Effect of rotation and magnetic field on thermal instability of a viscoelastic fluid permeated with suspended particles

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Abstract: - The effect of rotation and magnetic field on the thermal instability of Walters' (model B') viscoelastic fluid permeated with suspended particles is considered. For stationary convection, Walters' B' viscoelastic fluid behaves like a Newtonian fluid. It is found that, for the case of stationary convection, the rotation and magnetic field have stabilizing effect whereas the suspended particles has destabilizing effect on the system. The principle of exchange of stabilities is satisfied in the absence of magnetic field. The magnetic field, viscoelasticity and rotation introduce oscillatory modes in the systems which were non-existent in their absence.

Key Words: - Rotation, magnetic field, thermal instability, viscoelasticity, suspended particles.

1 Introduction

The theoretical and experimental results of the onset of thermal instability in a fluid layer under varying assumptions of hydrodynamics have been given by Chandrasekhar [1]. Scanlon and Segel [2] have considered the effect of suspended particles on the onset of Benard Convection and found that critical Rayleigh number was reduced solely because heat capacity of pure fluid was supplemented by that of the particles. The effect of suspended particles was thus found to destabilize the layer. Bhatia and Steiner [3] have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation. Sharma and Aggarwal [4] have studied the effect of compressibility and suspended particles on thermal convection in a Walters' B' elastico-viscous fluid in hydromagnetics. Spiegel and Veronis [5] have simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, if only motions of infinitesimal amplitude are considered. Scanlon and Segel [6] studied the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced, solely because the heat capacity of the pure gas was supplemented. Bhatia and Steiner [7] have also studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of a magnetic field. The importance of non-Newtonian fluids in modern technology and industries is ever increasing and the investigations on such fluids are desirable. One such class of non-Newtonian fluids is Walters' B' fluid. Walters [8] reported that the mixture of polymethyl methacrylate and pyridine at $25^0 C$ containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' B' elastico-viscous fluid. Aggarwal and Prakash [9] have studied the effect of suspended particles and rotation on thermal instability of ferromagnetic fluids.

In the present paper, we have studied the effect of suspended particles on the Walters' B' viscoelastic fluid heated from below in the presence of rotation and magnetic field.

2 Problem Formulation

Consider an infinite horizontal layer of electrically conducting Walters' B' elastico-viscous fluid layer of thickness d permeated with suspended particles, bounded by the planes $z = 0$ and $z = d$ in the presence of rotation. This layer is heated from below so that, the temperature and density at the bottom surface $z = 0$ are T_0, ρ_0 and at the upper surface $z = d$ are T_d, ρ_d, respectively, and that a uniform adverse temperature $\beta\left(=\left|dT/dz\right|\right)$ is maintained. A uniform magnetic field $\vec{H} = (0,0,H)$ and gravity field $\vec{g}(0,0,-g)$ pervades the system. The equations of motion and continuity for Walters' B' viscoelastic fluid in the presence of suspended particles and magnetic field with rotation are

$$\frac{\partial \vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}=-\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)$$
$$+\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H} \quad (1)$$
$$+\frac{KN_0}{\rho_0}(\vec{q}_d-\vec{q})+2\left(\vec{q}\times\vec{\Omega}\right)$$
$$\nabla.\vec{q}=0 \quad (2)$$

where $p, \rho, T, \vec{q}, \vec{q}_d, N_0, \nu, \nu'$ denote fluid pressure, density, temperature, fluid velocity, suspended particles velocity, suspended particles number density, kinematic viscosity and kinematic viscoelasticity respectively. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and $K=6\pi\mu\eta'$, η' being particle radius, is the Stokes' drag coefficient. Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equation of motion (1), proportional to the velocity difference between the particles and the fluid.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particle on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Interparticle reactions are ignored because the distance between particles are assumed to be quite large compared with their diameter. The effects due to pressure, gravity, Darcy's force and magnetic field on the particles are small and so are ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions are

$$mN\left\{\frac{\partial \vec{q}_d}{\partial t}+(\vec{q}_d.\nabla)\vec{q}_d\right\}=KN(\vec{q}-\vec{q}_d) \quad (3)$$
$$\frac{\partial N}{\partial t}+\nabla.(N\vec{q}_d)=0. \quad (4)$$

