)$ with s nodes is an $s \times s$ matrix, denoted by $C = (c_{ij})$, where $c_{ij}=1$ if E contains a directed link (j,i) (arrow pointing from node j to node i), and $c_{ij}=0$ otherwise.

Below are the examples of adjacency matrix drawn for Fig. 3.

(1)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
(a)	(b)	(c)

4 FACS of Fuzzy Graph Type-3

The main idea of the definition of FACS is the merger of fuzzy graph of type-3 to autocatalytic set [1]. The formal definition of FACS is given below.

Definition 4.1 ([1,2])

Fuzzy autocatalytic set (FACS) was defined as a subgraph where each of whose nodes has at least one incoming link with membership value

$$\mu(e_i) \in (0,1], \forall e_i \in E \tag{1}$$

The membership values for fuzzy edge connectivity for fuzzy graph are in the interval (0,1]. These values constitute the entries of the adjacency matrix for FACS as follows:

$$C_{F_{ij}} = \begin{cases} 0 & \text{for } i = j \text{ and } e_i \notin E \\ \mu(e_i) \in (0,1] & \text{for } i \neq j \end{cases} \tag{2}$$

As for incineration process, the membership values are determined through the chemical reaction taken place between six variables that play its vital roles in the clinical waste incinerator, namely waste, fuel, oxygen, carbon dioxide, carbon monoxide and other gases including water. The set of vertices in the graph of FACS of the incineration process, $V = \{v_1, v_2, ..., v_6\}$ is represented by these six variables.

When a fuzzy graph of type-3 is considered in the construction of FACS, the description of its fuzzy head, fuzzy tail and fuzzy edges connectivity of the edges are given as in [1,2].

From the explanation given in [1,2] pertaining to the construction of FACS of Fuzzy Graph of Type-3 for the incineration process, the graph is represented as in Fig. 4. and its adjacency matrix using (2) is represented as in Fig. 5. The different color signifies the different range of membership value for the fuzzy edge connectivity. The greater the value of connectivity between the vertices, the thicker is the link between them. The graph is strongly connected where each node in the graph has access to every other node.

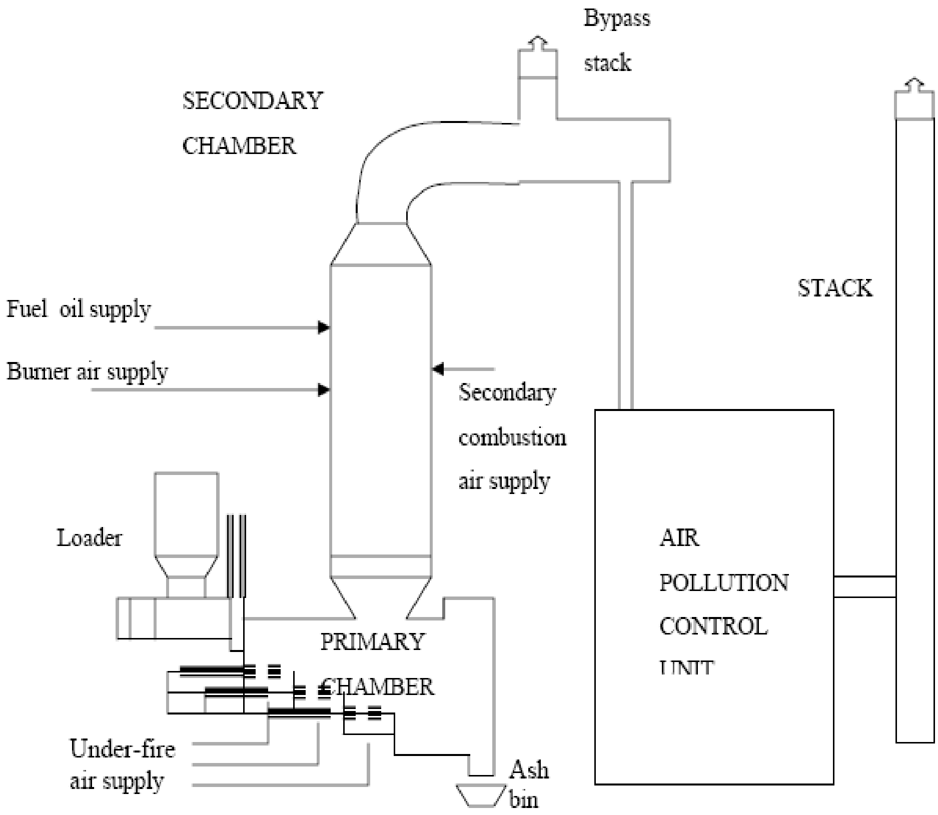
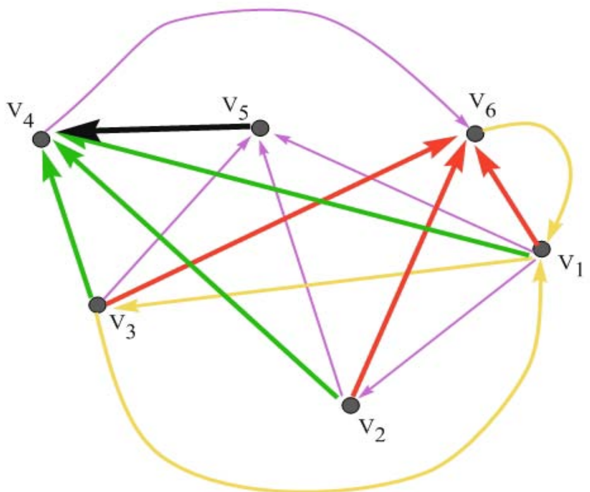