If C_v, C_{pt}, T and q' denote the heat capacity of fluid at constant volume, heat capacity of the particles, temperature and effective thermal conductivity of the pure fluid respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives

$$\rho_0 C_v\left(\frac{\partial}{\partial t}+\vec{q}.\nabla\right)T+mNC_{pt}\left(\frac{\partial}{\partial t}+\vec{q}_d.\nabla\right)T$$
$$=q'\nabla^2 T \quad (5)$$

The Maxwell's equations yield

$$\frac{\partial \vec{H}}{\partial t}=\left(\vec{H}.\nabla\right)\vec{q}+\eta\nabla^2\vec{H} \quad (6)$$
$$\nabla.\vec{H}=0. \quad (7)$$

The equation of state for the fluid is

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right] \quad (8)$$

where α is the coefficient of thermal expansion and the subscript zero refers to values at the reference level $z=0$. The kinematic viscosity ν, kinematic viscoelasticity ν', electrical resistivity η and coefficient of thermal expansion α are all assumed to be constants.

The basic motionless solution is

$$\vec{q}=(0,0,0),\ \vec{q}_d=(0,0,0),\ T=T_0-\beta z,$$
$$\rho=\rho_0(1+\alpha\beta z),\ N=N_0=\text{constant} \quad (9)$$

Assume small perturbations around the basic solution and let $\delta p, \delta\rho, \theta, \vec{q}(u,v,w), \vec{q}_d(l,r,s), N(\vec{x},t)$ and $h(h_x,h_y,h_z)$ denote respectively the perturbations in fluid pressure p, density ρ, temperature T, fluid velocity $(0,0,0)$, suspended particles velocity $(0,0,0)$, suspended particles number density N_0 and magnetic field $\vec{H}(0,0,H)$. The change in density $\delta\rho$ caused mainly the perturbation θ in temperature is given by

$$\delta\rho=-\alpha\rho_0\theta \quad (10)$$

Then the linearized perturbation equations of Walters' B' viscoelastic fluid become

$$\frac{\partial \vec{q}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p+\vec{g}\frac{\delta\rho}{\rho_0}+\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}$$
$$+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}+\frac{KN_0}{\rho_0}(\vec{q}_d-\vec{q})+2\left(\vec{q}\times\vec{\Omega}\right)$$
$$(11)$$

$$\nabla.\vec{q} = 0 \tag{12}$$

$$mN_0 \frac{\partial \vec{q}_d}{\partial t} = KN_0 (\vec{q} - \vec{q}_d) \tag{13}$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{q} + \eta \nabla^2 \vec{h} \tag{14}$$

$$(1 + h_1)\frac{\partial \theta}{\partial t} = \beta(w + h_1 s) + \kappa \nabla^2 \theta \tag{15}$$

$$\nabla.\vec{h} = 0 \tag{16}$$

where η stands for electrical resistivity, $\kappa = \dfrac{q}{\rho_0 C_v}$ and $h_1 = \dfrac{mN_0 C_{pt}}{\rho_0 C_v}$.

Eliminating $\vec{q}_d$ in equation (11) with the help of equation (13), we obtain

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\begin{array}{l} \dfrac{\partial q}{\partial t} + \dfrac{1}{\rho_0}\nabla \delta p - g\dfrac{\delta \rho}{\rho_0} \\ -\dfrac{\mu_e}{4\pi\rho_0}(\nabla \times h)\times \vec{H} - 2(q \times \Omega) \end{array}\right] + \frac{mN_0}{\rho_0}\frac{\partial q}{\partial t} = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 q \tag{17}$$

Writing the scalar components of the equations (17), (12) and (16), we obtain

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\begin{array}{l} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial}{\partial x}\delta p + \dfrac{\mu_e H}{4\pi\rho_0}\left(\dfrac{\partial h_z}{\partial x} - \dfrac{\partial h_x}{\partial z}\right) \\ -2v\Omega \end{array}\right] + \frac{mN_0}{\rho_0}\frac{\partial u}{\partial t} = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 u \tag{18}$$

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\begin{array}{l} \dfrac{\partial u}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial}{\partial x}\delta p + \dfrac{\mu_e H}{4\pi\rho_0}\left(\dfrac{\partial h_z}{\partial x} - \dfrac{\partial h_x}{\partial z}\right) \\ -2v\Omega \end{array}\right] + \frac{mN_0}{\rho_0}\frac{\partial u}{\partial t} = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 u \tag{19}$$

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\frac{\partial w}{\partial t} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\delta p - g\alpha\theta\right] + \frac{mN_0}{\rho_0}\frac{\partial w}{\partial t} = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2 w \tag{20}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{21}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0 \tag{22}$$

Multiplying (18) by $-\dfrac{\partial}{\partial x}$ and (19) by $-\dfrac{\partial}{\partial y}$ and adding the resulting equation, we get

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\begin{array}{l} \dfrac{\partial}{\partial t}\left(-\dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial y}\right) - \dfrac{1}{\rho_0}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\delta p \\ + \dfrac{\mu_e H}{4\pi\rho_0}\left(\begin{array}{l} -\dfrac{\partial^2 h_z}{\partial x^2} - \dfrac{\partial^2 h_z}{\partial y^2} \\ + \dfrac{\partial}{\partial z}\left(\dfrac{\partial h_x}{\partial x} + \dfrac{\partial h_y}{\partial y}\right) \end{array}\right) \\ + 2\Omega\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right) \end{array}\right] + \frac{mN_0}{\rho_0}\frac{\partial}{\partial t}\left(-\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2\left(-\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) \tag{23}$$

Eliminating u, v, h_x and h_y from equation (23) by using equation (21) and equation (22), we obtain

$$\left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left[\begin{array}{l} \dfrac{\partial}{\partial t}\left(\dfrac{\partial w}{\partial z}\right) - \dfrac{1}{\rho_0}\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\delta p \\ -\dfrac{\mu_e H}{4\pi\rho_0}\nabla^2 h_z + 2\Omega\xi \end{array}\right] + \frac{mN_0}{\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = \left(\frac{m}{k}\frac{\partial}{\partial t} + 1\right)\left(v - v'\frac{\partial}{\partial t}\right)\nabla^2\left(\frac{\partial w}{\partial z}\right) \tag{24}$$

where $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity.

Eliminating δp between equations (20) and (24), we obtain,

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\begin{bmatrix}\frac{\partial}{\partial t}\nabla^2 w - g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)\\ -\frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial z}\nabla^2 h_z + 2\Omega\frac{\partial\xi}{\partial z}\end{bmatrix} \tag{25}$$
$$+\frac{mN_0}{\rho_0}\frac{\partial}{\partial t}\nabla^2 w = \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^4 w$$

On substituting for s in terms of w with the help of equation (13), equation (15) becomes,

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[H_1\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta = \beta\left[\frac{m}{K}\frac{\partial}{\partial t}+H_1\right]w \tag{26}$$

z - component of equation (14) becomes

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z} \tag{27}$$

Equations (25)-(27) yield three perturbation equations in w, θ and h_z.

3 Problem Solution

3.1 Dispersion Relation

Analyze the perturbations into normal modes by seeking the solutions in the form

$$[w, \theta, \xi, \zeta, h_z] = [W(z), \Theta(z), Z(z), X(z), K(z)].\exp(ik_x x + ik_y y + nt) \tag{28}$$

where k_x, k_y are wave numbers along x and y directions respectively $k(=\sqrt{k_x^2+k_y^2})$ is the resultant wave number of the disturbances and n is, in general, the complex constant. Using expression (28), equations (25) - (27) in non-dimensional form become

$$\left(D^2-a^2\right)\left[\sigma-(1-F\sigma)\left(D^2-a^2\right)+\frac{M\sigma}{1+J_1\sigma}\right]W$$
$$-\frac{\mu_e H d}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK$$
$$+\frac{g\alpha d^2}{\nu}a^2\Theta + 2\frac{\Omega d^3}{\nu}DZ = 0 \tag{29}$$

Equation (26) yields,

$$(1+J_1\sigma)\left[D^2-a^2-H_1 p_1\sigma\right]\Theta = -\frac{\beta d^2}{\kappa}(H_1+J_1\sigma)W \tag{30}$$