Fig. 2. The shematic diagram of clinical waste incinerator.



V_1: Waste (particularly clinical waste)
V_2: Fuel
V_3: Oxygen
V_4:Carbon Dioxide
V_5:Carbon Monoxide
V_6: Other gases including water

Fig. 4. Fuzzy Graph of Type 3 for the clinical waste incineration process

$$C_{F_{ij}} = \begin{pmatrix} 0 & 0 & 0.06529 & 0 & 0 & 0.13401 \\ 0.00001 & 0 & 0 & 0 & 0 & 0 \\ 0.15615 & 0 & 0 & 0 & 0 & 0 \\ 0.51632 & 0.68004 & 0.63563 & 0 & 0.99999 & 0 \\ 0.00001 & 0.00001 & 0.00002 & 0 & 0 & 0 \\ 0.32752 & 0.31995 & 0.29906 & 0.00001 & 0 & 0 \end{pmatrix}$$

Fig. 5. Adjacency Matrix representing FACS for clinical waste incineration process

5 Markov Chains and Transition Probabilities for Directed Walk

Let $G = (V, E)$ be a directed graph with $V = \{1,...,n\}$. Pick any node to be the initial node. Suppose that at each time step, we choose at random a child of the current node and move towards this node. The resulting sequence of nodes is called a random walk. Recall that the transition probability matrix for a random walk on weighted directed graph is given as follows:

$$P(u,v) = \begin{cases} \dfrac{w_{uv}}{d_u} & \textit{if } u \to v \textit{ is an edge in } G \\ 0 & \textit{otherwise} \end{cases} \tag{3}$$

where d_u denotes the out-degree of vertex u and w_{uv} denotes the weight of edge from vertex u to v.