Equation (27) becomes

$$\left(D^2-a^2-p_2\sigma\right)K = -\frac{Hd}{\eta}DW \tag{31}$$

Operating equation (18) by $-\frac{\partial}{\partial y}$ and equation (19) by $\frac{\partial}{\partial x}$ and adding these two equations, we get

$$\left(\frac{m}{k}\frac{\partial}{\partial t}+1\right)\left[\frac{\partial\xi}{\partial t}+\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\zeta}{\partial z}-2\Omega\frac{\partial w}{\partial z}\right]+\frac{mN_0}{\rho_0}\frac{\partial\xi}{\partial t}$$
$$=\left(\frac{m}{k}\frac{\partial}{\partial t}+1\right)\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^2\xi$$

The non-dimensional form of this equation is

$$\left[(1-F\sigma)\left(D^2-a^2\right)-\sigma\left(1+\frac{M}{1+J_1\sigma}\right)\right]Z$$
$$=\frac{\mu_e H d}{4\pi\rho_0\nu}DX-\frac{2\Omega d}{\nu}DW \tag{32}$$

Now x and y components of equation (14) are

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_x = H\frac{\partial u}{\partial z}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_y = H\frac{\partial v}{\partial z}$$

Operating x-component by $-\frac{\partial}{\partial y}$ and y-component by $\frac{\partial}{\partial x}$ and adding these two equations, we get

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)\zeta = H\frac{\partial\xi}{\partial z}$$

The non-dimensional form of this equation is

$$\left(D^2-a^2-p_2\sigma\right)X = -\frac{Hd}{\eta}DZ \tag{33}$$

where we have expressed the co-ordinates x, y and z in the new unit of length d, time t in the new unit of length d^2/κ and put $a = kd$, $\sigma = \frac{nd^2}{\nu}$, $J = \frac{m}{k}$, $J_1 = \frac{J\nu}{d^2}$, $M = \frac{mN_0}{\rho_0}$, $p_1 = \frac{\nu}{\kappa}$ is the Prandtl number, $p_2 = \frac{\nu}{\eta}$ is the magnetic Prandtl number, $F = \frac{\nu}{d^2}$ is the dimensionless kinematic viscoelasticity, $H_1 = 1 + h_1$ and $D = \frac{d}{dz}$.

Eliminating Θ, Z, K and X from equations (29) to (33), we obtain

$$\begin{gathered}\left(D^2-a^2\right)\left[\sigma\left(1+\frac{M}{1+J_1\sigma}\right)+(\sigma F-1)(D^2-a^2)\right]W \\ -\frac{Ra^2\left(H_1+J_1\sigma\right)}{\left(1+J_1\sigma\right)\left(D^2-a^2-H_1p_1\sigma\right)}W \\ -T_A\left[\frac{\left(1+J_1\sigma\right)\left(D^2-a^2-p_2\sigma\right)}{\begin{array}{c}\left(1-F\sigma\right)\left(1+J_1\sigma\right)\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma\right)\\ -\sigma\left(1+J_1\sigma\right)-M\sigma+Q\left(1+J_1\sigma\right)D^2\end{array}}\right]D^2W \\ +Q\frac{\left(D^2-a^2\right)}{\left(D^2-a^2-p_2\sigma\right)}D^2W = 0\end{gathered} \tag{34}$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number.

Consider the case in which both the boundaries are free from stresses, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The case of two free boundaries is little artificial but allows us to have analytical solution. The boundary conditions, appropriate to the problem are

$$\begin{gathered}W = D^2W = 0,\ DZ = 0,\ \Theta = 0 \text{ at } z = 0, 1\\ \text{and } DX = 0,\quad K = 0.\end{gathered} \tag{35}$$

Using the above boundary conditions (35), it can be shown with the help of equations (29)-(33) that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$, hence the proper solution W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{36}$$

where W_0 is a constant.