The probability of moving from a node u to a node v is propotional to the weight of the edge $u \to v$. According to Rubinfeld [13], the concept of a random walk with weighted edges is equivalent to the concept of finite Markov chain. Here, a random process is defined by a finite set of states, $N = \{1,...,n\}$ and a sequence $X_0, X_1, X_2,...$ of random variables. This process is a (finite) Markov chain if the transition probabilities at step (k+1) depend only on the state at step k, that is

$$\mathrm{P}\left(X_{k+1} = j \middle| X_k = i_k,..., X_0 = i_0\right) = \mathrm{P}\left(X_{k+1} = j \middle| X_k = i_k\right) \tag{4}$$

Markov chain is also often described by transition probability matrix $P = \lfloor P_{ij} \rfloor_{i,j \in N}$ defined as

$$P_{ij} = \mathbb{P}\left(X_{k+1} = j \middle| X_k = i_k\right) \text{ for all } i, j \in N \tag{5}$$

Next, the transition probability matrix is a (row) stochastic matrix, that is, a square nonnegative matrix with all row sums equal to 1 and no value of its entries is negative. Note that every stochastic matrix represent a Markov chain. Since it is a nonnegative matrix, Perron-Frobenius theorem gives useful information about the eigenvalues of such matrices as described below.

Let $P \in \Re_{\geq 0}^{n \times n}$ be a stochastic matrix. Then

(a) $\rho(P) = 1$ is an eigenvalue of P;

(b) P has at least one invariant measure;

If P is irreducible, then

(c) P has exactly one invariant measure x^T, and x^T is positive;

Moreover, if P is primitive, then

(d) $\lim_{k\to\infty} P^k = 1x^T$.

Note that invariant measure of a stochastic matrix correspond exactly to its left Perron vector.

6 Transformation of FACS to Transition Probability Matrix

Since the concept of random walk with weighted edges is equivalent to the concept of finite Markov chain; i.e. it is a special case of Markov chain [13]. We can then apply the concepts in eq. (3), (4), (5) to the incineration process with the assumption that population size and the possible state are both constants. Here, population size denotes the six variables that plays vital role in the process whereas the possible states denote the possible condition or form of the variables in the system at a particular time. We assume that it takes the same amount of time to move from one state to another. Thus, the Markov chain is homogeneous and its dynamics are described by the transition probability matrix, P which is given in the following definition.

Definition 6.1 (Transition Matrix for FACS)

Suppose $G_{FT3}(V, E)$ is a no loop FACS of Fuzzy Graph Type-3. The transition matrix of FACS of

Fuzzy Graph Type-3 is P^*, with $P^*(u,v)$ is fuzzy value of moving from u to v as

$$P^*(u,v) = \begin{cases} \dfrac{\mu(u,v)}{d_{out}(u)} & if \ (u,v) \in E \\ 0 & otherwise \end{cases} \tag{6}$$

where for a vertex u, the out-degree of u is $d_{out}(u) = \sum \mu(u,t)$ and $\mu(u,v)$ is the ordinary membership value of an edge from u to v.

This matrix is not an ordinary transition matrix defined in [14-19] since it integrates the membership value of an edge into the calculation of its entries. Calculation of the membership value for every edges in the graph is formulated in [1] through several assumptions such as chemical reaction of the formation of the variables namely waste, fuel, oxygen, carbon dioxide, carbon monoxide and other gases including water or the relation between them. It takes values in the interval of [0,1]. Probability and fuzziness are two different concepts [20,21,22].

In our case, however, a membership value of an edge is regarded as the weight of the edge. Hence, the transition matrix for FACS formulated in eq. (6) of the incineration process is given as in Fig. 6:

$$P^* = \begin{bmatrix} 0 & \dfrac{0.00001}{1.00001} & \dfrac{0.15615}{1.00001} & \dfrac{0.51632}{1.00001} & \dfrac{0.00001}{1.00001} & \dfrac{0.32752}{1.00001} \\ 0 & 0 & 0 & 0.6800400 & 0.0000100 & 0.3199500 \\ 0.06529 & 0 & 0 & 0.6356300 & 0.0000200 & 0.2990600 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig. 6. Transition Probability Matrix representing FACS for clinical waste incineration process

Next, mapping of no-loop FACS of Fuzzy Graph Type-3, *G(V, E)* to transition probability matrix, P^* is established and as follows.

Corollary 1

Let $G_{k_{FT3}}$ (V, E) be a no loop fuzzy graph Type-3 which is autocatalytic; i.e FACS defined by

$$G_{k_{FT3}} = \begin{cases} 0 & when \quad i = j \quad and \quad e_i \notin E \\ \mu(e_i) \in (0,1] & when \ i \neq j \quad and \quad e_i \in E \end{cases}$$

for k = 1,2,3,4…,n.

Next define $G_{FT3} = \left\{ G_{k_{FT3}} ; k = 1,2,3,.....,n \right\}$ be finite set of all fuzzy graph type-3. Define

$$P_F^{n\times n} = \left\{ [p_{ij}]^{n\times n} : p_{ij} = \begin{cases} \dfrac{\mu(v_i,v_j)}{d_{out}(v_i)} & if \quad (v_i,v_j) \in E \\ 0 & otherwise \end{cases} \right\}$$

and $\sigma : G_{FT3} \to P_F^{n\times n} \ni \sigma(G_{i_{FT3}}) = [p_{ij}]$ then $\sigma : G_{FT3} \to P_F^{n\times n}$ is onto and one-to-one function.

Proof:

1) Let $G_{FT3}(V,E) = G'_{FT3}(V',E')$

$\Rightarrow V = \{v_1, v_2, v_3, ..., v_n\} = \{v_1', v_2', v_3', ..., v_n'\} = V'$

and

$E = \{\mu(v_i,v_j)\}_{i,j=1,2,3,...,n} = \{\mu'(v_i',v_j')\}_{i,j=1,2,3,...,n} = E'$

$\Rightarrow \mu(v_i,v_j) = P_{ij} = P_{ij}' = \mu(v_i',v_j')$

$\Rightarrow [p_{ij}] = [p_{ij}]'$

$\therefore \sigma$ is a function.

2) $f : A \to B$ is onto if $b \in B,$ then $\exists \ a \in A \ni f(a) = b$. So pick $[p_{ij}] \in P_F^{n\times n}$, then $\exists G_{i_{FT3}} \ni \sigma(G_{i_{FT3}}) = [p_{ij}]$ and $p_{ij} = \dfrac{\mu(v_i,v_j)}{d_{out}(v_i)}$ for $(v_i,v_j) \in G_{i_{FT3}}$

$\therefore \sigma$ is onto.

3) In the case of $\sigma(G_1(V,E)) = \sigma(G_2(V,E))$

$$\Rightarrow \begin{bmatrix} 0 & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & 0 & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & 0 & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & 0 \end{bmatrix} = \begin{bmatrix} 0 & p_{12}' & p_{13}' & \cdots & p_{1n}' \\ p_{21}' & 0 & p_{23}' & \cdots & p_{2n}' \\ p_{31}' & p_{32}' & 0 & \cdots & p_{3n}' \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1}' & p_{n2}' & p_{n3}' & \cdots & 0 \end{bmatrix}$$

It is the same matrix if

$$p_{12} = p_{12}' \Rightarrow \frac{\mu(v_1, v_2)}{d_{out}(v_1)} = \frac{\mu(v_1', v_2')}{d_{out}(v_1')}$$

$$\Rightarrow d_{out}(v_1')\mu(v_1, v_2) = d_{out}(v_1)\mu(v_1', v_2')$$

$$if \;\; d_{out}(v_1') = d_{out}(v_1) \Rightarrow \mu(v_1, v_2) = \mu(v_1', v_2')$$

$$\Rightarrow v_1 = v_1' \;\; and \;\; v_2 = v_2'$$

Generally,

$$p_{n(n-1)} = p_{n(n-1)}' \Rightarrow (v_n, v_{n-1}) = (v_n', v_{n-1}')$$

$$\Rightarrow v_n = v_n', \forall n = 1,2,3,\ldots, n$$

$$\Rightarrow G_1(V,E) = G_2(V,E)$$

$\therefore \sigma$ is one-to-one.