Substituting the proper solution (36) in equation (34), we obtain the dispersion relation

$$R_1 = \frac{\begin{gathered}\frac{Q_1\left(1+x\right)}{\left(1+x+ip_2\sigma_1\right)} - \left(1+x\right)\left[\begin{array}{c} i\sigma_1\left(1+\frac{M}{1+i\pi^2J_1\sigma_1}\right)\\ +iF_1\sigma_1\left(1+x\right)-\left(1+x\right)\end{array}\right]W \\ +T_{A_1}\left[\frac{\left(1+x+ip_2\sigma_1\right)}{\left(1-iF_1\sigma_1\right)\left(1+x\right)\left(1+x+ip_2\sigma_1\right)-\frac{i\sigma_1}{\pi^2}+Q_1}\right]\end{gathered}}{\frac{x\left(H_1+i\pi^2J_1\sigma_1\right)W}{\left(1+i\pi^2J_1\sigma_1\right)\left(1+x+iH_1p_1\sigma_1\right)}} \tag{37}$$

where,

$x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $R_1 = \frac{R}{\pi^4}$, $T_{A_1} = \frac{T_A}{\pi^4}$, $Q_1 = \frac{Q}{\pi^2}$, $F_1 = \pi^2 F$.

3.2 The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (37) reduces to

$$R_1 = \left(\frac{1+x}{xH_1}\right)\left[(1+x)^2 + Q_1 + T_{A_1}\left(\frac{(1+x)}{Q_1 + (1+x)^2}\right)\right] \tag{38}$$

To study the effects of magnetic field, suspended particles and rotation, we examine the natures of $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dH_1}$ and $\frac{dR_1}{dT_{A_1}}$ analytically. Equation (38) yields

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{xH_1}\right)\left[1 + T_{A_1}\frac{(1+x)}{\{Q_1 + (1+x)^2\}^2}\right] \tag{39}$$

$$\frac{dR_1}{dH_1} = -\left(\frac{1+x}{xH_1^2}\right)\left[\begin{array}{l}(1+x)^2 + Q_1 \\ + T_{A_1}\left(\dfrac{(1+x)}{Q_1 + (1+x)^2}\right)\end{array}\right] \tag{40}$$

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)}{xH_1}\left[\frac{(1+x)}{(1+x)^2 + Q_1}\right]. \tag{41}$$

It is clear from equations (39)-(41) that for stationary convection the magnetic field and rotation have a stabilizing effect whereas the suspended particles have destabilizing effect on the thermal convection.

Graphs have been plotted between the modified Rayleigh number R_1 and magnetic field parameter Q_1 for various values of wave number x, rotation parameter T_{A_1} and suspended particles parameter H_1. It is evident from fig. (1) and (2) that the magnetic field and rotation have a stabilizing effect whereas from fig. (3) that the suspended particles have a destabilizing effect on the thermal convection.

3.3 Stability of System and Oscillatory Modes

Multiplying equation (23) by W^*, the complex conjugate of W, integrating over the range of z and using equations (25)-(27) together with the boundary conditions (35), we obtain

$$\begin{aligned}
&\sigma\left(1 + \frac{M}{1+J_1\sigma}\right)I_1 + (1 - F\sigma)I_2 \\
&- \frac{g\alpha\kappa a^2}{\beta v}\frac{(1+J_1\sigma^*)}{(H_1 + J_1\sigma^*)}\left[I_3 + H_1 p_1\sigma^* I_4\right] \\
&- d^2\left[\begin{array}{l}\dfrac{\mu_e\eta}{4\pi\rho_0 v}I_5 - p_2\sigma^* I_6 + (1 - F\sigma^*)I_7 \\ + \sigma^*\left(1 + \dfrac{M}{1+J_1\sigma^*}\right)I_8\end{array}\right] \\
&+ \frac{\mu_e\eta}{4\pi\rho_0 v}\left[I_9 + p_2\sigma^* I_{10}\right] = 0
\end{aligned} \tag{42}$$

where,

$$\begin{aligned}
I_1 &= \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz, \\
I_2 &= \int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz, \\
I_3 &= \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz, \\
I_4 &= \int_0^1 \left(|\Theta|^2\right)dz, \\
I_5 &= \int_0^1 \left(|DX|^2 + a^2|X|^2\right)dz, \\
I_6 &= \int_0^1 \left(|DX|^2\right)dz, \\
I_7 &= \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz, \\
I_8 &= \int_0^1 \left(|Z|^2\right)dz, \\
I_9 &= \int_0^1 \left(|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz, \\
I_{10} &= \int_0^1 \left(|DK|^2 + a^2|K|^2\right)dz.
\end{aligned} \tag{43}$$