7 Characteristics of Transition Matrix of FACS of Fuzzy Graph of Type-3.

The above transition matrix P^* provides some basic facts or characteristics of FACS. There are:

Fact 1: For a strongly connected graph of FACS, the transition matrix is (row) stochastic matrix since the sum of the entries on each and every row is 1 and no value of its entries is negative.

Fact 2: For a strongly connected graph of FACS, the transition matrix is irreducible.

Eventhough transition matrix is not symmetric, we still can deduced some useful properties of the matrix since all of its entries are nonnegative.

The eigenvalues, λ and their respective eigenvectors, $\boldsymbol{x}$ of P^* using MATLAB version 7.0 is computed and given as below.

$\lambda =$ 1.00000000000000,-0.40154987138365+0.58558458293596i , -0.40154987138365-0.58558458293596i, -0.19686879108908, -0.00003146614362, 0.00000000000000

$$\boldsymbol{x} = \begin{pmatrix} -0.40824829046386 \\ -0.40824829046386 \\ -0.40824829046386 \\ -0.40824829046386 \\ -0.40824829046386 \\ -0.40824829046386 \end{pmatrix}, \begin{pmatrix} 0.22029544261060 + 0.05214951751047\boldsymbol{i} \\ 0.34881365313861 + 0.11848096015235\boldsymbol{i} \\ 0.31852679229419 + 0.09131911587833\boldsymbol{i} \\ -0.25394708515697 + 0.37033382039705\boldsymbol{i} \\ 0.63241729920602 \\ -0.11488935638125 - 0.29741499589890\boldsymbol{i} \end{pmatrix}, \begin{pmatrix} -0.00575720029008 \\ 0.46555029682935 \\ 0.43701540074308 \\ -0.14854484664238 \\ 0.75453730284334 \\ 0.02924384438100 \end{pmatrix}$$

$$\begin{pmatrix} 0.22029544261060 - 0.05214951751047\boldsymbol{i} \\ 0.34881365313861 - 0.11848096015235\boldsymbol{i} \\ 0.31852679229419 - 0.09131911587833\boldsymbol{i} \\ -0.25394708515697 - 0.37033382039705\boldsymbol{i} \\ 0.63241729920602 \\ -0.11488935638125 + 0.29741499589890\boldsymbol{i} \end{pmatrix}, \begin{pmatrix} -0.00000000000003 \\ 0.34057330989008 \\ 0.00001579972369 \\ -0.00002958503348 \\ 0.94021796380712 \\ 0.00000000093093 \end{pmatrix}, \begin{pmatrix} 0.00000000000000 \\ -0.99999999794938 \\ 0.00006404098610 \\ 0.00000000000000 \\ 0.00000000000000 \\ -0.00000000000000 \end{pmatrix}$$

For any nonnegative irreducible matrix, Perron-Frobenius (PF) theorem guarantees that there exists an eigenvalue which is real and larger than or equal to all other eigenvalues in magnitude. Here,

$$\lambda_1 = \rho(P^*) = 1$$

Fact 3: The largest eigenvalue of transition matrix of FACS is real and equal to 1 with algebraic multiplicity one.

Fact 4: Since $\lambda_1 = \rho(P^*) = 1$, then the strongly connected graph of FACS is aperiodic.

This fact follows from Lemma 2 by Costello [23]. Further, this matrix has a truly dominant eigenvalue using primitive matrix [24] since $(P^*)^k > 0$ for k = 6. Moreover, it is a regular transition matrix since for $k = 6$, $\left((P^*)^k\right)_{i,j} > 0,\ \forall i, j = 1,...,n$.

Fact 5: The regular transition matrix of FACS is primitive since

$$\boldsymbol{x} = \begin{pmatrix} 0.40824829046386 \\ 0.40824829046386 \\ 0.40824829046386 \\ 0.40824829046386 \\ 0.40824829046386 \\ 0.40824829046386 \end{pmatrix} = 0.40824829046386 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

and $P^*\boldsymbol{x} = \lambda_1\boldsymbol{x} = \boldsymbol{1}\boldsymbol{x} = \boldsymbol{x}$. Thus the 'all 1' vector is a right eigenvector of P^* with eigenvalue equal to 1.