The integrals $I_1, I_2, ..., I_{10}$ are all positive definite. Putting $\sigma = i\sigma_i$ and equating imaginary part of equation (42), we obtain

$$i\sigma_i \left[\begin{array}{l} \left(1+\dfrac{M}{1+iJ_1\sigma_i}\right) I_1 - FI_2 \\ + \dfrac{g\alpha\kappa a^2}{\beta\nu} \left\{ \begin{array}{l} \dfrac{J_1(H_1-1)}{H_1^2 + J_1^2\sigma_i^2} I_3 \\ + H_1 p_1 \dfrac{\left(H_1 + J_1^2\sigma_i^2\right)}{H_1^2 + J_1^2\sigma_i^2} I_4 \end{array} \right\} \\ - d^2 \left\{ p_2 I_6 + FI_7 - \left(1+\dfrac{M}{1-iJ_1\sigma_i}\right) I_8 \right\} \\ - \dfrac{\mu_e \eta}{4\pi\rho_0 \nu} p_2 I_{10} \end{array} \right] = 0 \tag{44}$$

Equation (42) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$, which means that modes may be non-oscillatory or oscillatory. In the absence of viscoelasticity, magnetic field and rotation, equation (44) reduces to

$$i\sigma_i \left[\begin{array}{l} \left(1+\dfrac{M}{1+iJ_1\sigma_i}\right) I_1 \\ + \dfrac{g\alpha\kappa a^2}{\beta\nu} \left\{ \begin{array}{l} \dfrac{J_1(H_1-1)}{H_1^2 + J_1^2\sigma_i^2} I_3 \\ + H_1 p_1 \dfrac{\left(H_1 + J_1^2\sigma_i^2\right)}{H_1^2 + J_1^2\sigma_i^2} I_4 \end{array} \right\} \end{array} \right] = 0 \tag{45}$$

and the quantity inside the brackets is positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The viscoelasticity, magnetic field and rotation introduce oscillatory modes (as σ_i may not be zero) in the systems which were non-existent in their absence.

4 Conclusions

With the growing importance of non-Newtonian fluids in chemical engineering, modern technology and industry, the investigations on such fluids are desirable. The Walters' B' fluid is one such important non-Newtonian fluid. Keeping in mind the importance of non-Newtonian fluids, the present paper considered the effect of suspended particles, rotation and magnetic field on the Walters' B' viscoelastic fluid heated from below. For stationary convection, Walters' B' viscoelastic fluid behaves like a Newtonian fluid. Here, we found that, rotation and magnetic field postpones the onset of convection whereas the suspended particles hasten the onset of convection. The principle of exchange of stabilities is satisfied in the absence of magnetic field. The magnetic field, viscoelasticity and rotation introduce oscillatory modes in the systems which were non-existent in their absence.

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Nomenclature

x - dimensionless wave number
C_v - heat capacity of fluid at constant volume
C_{pt} - heat capacity of suspended particles
d - depth of layer
F - dimensionless kinematic viscoelasticity
$\vec{g}$ - acceleration due to gravity
$\vec{H}$ - magnetic field
K - Stokes' drag coefficient
k - wave number
k_x - wave number along x-axis
k_y - wave number along y-axis
m - mass of suspended particle
N_0 - suspended particle number density
n - growth rate (complex constant)
p_1 - Prandtl number
p_2 - magnetic Prandtl number
Q - Chandrasekhar number
R - Rayleigh number
q' - effective thermal conductivity
$\vec{q}$ - fluid velocity
$\vec{q}_d$ - suspended particles velocity
T - temperature
t - time
α - coefficient of thermal expansion
β - uniform temperature gradient
η - electrical resistivity
η' - suspended particle radius
κ - thermal diffusivity
μ - viscosity of the fluid
μ_e - magnetic permeability
ν - kinematic viscosity
ν' - kinematic viscoelasticity
ρ - density of fluid.
p - fluid pressure
δp - perturbation in fluid pressure p
$\delta\rho$ - perturbation in fluid density ρ
θ - perturbation in fluid temperature T
$\vec{q}(u,v,w)$ - perturbation in fluid velocity $(0,0,0)$
$\vec{q}_d(l,r,s)$ - perturbation in suspended particles velocity $(0,0,0)$
$\vec{h}(h_x,h_y,h_z)$ - perturbation in magnetic field $(0,0,H)$
ξ - z component of vorticity
Q_1 - magnetic field parameter
H_1 - suspended particles parameter
T_{A1} - rotation parameter