Fact 6: The right eigenvector of transition matrix of FACS is column vector with all entries equal to 1.

We can also calculate the unique left Peron vector, x^T for matrix P^* given as

$$x^T = \big(0.36211364 \quad 0.362110019 \times 10^{-6} \quad 0.056543479 \quad 0.222912591 \quad 4.75200599 \times 10^{-6} \quad 0.358421916\big)$$

Note that $\sum_{j=1}^{6} x^T{}_{1j} = 1$.

Fact 7: The left eigenvector $x^T = (x_1,...,x_6)$ corresponds to the largest eigenvalue of transition matrix of FACS is a row vector which is unique and $\sum_{j=1}^{6} x^T{}_{1j} = 1$.

8 Steady State Vector for FACS of Fuzzy Graph Type-3.

The fundamental Ergodic Theorem for Markov chains in [25] states that the Markov chain represent by transition matrix. P has a stationary distribution vector under three conditions:

i. P is stochastic
ii. P is irreducible
iii. P is aperiodic

Thus, FACS of fuzzy graph type-3 of incineration process, P^* as discussed in Section 7 is an Ergodic Markov chain. It ensures the existence of steady state vector which in turn can be described the long term conditions of the system.

Two well-known methods were used in finding steady state vector for FACS. One of them by using standard matrix multiplication $(P^*)^{2n-1} = (P^*)^n (P^*)^{n-1}$.

For $n = 109$ we obtained the entries of our matrix as in Fig. 7:

Each entry of every column in this matrix is identical showing that the numbers in each column converges to a particular number. Elements in the row represent the steady state vector of P^*, that is

$$\Lambda^* = \big(0.36211364 \quad 0.362110019 \times 10^{-6} \quad 0.056543479 \quad 0.222912591 \quad 4.75200599 \times 10^{-6} \quad 0.358421916\big)$$

We interpret the i^{th} entries to be the the relative concentration of the variables at the end of the process as: waste (v_1) has relative concentration with the highest fuzziness as 0.36211364 and water and other pollutions (v_6) with second highest fuzziness as 0.358421916. This particular result consistent with Sabariah's [1] whereby only two variables left at the end of the the clinical waste incineration process namely waste (v_1) and other pollutions (v_6).

Similarly, we used Theorem 1 (pg.89) in [14] where $\lim_{n\to\infty} XP^n = Q$

In our case, the state vectors $\boldsymbol{Q}^{*(n)}$ for $n = 0, 1, 2, \ldots, 109$ can be calculated using initial relative

$$P^{*\,109} = \begin{bmatrix} 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \\ 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \\ 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \\ 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \\ 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \\ 0.36211364 & 0.00000362110019 & 0.056543479 & 0.222912591 & 0.00000475200599 & 0.358421916 \end{bmatrix}$$

Fig. 7: Transition probability matrix of FACS after n= 109 iteration

concentration, of i_{th} variable at initial time $t = 0$ where $\boldsymbol{X} = (0.269 \quad 0.031 \quad 0.645 \quad 0 \quad 0 \quad 0.055)$ as given in Sabariah [1]. The result is shown in Table 1. The result is given in Table 1 shows that regular markov chain of FACS of Fuzzy Graph Type-3 for incineration process converges to a steady state vector

$$(\boldsymbol{Q}^*)^{109} = (0.36211364 \quad 0.362110019\times10^{-6} \quad 0.056543479 \quad 0.222912591 \quad 4.75200599\times10^{-6} \quad 0.358421916)$$

This steady state vector of P^* is equivalent to the unique left Perron vector as discussed in Section 5.0.