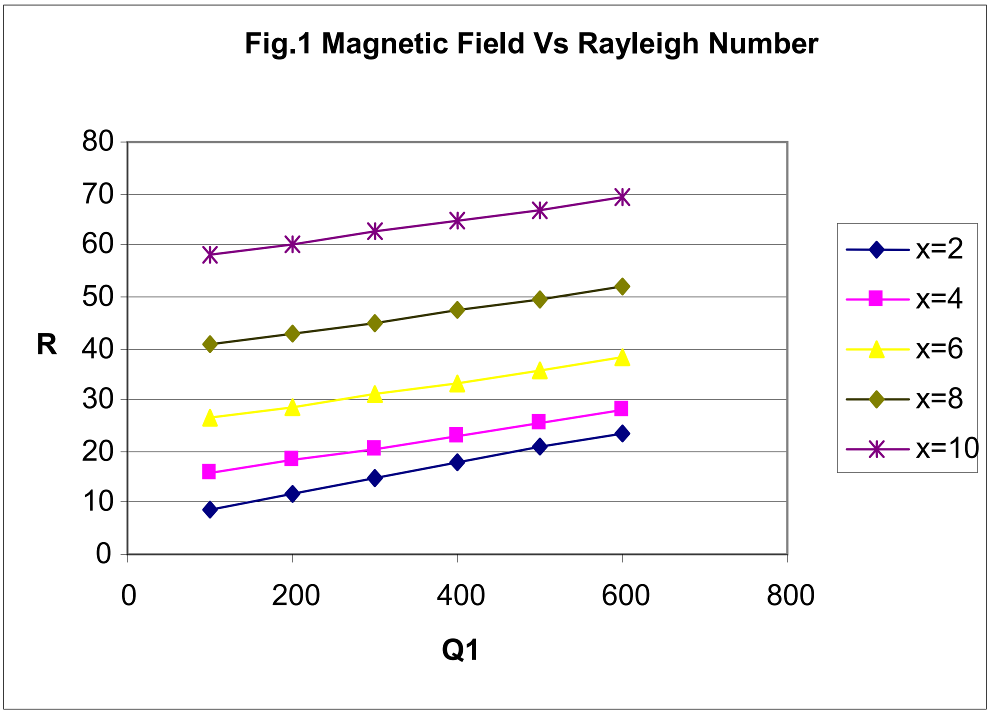

Fig. 1- Variation of R_1 with Q_1 for fixed $H_1 = 50,\ T_{A_1} = 20$, for different values of $x(=2,4,6,8,10)$ and $Q_1(=100,200,...,600)$.

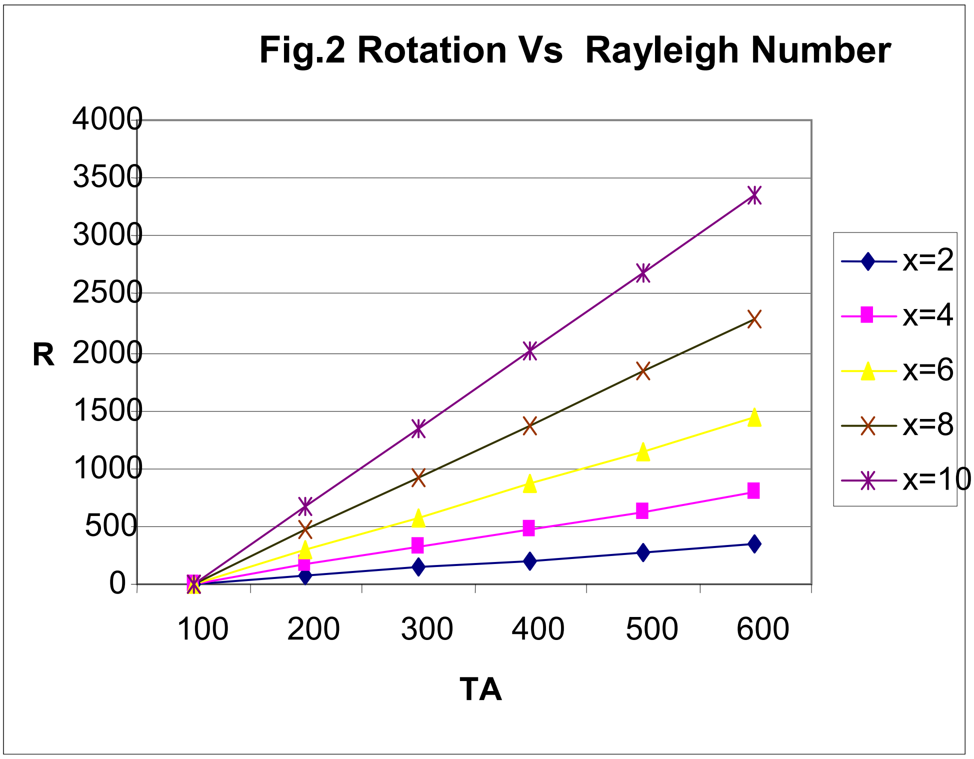

Fig. 2 – Variation of R_1 and T_{A_1} for a fixed $H_1 = 20,\ Q_1 = 60$, for different values of $x(=2,4,6,8,10)$ and $T_{A_1}(=100,200,...,600)$.

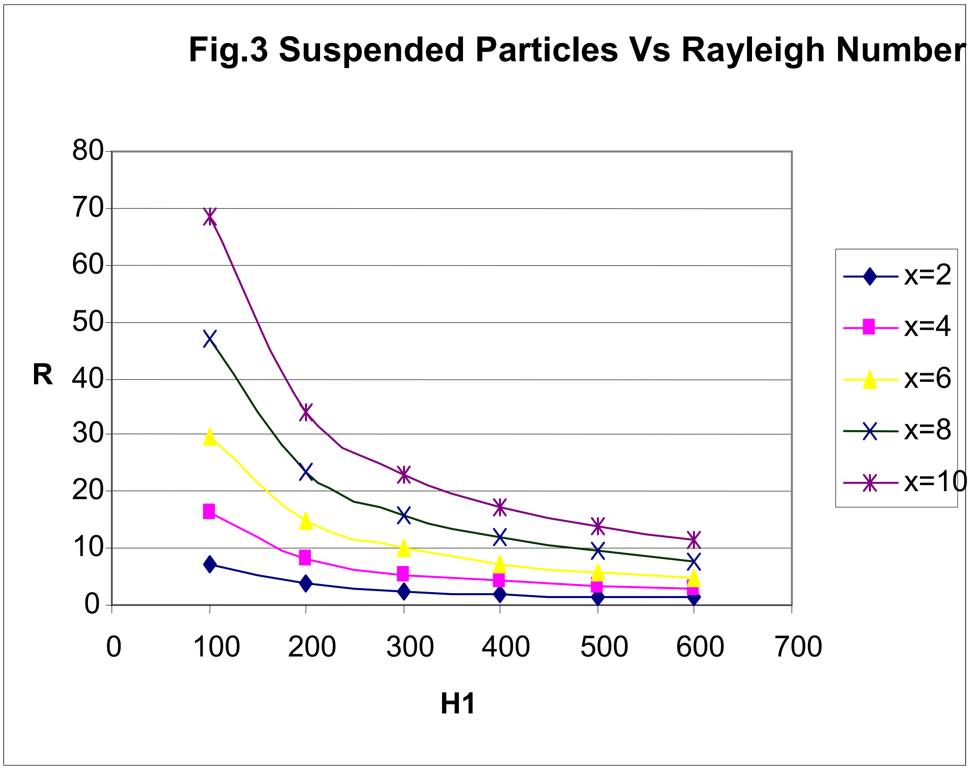

Fig. 3- Variation of R_1 and H_1 for a fixed $T_{A_1} = 20,\ Q_1 = 50,$ for different values of $x(=2,4,6,8,10)$ and $H_1(=100,200,...,600)$.