The convergence of vector $\boldsymbol{Q}^*$ is also visualized through its norm difference for 109 iterations as in Fig. 8. The graph indicated that the system is stable and no chemical reaction taken place at that particular time. Hence, the steady state vector of P^* of FACS is equivalent to the dominant left eigenvector of P^* of FACS.

Table 1: Convergence of a state vectors for clinical waste incineration process

$Q^{(n)}$ \ n	1	60	100	109
$q_1^{(n)}$	0.097112050000000	0.362113640171713	0.362113639969257	0.362113639969258
$q_2^{(n)}$	0.000002689973100	0.000003621100176	0.000003621100189	0.000003621100189
$q_3^{(n)}$	0.042003929960700	0.056543479254075	0.056543479446405	0.056543479446405
$q_4^{(n)}$	0.569951281113089	0.222912590800916	0.222912591281958	0.222912591281957
$q_5^{(n)}$	0.000015899973100	0.000004752005981	0.000004752005989	0.000004752005989
$q_6^{(n)}$	0.290914148980010	0.358421916667139	0.358421916196203	0.358421916196202

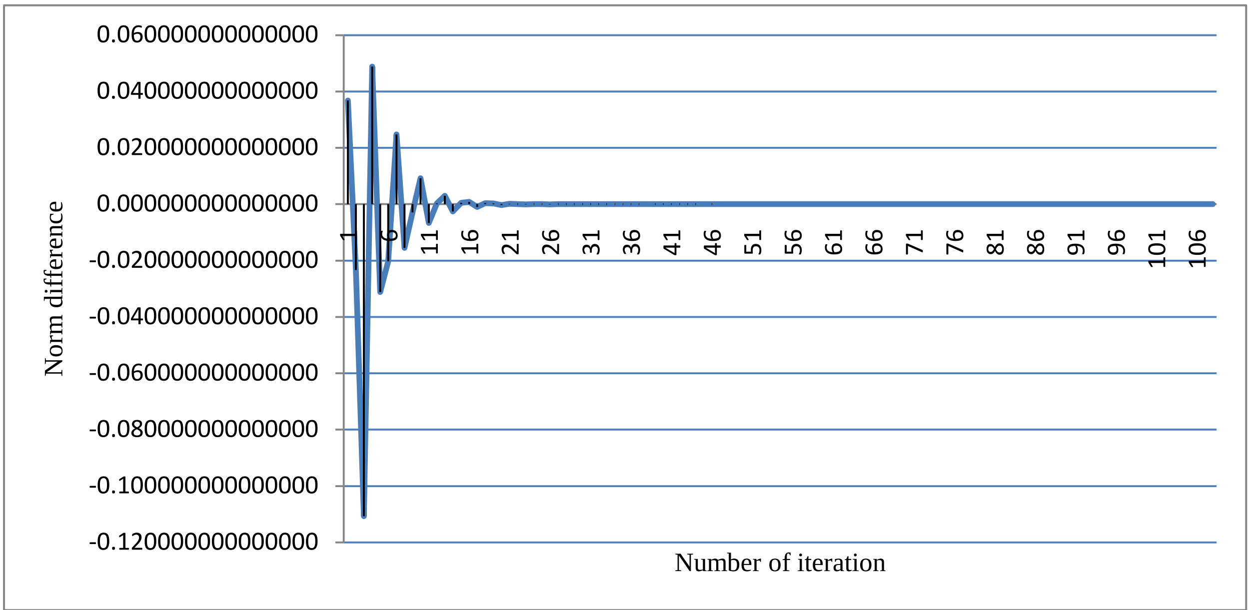


Fig. 8: Convergence of norm difference of vector $\boldsymbol{Q}^*$

9 Conclusion

In this paper, Markov chain of FACS of fuzzy graph type-3 represented by transition matrix, P^* is defined. This definition leads to the construction of transition matrix of FACS for the clinical waste incineration process. Fundamental characteristics of the matrix are highlited including its dominant eigenvalue and its right and left eigenvector (Perron vector). The Markov chain of FACS of fuzzy graph type-3 converged to a steady state vector.

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